Retirement Decision and Wage Bargaining: The Impact of Incentives to Work Longer Revisited

Arnaud Chéron*  Fouad Khaskhoussi†  François Langot‡

Abstract

This paper aims to better understand the impact of Nash bargaining of wages on labor market participation and retirement decisions. We consider a life-cycle model with labor market frictions in which people differ according to age and skill. The retirement decision is found to depend on the (i) worker’s skill (ii) degree of redistribution of the pension system (iii) wage bargaining power of workers. More precisely, a lower bargaining power of workers can lead even high-skilled unemployed workers to exit sooner the labor market. Furthermore, wage bargaining can affect the implications of pension reforms because this leads firms to extract part of the incentives to work longer.

Keywords: Retirement age, wage bargaining, life cycle, old workers.

JEL classification: H31, H55, J26

*GAINS-TEPP & EDHEC. Email: acheron@univ-lemans.fr
†GAINS-TEPP. Email: khaskhoussifouad@yahoo.fr. Corresponding author. Adress: Avenue O. Messiaen, 72085 Le Mans Cedex 9, France
‡GAINS-TEPP & ERMES & Cepremap & IZA. Email: Francois.Langot@univ-lemans.fr
1 Introduction

Many OECD countries experience not only an ageing of population but also sharp reductions in the labor force participation rates of older workers who decide to retire early (France, Belgique, Italy,..). Shorter working (and contributions) careers combined with longer periods of enjoyment of public pensions then put into question the financial sustainability of Pay-As-You-Go (PAYG) system and the economy as a whole (Gruber and Wise [1998]). Social security reform has therefore become an important public policy issue for many countries and various reform proposals have been recently put forward. Most of the countries have chosen to encourage the elderly to delay retirement by rewarding a longer working life with an increased pension. However, a large number of the existing theoretical and empirical studies of pensions reforms have been conducted using labor-supply models and without considering the presence of labor market frictions.

The primary goal of this paper is to study the implications of pension reforms aimed at delaying the retirement age by introducing incentives to work longer. The originality of our analysis is to take into account the interaction between labor market frictions and endogenous retirement decision. To that end, we use a life-cycle matching model and simultaneously include skill and age as important dimensions of heterogeneity across workers. We assume search frictions in line with the well known job search McCall model (see Hairault, Langot and Sopraseuth [2006]). But, in addition we consider Nash bargaining of wages. Because the link between labor demand and the age of the worker is somewhat disputed\(^1\), we leave for future research the interaction between labor demand and wage bargaining.

The contribution of this paper to the literature on actuarially fair pension policy is twofold because the formal analysis of retirement decisions is extended in two important ways. First, we derive the date of retirement taking into account all labor market transitions. In contrast, previous studies discussed retirement decisions assuming full employment. Secondly, pension reform is examined in the presence of wage bargaining. Our main results may be summarized as follows. First, we show that the outside options available to workers are crucially affected by their skill and the degree of redistribution of the pension system. In line with Cremer, Lozachmeur and Pestieau [2004], we find that the retirement age increases with the worker’s ability. Second,\(^{1}\)

\(^1\)Aging implies an increase in experience and then an increase in productivity (Mincer [1962]). Nevertheless, recent studies show that technological changes discriminate against older workers (Aubert, Caroli and Roger [2006]).
our model reveals a direct influence of wage bargaining on the worker’s retirement decision. A higher bargaining power gives some incentives to search for a job after the normal retirement age. Hence, wage bargaining can significantly affect the implications of pension reforms. Moreover, when bargaining takes place, the firm realizes that having a job today implies additional benefits to the employee; this leads firms to extract part of this surplus.

The paper is organized as follows. Section 2 describes the model. Section 3 solves the model with the conventional PAYG system. Section 4 evaluates the implications of introducing incentives to work longer. Finally, section 5 concludes.

2 The model

The basic framework is a life-cycle model à la Hairault, Langot and Sopraseuth [2006] with endogenous retirement decision and a finite horizon of five periods. There are, however, some important distinguishing features in our approach. Wages are determined by a specific sharing rule of the rent generated by a job. This rule consists in a period-by-period Nash bargaining. Finally we suppose that workers differ with respect to their individual ability\(^2\). To capture skill differences, we endow each individual worker with a measure of skill denoted \(h_j\) uniformly distributed over \([h_{\text{min}}, h_{\text{max}}]\). Thus, individuals differ in two ways: the age and the ability.

2.1 The Life-Cycle

We assume that there is no growth in the population size. At each period, some households are born and replace an equal number of dead workers (of size normalized to unity). Then, we consider that the population can be divided into 5 age groups denoted \(i = 1, \ldots, 5\). We also distinguish between three different notions of retirement age: the minimum retirement age, but especially the full rate age, fixed at the end of age 2, and the mandatory retirement age, at which workers must exit the labor market, fixed at the end of age 4. Those two ages are exogenous and perfectly known by agents. The effective retirement age is however endogenous and freely decided by workers. It is also perfectly observable by employers.

\(^2\)This ability captures either the human capital level or the productivity of workers.
2.2 The Labor Market

Contact rates between searchers and firms are assumed to be exogenous. The probability for unemployed workers of finding a job is denoted $\lambda$. At each period, firms and unemployed workers have at most one opportunity to meet and match. Furthermore, we assume that if a new job is matched with a worker of age $i$, production takes place in period $i + 1$. Finally, at the end of each period, the employment relationship between a worker and a firm may end involuntarily with an exogenous probability $\delta$.

2.3 The Behaviors

We suppose that all agents are risk-neutral and have the same rate of time preference, $\beta_i$. Agents do not have access to financial markets.

2.3.1 The Workers’ Behaviors

Before the full rate age, each unemployed worker receives an unemployment compensation which depends both on the age and the skill:

$$z_{ij} = z_i h_j$$

where $z_i$ can be interpreted as a replacement rate. However, at any age after the early retirement age, the pension benefits is only non-employment income. We assume that each worker gets utility from consumption and leisure denoted by $\ell > 0$. Finally, individuals are eligible for a constant pension, $p_j$, beyond the early retirement age which is given by the following expression:

$$p_j = \alpha p + (1 - \alpha) \rho h_j$$

with $0 < \alpha, \rho < 1$

where the parameter $\rho$ provides the degree of indexation of pensions on individual productivity. The parameter $\alpha$ captures the level of redistribution of the pension system.

For $i = 5$

All individuals of age 5 are retirees and receive their pension benefits. At the end of this period, they die. Let $R_{ij}$ be the value of a retired worker of age $i$ and productivity $h_j$. Then:

For $i = 3, 4$, an "unemployed" worker is a retiree who searches for a job.

Note that we abstract from a number of details of the pension and fiscal systems, in particular contributions and penalties for an insufficient number of contributive years. Another equivalent assumption would be that this model considers only workers with the right to retire.
\begin{align}
    \mathcal{R}_{5j} &= p_j + \ell \\
    \text{For } i = 4
\end{align}

At the beginning of this period, all workers are eligible for pension benefits and can choose to stay active or retire from the labor force. Employees must decide either to keep working at wage \( w_{4j} \) or to retire. Non-employed workers receive their total pension benefits. Furthermore, the latter should decide to keep searching or to retire. Let \( W_{4j} \) and \( U_{4j} \) denote the values of employed and non-employed workers of age \( i \) and skill \( j \), these values solve:

\begin{align}
    W_{4j} &= w_{4j} + \beta_4 \mathcal{R}_{5j} \\
    U_{4j} &= p_j + \beta_4 \mathcal{R}_{5j} \\
    \mathcal{R}_{4j} &= p_j + \ell + \beta_4 \mathcal{R}_{5j}
\end{align}

It is then straightforward to see that \( \mathcal{R}_{4j} = \max \{ U_{4j}, \mathcal{R}_{4j} \} \) \( \forall j \).

**Proposition 1.** There are no workers who continue searching in period 4. At this age, only employed individuals can choose to delay retirement. So, there is only one choice to do for all workers: keep job or retire.

**For \( i = 3 \)**

Workers of age 3 can choose between one of the three labor occupations: employment, retirement or search. The same options remain available for the next period. Then, currently employed workers have the option to retire immediately or to stay for one more period. In the latter case, the only risk they face in the future is the possibility of losing their jobs (with the exogenous probability \( \delta \)). This is easily reflected in the following value function:

\begin{align}
    W_{3j} &= w_{3j} + \beta_3 \left\{ (1 - \delta) \max \{ W_{4j}, \mathcal{R}_{4j} \} + \delta \max \{ U_{4j}, \mathcal{R}_{4j} \} \right\} \\
    U_{3j} &= p_j + \beta_3 \left\{ \lambda W_{4j} + (1 - \lambda) \max \{ U_{4j}, \mathcal{R}_{4j} \} \right\}
\end{align}

where \( \max \{ U_{4j}, \mathcal{R}_{4j} \} = \mathcal{R}_{4j} \) given the proposition 1. For unemployed workers who continue searching, and so keep the opportunity of being employed at the start of the period 4, the utility function writes as:

\begin{align}
    U_{3j} &= p_j + \beta_3 \left\{ \lambda W_{4j} + (1 - \lambda) \max \{ U_{4j}, \mathcal{R}_{4j} \} \right\}
\end{align}

However, for those who find it optimal to retire, the withdrawal from the labor force is assumed to be permanent. Consequently, from proposition 1, retirees of age 3 (who not search for jobs)
remain in this state (of retirement) for the rest of their life. Then, the value associated to a retired is given by:

\[ R_{3j} = p_j + \ell + \beta_3 R_{4j} \]  

(7)

**For \( i = 2 \)**

Individuals of age 2 are classified in one of two labor states: employed or unemployed. However, they have the additional option of retirement in the next period. Therefore, the values of employed and unemployed workers, including the option of (voluntarily) retirement in the following period, are respectively characterized by:

\[ W_{2j} = w_{2j} + \beta_2 \left\{ (1 - \delta) \max\{W_{3j}, R_{3j}\} + \delta \max\{U_{3j}, R_{3j}\} \right\} \]  

(8)

\[ U_{2j} = \max\left\{ z_{2j} + \beta_2 \left[ \lambda W_{3j} + (1 - \lambda) \max\{U_{3j}, R_{3j}\} \right]; z_{2j} + \ell + \beta_2 R_{3j} \right\} \]  

(9)

Equation (9) shows that non-employed workers of age 2, who have the right to retire at the next period, choose to search if and only if the employment value is larger than the retirement value at age 3.

**For \( i = 1 \)**

At age 1, each individual is either employed or unemployed. Note that, contrarily to age 2, all unemployed workers of age 1 search for jobs. Then, we have:

\[ W_{1j} = w_{1j} + \beta_1 \left\{ (1 - \delta) W_{2j} + \delta U_{2j} \right\} \]  

(10)

\[ U_{1j} = z_{1j} + \beta_1 \left\{ \lambda W_{2j} + (1 - \lambda) U_{2j} \right\} \]  

(11)

### 2.3.2 The Firms’ Behaviors

We assume that each firm has one job and the only factor of production is labor. If the job is filled by a worker of age \( i \) and productivity \( j \), the firm earns a positive profit denoted by \( \Pi_{ij} \). Finally, we suppose that the total output of the firm is simply equal to the productivity level of its employee.

**For \( i = 4 \)**

For a bargained wage \( w_{4j} \), the value \( \Pi_{4j} \) of a filled job with a worker of age 4 and ability \( j \) is defined as:

\[ \Pi_{4j} = h_j - w_{4j} \]  

(12)
This expression shows that a filled job with a worker of age 4 lasts only one period. Indeed, in the following period, the retirement becomes mandatory and all workers must therefore leave their jobs.

**For** $i = 2, 3$

Since workers of age $i = 2, 3$ have the choice to retire in the next period, the value of a filled job writes as follows:

$$
\Pi_{ij} = h_j - w_{ij} + \beta_i \psi_{ij} (1 - \delta) \Pi_{(i+1)j}
$$

where $\psi_{ij}$ is an indicator equal to unity when the employed worker decides to stay active in the following period and zero when he decides to retire at the end of the current period. $\psi_{ij}$ captures the endogenous job separations. It is important to note that expression (13) reflects that the value of a job today is greater than the instantaneous profit, $h_j - w_{ij}$. The term $\beta_i \psi_{ij} (1 - \delta) \Pi_{(i+1)j}$ represents the additional profit that the firm will obtain in the future if the match is not dissolved.

**For** $i = 1$

For workers of age 1, the retirement option is not available for the following period. Thus, all job separations are exogenous and outside the worker’s influence. The firm’s profit is given by:

$$
\Pi_{1j} = h_j - w_{1j} + \beta_1 (1 - \delta) \Pi_{2j}
$$

### 3 Wage Determination and Transitions in/out of the Labor Market by Age and Skill

We assume that wages are determined by the Nash solution to a bargaining problem. Let $\gamma \in [0, 1]$ denote the bargaining power of workers, considered as constant across ages.

**For** $i = 4$

Proposition 1 implies that the rent from the match for a worker of age 4 is: $W_{4j} - \mathcal{R}_{4j}$. Then the global surplus generated by a job is:

$$
S_{4j} = \Pi_{4j} + W_{4j} - \mathcal{R}_{4j}
$$

Nash bargaining implies that this surplus is shared by the firm and the worker according to the following rule:

$$
W_{4j} - \mathcal{R}_{4j} = \gamma S_{4j} \iff \Pi_{4j} = (1 - \gamma) S_{4j}
$$
We therefore obtain the following expression for the wage:

$$ w_{4j} = (1 - \gamma)(p_j + \ell) + \gamma h_j $$

(16)

This wage equation reveals that the pension system acts as a form of "unemployment insurance". Moreover, the wage that makes the worker indifferent between employment or retirement at age 4, is: $p_j + \ell$. This "reservation wage" exceeds the amount of the pension benefits because the employer must compensate the worker for the disutility of labor (the foregone leisure). Additionally, equation (16) shows that wages increase with workers’ skill.

**For $i = 3$**

In this period, the global surplus associated with a filled job is equal to: $S_{3j} = \Pi_{3j} + W_{3j} - \max\{U_{3j}, R_{3j}\}$, so that two cases should be distinguished:

- If $\max\{U_{3j}, R_{3j}\} = R_{3j}$, which implies that non-employed individuals do not search for jobs, only employed workers can choose to delay their retirement. It comes that:

$$ w_{3j} = (1 - \gamma)(p_j + \ell) + \gamma h_j $$

(17)

It is important to note that this wage value is the same as (16). In fact, if individuals have only the option to retire immediately or to stay employed, their threat point is the same at each period beyond the early retirement age.

- If $\max\{U_{3j}, R_{3j}\} = U_{3j}$, non-employed workers at period 3 stay active and continue searching. It follows that the wage is given by:

$$ w_{3j} = (1 - \gamma)p_j + \gamma(h_j + \beta_3 \lambda \Pi_{4j}) $$

(18)

By comparing the wage functions (17) and (18), it is worth emphasizing that the outside options available to workers crucially affects equilibrium wages.

**Proposition 2.** A necessary and sufficient condition implying that retirement is the optimal decision after the full rate age, $R_{ij} > W_{ij}$ for $i = 3, 4$, is that $$(1 - \gamma)(p_j + \ell) + \gamma h_j > p_j + \ell.$$ This condition implies that, a critical skill exists such that all workers with skill less than this critical value exit the labor force at the end of the period 2. This critical skill is defined as:

$$ (1 - \gamma)(p_j + \ell) + \gamma h_j \iff h = \frac{\alpha}{1 - (1 - \alpha)p} + \frac{1}{1 - (1 - \alpha)\ell} $$
Two important things deserve mention here. First, the critical skill \( h \) is independent of the bargaining power of the workers, \( \gamma \). Second, this critical skill is, however, crucially affected by the degree of redistribution of the pay-as-you-go system, captured by the parameter \( \alpha \). Then, under a beveridgian system (\( \alpha \to 1 \)), \( h = p + \ell \). In contrast, if the pension system is bismarckian (\( \alpha \to 0 \)), \( h = \frac{\ell}{1-\rho} \).

It remains to determine the characteristics of individuals who continue searching at age 3. Indeed, workers who find optimal to work after the full rate age (those with skill \( h_j > h \)), but are non-employed must decide whether to continue searching or to leave the labor force. The optimal behavior is derived by comparing \( U_{3j} \) with \( R_{3j} \). Hence, if \( U_{3j} - R_{3j} > 0 \) the non-employed worker decides to continue searching. Whereas, if \( U_{3j} - R_{3j} < 0 \) he decides to end searching and retire.

Then, we have:

\[
U_{3j} - R_{3j} = -\ell + \beta_3 \lambda \left[ w_{4j} - (p_j + \ell) \right]
\] (19)

This decision rule shows that workers who decide to continue searching in period 3 accept an immediate lost of utility (the cost of searching in terms of foregone leisure). However, they expect with a probability \( \lambda \) to become employed in the next period. The associated surplus of working in period 4 depends on the relative importance of the labor income, \( w_{4j} \), with respect to the utility derived from retirement, \( p_j + \ell \). Substituting out \( w_{4j} \) (from (16)), we find:

\[
U_{3j} - R_{3j} = -\ell + \beta_3 \lambda \xi (h_j - p_j - \ell)
\] (20)

It is clear that the worker’s decision is crucially affected by wage bargaining. More precisely, the value of staying active and searching increases with the bargaining power of workers.

**Proposition 3.**

- If \( \gamma = 0 \), then \( U_{3j} - R_{3j} < 0 \) \( \forall h_j \): which implies that there are no workers who continue searching beyond the early retirement age.

- For a positive value of bargaining power for workers, \( 0 < \gamma < 1 \), equation (20) implies that a critical skill exits such that only workers with a skill level higher than this critical value decide to continue searching. This latter value is defined as:

\[
\tilde{h} = h + \left( \frac{1}{\beta_3 \gamma \lambda} \right) \frac{\ell}{1 - (1 - \alpha) \rho}
\]
Proof.

\[ U_{3j} - R_{3j} = 0 \iff -\ell + \beta_3 \lambda \gamma [h_j - p_j] = 0 \]

\[ h_j - p_j - \ell = \frac{\ell}{\beta_3 \lambda \gamma} - \frac{\ell}{1 - (1 - \alpha) \rho} \]

\[ h_j = \frac{1}{\beta_3 \lambda \gamma} \times \frac{\ell}{1 - (1 - \alpha) \rho} + \frac{\alpha}{1 - (1 - \alpha) \rho} p + \frac{1}{1 - (1 - \alpha) \rho} \ell \]

It is straightforward to see that the lower the bargaining power of workers, the higher the skill needed to continue searching.

Summary

We briefly summarize our main results concerning the effects of human capital on the labor market decisions after the full rate age in the case of Nash bargaining. At this stage, we can distinguish between three types of workers:

- Unskilled workers, with upperscript "us", those with skill \( h_{\text{min}} < h_j < h \), find optimal to retire at the early retirement age.

\[ R_{ij}^{us} = \max\{W_{ij}^{us}, U_{ij}^{us}, R_{ij}^{us}\} \text{ for } i = 3, 4. \]

For those individuals things do not change after the full rate age and earn \( p_j^{us} + \ell \) at any age \( i = 3, 4, 5 \).

- Medium skilled workers, those with skill \( h_j < h < \tilde{h} \), with upperscript "s". At the begin of period 3, only currently employed workers stay active. However, those who are non-employed decide to retire (stop searching). There is no worker of this type who continues searching beyond the early retirement age. Then, for this type of workers there is only one choice after the full rate age: keep the job or retire. This implies that:

\[ W_{ij}^s = \max\{W_{ij}^s, U_{ij}^s, R_{ij}^s\} \text{ and } R_{ij}^s = \max\{U_{ij}^s, R_{ij}^s\} \text{ for } i = 3, 4. \]

It is important here to remember that this type of workers earn the same wage (see equation (16) or (17)).

- Finally, skilled individuals with skill \( \tilde{h} < h_j < h_{\text{max}} \), with upperscript "ss". That is in this case that non-employed workers at period 3 stay active and continue searching,

\[ U_{ij}^{ss} = \max\{U_{ij}^{ss}, R_{ij}^{ss}\}. \]

Employed workers of this type, earn a labor income according to (16) and (18) at age 4 and age 3, respectively.

Wage Bargaining at age 2 and 1 are presented in appendix. We now summarize our main results related to workers flows by age (see figure 1, where \( n_i^c, n_i^e \) and \( r_i^c \) denote respectively the measure of the unemployed, employed and retired workers of age \( i \) and type \( c \)).

At age 1, individuals of all skills are classified in one of two mutually exclusive labor states; employed or unemployed. Employed workers face an exogenous probability \( \delta \) of loosing their
Figure 1: Transitions in/out of the labor market by age and skill of workers.
jobs at the end of the current period. All unemployed workers search for jobs and have the same probability \( \lambda \) of being employed in the next period. At age 2, all employed workers choose to keep their jobs. However, for non-employed workers, only those with skill higher than \( h \), precisely those of type "s" and "ss", search for jobs for the next period. Then, at age 3, all workers of type "us" are retirees. For workers of type "s", only currently employed workers stay active. However, those who are non-employed at the beginning of this period decide to retire. In turn, all workers of group "ss" stay active in this period. In this case, non-employed workers continue searching. At age 4 there are only two labor states: employment or retirement; only employed workers at the begin of the current period stay active and continue working.

4 Introduction of Incentives Schemes

In this section we study the consequences of reforms aiming at delaying retirement by introducing incentive schemes beyond the early retirement age. We assume that only currently employed workers are eligible for this incentives. This means that non-working years are not accounted for incentives. More precisely, non-employed workers who continue searching after the full retirement age do not receive any incentive transfer. We suppose that eligible workers who work receive a fraction \( \kappa \) of pension benefits. \( \kappa \) provides a measure of the actuarially fairness of the social security system.

4.1 The Impact of Incentives Schemes on Retirement Decisions and Wage Determination

The introduction of incentives to delay retirement modifies only the problem of workers who are eligible for these incentives. Thus, given the analytical similarity between the two cases we focus exclusively on the workers in age-groups 3 and 4.

For \( i = 4 \)

Remember that according to proposition 1, non-employed workers of age 4 do not search for jobs and can choose only between employment and retirement. The values of employed and retired workers are respectively given by:

\[
W^+_{4j} = w^+_{4j} + \kappa p^c_j + \beta_4 R^c_{5j} \tag{21}
\]

\[
R^c_{4j} = p^c_j + \ell + \beta_4 R^c_{5j} \tag{22}
\]
where \( c = s, ss \). Equation (21) reflects the fact that the instantaneous value for an employed worker is greater than the current labor market income. Indeed, he receives a total income denoted \( \Omega^+_ij = w^+_ij + \kappa p_j \). From (22), it is also clear that for retired workers, the instantaneous value is the same as in a situation without incentives. Then, Nash bargaining implies:

\[
w^+_ij = (1 - \gamma)((1 - \kappa)p^+_j + \ell) + \gamma h^+_ij \tag{23}
\]

Compared to the situation without incentives, wages are reduced by the factor \( \kappa p^+_j \). Indeed, since the incentive transfers represent an additional revenue that workers obtains from working, firms extract a fraction of this additional revenue by paying them lower wages. The reservation wage is defined as: \( (1 - \kappa)p^+_j + \ell \). This expression shows that the employer must compensate the worker for both the disutility of labor, \( \ell \), and the part of the retirement benefits which is not adjusted in an actuarially fair way, \( (1 - \kappa)p^+_j \). As a borderline case, if the pension system is actuarially fair, \( \kappa = 1 \), and if there is no disutility of labor, \( \ell = 0 \), the reservation wage becomes null. In this case, workers accept to work "for free". Finally, given equation (23), we deduce:

\[
\Omega^+_ij = w^+_ij + \kappa p^+_j = w^+_ij + \gamma \kappa p^+_j \tag{24}
\]

where the term \( \gamma \kappa p^+_j \) represents the "effective" amount of incentive benefits obtained from working an additional year.

**For \( i = 3 \)**

For workers of group "s", who choose only between employment and retirement, the global surplus associated with a filled job in this period is: \( S^{+s}_{3j} = \Pi^{+s}_{3j} + W^{+s}_{3j} - R^{s}_{3j} \). Then, we deduce the following wage and total income functions which are completely analogous to those of age 4:

\[
w^{+s}_{3j} = (1 - \gamma)((1 - \kappa)p^+_j + \ell) + \gamma h^+_j \tag{25}
\]

\[
\Omega^{+s}_{3j} = w^+_j + \gamma \kappa p^+_j \tag{26}
\]

**Proposition 4.** A necessary and sufficient condition which ensures that retirement is the optimal decision after the full rate age, \( R^{s}_{ij} > W^{+s}_{ij} \) for \( i = 3, 4 \), is that \( (1 - \gamma)((1 - \kappa)p^+_j + \ell) + \gamma h^+_j > p^+_j + \ell \).

This condition implies that, a critical skill level exists such that workers with a skill level bellow the critical value exit from the labor force at the full rate age. This critical skill is defined as:

\[
h^+ = \frac{\alpha}{1 - (1 - \alpha)(1 - \kappa)p^+_j} + \frac{1}{1 - (1 - \alpha)(1 - \kappa)p^+_j} \ell - \frac{\alpha \kappa}{1 - (1 - \alpha)(1 - \kappa)p^+_j}
\]
It is clear that working becomes more attractive for a large number of workers as $h^+ < h$.

Two important things deserve mention here. First, if the incentives are actuarially fair, $\kappa = 1$, then $h^+ = \ell$: the critical skill must compensate the disutility of labor in terms of the foregone leisure, $\ell$, whatever the nature of the pension system (beveridgian or bismarckian). However, if the incentives are not actuarially fair, the bismarckian pension system ($\alpha = 0$) verifies: $h^+ = (1-\kappa)p + \ell < h = p + \ell$, which implies that incentives to work longer have a positive impact on the labor market participation of older workers.

For workers of group "ss", the global surplus associated with a job in this period is: $S^{ss}_{ss} = \Pi^{ss}_{ss} + W^{ss}_{ss} - U^{ss}_{ss}$. It is then straightforward to deduce the following wage function:

$$w^{ss}_{3j} = (1-\gamma)(1-\kappa)p^{ss}_{j} + \gamma[h^{ss}_{j} + \beta\lambda\Pi^{ss}_{4j}]$$

(27)

It follows that the total gain for the employed worker of type "ss" is:

$$\Omega^{ss}_{3j} = w^{ss}_{3j} + \gamma\kappa p_{j}[1 + \beta\lambda(1-\gamma)]$$

(28)

This equation shows that, at age 3, workers of type "ss" are able to extract a higher fraction of the incentives, equal to $\gamma[1 + \beta\lambda(1-\gamma)]$, than those of type "s", who receive only a fraction $\gamma$ of these incentives. However, at age 4, as the two types of workers have only one choice: to keep job or to retire, their threat point is the same, then they equally enjoy the incentive benefits at a level $\gamma$.

We now turn to characterize individuals who continue searching at age 3. Indeed, the non-employed worker of type "ss" decides to continue searching only if:

$$U^{ss}_{3j} - R^{ss}_{3j} = -\ell + \beta\lambda\gamma[h^{ss}_{j} - (1-\kappa)p^{ss}_{j} - \ell] \geq 0$$

(29)

**Proposition 5.**

- If $\gamma = 0$, then $U^{ss}_{3j} - R^{ss}_{3j} < 0 \ \forall h_{j}$: which implies that no worker continues searching beyond the early retirement age. The introduction of incentives to work longer does not affect the labor market decisions of workers.

- For $0 < \gamma < 1$, equation (29) implies the following value of skill required to continue searching after the full rate age:

$$U^{ss}_{3j} - R^{ss}_{3j} = 0 \iff \tilde{h}^+ = h^+ + \ell \left( \frac{1}{\beta\gamma\lambda} \right) \left[ \frac{1}{1 - (1 - \alpha)(1 - \kappa)p} \right]$$
It is straightforward to verify that: \( \tilde{h}^+ < \tilde{h} \); there are more individuals who continue searching beyond the full rate age.

5 Conclusion

This paper proposes an original model to investigate the interaction between the endogenous retirement decision and the wage bargaining. We find that the degree of redistribution of the pension system, the human capital and the wage bargaining power of workers crucially affect the labor market participation decisions of older workers. Our main results are the following:

1. The skill level required to continue working after the full rate age is independent of the bargaining power of workers. However, this level is crucially affected by the degree of redistribution of the pay-as-you-go system: a mixed and generous pension system discourages the labor market participation of workers.

2. For unemployed workers who prefer to work after the full rate age, the decision of staying active and continue searching is highly depends on their bargaining power.

3. The retirement age recursively affects the individual job search decisions on the labor market and the bargained wage.

4. The wage depends on age because the outside options available to workers are dependent on worker’s distance to retirement. In particular, the wage is affected not only by the retirement age but also by (non-)search decision of non-employed workers.

5. Incentives schemes not only encourage individuals to keep their jobs, but they also make searching more attractive to unemployed workers not only after the full rate age but also before this age.

6. In the context of bargaining, firms naturally extract a part of financial incentives for delayed retirement.

Overall, we think that it could interesting to endogenize both job creation and job destruction and investigates the impact of pension reforms and tries to draw some qualitative and quantitative insights about the impact of social security reforms.
References


Langot F. and Moreno-Galbis E. [2008], "Does the Growth Process Discriminate Against Older Workers?" mimeo, University of Le Mans.

McCall J. [1970], "Economics of Information and Job Search" Quarterly Journal of Economics, 84, 113-126.


Pissarides C. [2000], "Equilibrium Unemployment" MIT Press.
Appendix

We now turn to the decisions of non-eligible individuals and investigate the interaction between the endogenous market participation decisions of workers before and after the full rate age.

For $i = 2$

To simplify the analyze, we choose to derive the endogenous labor market participation decisions and wage determination by group of workers. First, taking into account that all workers of type "us", both currently employed an unemployed, will exit the labor force at age 3, we obtain:

\[ W_{2j}^{us} = w_{2j}^{us} + \beta_2 R_{3j}^{us} \]  
\[ U_{2j}^{us} = z_2 h_{j}^{us} + \ell + \beta_2 R_{3j}^{us} \]  

It is important to note here that unemployed workers of type "us" consume leisure as they do not participate in the labor market. The value of a job filled by this type of workers is given by:

\[ \Pi_{2j}^{us} = h_j^{us} - w_{2j}^{us} \]  

Then, we can derive the following expression for the wage:

\[ w_{2j}^{us} = (1 - \gamma)(z_2 h_{j}^{us} + \ell) + \gamma h_{j}^{us} \]  

For workers of type "s", who prefer to extend their working period after the early retirement age conditional on beginning the period 3 as employed, we have:

\[ W_{2j}^{s} = w_{2j}^{s} + \beta_2 \left\{ (1 - \delta)W_{3j}^{s} + \delta R_{3j}^{s} \right\} \]  
\[ U_{2j}^{s} = z_2 h_{j}^{s} + \beta_2 \left\{ \lambda W_{3j}^{s} + (1 - \lambda) R_{3j}^{s} \right\} \]  

Note that, contrary to unemployed workers of type "us", the instantaneous value of those of type "s" is \( z_2 h_{j}^{s} \), as they continue searching. On the other hand, when the firm is employing someone of type "s", the job lasts until an exogenous process destroys it, an event that takes place at rate \( \delta \), or until the retirement becomes mandatory \(^5\). Then, the value of this job writes as:

\[ \Pi_{2j}^{s} = h_j^{s} - w_{2j}^{s} + \beta_2 (1 - \delta) \Pi_{3j}^{s} \]  

The Nash bargained wage is given by:

\[ w_{2j}^{s} = (1 - \gamma) z_2 h_{j}^{s} + \gamma (h_{j}^{s} + \beta_2 \lambda \Pi_{3j}^{s}) \]  

\(^5\)There is no endogenous job destruction, \( \psi_{2j}^{s} = \psi_{3j}^{s} = 1. \)
Finally, consider the case in which workers continue searching at age 3 (workers of type "ss"), we have:

\[ W_{2j}^{ss} = w_{2j}^{ss} + \beta_2 \left\{ (1 - \delta)W_{3j}^{ss} + \delta U_{3j}^{ss} \right\} \]  \hspace{1cm} (38)

\[ U_{2j}^{ss} = z_2 h_j^{ss} + \beta_2 \left\{ \lambda W_{3j}^{ss} + (1 - \lambda)U_{3j}^{ss} \right\} \]  \hspace{1cm} (39)

For the firm, we have:

\[ \Pi_{2j}^{ss} = h_j^{ss} - w_{2j}^{ss} + \beta_2 (1 - \delta) \Pi_{3j}^{ss} \]  \hspace{1cm} (40)

Then, the negotiation problem is identical to the one of workers of type "s", given by:

\[ w_{2j}^{ss} = (1 - \gamma)z_2 h_j^{ss} + \gamma (h_j^{ss} + \beta_2 \lambda \Pi_{3j}^{ss}) \]  \hspace{1cm} (41)

For \( i = 1 \):

At this age, the option of retirement is not open in the next period, and, therefore, all unemployed workers search for jobs. The values of employed and unemployed workers of any skill, are respectively given by:

\[ W_{1j}^{c} = w_{1j}^{c} + \beta_1 \left\{ (1 - \delta)W_{2j}^{c} + \delta U_{2j}^{c} \right\} \]  \hspace{1cm} (42)

\[ U_{1j}^{c} = z_1 h_j^{c} + \beta_1 \left\{ \lambda W_{2j}^{c} + (1 - \lambda)U_{2j}^{c} \right\} \]  \hspace{1cm} (43)

where \( c = us, s, ss \). For the firm, the value of a filled job writes as:

\[ \Pi_{1j}^{c} = h_j^{c} - w_{1j}^{c} + \beta_1 (1 - \delta) \Pi_{2j}^{c} \]  \hspace{1cm} (44)

Analogously, it follows that the wages of workers of age 1 is given by:

\[ w_{1j}^{c} = (1 - \gamma)z_1 h_j^{c} + \gamma (h_j^{c} + \beta_1 \lambda \Pi_{2j}^{c}) \]  \hspace{1cm} (45)