Some New Results about Backward Stochastic Differential Equations with Singular Terminal Condition.

Alexandre Popier

Le Mans Université, France

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About the talk

Based on joint works with

- Paulwin Graewe (Berlin, Germany),
- Thomas Kruse (Gießen, Germany),
- Dmytro Marushkevych (Le Mans, France).

Outline

Outline

Definition of a BSDE

Equation of the following type: $\forall t \in [0, T]$

$$\begin{aligned} Y_t &= \xi + \int_t^T f(r, Y_r, Z_r, U_r) dr \\ &- \int_t^T Z_r dW_r - \int_t^T \int_{\mathcal{E}} U_r(e) \widetilde{\pi}(de, dr) - \int_t^T dM_r. \end{aligned}$$

with adapted unknowns $(Y_t, Z_t, U_t, M_t)_{0 \leq t \leq T}$ and data:

- T: deterministic terminal time.
- $f : \Omega \times [0, T] \times \mathbb{R} \times \mathbb{R}^d \times \mathcal{B}^2_\mu \to \mathbb{R}$: generator.
- ξ: terminal condition $=$ an $\mathcal{F}_T$-measurable random variable.

(Very very partial) related literature:

- J.M. Bismut (1973): Stochastic Pontryagin maximum principle, linear or Ricatti BSDE.
- É. Pardoux & S. Peng (1990): Non linear BSDE in Brownian setting.
- ...

Existence and uniqueness for L^p data ($p > 1$).

Assumptions:

- $\mathbb{F} = (\mathcal{F}_t, t \geq 0)$: right-continuous and complete filtration.
- $\xi \in L^p(\Omega)$ and $f(t,0,0,0) \in L^p([0,T] \times \Omega)$, for $p > 1$.
- f is Lipschitz continuous w.r.t. z and u

$$|f(t,y,z,u) - f(t,y,z',u')| \leq K(|z - z'| + \|u - u'\|_{\mathcal{B}^2_\mu}).$$

- f continuous and "monotone" in y: $\exists \chi \in \mathbb{R},\ \forall (t,y,y',z,u)$

$$\langle\ y - y',\ f(t,y,z,u) - f(t,y',z,u)\ \rangle \leq \chi |y - y'|^2;$$

- Growth condition on f: for all $r > 0$:

$$\sup_{|y| \leq r} |f(t,y,0,0) - f(t,0,0,0)| \in L^1([0,T] \times \Omega, \mathrm{Leb} \otimes \mathbb{P}).$$

Typical example: $f(t,y) = -\dfrac{1}{\eta_t} y|y|^q$ for some non negative process η and $q \geq 0$.

Existence and uniqueness for L^p data ($p > 1$).

- N. El Karoui, S. Peng & M.-C. Quenez (1997), Ph. Briand et al. (2003), $\cdots$, T. Kruse & A.P. (2016 & 2019).

Theorem (existence and uniqueness)

Under the previous conditions, the BSDE

$$Y_t = \xi + \int_t^T f(r, Y_r, Z_r, U_r)dr - \int_t^T Z_r dW_r - \int_t^T \int_{\mathcal{E}} U_r(e)\widetilde{\pi}(de, dr) - \int_t^T dM_r$$

has a unique solution (Y, Z, U, M) with suitable integrability conditions.

Case $p = 1$ in the Brownian setting: Ph. Briand & Y. Hu (2006), $\cdots$, S. Fan (2016).

Remark: a.s.

$$\lim_{t \to T} Y_t = \xi - (M_T - M_{T-}) = \xi - \Delta M_T = Y_T - \Delta M_T.$$

Assumption: the filtration is left-continuous at time T (avoid thin time case).

Singularity at time T.

Ordinary differential equation:

$$\dot{y}(t) = -g(y(t)), \quad y(T) = +\infty$$

- Has a finite solution provided that

$$\int_{\cdot}^{\infty} \frac{1}{-g} < +\infty.$$

- Given by:

$$y(t) = \Gamma(T-t), \quad \Gamma = G^{-1}, \quad G(x) = \int_x^{\infty} \frac{1}{-g}.$$

$\rightsquigarrow$ Extension to BSDE with generator f "bounded from above by g".

Singularity at time T for PDE

Initial trace for parabolic equation: consider the PDE on $[0, T] \times \mathbb{R}^d$

$$\frac{\partial v}{\partial t} + \frac{1}{2}\Delta v + g(v) = \frac{\partial v}{\partial t} + \frac{1}{2}\Delta v - v|v|^q = 0$$

- M. Marcus & L. Véron (1999): trace of $v \approx v(T, \cdot)$

$$\mathcal{R} = \left\{ y \in \mathbb{R}^d,\ \exists U \text{ neighbourhood of } y \text{ s.t. } \limsup_{t \to T} \int_U v(t,x)dx < +\infty \right\},$$

μ measure on $\mathcal{R}$,

$\mathcal{S} = \mathbb{R}^d \setminus \mathcal{R}$ (closed set of the singular points).

$\rightsquigarrow$ trace of v = measure ν

$$\forall \text{ Borel set } A,\ \nu(A) = \begin{cases} \infty & \text{if } A \cap \mathcal{S} \neq \emptyset, \\ \mu(A) & \text{if } A \subseteq \mathcal{R}. \end{cases}$$

- É. Pardoux & S. Peng (1992): $Y_t := v(t, W_t)$: BSDE with “terminal condition $v(T, W_T)$”.

Stochastic control with constraint

Calculus of variations, optimal liquidation.

$$v(t,x_0) \quad = \quad \inf_{X\in\mathcal{A}(t,x_0)} \mathbb{E}\left[\int_t^T \eta_s|\dot{X}_s|^p ds + \xi|X_T|^p \middle| \mathcal{F}_t\right]$$

with

$$X_s = x_0 + \int_t^s \dot{X}_u du, \quad X_T \mathbf{1}_{\xi=+\infty} = 0.$$

- S. Ankirchner, M. Jeanblanc & T. Kruse (2013):

mandatory closure ($\xi = +\infty$ a.s.) $\rightsquigarrow v(t,x_0) = |x_0|^p Y_t$, with

$$Y_t = +\infty - \int_t^T \frac{1}{q\eta_s^q}|Y_s|^q Y_s ds - \int_t^T Z_r dW_r.$$

- T. Kruse & A.P. (2016).

Outline

The setting

ξ $\mathcal{F}_T$-measurable and

$$\mathbb{P}(\xi = +\infty) > 0.$$

Assumptions:

- Left continuity of the filtration $\mathbb{F}$ at time T (avoid thin time case).
- Integrability condition on ξ^- and $(f^0(t))^- = f(t,0,0,0)^-$: in L^ℓ, $\ell > 1$.
- Lipschitz continuous of f w.r.t. z.
- “Comparison” condition w.r.t. the jump component:

$$f(t,y,z,u) - f(t,y,z,v) \leq \int_{\mathcal{E}} (u(e) - v(e)) \kappa_t^{y,z,u,u'}(e) \mu(de)$$

with $\kappa_t^{y,z,u,u'}(e) \geq -1$ and some integrabilty condition.

$\rightsquigarrow$

- Lipschitz continuity of f w.r.t. u.
- Comparison principle for BSDEs with jumps.

Growth of f w.r.t. y

On the generator f:

- Continuous and monotone in y with one-sided growth condition:

$$f(t,y,z,u) - f(t,0,z,u) \leq \frac{1}{\eta_t} g(y).$$

with

$$\mathbb{E}\int_0^T \frac{1}{\eta_s} ds < +\infty.$$

Assumption on g:

- Continuous and non-increasing.
- For some $c > 0$, $\displaystyle\int_c^\infty \frac{1}{-g(x)} dx < +\infty$ ($G(x) = \int_x^\infty \frac{1}{-g}$, $\Gamma = G^{-1}$).

Typical examples ($q > 0$)

- Power case: $g(y) = -y|y|^q$.
- Logarithmic case: $g(y) = -(y+1)|\log(y+1)|^{q+1}$.

Construction of a solution

By truncation:

$$Y_t^L = \xi \wedge L + \int_t^T \left[\left(f(s, Y_s^L, Z_s^L, U_s^L) - f^0(s) \right) + (f^0(s) \wedge L) \right] ds \quad - \quad \text{martingale}$$

Comparison principle: $\bar{Y}_t \leq Y_t^L \leq Y_t^{L'} \leq \widehat{Y}_t^{L'}$ where

$$\widehat{Y}_t^L = (\xi^+ \wedge L) + \int_t^T \left[\frac{1}{\eta_s} g(\widehat{Y}_s^L) + f^{L,+}(s, \widehat{Z}_s^L, \widehat{U}_s^L) \right] ds \quad - \quad \text{martingale}$$

and

$$f^{L,+}(t,z,u) = \left[f(t,0,z,u) - f^0(t) \right] + ((f^0(t))^+ \wedge L).$$

Construction of a solution

Definition of a solution by monotonicity

$$Y_t = \lim_{L \to +\infty} Y_t^L \leq \widehat{Y}_t = \lim_{L \to +\infty} \widehat{Y}_t^L.$$

Problem: a.s. for $t < T$, $\widehat{Y}_t < +\infty$? $\mathbb{E}\left[(\widehat{Y}_t)^\ell\right] < +\infty$?

If yes, then we get the (minimal) super-solution of the BSDE with singular terminal condition !

- Requires some a priori estimate on $\widehat{Y}^L$.

A candidate

ODE:

$$\dot{y} = -\frac{1}{\eta_t} g(y), \quad y(T) = +\infty \Longrightarrow y(t) = \Gamma\left(\int_t^T \frac{1}{\eta_s} ds\right)$$

Jensen's inequality (Γ convex) + conditional expectation

$$y(t) \leq \frac{1}{T-t} \mathbb{E}\left[\int_t^T \Gamma\left(\frac{T-t}{\eta_s}\right) ds \middle| \mathcal{F}_t\right] = \mathcal{Y}(t).$$

Questions:

- Dynamics of $\mathcal{Y}$?
- $\widehat{Y}^L \leq \mathcal{Y}$?

Special case: If η bounded from above by $\eta^\star$ ($f^0 = 0$), deterministic a priori estimate:

$$\widehat{Y}^L_t \leq \Gamma\left(\frac{T-t}{\eta^\star}\right).$$

The power case $g(y) = -y|y|^q$

Then $\Gamma(x) = (qx)^{-1/q}$ and

$$\mathcal{Y}(t) = \frac{1}{(T-t)^{1+\frac{1}{q}}} \mathbb{E}\left[\int_t^T \left(\frac{\eta_s}{q} \right)^{\frac{1}{q}} ds \Big| \mathcal{F}_t \right].$$

- Dynamics of $\mathcal{Y}$: a BSDE.
- Comparison principle for BSDEs:

$$-\frac{1}{\eta} y^{1+q} \leq -p \frac{1}{(T-t)} y + \left(\frac{\eta}{q(T-t)} \right)^p.$$

- Explicit formula for linear (or linearized) BSDEs.

A priori estimate: for some $\ell > 1$

$$Y_t^L \leq \frac{C_{\ell,K_z,K_u}}{(T-t)^{1+\frac{1}{q}}} \left[\mathbb{E} \left(\int_t^T \left[\left(\frac{\eta_s}{q} \right)^{\frac{1}{q}} + (T-s)^{1+\frac{1}{q}} (f^0(s))^+ \right]^\ell ds \Big| \mathcal{F}_t \right) \right]^{1/\ell}.$$

The power case $g(y) = -y|y|^q$

Theorem (T.K. & A.P., SPA 2016)

There exists a process (Y, Z, U, M) s.t.

1. Integrability on $[0, t]$, for all $0 \leq t < T$

$$\mathbb{E}\left(\sup_{s\in[0,t]} |Y_s|^\ell + \left(\int_0^t |Z_r|^2 dr \right)^{\frac{\ell}{2}} + \left(\int_0^t \int_{\mathcal{E}} |U_r(e)|^2 \pi(de, dr) \right)^{\frac{\ell}{2}} + [M]_t^{\frac{\ell}{2}} \right) < +\infty;$$

2. Y is bounded from below by some $\bar{Y} \in \mathbb{S}^\ell(0, T)$.
3. $\mathbb{P}$-a.s. for all $0 \leq s \leq t < T$

$$Y_s = Y_t + \int_s^t f(r, Y_r, Z_r, U_r)\, dr - \int_s^t Z_r dW_r - \int_s^t \int_{\mathcal{E}} U_r(e) \widetilde{\pi}(de, dr) - \int_s^t dM_r.$$

4. $\mathbb{P}$-a.s. $\liminf_{t\to T} Y_t \geq \xi = Y_T$.
5. Minimality: for another solution (Y', Z', U', M'), $Y_t \leq Y'_t$ a.s. for any $t \in [0, T)$.

The power case $g(y) = -y|y|^q$

Remark: under some particular setting, a priori estimate = solution of the singular BSDE.

- Linear closure for portfolio liquidation with martingale illiquidity.
- Best estimate !

Extension to:

- Backward doubly SDEs and SPDEs, A. Matoussi, L. Piozin & A.P. (2017).
- Second-order BSDEs, A.P. & C. Zhou (2019).

For general function g

Adapted candidate:

$$\mathcal{Y}(t) = \frac{1}{T-t}\mathbb{E}\left[\int_t^T \Gamma\left(\frac{T-t}{\eta_s}\right) ds \middle| \mathcal{F}_t\right].$$

Questions:

- Dynamics of $\mathcal{Y}$? We cannot separate t and s ! $\rightsquigarrow$ BSVIE !
- $\widehat{Y}^L \leq \mathcal{Y}$? Comparison principle for BSVIE ?

Related literature on existence and uniqueness for BSVIE:

- J. Lin (2002), J. Yong (2005 and 2008),
- Z. Wang & X. Zhang (2007) (jump case),
- ...
- A.P. (2019).

Backward stochastic Volterra integral equations

Formal dynamics of $\mathcal{Y}$ on $[0, T-\varepsilon]$

$$\mathcal{Y}(t) = \Psi^\varepsilon(t) + \int_t^{T-\varepsilon} \mathfrak{f}(t, s, \mathcal{Y}(s), \mathcal{Z}(t,s), \mathcal{U}(t,s)) ds$$
$$- \int_t^{T-\varepsilon} \mathcal{Z}(t,s) dW_s - \int_t^{T-\varepsilon} \int_{\mathcal{E}} \mathcal{U}(t,s,e) \widetilde{\pi}(de, ds) - \int_t^{T-\varepsilon} d\mathcal{M}(t,s).$$

where

$$\Psi^\varepsilon(t) = \mathbb{E}(\kappa(t, T-\varepsilon) | \mathcal{F}_{T-\varepsilon})$$

and

$$\mathfrak{f}(t, s, y, z, u) = -\frac{1}{T-s} y + \gamma(t,s) + \left[f(s, 0, z, u) - f^0(s) \right]$$

and $\kappa(\cdot, \cdot)$ and $\gamma(\cdot, \cdot)$ depend on g, η and $(f^0)^+$ and are explicitly known.

Backward stochastic Volterra integral equations

Formal dynamics of $\mathcal{Y}$ on $[0, T-\varepsilon]$

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Integrability condition: There exists $\ell > 1$ such that for any $\varepsilon > 0$:

$$\mathbb{E}\left[\left(\int_0^T \Gamma\left(\frac{\varepsilon}{\eta_s}\right) + \Gamma\left(\frac{\varepsilon}{T-s}\right) (T-s)(f_s^0)^+ ds \right)^{\ell} \right] < +\infty$$

Proposition

There exists a unique solution $(\mathcal{Y}^{\varepsilon}, \mathcal{Z}^{\varepsilon}, \mathcal{U}^{\varepsilon}, \mathcal{M}^{\varepsilon})$ to the BSVIE.

If f is linear w.r.t. z and u, $\mathcal{Y}^{\varepsilon}$ does not depend on ε and $\mathcal{Y}^{\varepsilon} = \mathcal{Y}$.

Comparison principle for BSVIEs

Goal: $\widehat{Y} \leq \mathcal{Y}$ a.s.

Much more delicate than for BSDEs !!! One reference:

- T. Wang & J. Yong (2015), extended to general filtration in A.P. (2019).

Two sufficient conditions:

- Separation by a generator which is non-decreasing w.r.t. y.
- Linearity w.r.t. z and u (without the presence of t).

Comparison of the generators:

$$\begin{aligned} \mathfrak{f}(t,s,y,z,u) &= -\frac{1}{T-s}y + \widehat{\gamma}(t,s) + f^0(s)^+ \widetilde{\gamma}\left(\frac{T-t}{T-s}\right) + [f(s,0,z,u) - f^0(s)] \\ &\geq \frac{g(y)}{\eta_t} + (f^0(s)^+ \wedge L) + [f(s,0,z,u) - f^0(s)]. \end{aligned}$$

Holds if for any $y > c$, $-\int_c^y \frac{1}{G(w)} dw \geq g(y)$, where $G(y) = \int_y^\infty \frac{1}{-g}$.

- True for most examples ($g(y) = -y|y|^q$), but not in general !

Comparison principle for BSVIEs

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Comparison of the generators:

$$\begin{aligned}\mathfrak{f}(t,s,y,z,u) &= -\frac{1}{T-s}y + \widehat{\gamma}(t,s) + f^0(s)^+\widetilde{\gamma}\left(\frac{T-t}{T-s}\right) + \left[f(s,0,z,u) - f^0(s)\right] \\ &\geq \frac{g(y)}{\eta_t} + (f^0(s)^+ \wedge L) + \left[f(s,0,z,u) - f^0(s)\right].\end{aligned}$$

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- True for most examples ($g(y) = -y|y|^q$), but not in general !

Our main result

Assumptions on the coefficients:

- ξ $\mathcal{F}_T$-measurable and $\mathbb{P}(\xi = +\infty) > 0$.
- Integrability condition on ξ^- and $(f^0(t))^- = f(t,0,0,0)^-$: in L^ℓ, $\ell > 1$.
- Left continuity of the filtration $\mathbb{F}$ at time T (avoid thin time case).

- f continuous and monotone in y with growth condition:

$$f(t,y,z,u) - f^0(t) \leq \frac{1}{\eta_t} g(y) + h(t)z + \int_{\mathcal{E}} \widehat{\kappa}_t(e)u(e)\mu(de)$$

  and for some $c > 0$, $-\int_c^\infty \frac{1}{g(x)} dx < +\infty$.
- For any $y > c$:

$$-\int_c^y \frac{1}{G(w)} dw \geq g(y).$$

- There exists $\ell > 1$ such that for any $\varepsilon > 0$:

$$\mathbb{E}\left[\left(\int_0^T \Gamma\left(\frac{\varepsilon}{\eta_s}\right) ds \right)^\ell + \left(\int_0^T \Gamma\left(\frac{\varepsilon}{T-s}\right)(T-s)(f_s^0)^+ ds \right)^\ell \right] < +\infty.$$

Our main result

Theorem

For any L, $\mathbb{P}$-a.s. for any $t \in [0, T]$

$$Y_t^L \leq \widehat{Y}_t^L \leq \frac{1}{T-t} \mathbb{E}^{\mathbb{Q}} \left[\int_t^T \left(\Gamma\left(\frac{T-t}{\eta_s}\right) + \Gamma\left(\frac{T-t}{T-s}\right) \frac{(T-s)}{\mathfrak{I}} (f^0(s))^+ \right) ds \middle| \mathcal{F}_t \right].$$

- $\mathbb{Q} \sim \mathbb{P}$: linearity w.r.t. z and u and Girsanov's theorem.
- $\mathfrak{I} = \int_1^\infty \frac{\Gamma(a)}{a^2} da$.

In particular for any $\varepsilon > 0$ and any $1 < \varrho < \ell$

$$\mathbb{E}\left[\sup_{t\in[0,T-\varepsilon]} (\widehat{Y}_t^+)^\varrho \right] < +\infty.$$

- Existence of a minimal super-solution of the BSDE with generator f and terminal condition ξ.

Outline

Bounded coefficients

Generator f of the form:

$$(\omega, t, y) \mapsto f(\omega, t, y) = \frac{1}{\eta_t(\omega)} g(y) + \lambda_t(\omega).$$

with

1. a.s. for any t, $0 < \eta_\star \leq \eta_t(\omega) \leq \eta^\star, \qquad 0 \leq \lambda_t(\omega) \leq \|\lambda\|$.
2. g is of class C^1 and non increasing, with $g(0) = 0$.
3. For any $x > 0$, $G(x) := \int_x^\infty \frac{1}{-g(t)} dt$ is well-defined.

Proposition

The BSDE with singular terminal condition has a minimal non-negative solution (Y, Z).

- Deterministic (and explicit) a priori upper-bound of the form $\vartheta(T - t)$.

Asymptotic behavior of Y

Now $\xi = +\infty$ a.s.

- $\lim_{t \to T} Y_t = +\infty$ a.s.
- Lower bound:

$$Y_t \geq \Gamma \left(\mathbb{E} \left[\int_t^T \frac{1}{\eta_s} ds \middle| \mathcal{F}_t \right] \right).$$

Question: what is the behavior of

$$Y_t - \Gamma \left(\mathbb{E} \left[\int_t^T \frac{1}{\eta_s} ds \middle| \mathcal{F}_t \right] \right) ?$$

Asymptotic behavior of Y

Theorem

Under some technical conditions, one-to-one correspondence

$$Y_t = \Gamma\left(\mathbb{E}\left[\int_t^T \frac{1}{\eta_s} ds \Big| \mathcal{F}_t\right]\right) + \Upsilon\left(\frac{T-t}{\eta^\star}\right) H_t,$$

where

- $\Gamma = G^{-1}$, $\Upsilon = -\Gamma'$
- H is the minimal non-negative solution of a BSDE with terminal condition 0 and with a singular generator F in the sense of M. Jeanblanc & A. Réveillac (2014):

$$F(t,h) = a_t + b_t h + c_t\left[g(\Gamma_t + \Upsilon_t h) - g(\Gamma_t)\right]\mathbf{1}_{h\geq 0}, \quad \int_0^T b_t dt = +\infty.$$

a, b, c depend explicitely on Γ, Υ, η and λ.

BSDE with singular generator

(H, Z^H) satisfies:

- For any $0 \leq t \leq T$

$$H_t = \int_t^T F(s, H_s)ds - \int_t^T Z_s^H dW_s \qquad \Rightarrow \lim_{t \to T} H_t = 0.$$

- For any $0 \leq t < T$, $0 \leq \sup_{s \in [0,t]} H_s < +\infty$ a.s. and

$$\mathbb{E} \int_0^T |F(s, H_s)| ds < +\infty.$$

- The process Z^H belongs to $\mathbb{H}^1(0, T) \cap \mathbb{H}^p(0, T - \theta)$ for any $\theta > 0$ and $p > 1$.

Note that H is constructed independently of Y.

Asymptotic behavior

$$\Gamma\left(\mathbb{E}\left[\int_t^T \frac{1}{\eta_s} ds \middle| \mathcal{F}_t\right]\right) \leq Y_t \leq (1+\kappa)\Gamma\left(\mathbb{E}\left[\int_t^T \frac{1}{\eta_s} ds \middle| \mathcal{F}_t\right]\right),$$

where κ depends on the coefficients η, λ and g.

More accurate properties

- If g is concave, then symmetric development

$$Y_t = \Gamma(A_t) + \Upsilon(A_t)\widehat{H}_t, \qquad A_t = \mathbb{E}\left[\int_t^T \frac{1}{\eta_s} ds \middle| \mathcal{F}_t\right].$$

  - Uniqueness of $\widehat{H}$ and thus of Y.
  - Extension of uniqueness result of P. Graewe, U. Horst & É. Séré (2018).
- If $g(y) = -y|y|^q$, then H can be obtained by Picard iterations in the space

$$\mathcal{H}^\delta := \{H \in L^\infty(\Omega; C([0,T]; \mathbb{R})) : \|H\|_{\mathcal{H}} < +\infty\}$$

endowed with the weighted norm

$$\|H\|_{\mathcal{H}} = \left\| \sup_{t\in[0,T)} (T-t)^{-2}|H_t| \right\|_\infty .$$

  - Numerics ?

Thank you very much !

多谢 (computer translation)

Joyeux anniversaire, Rainer.

Based on:

- T. Kruse, A.P. Minimal supersolutions for BSDEs with singular terminal condition and application to optimal position targeting. Stochastic Processes and their Applications, 2016.
- T. Kruse, D. Marushkevych, A.P. Singular BSDE and backward stochastic Volterra integral equations. Work in progress, submitted soon.
- P. Graewe, A.P. Asymptotic approach for backward stochastic differential equation with singular terminal condition. hal-02152177 or Arxiv 1906.05154.