Equilibrium Wage Dispersion and the Role of Endogenous Search Effort Revisited

Arnaud CHERON[*]
Université du Maine (GAINS-TEPP) and EDHEC

Guoqing DING
Université du Maine (GAINS-TEPP) and Bank of China

November 12, 2008

Abstract

This paper put forward the idea that extending the on-the-job search model to account for endogenous search effort can help explaining the shape of the wage distribution. More precisely, we show that it can be the case the model generates hump-shaped wage distributions if preferences are non-separable between consumption and leisure.

Code JEL: J31, J41, J64.

1 Introduction

Since Burdett and Mortensen [1998] (BM henceforth), it is well known that the on-the-job search framework typically features equilibrium wage dispersion even though workers are *ex-ante* homogenous. Nevertheless, this theory typically predicts that the equilibrium wage density is monotonically increasing in wages in apparent contradiction with the observed hump-shaped wage distribution.

[*]Correspondance: Avenue Olivier Messiaen, Faculté de Droit et de Sciences Economiques, 72085 Le Mans Cedex 9, France. E-mail: acheron@univ-lemans.fr

Then, it is soon appeared that *ex-ante* exogenous heterogeneity related to job productivity and worker ability helps explaining the shape of the wage dispersion (see Bontemps, Robin and Van den Berg [2000] and Postel-Vinay and Robin [2002]). *Ex-post* heterogeneity due to specific human capital investments is also likely to generate hump-shaped equilibrium wage distributions (see Mortensen [1998], Quercioli [2005], Rosholm and Svarer [2004] or Chéron, Hairault and Langot [2008]). But, on the opposite, *ex-post* heterogeneity due to endogenous search effort does not help explaining to circumvent the empirical shortcomings of the BM model (see Mortensen [2003]). The intuition is as follows. Because the likelihood of finding a better offer decreases with the wage earned, search intensity is found to decrease with employed worker's current earnings and ultimately falls to zero at the top of the wage distribution. This gives firms additional incentives to post high wages, because the decrease in search effort raises job duration.

In this paper, it is our contention that allowing the BM model to feature non-strictly decreasing search effort may help explaining the shape of the wage distribution even in the context of *ex-ante* homogenous workers. To that end, we consider an on-the-job search model with two types of jobs and non-separable preferences -as simple as possible- to put emphasize on the potential explaining role of a rising search effort with wage on the shape of the wage distribution.

2 Model

2.1 Environment and labor market flows

We distinguish two types of jobs which can be think of as corresponding to differences in occupational categories. Let p_i be the marginal productivity in occupational category $i = 1, 2$, and assume $p_1 < p_2$.

Homogenous workers who exit unemployment at rate λ_0, start working in type-1 jobs. Then, they work h hours, face an exogenous risk of job destruction whose rate is denoted as δ, and search on-the-job not only for better opportunities in type-1 jobs, but also type-2 jobs, according to a search intensity $s = s(w)$, which in equilibrium depends on the current wage earnings w. Transition from p_1-job to p_2-job typically features a promotion by switching jobs (for instance a Technician who find a Manager position in another firm). Search is sequential and non-directed, and we let $s\lambda_i$, $i = 1, 2$ denote the arrival rates of offers, either of type-1 or of type-2, both increasing with search effort. In the p_1 jobs segment, wage is posted by firms, according to the wage posting game as described by BM. Cahuc, Postel-Vinay and Robin

[2006] emphasize that low-skilled workers' earnings are indeed mainly related to on-the-job search opportunity and wage posting game among firms, whereas the impact of bargaining power is much more important for high-skilled workers. We then consider that once an employed switch to a high productivity job, he works $\overline{h} > h$ hours and earns productivity p_2. That is, we are assuming that the bargaining power of workers in type-2 jobs is one and, as a consequence, the worker stops searching on-the-job. p_2-jobs destroy at rate δ and workers then reinter the homogenous pool of unemployed workers. Allowing for heterogeneous unemployed positions and/or lower bargaining power for workers on p_2-job position would add some complexities but without modifying, at least qualitatively, our analysis of the wage distribution impact of the search effort function.

Let normalize to one the sum of unemployed workers and employees in type-1 jobs, the steady state unemployment rate is given by $u = \frac{\delta}{\delta+\lambda_0}$.[1] Then, let G and F denote the cumulative distribution function of earnings and wage offers in p_1-jobs, the stock of employees earning w or less in p_1-jobs is $(1-u)G(w)$. Workers leave this stock because they are laid off, because they receive an outside p_2-job offer with associated wage greater than w, or because they receive a p_2-job offer. On the other side, the inflows into the stock $(1-u)G(w)$ consist of unemployed workers who draw a wage offer below w. In steady-state, $G(w)$ is thus derived from the following condition:

$$u\lambda_0 F(w) = (1-u)\left\{G_1(w)\delta + \lambda_1[1-F(w)]\int_{\underline{w}}^{w} s(y)g(y)dy + \lambda_2\int_{\underline{w}}^{w} s(y)g(y)dy\right\}$$

where $g(w) \equiv G'(w)$ is the density of wage earnings.

2.2 Intertemporal values of the workers

Let $\mathcal{U}$, W_1 and W_2 denote the expected discounted lifetime income of unemployed workers, employees in p_1 and p_2-jobs, respectively. Unemployed workers receive unemployment benefits z and search for p_1-job offers. For the sake of simplicity, we are considering a binding minimum wage $\underline{w}$ so that it is always unemployed workers interest to accept job offers. These value

[1] Because all equilibrium wage o ers are greater than unemployed worker's reservation wage, the contact rate λ_0 gives the exit rate of unemployment.

functions are written as follows:

$$
\begin{aligned}
r\mathcal{U} &= z + \lambda_0 \int_{\underline{w}}^{\overline{w}} [W_1(y) - \mathcal{U}] f(y)dy \\
rW_1(w) &= \max_{s\geq 0} \{V(w, 1-h-s) + \delta [\mathcal{U} - W_1(w)] \\
&\quad + s\lambda_1 \int_{w}^{\overline{w}} [W_1(y) - W_1(w)] f(y)dy + s\lambda_2 [W_2(p_2) - W_1(w)] \Big\} \\
rW_2(p_2) &= V(p_2, 1-\overline{h}) + \delta [\mathcal{U} - W_2(p_2)]
\end{aligned}
$$

where r denotes the interest rate, the function V is increasing and concave in both arguments, and $f(w) \equiv F'(w)$ is the density of wage offers. Obviously, it is always worker's interest to accept a wage greater than its current earnings whenever it refers to a p_1-job position. Because $p_2 > p_1$, it also comes that $\overline{w} < p_2$ which implies $W_2(p_2) > W_1(\overline{w})$, so that it always worker's interest to accept a p_2-job offer.

2.3 The wage setting process

Since an unemployed worker accepts all wage offers above the minimum wage, whereas an employee accepts an offer only if the latter is greater than worker's current wage, the unconditional probability that an offer w will be accepted by a randomly contacted worker, denoted by $h_1(w)$, is defined by:

$$
h_1(w) \equiv \frac{\lambda_0 u + (1-u)\lambda_1 \int_{\underline{w}}^{w} s(y)g(y)dy}{\lambda_0 u + (1-u)\lambda_1 \int_{\underline{w}}^{\overline{w}} s(y)g(y)dy} = \frac{\delta + \lambda_1 \int_{\underline{w}}^{w} s(y)g(y)dy}{\delta + \lambda_1 \int_{\underline{w}}^{\overline{w}} s(y)g(y)dy}
$$

On the other side, a match separation occurs because of a job destruction, or because of job-to-p_1/p_2-job transitions. Hence, the employer's value of a continuing match, $J_1(w)$, solves the following asset pricing equation:

$$
rJ_1(w) = p_1 - w - \{\delta + \lambda_1[1 - F(w)]s(w) + \lambda_2 s(w)\} J_1(w)
$$

It is worth emphasizing that the expected job value for firms highly depend on the way search effort reacts to p_2-jobs opportunities. Importantly, there is still an incentive for the highest-paid workers to search on-the-job for the p_2-position.

The wage posting policy of the firm can then be stated from the conventional optimization problem $w = \arg\{\max_{w\geq\underline{w}} h_1(w)J_1(w)\}$ from which we derive the optimal distribution of wage offers in p_1-jobs, $F(w)$.

2.4 Optimal search effort

The optimal search effort of the worker is characterized by the following first-order condition:

$$V_2(w, 1-h-s) = \lambda_1 \int_w^{\overline{w}} [W_1(y) - W_1(w)] f(y)dy + \lambda_2 [W_2(p_2) - W_1(w)]$$

so that $s = s(w)$. Differentiating this optimal condition once again with respect to w yields:

$$V_{22}(w, 1-h-s(w))s'(w) = V_{21}(w, 1-h-s(w)) + (\lambda_2 + \lambda_1[1 - F(w)]) W_1'(w) \tag{1}$$

where $V_{22} \leq 0$ and $V_{21} \gtrless 0$. Usually (see Mortensen [2003]), it is both assumed $\lambda_2 = 0$ and a separable utility function, that is $V_{21}(w, 1-h-s) = 0$; in this case it unambiguously comes that $s(\overline{w}) = 0$ and $s'(w) \leq 0 \, \forall w$. We here depart from these two restrictions. This implies that it is still of interest for the highest paid workers on p_1-jobs to search on-the-job, since they expect a p_1-job to a p_2-job transition. Furthermore, according to $V_{21} \gtrless 0$, the relation between search effort and wages becomes unclear:

- On the one hand, a higher wage decreases the expectation of wage increases in p_1-jobs (as usual).
- On the other hand, if $V_{21}(w, 1-h-s) < 0$, a higher wage decreases the marginal value of leisure, that is it reduces the marginal cost of search. Then, it could be the case that this effect is large enough to induce an increasing relationship between search effort and wage earnings.[2]

3 Equilibrium Search Effort and the Shape of the Wage Distribution

3.1 Labor market equilibrium and calibration

The labor market equilibrium can be summarized by a system which jointly defines $\{F(w), s(w)\}$. Let consider the particular case where $r \to 0$, we have:

[2] If $V_{21}(w, 1-h-s) > 0$, non-separability in preferences also contribute to increasing search effort with wages.

$$
\begin{aligned}
f(w) &= \frac{\delta + \lambda_1[1-F(w)]s(w) + s(w)\lambda_2}{2s(w)\lambda_1(p_1 - w)} + \frac{s'(w)[\lambda_2 + \lambda_1(1-F(w))]}{2s(w)\lambda_1} \\
s'(w) &= \frac{V_{21}(w, 1-h-s(w)) + \frac{\lambda_1[1-F(w)]}{\delta + s(w)\lambda_1[1-F(w)] + s(w)\lambda_2} V_1(w, 1-h-s(w))}{V_{22}(w, 1-h-s(w))}
\end{aligned}
$$

where $F(w)$ and $s(w)$ satisfy the following boundary conditions:

$$
\begin{aligned}
F(\underline{w}) &= 0 \\
V_2(\underline{w}, 1-h-s(\underline{w})) &= \left(\frac{\lambda_2}{\delta + s(\underline{w})\lambda_2}\right)\left[V(p_2, 1-\overline{h}) - V(\underline{w}, 1-h-s(\underline{w})\right] \\
&\quad + \lambda_1 \left(1 - \frac{s(\underline{w})\lambda_2}{\delta + s(\underline{w})\lambda_2}\right) \int_{\underline{w}}^{\overline{w}} \left(\frac{[1-F(y)]V_1(y, 1-h-s(y)}{\delta + s(y)\lambda_1 + s(y)\lambda_2}\right) dy
\end{aligned}
$$

and $\overline{w}$ is given by $F(\overline{w}) = 1$.

A parametric specification of the utility function V is required. In order to allow the marginal cost of search to be either increasing or decreasing with wages, we consider the following CRRA specification:

$$
V(w, 1-h-s) = \left| \begin{array}{ll} \frac{(w^\alpha(1-h-s)^{1-\alpha})^{1-\rho}}{1-\rho} & (\text{if } \rho \neq 1) \\ \\ \alpha \log(w) + (1-\alpha)\log(1-h-s) & (\text{if } \rho = 1) \end{array} \right.
$$

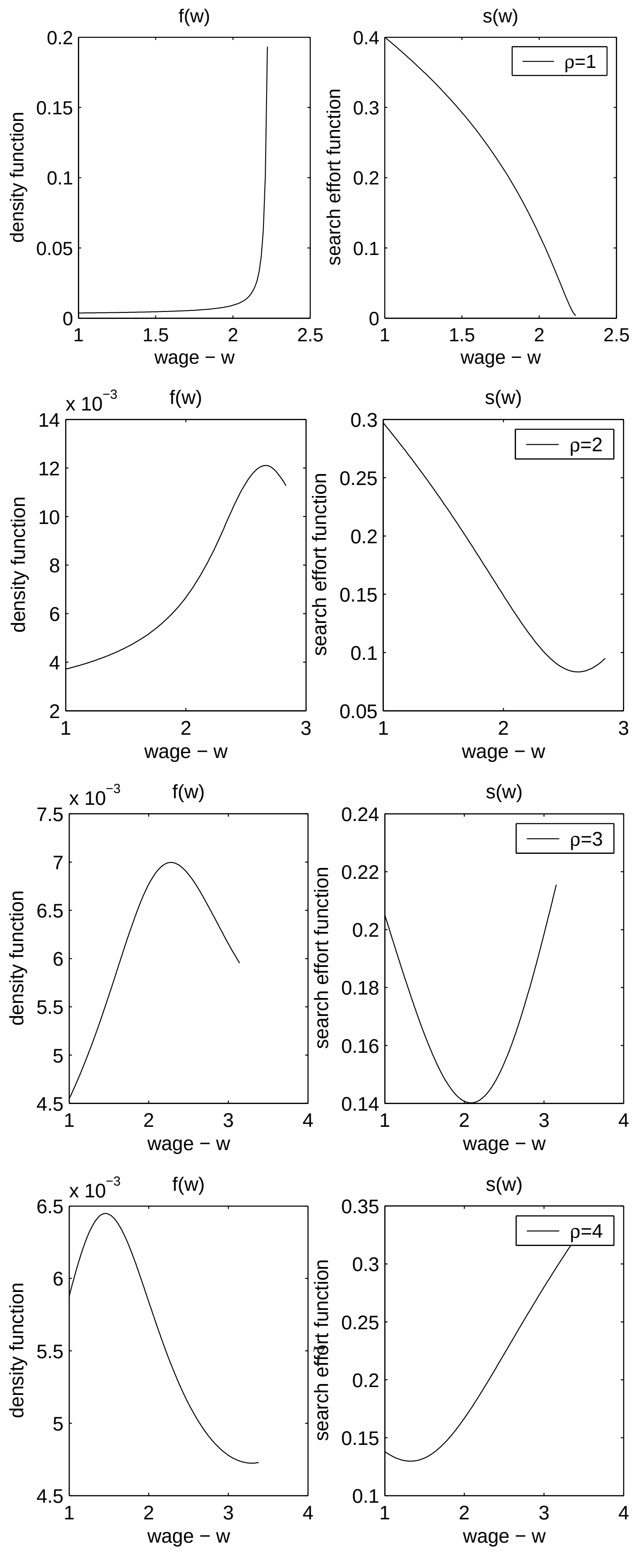
with $\rho \geq 0, \alpha \in [0, 1]$, and where the value of ρ with respect to 1 determines the sign of the cross-partial $V_{12}(w, 1-h-s)$. Our main purpose is then to put emphasize on the potential explaining role of the wage distribution played by search effort. Therefore, simulations aim at showing how sensitive is the equilibrium wage offer distribution with respect to parameter ρ. Other parameters are set in a fairly standard way to provide illustrative simulations (see table 1).

h	$\overline{h}$	α	$\underline{w}$	p_1	p_2	δ	λ_0	λ_1	λ_2
1/3	1.2*h	2/3	1	5	10	0.1	1	0.3	$\lambda_1/10$

3.2 Computational experiments

Figure 1 reports our simulation results for search effort and wage offer density functions according to plausible empirical values of risk-aversion parameter

Figure 1. Illustrative simulations



ρ[3]. When $\rho = 1$, separability between consumption and leisure leads to the conventional Burdett-Mortensen outcome: search effort decreases with wage and the wage density is unambiguously increasing with wages. Then, the point is that if we consider $\rho > 1$, it can be the case that search effort increases with wages, at least for high level of earnings. In words, although the likelihood to get an acceptable p_1-job offer does decrease with worker's current wage, higher earnings (hence consumption) give workers incentive to increase search effort as mean to get the p_2-job position.

Interestingly, if the strength of that mechanism is large enough, that is if ρ is high enough, the wage offer density turns out to be hump-shaped.[4] Indeed, because for high level of wages, workers raise their search effort, this increases the probability for firms to face a job destruction (reduces job duration). Everything else being equal, this gives firms additional incentives to post lower wages with respect to an equilibrium with a decreasing search effort function.

4 Concluding remarks

To conclude, it is seems worth discussing briefly a direct extension of this work. In particular, one may indeed wonder to what extent labor market equilibrium properties depend on bargaining power of workers in the p_2-job position, which here is assumed to be one.[5] More generally, one would obtain that worker's p_2-job earning is a weighted average of worker's productivity and its reservation wage. It then seems comprehensive that if one succeeds to generate a hump-shaped wage distribution in p_1-jobs this would imply a hump-shaped distribution of reservation wages for workers who bargain their wages when contacting p_2-jobs. As consequence, it could the case that both wage distributions in p_1 and p_2-jobs be hump-shaped. We leave for future research this extension of our canonical model.

[3] For instance King and Rebelo [1999] consider $\rho = 3$ in their Handbook of Macroeconomics' paper).

[4] Higher values for ρ could also imply a strictly decreasing wage o□er density.

[5] As emphasized by Cahuc, Postel-Vinay and Robin [2006], worker's bargaining power plays an important role on the determination of wage earnings.

5 References

Bontemps, C. and Robin, J-M. and Van den Berg, G.J. (2000), "Equilibrium search with continuous productivity dispersion: theory and non-parametric estimation", *International Economic Review*, **41**(2), 305-358.

Burdett, K. and Mortensen, D.T. (1998), "Wage Differentials, Employer Size, and Unemployment ", *International Economic Review*, **39**(2), 257-273.

Cahuc, P. Postel-Vinay, F. and Robin, J.-M. (2006), "Wage bargaining with on-the-job search: theory and evidence", *Econometrica*, **74**(2), 323-364.

Chéron, A. and Hairault, J-O. and Langot, F. (2008), "A quantitative evaluation of payroll tax subsidies for low-wage workers: an equilibrium search approach", *Journal of Public Economics*, **92**(3-4), 817-843.

King, R. and Rebelo, S. (1999), "Rescuscitating Real Business Cycles", Handbook of Macroeconomics, North-Holland, Amsterdam.

Mortensen, D.T. (2003), "Wage dispersion: why are similar workers paid differently?", Cambridge: MIT Press.

Postel-Vinay, F. and Robin, J.-M. (2002), "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity", *Econometrica*, **70**(6), 2296-2350.

Quiercioli, E. (2005), "Training, Turnover and Search", *International Economic Review*, **46**(1), 133-143.

Rosholm, M. and Svarer, M. (2004), "Endogenous wage dispersion in a search-matching model", *Labour Economics*, **11**, 623-645.