

Turbulence, Training and Unemployment: Do we need higher training subsidies?

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Abstract

In this paper, we develop a matching model where firms invest in transferable human capital. Workers are endowed with heterogeneous abilities and, as a result of economic turbulence, can undergo a depreciation of their human capital during unemployment spells. We find that an increase in the probability of experiencing human capital depreciation raises unemployment and decreases training investments. However, we show that higher turbulence does not necessarily require higher training subsidies. Reduction in optimal training subsidies arises in economies where the average unemployment durations of low-skilled and high-skilled workers are close enough. Lastly, we show the robustness of our main results considering alternative wage setting processes and endogenous contact rates.

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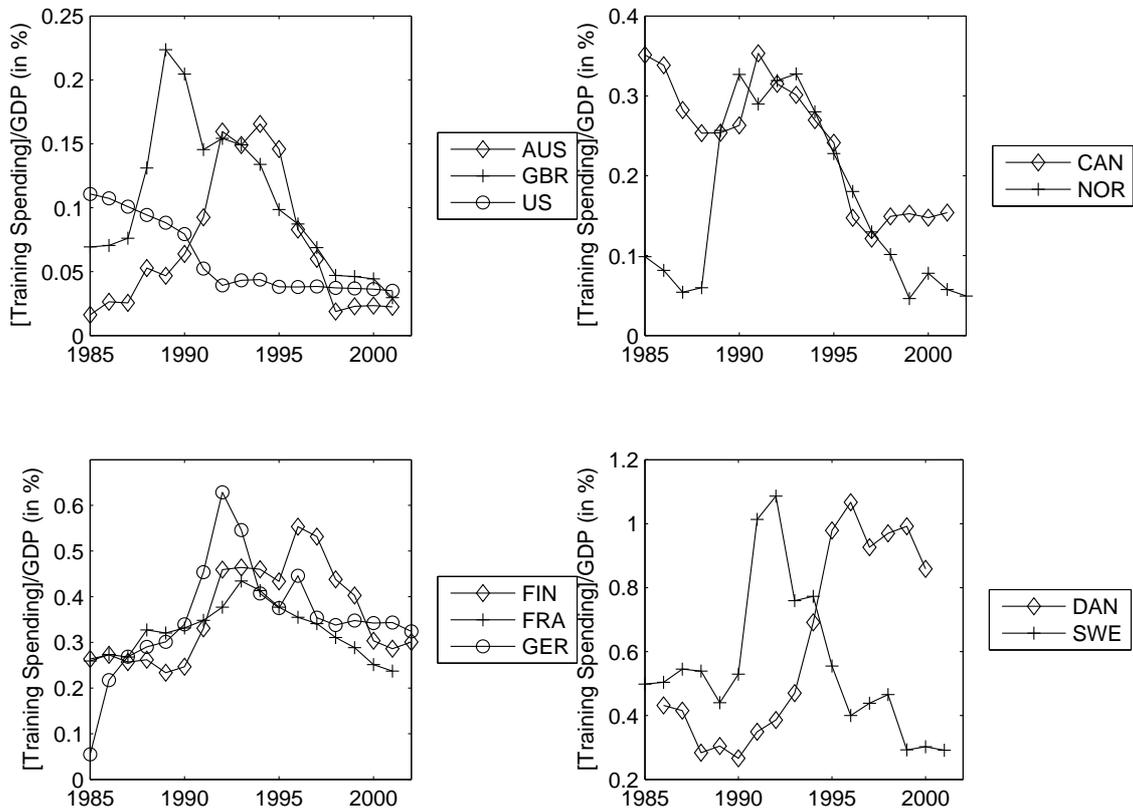
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1 Introduction

During the mid-nineties and early two thousand, some major OECD countries have faced a decrease in public expenditures on labor market training programmes as a share of GDP (see Figure 1)¹. In this paper, we argue that this quite surprising observation is not necessarily at odds with normative theoretical findings. We derive this result by considering a model with matching frictions and endogenous training decisions where the optimal training subsidy rate is found to depend on the probability of human capital loss during unemployment spells.

Figure 1: Evolution of the ratio of public expenditures on labor market training programmes as a share of GDP.



Ljungqvist and Sargent (LS henceforth) have indeed emphasized the significant role of turbulence to explain the rise of unemployment over more

¹We use the OECD database provided by Bassanini and Duval [2006]. This includes expenditures related to training for employed and unemployed adults (apart from specific training programmes for youth and disabled).

than twenty years in European countries with generous unemployment benefit systems (see among others Ljungqvist and Sargent [1998, 2007]). A period of turbulence is characterized by an increase in the probability of losing human capital during an unemployment spell. This analysis relies first on the observation that long-tenured displaced workers experience large and enduring earning losses which can be thought of as corresponding to worker's skill depreciation during unemployment spell.² Second, the 1980's was accompanied by an increase in the dispersion of earnings and the intertemporal volatility of individual earnings.

This paper aims at examining the optimal design of training subsidies in the context of a labor market search model with turbulence. Faced with skill depreciation during unemployment spell, should the government implement training subsidies? If the answer is yes, does the government has to increase subsidies of the training costs in case of higher turbulence? Our answers to these two questions are: yes and not necessarily.

From a first-best perspective, the usefulness of training subsidies is obvious in the context of externality and inefficiencies. At the end of the 1990's, Acemoglu [1997], Acemoglu and Pischke [1998, 1999a, 1999b] and Acemoglu and Shimer [1999] put emphasis on this point by focusing on inefficient training in the context of frictional labor markets. More precisely, Acemoglu [1997] points out the possibility of an externality between the worker and his future employers (poaching externality). In competitive equilibrium, the worker obtains 100% of the increase in productivity due to training in general human capital, and was therefore willing to pay the cost through wage cuts.³ On the contrary, a frictional labor market may explain the willingness of employers to bear part of the costs of general training. Wage bargaining implies that a fraction of additional productivity obtained from worker's training goes to the firm. But training investment may also benefit future employers, that is, with some probability, an unknown party (the future employer) is getting a proportion of the training benefit when the worker is displaced. This results in underinvestment because the rents accruing to this third party do not fea-

²As emphasized by Neal [1995], subsequent earnings of displaced workers are an indicator of human capital surviving beyond the old match.

³By distinguishing general and specific training, Becker [1964] emphasizes that a worker should pay for any general training that may improve his productivity in future jobs. Therefore, in a competitive framework, inefficiency in training investment is the consequence of incomplete contracts or credit market imperfections between the worker and his current employer (see Becker [1964] and Grout [1984]). Contractual arrangements, as exit penalties for trained workers who quit their firm, can restore efficiency. Government intervention should also be mostly limited to improve loan markets; training subsidies are unnecessary.

ture in the calculations of the worker's current employer.⁴ It is then obvious that the size of these externalities should be related to the probability of human capital depreciation (turbulence as defined by LS). A first contribution of our paper is to examine the interplay between turbulence and poaching externalities, by focusing on the optimal design of training subsidies.

Beyond this, our analysis considers a new externality related to the social unemployment gain of training, called hereafter steady-state unemployment externality. Indeed, training investments may increase the probability of leaving unemployment and contribute to raise steady-state employment, hence output. Firms do not take into account the consequence of their own training decisions on unemployment level. This results in lower training investments than is required by the first-best allocation. To the best of our knowledge, this issue has not yet been addressed (see Leuven [2005] for a survey).

The immediate consequence is that both externalities (poaching and steady-state unemployment) make training subsidies efficient. Nevertheless, we point out that the way turbulence interacts with training subsidies differs according to the externality that plays the dominant role. In particular, despite a higher turbulence leads to increase unemployment and decrease training investments, it can be the case that training subsidies should be reduced.

In Section 2, we develop a frictional labor market model with heterogeneous workers according to observable characteristics, where firms can invest in training that brings up-to-date knowledge to workers and raises their productivity. During unemployment spells, the worker may lose his up-to-date knowledge, but firms cannot know *ex ante* who among the unemployed underwent such a depreciation of their human capital. Once hiring decision is taken, knowledge of the worker is revealed and the firm chooses to pay the training cost or not. Turbulent times mean that depreciation of transferable knowledge during unemployment spell occurs with higher probability. The key variable is the share of workers that firms will be ready to train once matched. We first show that the partial equilibrium with exogenous contact rates is characterized by an increasing (decreasing) relationship between the probability of losing human capital and unemployment rate (resp. share of trained workers).

Section 3 is devoted to the presentation of the efficient training system. The optimal subsidy rate of training cost is characterized. We show that the higher the turbulence, the lower the size of the poaching externality. For this reason turbulence has a negative effect on the optimal subsidy rate

⁴Actually, the equilibrium could be efficient only if the bargaining power of workers was 100%.

of training costs. On the contrary, the higher the turbulence, the higher the size of steady-state unemployment externality. This second effect of turbulence increases the optimal subsidy rate of training costs. Overall, we stress that the respective impacts of both externalities depend on the gap between the contact rates of low-skilled and high-skilled workers. It turns out that in economies where the average unemployment durations of low-skilled and high-skilled workers are close enough, it is optimal to reduce training subsidies when turbulence is rising.

In Section 4, the robustness of the results with respect to the wage setting process is considered. The results that have been derived under the conventional Nash bargaining are compared with what is obtained in the context of a strategic alternating wage bargaining game in line with Hall and Milgrom [2006,2008]. Additionally, we find that, if a hold-up problem arises, higher economic turbulence unambiguously leads to lower optimal training subsidies.

In Section 5, we generalize the analysis by considering endogenous contact rates. This allows to show that both poaching and steady-state externalities do exist at the general equilibrium level. We also find that the Hosios condition is no longer sufficient to achieve efficiency and turbulence still has an ambiguous effect on optimal training subsidies. Section 6 concludes.

2 Turbulence, Training and Partial Equilibrium

2.1 Environment and labor market flows

Time is continuous. The population of workers is a continuum of unit mass. Workers look for jobs and are randomly matched with employers looking for workers to fill vacant units of production. A productive unit is the association of one worker and one firm. Workers are heterogeneous with respect to ability a , distributed on the interval $[\underline{a}, \bar{a}]$ according to p.d.f. $f(a)$. Ability a is a general human capital index, perfectly observable by firms, due to certification through school or university diplomas.

An individual at skill level a can reach two levels of productivity, $(1 + \Delta)a$ or a , with $\Delta > 0$, according to the fact that his knowledge is respectively still up-to-date or not. Additionally, firms can pay for a fixed training cost γ_F in order to provide up-to-date knowledge to the worker.⁵ Training brings

⁵In the context of Nash bargaining of wages, no matter who pays the training cost if both parties cannot renege on the wage contract once the match is formed (no hold-up).

additional transferable job skills which can be used in present and any future occupation. For instance, workers can be trained in order to use new technologies and be aware of recent innovations in their field. These skills can be used in other jobs, but are not certified by any diploma and is therefore not observable by future employers.

Moreover, during unemployment spell, the worker may lose the benefits of past firms training, and will then need a new formation to recover his up-to-date knowledge when matched with a new job. The latter assumption introduces obsolescence of human capital as a result of what Ljungqvist and Sargent [1998] have called turbulence. It embodies the possibility of substantial human capital destruction after job loss (Jacobson & al. [1993], Farber [2005]).

Therefore, the combination of firm training and human capital obsolescence gives rise to informational asymmetry between workers and firms during the matching process. Once the match is formed, the information on worker's skill is revealed and the negotiated wage takes account of the training cost that the firm has to pay if the worker has obsolete knowledge.

For the sake of simplicity, it is assumed that workers cannot accumulate skills according to tenure and experience: either the worker has up-to-date knowledge which improves its efficiency on the job according to its ability (with an additional output equal to Δa), or his knowledge has become obsolete or depreciated which precludes any additional output.

Training policy of a firm simply consists in determining a threshold ability \tilde{a} above which the worker is trained if he faced human capital depreciation. It follows that any unemployed individual belongs to one of the three following categories: (1) type-0 individuals: unable for training ($a < \tilde{a}$); (2) type-1 individuals: able enough for training ($a \geq \tilde{a}$), but with obsolete knowledge or not previously trained; (3) type-2 individuals: able enough for training ($a \geq \tilde{a}$), previously trained and still highly productive. In steady state, type-1 unemployed is always an individual whose general human capital became obsolete.

At a first (partial equilibrium) stage, we consider exogenous heterogenous contact rates for workers leaving the possibility of a discontinuity at $a = \tilde{a}$:

$$p(a) = \begin{cases} p_0 & \text{if } a < \tilde{a} \\ p & \text{for type-1 and type-2 individuals with } a \geq \tilde{a} \end{cases}$$

with $p_0 \leq p$. As will be stated in Section 5, such a discontinuity point will typically arise in the context of endogenous contact rates with free entry conditions. It results from imperfect information of firms about workers' productivity. Since firms cannot *ex ante* observe whether an unemployed worker has up-to-date knowledge or not, they cannot discriminate between

type-1 and type-2 workers before entering the matching process. At the ability level \tilde{a} , a worker either have up-to-date knowledge and productivity $(1 + \Delta)\tilde{a}$, or he will be trained in his future position at cost γ_F in order to reach this same level of productivity. By contrast, for ability levels a just below \tilde{a} , worker's productivity is a and firm pays no training cost. We will show that this results in a discontinuity of the contact rate function in \tilde{a} . At this stage, for simplicity, we only consider this discontinuity and assume homogenous contact rates within both segments on each side of the threshold \tilde{a} .

All jobs are assumed to separate at rate $\delta > 0$. The number of unemployed with up-to-date knowledge who experience a human capital depreciation follows a Poisson process with rate $\pi > 0$. We denote respectively employment and unemployment levels of a -ability population by $e(a)$ and $u(a)$, with $e(a) = f(a) - u(a)$. In steady state, for type-0 workers ($a < \tilde{a}$), inflow into unemployment $\delta(f(a) - u(a))$ is equal to outflow $p_0 u(a)$, so that

$$u(a) = f(a) \frac{\delta}{p_0 + \delta}, \quad \text{for all } a < \tilde{a}.$$

For $a \geq \tilde{a}$, the population splits into type-1 and type-2 workers. Let $u_1(a)$ and $u_2(a)$ be the respective unemployment levels. In steady state,

- inflow into type-1 unemployment $\pi u_2(a)$ is equal to outflow $pu_1(a)$.
- inflow into type-2 unemployment $\delta(f(a) - u_2(a) - u_1(a))$ is equal to outflow $(p + \pi)u_2(a)$.

The resulting unemployment levels are

$$u_1(a) = f(a) \frac{\delta\pi}{(p + \pi)(p + \delta)} \quad \text{and} \quad u_2(a) = f(a) \frac{\delta p}{(p + \pi)(p + \delta)} \quad (1)$$

so that, for $a \geq \tilde{a}$,

$$u(a) = u_1(a) + u_2(a) = f(a) \frac{\delta}{p + \delta} \quad (2)$$

Lastly, overall unemployment is

$$u = F(\tilde{a}) \frac{\delta}{p_0 + \delta} + (1 - F(\tilde{a})) \frac{\delta}{p + \delta} \quad (3)$$

where F is the c.d.f. associated to f . This shows that unemployment is negatively related to the share of workers who are eligible for training if $p > p_0$. The effect of turbulence on unemployment therefore depends on the relationship between \tilde{a} and π .

2.2 Training decision of the firm

For a firm, the intertemporal value of a filled job depends on worker's ability and worker's type. We denote this value by $J_i(a)$, $i \in \{0, 1, 2\}$. If the worker is of type 0 (i.e. $a < \tilde{a}$), instantaneous production is a . If the worker is of type 1 or 2 (i.e. $a \geq \tilde{a}$), training in past or current job increases his productivity and instantaneous production becomes $(1 + \Delta)a$. Further, we consider that the government can subsidy training at the time of job creation, by paying a fraction s of the training cost γ_F , so that the net training cost is $\hat{\gamma}_F \equiv \gamma_F(1 - s)$.

The Bellman equations for jobs therefore write as follows:

$$rJ_0(a) = a - w_0(a) - \delta J_0(a) \quad (4)$$

$$rJ_i(a) = (1 + \Delta)a - w_i(a) - \delta J_i(a), \quad i \in \{1, 2\} \quad (5)$$

where r is the interest rate, and $w_i(a)$, $i \in \{0, 1, 2\}$, the wage rate. The training policy consists in determining the ability threshold \tilde{a} above which it is in the interest of firms to finance the training of workers who do not have up-to-date knowledge. The threshold ability \tilde{a} is then defined by

$$J_1(\tilde{a}) = J_0(\tilde{a}) + \hat{\gamma}_F \quad (6)$$

which implies

$$\Delta\tilde{a} = w_1(\tilde{a}) - w_0(\tilde{a}) + (r + \delta)\hat{\gamma}_F \quad (7)$$

This condition states that a firm trains a worker if the present value of the productivity gain $\Delta a / (r + \delta)$ is larger or equal to the present value of the wage gap plus the training cost. Since additional output Δa increases with workers' ability, firms may choose not to train low-ability workers.

2.3 Nash bargaining of wages

At this stage, we consider the standard Nash bargaining of wages.⁶ The respective intertemporal values of employment and unemployment are denoted by $E_i(a)$ and $\mathcal{U}_i(a)$, $i \in \{0, 1, 2\}$. For individuals with $a < \tilde{a}$, steady-state Bellman equations write

$$r\mathcal{U}_0(a) = b + p_0(E_0(a) - \mathcal{U}_0(a)) \quad (8)$$

$$rE_0(a) = w_0(a) - \delta(E_0(a) - \mathcal{U}_0(a)) \quad (9)$$

⁶We devote a specific attention to the robustness of our results with respect to the wage setting assumption in Section 4.

where b represents home production.

For individuals with ability $a \geq \tilde{a}$, Bellman equations turn out to be

$$r\mathcal{U}_1(a) = b + p(E_1(a) - \mathcal{U}_1(a)) \quad (10)$$

$$r\mathcal{U}_2(a) = b + p(E_2(a) - \mathcal{U}_2(a)) - \pi(\mathcal{U}_2(a) - \mathcal{U}_1(a)) \quad (11)$$

$$rE_1(a) = w_1(a) - \delta(E_1(a) - \mathcal{U}_2(a)) \quad (12)$$

$$rE_2(a) = w_2(a) - \delta(E_2(a) - \mathcal{U}_2(a)) \quad (13)$$

Equation (12) deserves some attention. Since any type-1 worker when employed benefits from training provided by his new employer, he becomes a type-2 unemployed in the event of job destruction.

Wages are the solutions of the following Nash-sharing rules:

$$\beta J_0(a) = (1 - \beta)(E_0(a) - \mathcal{U}_0(a)) \quad (14)$$

$$\beta(J_1(a) - \hat{\gamma}_F) = (1 - \beta)(E_1(a) - \mathcal{U}_1(a)) \quad (15)$$

$$\beta J_2(a) = (1 - \beta)(E_2(a) - \mathcal{U}_2(a)) \quad (16)$$

Let us define $x = \beta \left(\frac{r+\delta+p}{r+\delta+\beta p} \right)$ and $x_0 = \beta \left(\frac{r+\delta+p_0}{r+\delta+\beta p_0} \right)$. Wage equations write

$$w_0(a) = x_0 a + (1 - x_0)b \quad (17)$$

$$w_1(a) = x[(1 + \Delta)a - (r + \delta)\hat{\gamma}_F] + (1 - x)[b - \delta(\mathcal{U}_2(a) - \mathcal{U}_1(a))] \quad (18)$$

$$w_2(a) = x(1 + \Delta)a + (1 - x)[b - \pi(\mathcal{U}_2(a) - \mathcal{U}_1(a))] \quad (19)$$

where

$$\mathcal{U}_2(a) - \mathcal{U}_1(a) = \frac{\beta p}{r + \pi + \beta p} \hat{\gamma}_F \quad (20)$$

Property 1. *Human capital depreciation is associated with a wage loss that amounts to*

$$w_2(a) - w_1(a) = \hat{\gamma}_F \frac{\beta(r + \delta)(r + \pi + p)}{(r + \pi + \beta p)} > 0, \quad \forall a \geq \tilde{a}.$$

Proof. By substituting out for $\mathcal{U}_2(a) - \mathcal{U}_1(a)$ from (20) into (18) and (19), the result follows. \square

Property 1 states that holding up-to-date knowledge brings a wage premium to the worker. This is consistent with Ljungqvist and Sargent's analysis: turbulence entails substantial wage loss for workers.

Wages actually correspond to a weighted average of worker's net contribution to output and reservation wages. Importantly, the reservation wages of type-1 and type-2 workers are negatively related to the unemployment

gap $\mathcal{U}_2(a) - \mathcal{U}_1(a)$ which is due to higher wage prospects ($w_2(a) > w_1(a)$), as shown by the following equality:

$$\mathcal{U}_2(a) - \mathcal{U}_1(a) = \frac{p(w_2(a) - w_1(a))}{(r + \delta)(r + \pi + p)}$$

When matched with a job, type-1 workers expect that, in the event of job destruction at rate δ , they will enter the pool of type-2 unemployed instead of their actual type-1 position, and then can earn a higher wage in some future type-2 position. This reduces type-1 reservation wages at the time of bargaining. Furthermore, type-2 workers expect that if the bargaining process fails they face a risk of human capital depreciation, occurring at rate π , which accounts for a loss $\mathcal{U}_2(a) - \mathcal{U}_1(a)$. This lowers type-2 reservation wages.

2.4 The impact of turbulence on training

At this stage, the (partial) labor market equilibrium is mainly characterized by the equilibrium value of the ability threshold \tilde{a} . Our goal here is to show how turbulence affects this threshold. In order to avoid corner equilibrium with $\tilde{a} = \underline{a}$, we introduce the following assumption:

Assumption 1. (i) $\underline{a} = b$; (ii) $(1 - s)\gamma_F \left(r + \frac{(r+\pi)\delta}{r+\pi+p} \right) > \Delta b$.

Proposition 1. *Under Assumption 1, the equilibrium ability threshold with Nash bargaining is characterized by:*

$$\Delta \tilde{a} = \hat{\gamma}_F \left(r + \frac{(r + \pi)\delta}{r + \pi + \beta p} \right) + (\tilde{a} - b) \left(\frac{x - x_0}{1 - x} \right) \quad (21)$$

which, for $r \rightarrow 0$, collapses to

$$\Delta \tilde{a} = \hat{\gamma}_F \left(\frac{\delta \pi}{\pi + \beta p} \right) + (\tilde{a} - b) \beta \left(\frac{p - p_0}{\delta + \beta p_0} \right) \quad (22)$$

Proof. Combining equation (7) with (17), (18) and (20) leads to equation (21). Assumption 1 implies that $\tilde{a} > \underline{a} = b$. \square

Property 2. *Under Assumption 1 and condition $(1 - x)\Delta > (x - x_0)$, a higher turbulence reduces the share of trained workers and, for $p > p_0$, raises the unemployment rate.*

Proof. From Proposition 1, it comes that

$$\tilde{a} = \frac{\hat{\gamma}_F \left(r + \frac{(r+\pi)\delta}{r+\pi+\beta p} \right) - b \left(\frac{x-x_0}{1-x} \right)}{\Delta - \frac{x-x_0}{1-x}}$$

Then, if $(1-x)\Delta > (x-x_0)$, the threshold \tilde{a} is unambiguously increasing with respect to π . Moreover, equation (3) implies that overall unemployment is positively related to the proportion of type-0 workers, $F(\tilde{a})$, if $p > p_0$. This concludes the proof. \square

The condition $(1-x)\Delta > (x-x_0)$ (equivalent to $\Delta > \beta(p-p_0)/(\delta+\beta p_0)$ if r is assumed to be close to zero) is necessary for the solution to be interior. It is actually equivalent to $(1-x)(1+\Delta)a > (1-x_0)a$ which means that, once the training cost has been paid, the instantaneous profit value of training is positive.⁷

To understand Property 2, it is worth emphasizing that the equilibrium training rule highly depends on the expected unemployment surplus related to training $\mathcal{U}_2(a) - \mathcal{U}_1(a)$. Formally, the derivation of Proposition 1 indeed relies on the fact that the equilibrium ability threshold solves:

$$\Delta \tilde{a} = (r + \delta) \hat{\gamma}_F - \delta (\mathcal{U}_2(a) - \mathcal{U}_1(a)) + (\tilde{a} - b) \left(\frac{x - x_0}{1 - x} \right) \quad (23)$$

where the unemployment gain $\mathcal{U}_2(a) - \mathcal{U}_1(a)$ is defined by (20). The intuition relies on the fact that higher turbulence reduces the relative unemployment gain associated with up-to-date knowledge $\mathcal{U}_2(a) - \mathcal{U}_1(a)$, because workers expect to switch more quickly from type-2 to type-1 unemployment. Therefore, turbulence increases the reservation wage of type-1 workers and raises the ability threshold \tilde{a} . Otherwise stated, turbulence discourages firms to train by increasing threat points of type-1 workers.⁸ Therefore, the fraction of untrained workers $F(\tilde{a})$ who face a lower probability of exiting unemployment ($p_0 < p$) increases with π . From equation (3), this results in a rise in the overall unemployment rate, as Property 2 states.

On the contrary, it should also be added that the expected unemployed surplus related to training is increasing with respect to worker's bargaining

⁷Since $p \geq p_0$, the "effective" bargaining powers of the workers are such that $x \geq x_0$. Therefore, Δ may not be large enough to imply that the instantaneous profit of the firm increases with training. Assumption $(1-x)\Delta > (x-x_0)$ rules out this possibility which would lead to a corner equilibrium without training ($\tilde{a} = \bar{a}$).

⁸It should be emphasized that this result is consistent with an average wage that decreases with turbulence. Indeed, the share of untrained workers who earn the lowest wage $w_0(a)$ is increasing with π .

power β . Higher bargaining power for workers means that workers internalize a higher wage cut due to the cost of training (see equation (18)). Hence, the relative value of having up-to-date knowledge (type-2 position) is higher.

3 Turbulence, Efficient Training and Optimal Subsidy

3.1 The efficient training policy

The problem of the planner consists in maximizing the steady-state average output value net of turnover costs by choosing the optimal ability threshold below which workers are not trained (throughout we consider $r = 0$). This problem can be stated as follows:

$$\max_{a^*} \int_{\underline{a}}^{a^*} S_L(a) da + \int_{a^*}^{\bar{a}} S_H(a) da$$

where

$$\begin{aligned} S_L(a) &= a[f(a) - u(a)] + bu(a) \\ S_H(a) &= (1 + \Delta)a[f(a) - u(a)] + bu(a) - \gamma_F p u_1(a) \end{aligned}$$

and subject to

$$u(a) = \begin{cases} f(a) \frac{\delta}{\delta + p_0} & \text{if } a < a^* \\ f(a) \frac{\delta}{\delta + p} & \text{if } a \geq a^* \end{cases} ; u_1(a) = f(a) \frac{\delta \pi}{(p + \pi)(p + \delta)} \text{ if } a \geq a^*$$

Proposition 2. *Under Assumption 1, the efficient training ability threshold is characterized by*

$$\Delta a^* = \gamma_F \frac{\delta \pi}{\pi + p} - (a^* - b) \frac{\delta}{\delta + p_0} \left(\frac{p - p_0}{p} \right) \quad (24)$$

and is increasing with turbulence.

Proof. Straightforward by taking into account of the discontinuity of $u(a)$ at $a = a^*$. \square

Turbulence has the same effect on the optimal ability threshold as on the equilibrium one. From the social planner's point of view, it is indeed less worthwhile to train workers since turbulence increases the probability of losing training investment costs. Hence, low-ability workers who could

be trained in the context of low turbulence, are no longer trained if the turbulence is high.

The contact rates p and p_0 affect the efficient ability threshold, but not in the same way as they affect the equilibrium threshold. In order to understand these differences, we need to disentangle the externalities that make equilibrium inefficient, focusing on the two following extreme configurations: (i) $p_0 = p$ and (ii) $p_0 = 0$ and β close to one.

3.2 The poaching externality

Property 3. Consider $p_0 = p$ and $r = 0$. The efficient training ability threshold is characterized by

$$\Delta a^* = \gamma_F \frac{\delta\pi}{\pi + p}$$

and is lower than the equilibrium threshold \tilde{a} for $s = 0$, which satisfies $\Delta\tilde{a} = \gamma_F \frac{\delta\pi}{\pi + \beta p}$.

This result is in line with Acemoglu [1997]: imperfect labor market implies that firms do not internalize the entire social gain of their own training decision. Training does not only increase productivity of the worker in the current firm but also increases productivity in future jobs if his human capital does not depreciate during unemployment spells. Unlike firms, workers may internalize this value of training (higher expected wages in other firms). Nevertheless, since $\beta < 1$, they only get a fraction of the additional productivity related to training when they move to another job. Thus, the relative unemployed value of having up-to-date knowledge ($\mathcal{U}_2(a) - \mathcal{U}_1(a)$) is not high enough. Consequently, the reservation wage of type-1 workers is too high and the number of people kept out of training is higher than the efficient level. Of course, as β goes to unity, workers tend to capture the total gain related to training. The gap between equilibrium and efficient thresholds would then tend to zero (assuming $p_0 = p$).

3.3 The steady-state unemployment externality

Let us now abstract from the poaching externality by assuming that β is arbitrarily close to unity, and consider the extreme case where workers who have never been trained face a zero job finding rate ($p_0 = 0$).

Property 4. Consider $p_0 = 0$, $\beta \rightarrow 1$, $s = 0$ and $r = 0$. The efficient training ability threshold is characterized by:

$$\Delta a^* = \gamma_F \frac{\delta\pi}{\pi + p} - (a^* - b) < \Delta\tilde{a} = \gamma_F \frac{\delta\pi}{\pi + p} + (\tilde{a} - b) \frac{p}{\delta} \quad (25)$$

and the equilibrium unemployment is higher than its efficient value.

From the point of view of the social planner, equation (24) already shows that a lower contact rate for type-0 workers reduces the efficient ability threshold a^* , so that it increases the fraction of workers who should be trained. This reflects the fact that firms do not internalize the impact of training on (un)employment rate. If $p_0 = 0$, keeping a worker of ability a out of training leads to a social output loss that amounts to $a - b$. Firms do not value this loss, whereas the planner does.⁹ Then, obviously, the size of the externality is greater when the difference between market and home productions is large, *i.e.* the social cost of a rise in unemployment increases with $a^* - b$.

In addition, from the equilibrium's perspective, it also appears that assuming $p > p_0$ contributes to increase the ability threshold by raising the "effective" bargaining power of workers (x instead of x_0), hence wages.¹⁰

Accordingly, from both arguments, firms underinvest in training. Then, too many workers face a low probability to exit unemployment. Therefore, the equilibrium unemployment rate is higher than the efficient one.

3.4 Turbulence and the optimal training subsidies

Differences between the equilibrium and the optimum come from the gap between the efficient ability threshold and its equilibrium value. We now address the issue of the optimal policy. Does a subsidy of the training cost allow to decentralize the optimum? Moreover, how should it evolve according to the level of turbulence? Otherwise stated, should the subsidy rate increase or decrease when the probability of losing human capital during unemployment spell raises? Our point is that there exists two offsetting effects according to the externality we take into consideration, either "poaching" or "steady-state unemployment".

Proposition 3. *Consider $r = 0$. Under Assumption 1, the optimal subsidy rate of training is:*

$$s^* = (1 - \beta) \left(\frac{p}{\pi + p} \right) + \left[\frac{1 + \beta \left(\frac{\delta + p_0}{\delta + \beta p_0} \right) \frac{p}{\delta}}{1 + \Delta \left(\frac{\delta + p_0}{p - p_0} \right) \frac{p}{\delta}} \right] \left[\frac{\pi + \beta p}{\pi + p} - \frac{\Delta b}{\delta \gamma_F} \left(\frac{\pi + \beta p}{\pi} \right) \right]$$

⁹For $0 < p_0 < p$, the externality consists in having not internalized the fact that workers who are not trained face a longer unemployment spell than those who can benefit from up-to-date knowledge.

¹⁰As stated in the sequel this effect vanishes in the context of alternating bargaining of wages *à la* Hall-Milgrom.

Proof. Using Propositions 1 and 2 with $\widehat{\gamma}_F = (1 - s^*)\gamma_F$, s^* is the value of the subsidy rate such that $\tilde{a} = a^*$. \square

In a preliminary step we go back to the two particular specifications $\{\beta < 1, p_0 = p\}$ and $\{\beta = 1, p_0 = 0\}$ allowing to focus separately on each externality at work. A more general analysis will then be provided at the end of this section.

Corollary 1. *Consider $p_0 = p$ and $r = 0$. The optimal rate of training subsidy is:*

$$s^* = (1 - \beta) \left(\frac{p}{\pi + p} \right) \quad (26)$$

and is decreasing with the Poisson rate of human capital depreciation π .

For $\beta < 1$, it is efficient to subsidize the training cost due to the poaching externality. Corollary 1 states that the optimal subsidy rate should decrease with turbulence. Both equilibrium and efficient ability thresholds are increasing with π but, the higher the turbulence, the lower the size of the poaching externality. Indeed, \tilde{a} tends to a^* as the Poisson rate π goes to infinity. When the turbulence is high, a firm would less likely benefit from training decision of other firms; the social return to training investment converges to its private return. Ultimately, if the rate π goes to infinity, the probability of experiencing a human capital depreciation tends to unity. Then, training investment collapses to job-specific human capital, so that both equilibrium training policy and unemployment rate are optimal and no subsidy is required. From that point of view, we need less training subsidies in the context of high turbulence.

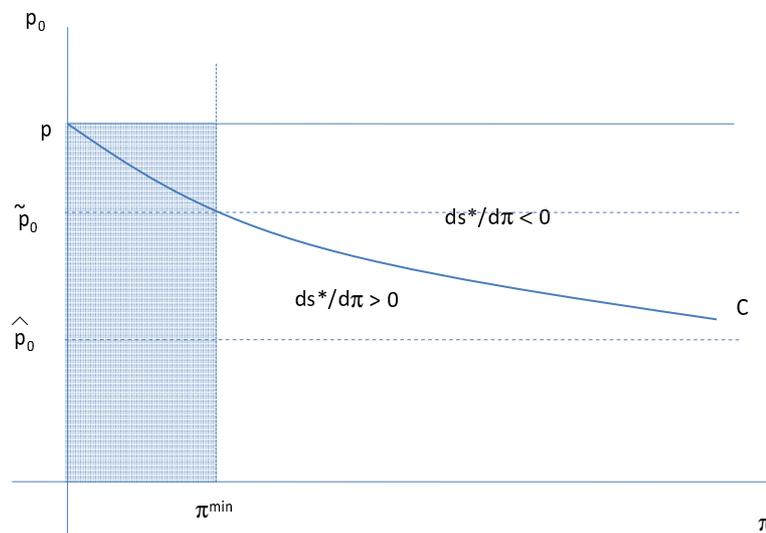
Corollary 2. *Consider $p_0 = 0$, $\beta = 1$ and $r = 0$. The optimal subsidy rate of training is:*

$$s^* = \frac{1 + \frac{p}{\delta}}{1 + \Delta} \left[1 - \frac{\Delta b}{\delta \gamma_F} \left(\frac{\pi + p}{\pi} \right) \right]$$

and is increasing with the Poisson rate of human capital depreciation π .

Assuming $p_0 = 0 < p$ and $\beta = 1$ eliminates the poaching externality. Corollary 2 only deals with what we have labeled “steady-state unemployment” externality. It turns out that a higher rate π of human capital depreciation leads to implement a higher training subsidy rate in order to restore efficiency. Both efficient and equilibrium training ability thresholds are still increasing with economic turbulence, but the gap between them also increases with respect to π . Indeed, any increase in the ability thresholds a^* raises the

Figure 2: Turbulence and the optimal subsidy rate



size of the output loss $a^* - b$ related to higher unemployment duration of type-0 workers (this duration is actually infinite when $p_0 = 0$). Unlike the central planner, firms do not internalize the social incidence of an increase in ability threshold. Therefore, the higher the turbulence, the larger the gap between equilibrium and efficient thresholds. Consequently, training subsidies increases with π .

As a result, the overall impact of turbulence on the subsidy rate of training cost is ambiguous. In response to higher turbulence, the increase in steady-state unemployment externality introduces an opposite force to the reduction of inefficiencies related to the poaching externality. It is thus unclear whether decentralization of the efficient threshold leads to greater training subsidies in the context of higher economic turbulence or not.

Let us now consider the intermediate case where both externalities are simultaneously at work. Figure 2 reports the sign of the marginal impact of an increase in turbulence on the optimal subsidy rate of training, according to the Poisson rate of human capital depreciation and the contact rate for type-0 workers, (π, p_0) . It shows that three regimes typically emerge, according to the value of p_0 :¹¹

¹¹On Figure 2, π^{\min} corresponds to the lower bound of the Poisson rate π that satisfies

- If $p_0 \leq \hat{p}_0$, then the steady-state unemployment externality is so high that an increase in turbulence always requires to increase the subsidy rate of training.
- If $p_0 \geq \tilde{p}_0$, then the steady-state unemployment externality is so low that an increase in turbulence always requires to reduce the subsidy rate of training.
- For intermediate values $p_0 \in (\hat{p}_0, \tilde{p}_0)$, there exists a threshold initial value for π (defined by C) above which an increase in turbulence requires a cut in the subsidy rate of training.

This suggests that the gap $p-p_0$ is crucial. In economies where unemployment durations of low-skilled workers ($1/p_0$) and high-skilled workers ($1/p$) are close, it is optimal to reduce the subsidy rate of training when economic turbulence is rising. On the contrary, in economies where low-skilled workers face low contact rates, training subsidies are increasing with turbulence.

4 Robustness to the Wage Setting Process

Before turning to a general equilibrium analysis with endogenous contact rates, we consider robustness of the preceding results to alternative wage bargaining processes.

4.1 Strategic alternating bargaining

Since Shimer [2005] and Hall and Milgrom [2008], Nash-bargaining of wages is a somewhat disputed assumption, at least from an empirical perspective. According to Shimer [2005], the standard matching model does not explain the volatility in the ratio of job vacancies to unemployment. The role of real wage rigidity is pointed out, challenging the Nash-bargaining assumption for wage determination. The strategic alternating wage bargaining game, introduced by Hall and Milgrom,¹² is a way to reconcile the model with the data.

In our framework, the main difference between the standard Nash bargaining and the strategic alternating bargaining lies in the threat points.

Assumption 1 (ii), for $s = 0$ and $r = 0$. Considering π larger than π^{\min} means that \tilde{a} and a^* are both greater than b . The curve C is the set of values of π and p_0 such that $\frac{ds}{d\pi} = 0$. C is decreasing, passes through the point $(0, p)$ with an asymptote $p_0 = \hat{p}_0$ for $\pi \rightarrow +\infty$. Finally, \tilde{p}_0 is the value of p_0 where the curve C intersects the vertical line $\pi = \pi^{\min}$.

¹²See, e.g., Nagypal [2007] for an application of this wage setting game to a search-matching model

Following Hall and Milgrom, we now consider that during the bargaining process the threat point of employed workers is not the unemployed position but the so-called *delay* D_i ($i \in \{0, 1, 2\}$), which satisfies

$$rD_i(a) = b + \delta (\mathcal{U}_i(a) - D_i(a)), \quad \forall i = 0, 1, 2.$$

Importantly, for type-1 workers, we assume that bargaining takes place before worker's training. Therefore, if bargaining fails, the worker goes back to the type-1 unemployed position.

Accordingly, the sharing rules can be re-stated as follows

$$\begin{aligned} \beta J_0(a) &= (1 - \beta) (E_0(a) - D_0(a)) \\ \beta (J_1(a) - \widehat{\gamma}_F) &= (1 - \beta) (E_1(a) - D_1(a)) \\ \beta J_2(a) &= (1 - \beta) (E_2(a) - D_2(a)) \end{aligned}$$

It is then straightforward to derive the following wage equations

$$w_0(a) = \beta a + (1 - \beta) b \quad (27)$$

$$w_1(a) = \beta [(1 + \Delta) a - (r + \delta) \widehat{\gamma}_F] + (1 - \beta) [b - \delta (\mathcal{U}_2(a) - \mathcal{U}_1(a))] \quad (28)$$

$$w_2(a) = \beta (1 + \Delta) a + (1 - \beta) b \quad (29)$$

where

$$\mathcal{U}_2(a) - \mathcal{U}_1(a) = \frac{\beta p}{r + \pi + \beta p + \frac{r(1-\beta)p}{r+\delta}} \widehat{\gamma}_F \quad (30)$$

Wages are still a weighted average of productivity and reservation wage. But, unlike the conventional Nash-bargaining, weights obtained with the strategic alternating wage setting game do not depend on the contact rates. The share of the surplus that goes to the worker is now lower ($\beta < x$).

Proposition 4. *Consider $r = 0$. Under Assumption 1, the equilibrium ability threshold with strategic alternating bargaining is characterized by:*

$$\Delta \tilde{a} = \widehat{\gamma}_F \frac{\delta \pi}{\pi + \beta p} \leq \Delta \tilde{a} \quad (31)$$

Proof. Combining equation (7) with (27), (28) and (30) leads to equation (31). \square

Comparing equations (22) and (31) shows that the equilibrium ability thresholds are the same under the restriction $p_0 = p$; the optimal subsidy rate of training is then given by equation (26). Otherwise (for $p > p_0$), the

threshold with strategic bargaining is lower than with Nash bargaining. This is indeed due to the fact that, under strategic bargaining, wages no longer depend on the contact rates of the unemployed. The wage gap related to training, *i.e.* to the switch from type-0 to type-1 status for a given a , is then lower.

Interestingly, the ambiguity of the relationship between the subsidy rate of training and turbulence remains however unchanged. On the one hand, for $p_0 = p$, the optimal subsidy rate, still given by (26), is decreasing with π . On the other hand, if we revisit the optimal subsidy rate of training when only the unemployment externality exists, the optimal subsidy rate of training is lower than with Nash bargaining, but is still positively related to turbulence. Corollary 3 states this point.

Corollary 3. *Consider $p_0 = 0$, $\beta = 1$ and $r = 0$. Under Assumption 1, in the context of strategic alternating bargaining, the optimal subsidy rate of training is:*

$$s^{**} = \frac{1}{1 + \Delta} \left[1 - \frac{\Delta b}{\delta \gamma_F} \left(\frac{\pi + p}{\pi} \right) \right] = \frac{s^*}{1 + \frac{p}{\delta}}$$

and is increasing with π .

4.2 Bargaining with hold-up

With both Nash bargaining and strategic alternating wage setting processes, it is obvious that workers have ex-post a clear incentive to renege on the initial wage agreement. Without enforceable contractual arrangement of the type developed by Malcomson [1997], a hold-up problem may then arise.¹³ In other words, we consider here that type-1 workers are able to bargain the same earnings as type-2 workers. Since we assume homogenous contact rate p for type-1 and type-2 unemployed workers, it comes that $\mathcal{U}_1 = \mathcal{U}_2$.

Then, denoting by $w^h(a)$ the wage for workers with ability $a \geq \tilde{a}$, it is now straightforward to see that:

$$w^h(a) = \begin{cases} x(1 + \Delta)a + (1 - x)b & \text{if Nash bargaining} \\ \beta(1 + \Delta)a + (1 - \beta)b & \text{if strategic alternating} \end{cases}$$

Proposition 5. *Consider $r = 0$. Under Assumption 1, in the context of un-enforceable wage contracts, the equilibrium training ability threshold is char-*

¹³This point has already been emphasized in a context where firms invest in physical capital by Acemoglu and Shimer [1999]. Chéron [2005] also considers the case of training costs.

acterized by

$$\Delta a^h = \begin{cases} \widehat{\gamma}_F \frac{\delta + \beta p}{1 - \beta} + (a^h - b)\beta \left(\frac{p - p_0}{\delta + \beta p_0} \right) & \text{if Nash bargaining} \\ \widehat{\gamma}_F \frac{\delta}{1 - \beta} & \text{if strategic alternating} \end{cases}$$

In the context of contractual incompleteness, each type-1 worker has ex-post incentives to renege on the wage contract and claims for type-2 wage level. Accordingly, firms face an additional wage cost that moves upward the training threshold: either $a^h > \tilde{a}$ or $a^h > \tilde{\tilde{a}}$ according to the wage setting process we consider.

Property 5. *If wage contracts are unenforceable, the share of trained workers does not depend on turbulence.*

Otherwise stated, since wages are (re)negotiated after training, turbulence has no longer impact on wages and equilibrium training rules. Furthermore, it also appears that the sign of the relationship between the optimal subsidy rate of training and turbulence is now clear cut even though $p > p_0$.

Proposition 6. *Consider $r = 0$ and $p_0 = 0$. Under Assumption 1, the optimal rate of training subsidy is:*

$$s^* = \begin{cases} 1 - \frac{\Delta}{1 + \Delta} (1 - \beta) \left[\frac{\pi}{\pi + p} \left(\frac{\delta}{\delta + \beta p} \right) \left(1 - \frac{\beta p}{\Delta \delta} \right) + \frac{b}{\delta \gamma_F} \right] & \text{if Nash bargaining} \\ 1 - \frac{\Delta}{1 + \Delta} (1 - \beta) \left(\frac{\pi}{\pi + p} + \frac{b}{\delta \gamma_F} \right) & \text{if strategic alternating} \end{cases}$$

and is decreasing with π .

An increase in the Poisson rate of human capital depreciation π unambiguously implies a decrease in the optimal subsidy rate of training, whatever the wage setting process we consider, and despite we are looking at a parameter configuration where the steady-state unemployment externality is at its highest value (with $p_0 = 0$).

The intuition behind this result is as follows. On the one hand, the equilibrium ability threshold a^h is no longer related to the Poisson rate of human capital loss. This occurs because, in the context of hold-up, workers do not expect any wage loss in the event of human capital depreciation. On the other hand, the efficient ability threshold still increases with turbulence, because it is less worthwhile from the planner's point of view to pay the training costs which are more likely to become unproductive. As a consequence, the gap between efficiency and equilibrium thresholds unambiguously decreases when the Poisson rate of human capital depreciation raises. The higher the turbulence, the lower is the optimal subsidy rate of training.

5 Optimal Training Policy with Endogenous Matching

Until now, results have been derived under the assumption of exogenous matching probabilities. Our purpose is now to show that both externalities (poaching and steady-state unemployment) are still at work in a more general framework with endogenous contact rates.

5.1 Labor market equilibrium with Nash bargaining and endogenous matching

As in the partial equilibrium framework, labor market is segmented. Firms can direct their search according to the observable component of worker's ability, a , which represents for instance diploma and is assumed to be constant over time. But, firms cannot know who are the unemployed workers that has been hit by a human capital depreciation shock. Otherwise stated, we assume that firms cannot discriminate *ex ante* between workers with up-to-date knowledge and workers without.

Let $\theta(a)$ denote the labor market tightness associated with ability a , and $q(\theta(a))$, the contact rate for a vacant job directed toward individuals with ability a to be matched with a worker. Expected values $V(a)$ of vacancies satisfy:

$$rV(a) = -c + q(\theta(a)) \{J_0(a) - V(a)\}, \quad \text{if } a < \tilde{a} \quad (32)$$

$$rV(a) = -c + q(\theta(a)) \left\{ \frac{(J_1(a) - \hat{\gamma}_F)u_1(a) + J_2(a)u_2(a)}{u_1(a) + u_2(a)} - V(a) \right\}, \quad \text{if } a \geq \tilde{a} \quad (33)$$

where c stands for the instantaneous flow of recruitment cost. Expected returns of a filled job $J_i(a)$ satisfy equations (4)-(5), and the cut-off point in ability space is still characterized by (6).

Free-entry conditions for each ability level, $V(a) = 0$, then allows to determine the equilibrium labor market tightness and therefore the endogenous contact rate for the workers. With a constant-returns-to-scale matching function, the contact rate $p(a) \equiv \theta(a)q(\theta(a))$ is increasing and concave with respect to tightness $\theta(a)$. As stated hereafter, one feature of this equilibrium is that there exists a discontinuity of $p(a)$ at \tilde{a} , which is consistent with our previous assumption on exogenous contact rates.

Proposition 7. Consider $r = 0$. The labor market equilibrium is characterized by a tightness function $\theta(a)$ and an ability threshold \tilde{a} that satisfy:

$$\frac{c\delta}{q(\theta(a))} = (1 - \beta)(a - b) - \beta c\theta(a), \quad \forall a < \tilde{a} \quad (34)$$

$$\begin{aligned} \frac{c\delta}{q(\theta(a))} &= (1 - \beta)((1 + \Delta)a - b) - \beta c\theta(a) \\ &\quad - \frac{\hat{\gamma}_F(1 - \beta)\pi}{\pi + p(a)} \left[\delta - \frac{(p(a) + \delta)\beta p(a)}{\pi + \beta p(a)} \right], \quad \forall a \geq \tilde{a} \end{aligned} \quad (35)$$

$$\Delta\tilde{a} = \hat{\gamma}_F \frac{\delta\pi}{\pi + \beta p(\tilde{a})} + (\tilde{a} - b)\beta \left(\frac{p(\tilde{a}) - p(\tilde{a}_-)}{\delta + \beta p(\tilde{a}_-)} \right) \quad (36)$$

where $p(a) = \theta(a)q(\theta(a))$ and $p(\tilde{a}_-) = \lim_{a \rightarrow \tilde{a}, a < \tilde{a}} p(a)$.

Proof. The characterization of the labor market equilibrium with endogenous matching probabilities is obtained using the wage equations (17)-(19), with $p(a)$ instead of p_0 and p , the training condition (7) and the free-entry conditions $V(a) = 0, \forall a$. \square

Property 6. The labor market equilibrium is characterized by a discontinuity of the contact rate function $p(a)$ at \tilde{a} such that $p(\tilde{a}) > p(\tilde{a}_-) \equiv \lim_{a \rightarrow \tilde{a}, a < \tilde{a}} p(a)$.

Proof. Let us define

$$\Psi(\theta) \equiv \frac{c\delta}{q(\theta)} - (1 - \beta)(a - b) + \beta c\theta$$

It comes that (35) at $a = \tilde{a}$ is equivalent to

$$\Psi(\theta(\tilde{a})) = \frac{(J_1(\tilde{a}) - \hat{\gamma}_F - J_0(\tilde{a}))u_1(\tilde{a}) + (J_2(\tilde{a}) - J_0(\tilde{a}))u_2(\tilde{a})}{u_1(\tilde{a}) + u_2(\tilde{a})}$$

where the right-hand side is positive since $J_2(\tilde{a}) > J_1(\tilde{a}) - \hat{\gamma} = J_0(\tilde{a})$. It is straightforward to see that Ψ is increasing with θ . Accordingly, this shows that $a = \tilde{a}$ is a discontinuity point of the tightness function $\theta(a)$, i.e. $\theta(\tilde{a}) > \theta(\tilde{a}_-) \equiv \lim_{a \rightarrow \tilde{a}, a < \tilde{a}} \theta(a)$. Hence $\theta(\tilde{a})q(\theta(\tilde{a})) = p(\tilde{a}) > \theta(\tilde{a}_-)q(\theta(\tilde{a}_-)) = p(\tilde{a}_-)$. This concludes the proof. \square

By posting a vacant position directed toward workers with ability higher or equal to \tilde{a} , the firm expects that it may meet a type-2 worker whose up-to-date knowledge allows to reach a higher productivity without any additional

cost. This implies that, everything else being equal, more firms would like to enter. Therefore, this provides general equilibrium foundations to our former assumption on contact rates and gives rise to the steady-state unemployment externality related to the training decision of the firms.

In addition, from Proposition 7, it is clear that there is no impact of turbulence on labor market tightness for workers with ability $a < \tilde{a}$. For workers with ability $a \geq \tilde{a}$, one can suspect that the higher the turbulence the lower the labor market tightness.¹⁴ To understand this, it should be first noticed that the impact of turbulence on equilibrium labor market tightness relies on wage costs variations. The point is that an increase in turbulence raises reservation wages of type-1 and type-2 workers by reducing the relative value of having up-to-date knowledge when unemployed (the gap $\mathcal{U}_2(a) - \mathcal{U}_1(a)$). Hence, this entails an increase in wages $w_1(a)$ and $w_2(a)$, which in turn implies a rise of the expected wage costs for the firms so that fewer firms choose to post vacancies directed toward workers with ability high enough to be trained ($a \geq \tilde{a}$). Interestingly, this suggests that this additional mechanism (with respect to the exogenous rates case) accounts for a decrease in the gap between contact rates $p(\tilde{a}) - p(\tilde{a}_-)$ (previously $p - p_0$) when turbulence is rising.

5.2 Efficient allocation and policy revisited with endogenous matching

The problem of the planner now consists to choose the labor market tightness for each ability level and the ability cut-off for training that maximize the steady-state average output value net of turnover costs:

$$\max_{a^*, \theta^*(a)} \int_a^{a^*} S_L(a) da + \int_{a^*}^{\bar{a}} S_H(a) da$$

with

$$\begin{aligned} S_L(a) &= a[f(a) - u(a)] + (b - c\theta^*(a))u(a) \\ S_H(a) &= (1 + \Delta)a[f(a) - u(a)] + (b - c\theta^*(a))u(a) - \gamma_{FP} p^*(a) u_1(a) \end{aligned}$$

¹⁴A formal proof of this latter statement is not trivial, because equation (35) which can be written as $f(a, \theta) = 0$ does not imply in general a clear cut sign of the derivative $\frac{\partial f(a, \theta)}{\partial \theta}$. This is actually due to the fact that the gap $\mathcal{U}_2(a) - \mathcal{U}_1(a)$ is increasing with p , hence θ . As a consequence, reservation wages for workers with ability $a \geq \tilde{a}$ are decreasing with labor market tightness and this introduces an opposition force to the conventional positive wage impact of an increase in search costs ($c\theta$). During our discussion, we consider that this positive wage impact of labor market tightness *via* search costs still dominates its negative impact *via* reservation wages.

where $p^*(a) = \theta^*(a)q(\theta^*(a))$, and subject to

$$\begin{aligned} u(a) &= f(a) \frac{\delta}{\delta + p(a)}, \quad \forall a \in [\underline{a}, \bar{a}] \\ u_1(a) &= f(a) \frac{\delta\pi}{(p(a) + \pi)(p(a) + \delta)}, \quad \forall a \geq a^* \end{aligned}$$

Proposition 8. *Let $\psi = 1 - \frac{\theta p'(\theta(a))}{p(\theta(a))}$. The efficient allocation is characterized by:*

$$\frac{c\delta}{q(\theta^*(a))} = (1 - \psi)(a - b) - \psi c\theta^*(a), \quad \forall a < a^* \quad (37)$$

$$\begin{aligned} \frac{c\delta}{q(\theta^*(a))} &= (1 - \psi)((1 + \Delta)a - b) - \psi c\theta^*(a) \\ &\quad - \gamma_F \frac{\pi}{\pi + p^*(a)} \left[\delta - \frac{\delta + p^*(a)}{\pi + p^*(a)} p^*(a) \right], \quad \forall a \geq a^* \end{aligned} \quad (38)$$

$$\Delta a^* = \widehat{\gamma}_F \frac{\delta\pi}{\pi + p^*(a^*)} - \frac{(a^* - b)\delta}{\delta + p^*(a^*)} \left(\frac{p^*(a^*) - p^*(a^*_-)}{p^*(a^*)} \right) \quad (39)$$

Property 7. *The Hosios condition $\psi = \beta$ does not achieve efficiency.*

Proof. Assuming $\psi = \beta$ in Propositions 7 and 8 leads to the result. \square

First, it is obvious that, due to poaching and steady-state externalities, the equilibrium ability threshold does not correspond to its efficient counterpart ($\tilde{a} \neq a^*$). But importantly, even if training were optimal ($\tilde{a} = a^*$), efficiency would still not be achieved. Indeed, with positive investment costs γ_F , equation (38) and equation (35) are still different. This is due to the fact that the valuation of the gap $\mathcal{U}_2 - \mathcal{U}_1$, which is related to training costs paid through wage cuts by type-1 unemployed when they come back to employment, remains lower than the corresponding valuation by the planner. Accordingly, reservation wages of type-1 and type-2 workers are too high and this pushes down labor market tightness for workers with ability $a \geq \tilde{a}$.

Two policy tools are then required to restore equilibrium efficiency. The training subsidy at rate s should be completed by another instrument such as an employment subsidy that can take the form of a transfer to workers, $\mu(a)$, $\forall a \in [\tilde{a}, \bar{a}]$. The transfer $\mu(a)$ is actually shared with the firm according

to the wage bargaining process and modifies the equilibrium allocation as follows:

$$\begin{aligned} \frac{c\delta}{q(\theta(a))} &= (1 - \beta)((1 + \Delta)a - b + \mu(a)) - \beta c\theta(a) \\ &\quad - \frac{(1 - s^*)\gamma_F(1 - \beta)\pi}{\pi + p(a)} \left[\delta - \frac{(p(a) + \delta)\beta p(a)}{\pi + \beta p(a)} \right], \quad \forall a \geq \tilde{a} \\ \Delta\tilde{a} &= \frac{(1 - s^*)\gamma_F\delta\pi}{\pi + \beta p(\tilde{a})} + (\tilde{a} - b)\beta \left(\frac{p(\tilde{a}) - p(\tilde{a}_-)}{\delta + \beta p(\tilde{a}_-)} \right) - \mu(\tilde{a}) \end{aligned} \quad (40)$$

Then, the optimal policy $\{s^*, \mu^*(a), \forall a \in [\tilde{a}, \bar{a}]\}$ can be derived from equalities $\tilde{a} = a^*$ and $\theta(a) = \theta^*(a)$. Since the two externalities we discussed in the partial equilibrium framework do exist in the general equilibrium context,¹⁵ it is still not clear-cut whether it is optimal to increase or to reduce the subsidy rate of training when turbulence is rising.

6 Conclusion

The main goal of this paper was to examine the role of training subsidies in an economy where workers face a risk of human capital depreciation. The idea that an increase of this risk (turbulence) was the main driving force behind the rise of European unemployment due to its interplay with generous unemployment benefits system was recently developed by Ljungqvist and Sargent. Our main result is that despite training and employment are lowered in a context of high turbulence, this does not necessarily involve a higher subsidy rate of human capital investments (training). The optimal subsidy rate indeed depends on the relative size of the standard poaching externality and the steady-state unemployment externality.

To some extent we think that these results can give some theoretical grounds to the observed decrease in public expenditures in some major OECD countries from the mid-nineties (Figure 1). However, it is obvious that to precisely deal with this issue our work should be extended to allow for a rigorous quantitative investigation. We should extend our work to account for a second-best analysis. Taking into account distortions related to welfare state is also an important issue and it should be interesting to examine the design of training subsidies from that point of view.

¹⁵One should also have noticed that a positive optimal level of the employment subsidy ($\mu^*(a) > 0$), which arises for instance when $\beta = \psi$ (the Hosios condition), contributes to reduce the optimal subsidy rate of training s^* because this employment subsidy reduces wages of type-1 workers, and therefore tends to decrease the equilibrium ability threshold (see equation (40)).

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