General Equilibrium Approach and Rationnal Expectations

- We explore some of the reasons why the AD-AS model is not appropriated for policy analysis:

  → lack of micro foundations in general equilibrium

  → lack of proper modeling of expectations

  → lack of dynamics

Plan of the talk

1. The general equilibrium approach
2. The Lucas critique
3. The Lucas Island Model

1.The General Equilibrium Approach

- General equilibrium is an important requirement.

  → This can be illustrated by a very simple general equilibrium model

  → This allows to show that interactions between markets are crucial.

  → We take into account optimizing behaviors of both firms and household (in opposition to the standard AS-AD model)

- We consider an example related to theory of unemployment where the economy is composed by two atomistic agents, one representative firm and one representative household.

Preferences and technology

- Firm:

  → Output $y = \ell^{\beta}, 0 < \beta < 1$

  → Profits $\pi = py - w\ell.$

- Household:

  → supply inelastically ℓ_0

  → Utility function: $U = \alpha \log(c) + (1-\alpha)\log(m/p) \quad , 0 < \alpha < 1$

  → is endowed with money and receive profits:

  >budget constraint $pc + m \leq w\ell + \pi + m_0$

Household's behaviors

- The household maximizes U st. the budget constraint:

$$\begin{aligned} \max_{c,m,\ell} \quad & \alpha \log(c) + (1-\alpha)\log(m/p) \\ st \quad & w\ell + \pi + m_0 \leq pc + m \\ & \ell \leq \ell_0 \end{aligned}$$

- Forming the lagrangian,

$$\mathcal{L} = \alpha \log(c) + (1-\alpha)\log(m/p) + \lambda(w\ell + \pi + m_0 - pc - m) + \mu(\ell_0 - \ell)$$

- With $\lambda, \mu \geq 0$ being the Lagrange multipliers
- From the FOC, one gets

$$\begin{aligned} c &= \alpha \left(\frac{m_0}{p} + y\right) && (a) \\ m^d &= (1-\alpha)(m_0 + py) && (b) \\ \ell^s &= \ell_0 \end{aligned}$$

Firm's behaviors

- Firm's profit maximization yields

$$l^d = \left(\frac{1}{\beta} \times \frac{w}{p}\right)^{\frac{-1}{1-\beta}}$$

$$y^s = \left(\frac{1}{\beta} \times \frac{w}{p}\right)^{\frac{-\beta}{1-\beta}}$$

→ labor demand and the supply of goods are decreasing functions of the real wage

The case of Walrasian equilibrium

- 3 markets: labor, good, money, 2 relative prices (w and p), money being the numeraire
- A Walrasian equilibrium of this economy is a set of prices (w, p) and quantities (c, m, ℓ, y) such that those quantities maximize utility and profit for thoses prices and markets clear.
- Labor market equilibrium yields $\ell^\star = \ell_0,\ y^\star = y_0 = \ell_0^\beta$ and $w^\star / p^\star = \beta \ell_0^{\beta-1}$
- Then the good market equilibrium condition c=y allow to get prices $p^\star = \dfrac{\alpha m_0}{(1-\alpha) y_0}$ $w^\star = \beta \ell_0^{\beta-1} \times \dfrac{\alpha m_0}{(1-\alpha) y_0} = \dfrac{\alpha\beta}{1-\alpha} \ell_0^{-1}$

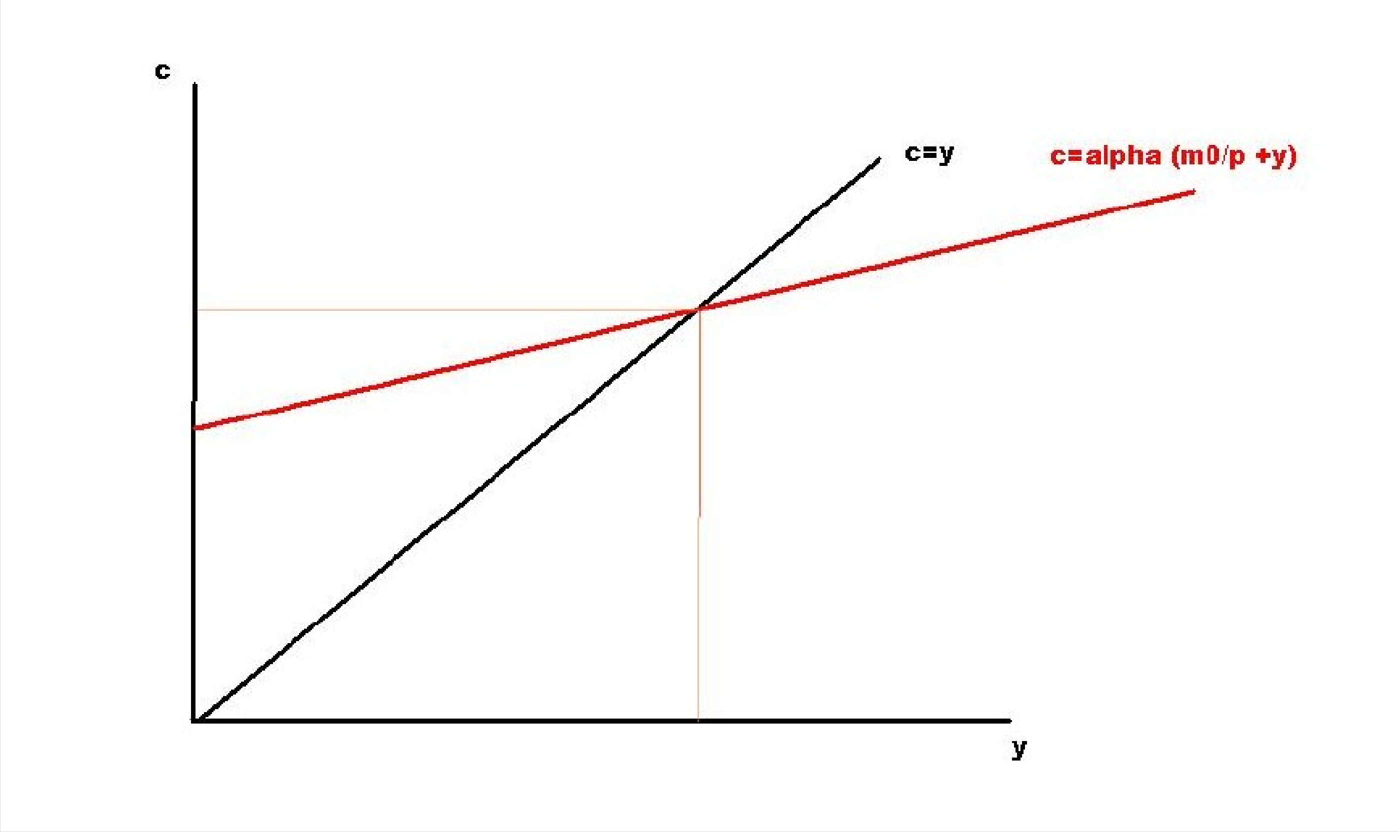
A graphical interpretation

- The model equilibrium can be otained by solving the following system of four equations we use are:

$$
\begin{aligned}
c &= \alpha\left(\frac{m_0}{p} + y\right) \quad \& \quad c = y \\
y &= \left(\frac{1}{\beta} \times \frac{w}{p}\right)^{\frac{-\beta}{1-\beta}} \\
\ell^d &= \left(\frac{1}{\beta} \times \frac{w}{p}\right)^{\frac{-1}{1-\beta}} \\
\ell^s &= \ell_0
\end{aligned}
$$

- For a given p, the first equation is an IS curve (planned expenditures c equal to actual ones y)

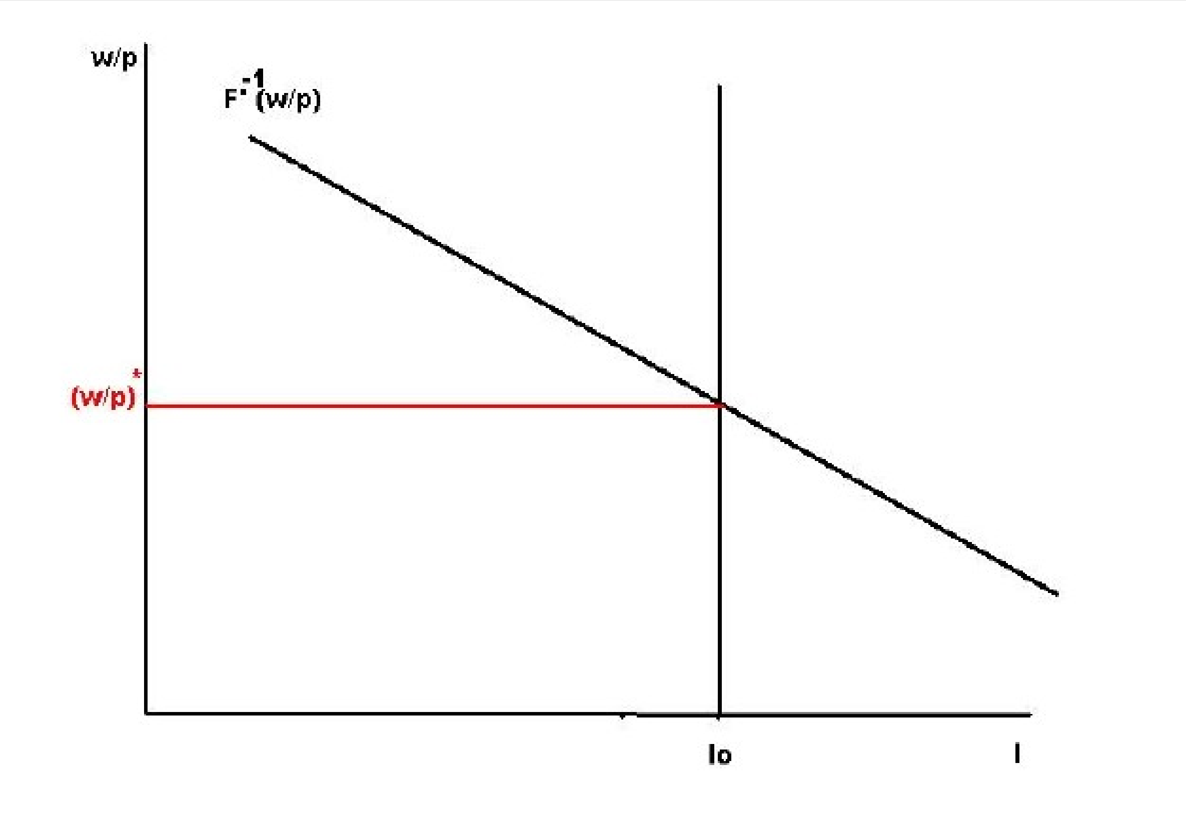
- The IS curve



- When p varies this figures describes an AD curve → no need to take into account the LM curve (no bond market here, and use of the Walras law)

Labor market equilibrium

- We can determine the real wage that clears the labor market:



- Given this real wage, we find the value of output and prices at Walrasian equilibrium

The case of Classical Unemployment

- Assume that w/p is set rigid above its Walrasian value and that voluntary exchange prevails. $(w/p = \overline{w/p})$
- Employment (transactions on the labor market) will be given by $\min(\ell^s, \ell^d) = \ell^d = \left(\frac{1}{\beta} \times \overline{w/p}\right)^{\frac{-1}{1-\beta}} = \overline{\ell}$
- Households are the constrained on their labor supply and now solve:

$$\begin{array}{rl} \max_{c,m,\ell} & \alpha \log(c) + (1-\alpha)\log(m/p) \\ st & w\ell + \pi + m_0 \leq pc + m \\ & \ell \leq \overline{\ell} \end{array}$$

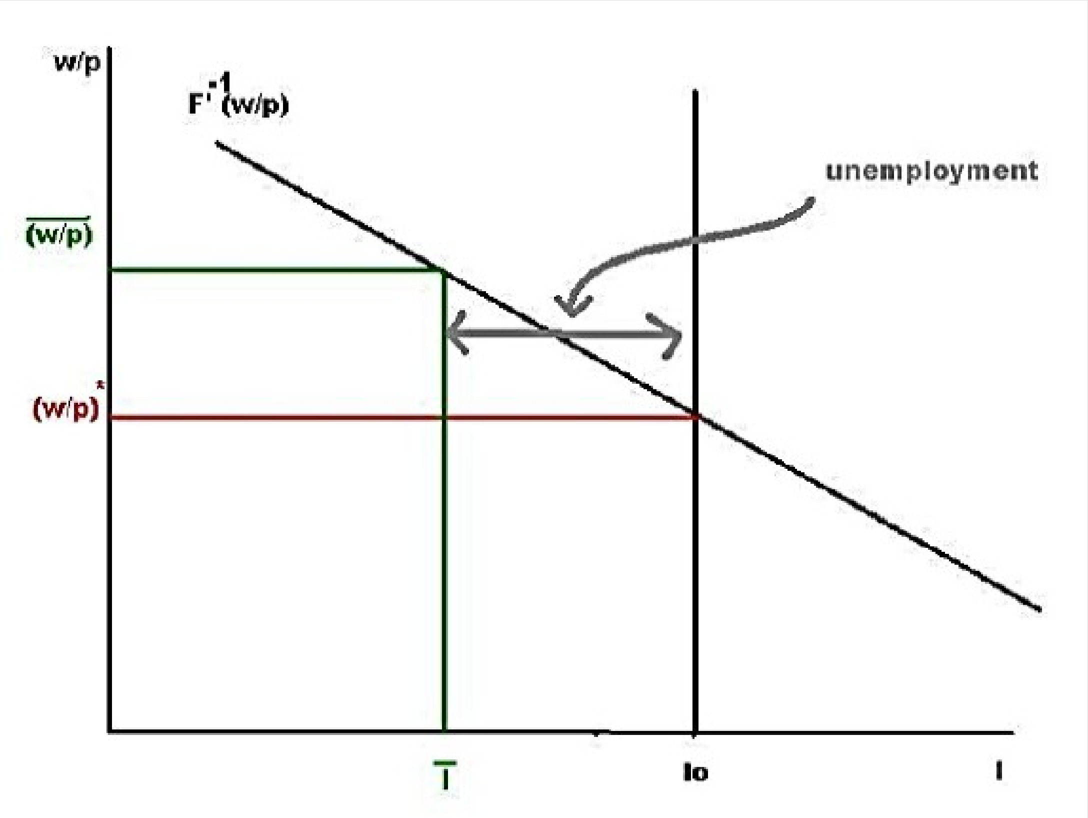
- The solution for the consumption is

$$c = \alpha\left(\frac{m_0}{p} + \frac{w\overline{\ell}+\pi}{p}\right) = \alpha\left(\frac{m_0}{p} + \overline{y}\right)$$

- Then p will adjust such that $c = \overline{y}$: $\overline{p} = \dfrac{\alpha m_0}{(1-\alpha)\overline{y}}$

  and $\overline{w} = \overline{w/p} \times \overline{p}$
- This is the view according to which high real wages are the cause of unemployment and that the problem comes from the labor market

Graphical representation

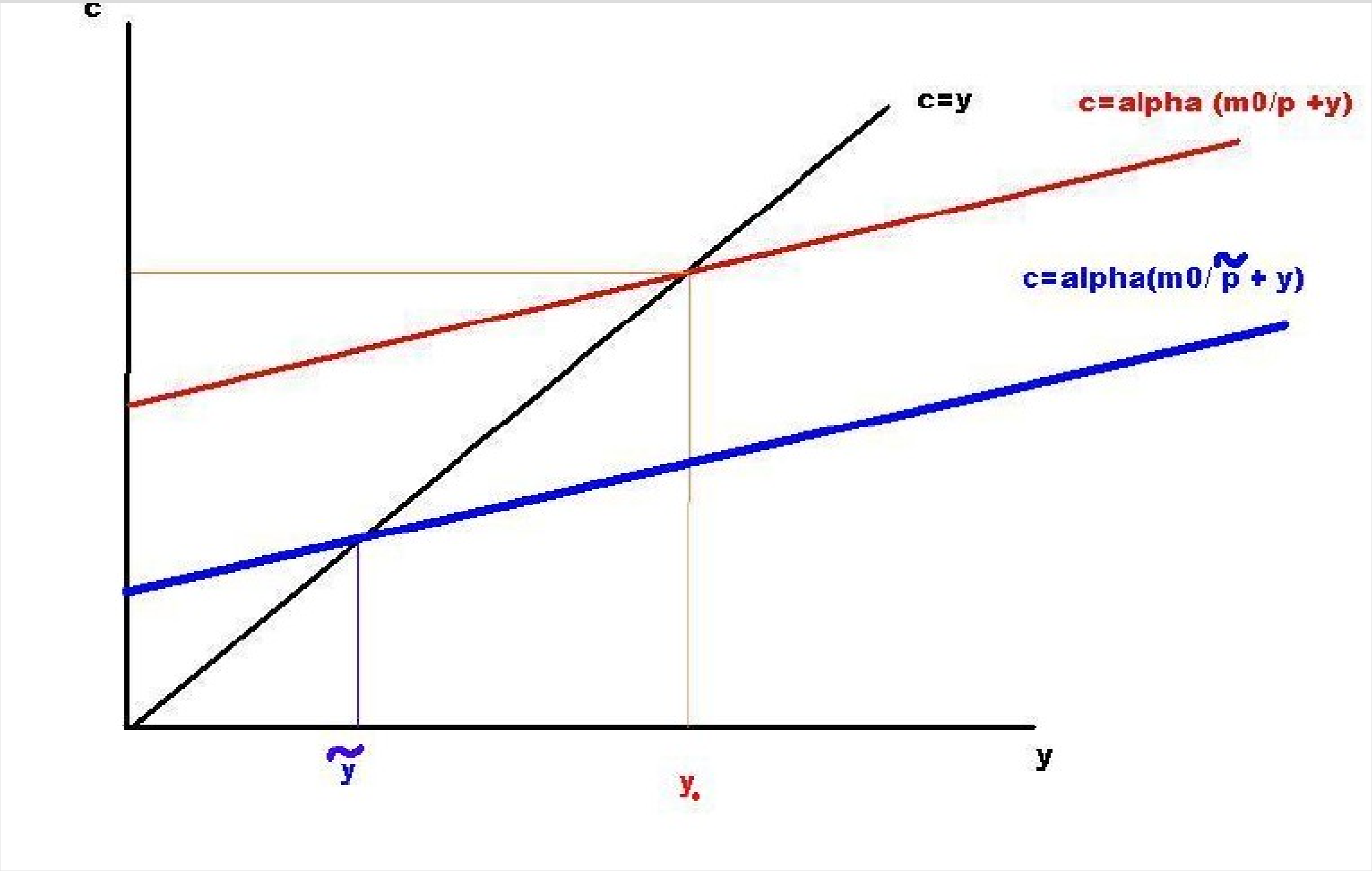


The case of Keynesian unemployment

- Assume that prices are rigid and set to $\widetilde{p} > p^\star$
- Consider a nominal wage $\widetilde{w}$ such that $\widetilde{w}/\widetilde{p} \leq w^\star/p^\star$
- Assume that the nominal wage does not decrease if there is unemployment (if there were not the case, excess supply on the labor market would drive the wage down until the equality holds)
- At this price households are expressing a demand

$$c = \alpha \left(\frac{m_0}{\widetilde{p}} + \overline{y} \right)$$

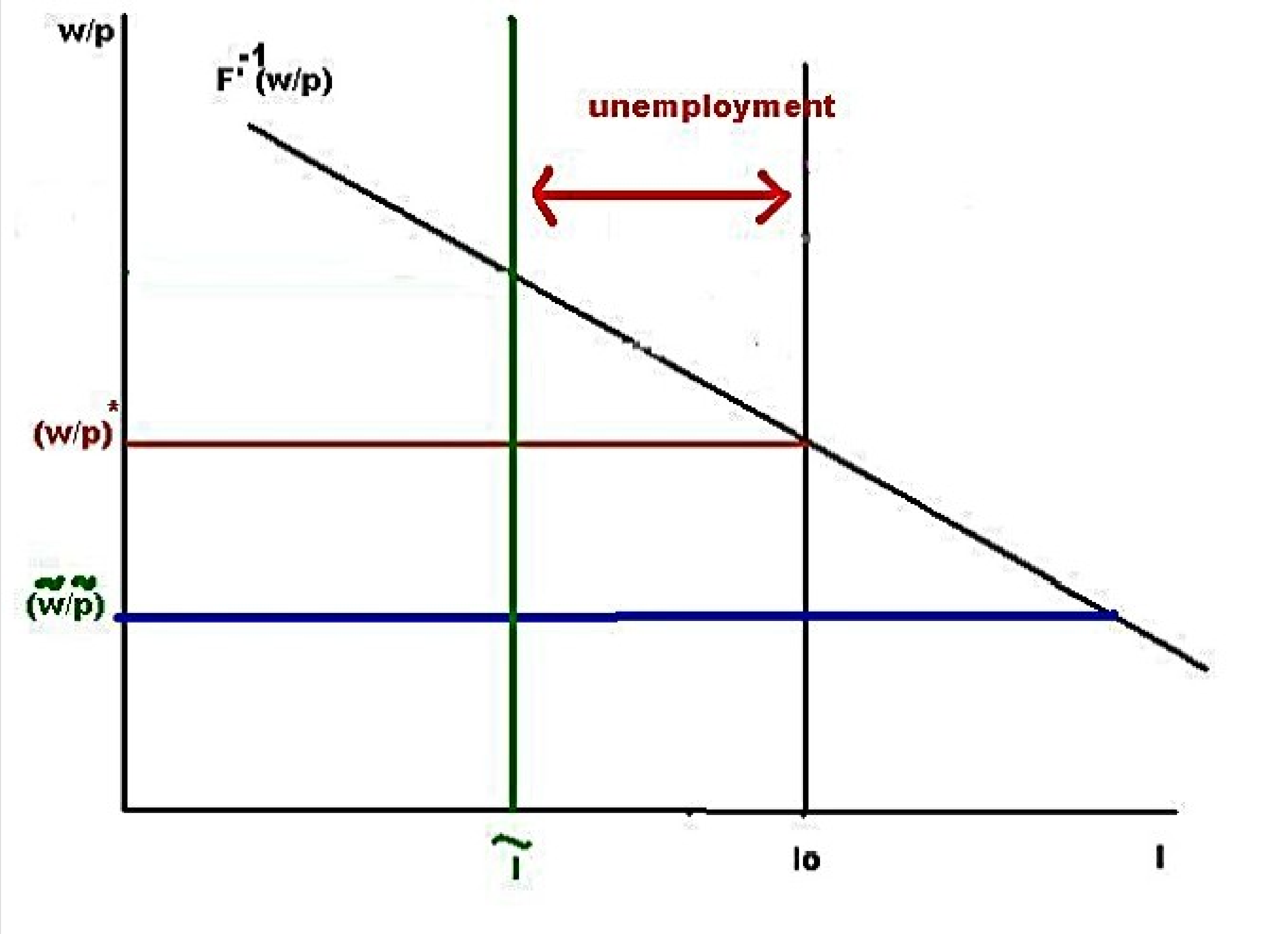
- This implies that c is smaller than $y^\star = y_0$ if $\widetilde{p} > p^\star$


c
c=y
c=alpha (m0/p +y)
c=alpha(m0/p̃ + y)
ỹ
y
y_0

- We therefore have at equilibrium of the good market:

$$\widetilde{c} = \widetilde{y} = \frac{\alpha m_0}{(1-\alpha)\widetilde{p}} < y^\star$$

- Facing a demand $\widetilde{c} < y^\star$ it is not optimal for the firm to demand the full employment quantity of labor, $\ell^\star = \ell_0$
- Rather, the firm will limit production to the level demanded $\widetilde{y}$ and therefore hires $\widetilde{\ell} = \widetilde{y}^{1/\beta}$
- Employment is below the full employment level although the real wage is lower than its walrasian level → the root of unemployment is the malfunctioning of the good market → importance of markets interactions and general equilibrium


w/p
F'-1(w/p)
unemployment
(w/p)*
(w̃/p̃)
l̃
lo
l

2.The Lucas critique

- We want to show that if expectations are not properly taken into account, models predictions about the effect of economic policy can be misleading.

  → This is the Lucas critique, that shows that estimated parameters of models where expectations are not properly modelled are not structural ones.

2.1 Benchmark model

- The model is given by two equations.
- Private agents behavior: $n_t = \alpha c_t + \beta g_t$

  with $\alpha > 0$ and $\beta > 0$. n is employment, c consumption and g government expenditures;

  $$c_t = \gamma n^e_{t+1}, \qquad \gamma > 0$$

  where n^e_{t+1} is the expectation of period t+1 employment based on the information of period t.

- Information embodies: variables at date t, knowledge of the model equations, parameters and the process of the government spending shocks

  ↔ rational expectations

- The model reduces to:

$$n_t = \alpha\gamma n^e_{t+1} + \beta g_t$$

$$g_t = \rho g_{t-1} + \varepsilon_t$$

  with $\alpha\gamma < 1$

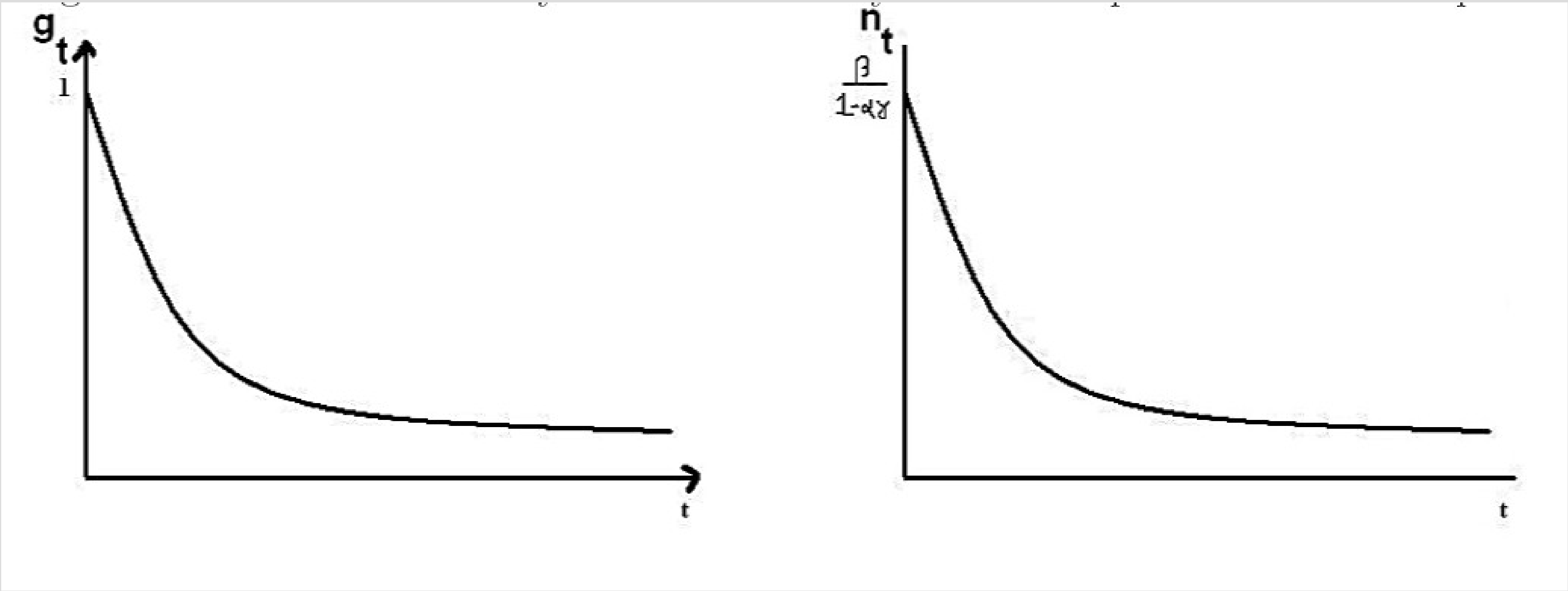
- We want to compute the IRF of the employment to a government spending shock
- In order to solve the model one needs to specify the formation of expectations

2.2 The case of naive expectations

- Assume that $n^e_{t+1} = n_t$

  → agents are always wrong in their forecast, except in the very long run (asymptotically).
- Then the model solution is $$n_t = \frac{\beta}{1-\alpha\gamma} g_t$$
- The instantaneous multiplier is $\mu^N = \frac{\beta}{1-\alpha\gamma}$ and it does not depend on ρ
- Assume $g_{-1} = 0$ and consider a policy shock $\varepsilon_1 = 1$ and $\varepsilon_t = 0$ for t>1, together with a change in the policy rule (this has no incidence on the multiplier)

- Impact of the policy shock:



2.3 The case of rational expectations

- Let now consider that agents form rational expectations, conditionally on the information available at t: $n^e_{t+1} = E_t n_{t+1}$ where E_t is the conditional mathematical expectation.

  → the expectation is endogenous, so that we have to solve the model in order to determine this expectation

$$n_{t+1} = \alpha\gamma E_{t+1} n_{t+2} + \beta g_{t+1}$$

$$E_t n_{t+1} = \alpha\gamma E_t E_{t+1} n_{t+2} + \beta E_t g_{t+1}$$

- Using the law of iterated expectations we get

$$E_t n_{t+1} = \alpha\gamma E_t n_{t+2} + \beta\rho g_t$$

- Repeating this calculation gives:

$$E_t n_{t+n} = \alpha\gamma E_t n_{t+n+1} + \beta\rho^n g_t$$

$$\begin{aligned} E_t n_{t+1} &= \alpha\gamma^n E_t n_{t+n+1} \\ &\quad + \beta((\alpha\gamma)^{n-1}\rho^n + (\alpha\gamma)^{n-2}\rho^{n-1} + \cdots + \alpha\gamma\rho^2 + \rho)g_t \end{aligned}$$

and plus this expectation into the expression of employment at date t yields:

$$\begin{aligned} n_t &= (\alpha\gamma)^{n+1} E_t n_{t+n+1} \\ &\quad + \beta((\alpha\gamma)^n\rho^n + (\alpha\gamma)^{n-1}\rho^{n-1} + \cdots + (\alpha\gamma)^2\rho^2 + \alpha\gamma\rho + 1)g_t \end{aligned}$$

- Taking the limit when n goes to infnity yields:

$$n_t = \frac{\beta}{1-\alpha\gamma\rho} g_t$$

- The instantaneous multiplier is now: $\mu^{RE} = \frac{\beta}{1-\alpha\gamma\rho}$
- In opposition to the naive case, the policy rule now enters in the value of the multiplier → the multiplier is not invariant to policy shock

2.4 Definition of Lucas critique

- Assume that agents are forming rational expectations
- Assume that for the last 50 years the government persistence of spending has been $\rho = .95$ (very high persistency)
- An econometrician that would estimate the impact effect of government spending shocks would find a multiplier $\widehat{\mu}$ = $\frac{\beta}{1-.95\times\alpha\gamma}$
- In order to economize on spending the government can decide to reduce the persistence paramter to $\rho = .5$

  → according to the gvt believes, the change in the policy rule will affect the persistence of the response of the economy but not its impact effect.

- In reality μ will be affected by the change of the policy rule, because agents are rational in their expectations: μ will be reduced from $\frac{\beta}{1-.95\times\alpha\gamma}$ to $\frac{\beta}{1-.5\times\alpha\gamma}$
- This suggests that the estimated multiplier cannot be used for evaluating changes of policy because it depends on agents reactions to this policy change.

  → non structural econometric models cannot be used for policy evaluation: this is the Lucas critique

3. The Lucas Island Model

- A model with rational expectations and yet one where policy does matter
- Lucas (1978) : agents live in separate isolated islands, and have information about the price of the good only in their island => unable to form accurate expectations as regads overall inflation even though they are rationnal.
- The price of the good being produced and sold in island *i* is given by : $$p_t^i = p_t + z_t^i$$

  where z_t^i is an idiosyncratic shock with variance σ_z^2
- Agents only observe the price in their island and dot not know separate components (aggregate price / shock)

- The aggregate price is defined as :

$$p_t = \gamma + \varepsilon_t$$

  where the variance of the aggregate shock is σ_ε^2

- This implies that an agent does not know if the price changes in his island because of the aggregate or the idiosyncratic shock (both shocks have zero mean).
- Why should that matter ? Because producers do care about the relative price : want to produce more only if the good becomes relatively more expensive, *ie.* It is assumed :

$$Y_t^i = Y + a\left[p_t^i - E\{p_t \mid p_t^i\}\right]$$

How to solve $E\{p_t \mid p_t^i\}$?

- We use the Best Linear Unbiased Estimator (BLUE)

BLUE will minimize the mean squared error $\Sigma\eta_t^2$ in

$$E\{p_t \mid p_t^i\} = \alpha + \beta p_t^i + \eta_t$$

The expression reflects a forecast about p_t that is formed linearly on the basis of observed island prices p_t^i. We want this linear forecast to be as accurate as possible, i.e. minimize the mean squared error.

Minimizing the mean squared error is precisely what is asked from a classic estimator, OLS. The residuals must verify two conditions:

$$\begin{aligned} E\{\eta_t\} &= E\{p_t - \alpha - \beta p_t^i\} = 0 \\ E\{\eta_t p_t^i\} &= E\{\left(p_t - \alpha - \beta p_t^i\right) p_t^i\} = 0 \end{aligned}$$

The first condition imposes the residual be zero in expectations. The second imposes the residual be orthogonal to the regressor. Note these are UNCONDITIONAL expectations, unlike $E\{p_t \mid p_t^i\}$, which is the expectation of p_t conditional on p_t^i

The two conditions provide a system of two equations in two unknowns, α and β.

We have

$$\begin{aligned} E\{p_t\} &= \alpha + \beta E\{p_t^i\} & (1) \\ E\{p_t p_t^i\} &= \alpha E\{p_t^i\} + \beta E\{\left(p_t^i\right)^2\} & (2) \end{aligned}$$

All we have to do now is to solve for the unknowns using what we know about p_t and p_t^i.

Firstly, $E\{p_t^i\} = E\{p_t + z_t^i\} = E\{p_t\} = \gamma$. Therefore equation (1) implies

$$\gamma = \frac{\alpha}{1-\beta}$$

Secondly,

$$
\begin{aligned}
E\{p_t p_t^i\} &= E\{(\gamma + \varepsilon_t)(\gamma + \varepsilon_t + z_t^i)\} = E\{(\gamma + \varepsilon_t)^2\} = \gamma^2 + \sigma_\varepsilon^2 \\
E\{(p_t^i)^2\} &= E\{(\gamma + \varepsilon_t + z_t^i)^2\} = \gamma^2 + \sigma_\varepsilon^2 + \sigma_z^2
\end{aligned}
$$

Using these in equation (2) gives

$$\gamma^2 + \sigma_\varepsilon^2 = \alpha\gamma + \beta\left(\gamma^2 + \sigma_\varepsilon^2 + \sigma_z^2\right)$$

Now combining both equations, we have

$$\gamma^2 + \sigma_\varepsilon^2 = \gamma(1-\beta)\gamma + \beta\left(\gamma^2 + \sigma_\varepsilon^2 + \sigma_z^2\right)$$

which simplifies into

$$\beta = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_z^2}$$

And

$$\alpha = \gamma(1-\beta) = \frac{\gamma\sigma_z^2}{\sigma_\varepsilon^2 + \sigma_z^2}$$

Classic results of a signal extraction problem. BLUE is given by

$$
\begin{aligned}
E\{p_t \mid p_t^i\} &= \frac{\gamma\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2} + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_z^2} p_t^i \\
&= \gamma + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_z^2}\left(p_t^i - \gamma\right)
\end{aligned}
$$

I try and predict p_t, but I only observe p_t^i. Suppose I observe p_t^i above its unconditional mean γ. It is somewhat surprising, as I know prices in all island should be γ on average. So I know some shocks have happened - but I don't know which, z_t^i or ε_t. The only thing I know are the variances of both shocks, and I will try and use that information.

In particular, if I know the aggregate shocks ε_t are much more volatile than the idiosyncratic shocks z_t^i, I will tend to ascribe most of the observed discrepancy $p_t^i - \gamma$ to an aggregate shock. In the limit, suppose idiosyncratic shocks to not exist, i.e. $\sigma_z^2 = 0$. Then BLUE implies that $E\{p_t \mid p_t^i\} = p_t^i$. Observing p_t^i is exactly identical to observing p_t, since there are no island specific shocks.

A contrario, if I know the the idiosyncratic shocks z_t^i are much more volatile than the aggregate shocks ε_t, I will tend to ascribe most of the observed discrepancy $p_t^i - \gamma$ to an idiosyncratic shock. In the limit, suppose aggregate shocks to not exist, i.e. $\sigma_\varepsilon^2 = 0$. Then BLUE implies that $E\{p_t \mid p_t^i\} = \gamma$. Observing p_t^i carries absolutely no additional information about the aggregate price, since p_t^i is exclusively driven by island shocks.

So now back to the supply function:

$$
\begin{aligned}
Y_t^i &= Y + a\left[\varepsilon_t + z_t^i - \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_z^2}\left(p_t^i - \gamma\right)\right] \\
&= Y + a\frac{\sigma_z^2}{\sigma_\varepsilon^2 + \sigma_z^2}\left(p_t^i - \gamma\right)
\end{aligned}
$$

Output only responds to idiosyncratic shocks: in the absence of any island specific shock, $\sigma_z^2 = 0$ and producers simply do not alter their production plans, leaving them at Y. Why? Because they know any observed deviation between the price on their island and what they expect it to be (γ) comes from an aggregate shock (remember: they know the variances of the shocks). In other words, they know all prices on all islands have shifted identically. No reason to produce more.

Now we can aggregate across islands, to obtain

$$
\begin{aligned}
Y_t &= \frac{1}{N}\sum_n Y_t^i = Y + \frac{1}{N}\frac{a\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2}\sum_n \left(\varepsilon_t + z_t^i\right) \\
&= Y + \frac{a\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2}\left(\varepsilon_t + \frac{1}{N}\sum_n z_t^i\right) = Y + \frac{a\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2}\varepsilon_t \\
&= Y + \frac{a\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2}\left(p_t - E\{p_t\}\right)
\end{aligned}
$$

Across the MACROeconomy, production only responds to changes in aggregate prices that were not expected. It does so to an extent that increases with the variance of idiosyncratic shocks.

Conclusion

- The form of Household's expectations is crucial to understand the functionning of the economy and macroeconomic policy
- With rationnal expectations agents behaviors do depends on macroeconomic policy.
- What are the implications for monetary and government spending policies ?