Job Creation and Job Destruction over the Life Cycle: the Older Workers in the Spotlight

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Abstract

This paper extends the job creation - job destruction approach to the labor market to take into account for the life-cycle of workers. Forward looking decisions about hiring and firing thus depend on the time over which to recoup adjustment costs. The equilibrium is typically featured by increasing (decreasing) firing (hiring) rates with age, and an hum-shaped age-dynamics of employment. The empirical plausibility of the model is more precisely assessed using the French older workers experience by incorporating existing age-specific labor market policies. We finally show that age-discrimination against older workers is optimal and we design the optimal age-pattern for employment policies.

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1 Introduction

Since Oi [1962], labor has been conventionally viewed as a quasi-fixed input factor. The hiring process is costly and this leads firms to implement a labor-hoarding strategy. The major contribution of the extensively used framework of Mortensen and Pissarides [1994] (MP hereafter) is to provide theoretical foundations for these mechanisms in an overall theory of equilibrium unemployment. In that context, endogenous hirings and separations depend on the expected duration of jobs. Surprisingly enough, the life cycle of workers has not been yet incorporated into equilibrium unemployment models although the job duration is obviously closely related to the distance from the worker’s retirement. This is surprising inasmuch as that the relation between the labor supply and the life cycle has been pointed out since the seminal Heckman [1974] and MaCurdy [1981] papers. Seater [1974] and more recently Bettendorf and Broer [2003], Lunqvist and Sargent [2002] or Hairault, Langot and Sopruseuth [2006] have also emphasized the role played by the life cycle in the job-search decision of unemployed workers.

This paper aims at understanding job creation and job destruction in a life cycle version of the canonical model of Mortensen and Pissarides [1994]. From our point of view, this approach could help to better understand the economic rationale under the discrimination against older workers in the process of hirings and firings. It could also provide a framework to examine the optimality and the impact of policies specific to older workers. Because the horizon of older workers is shorter, firms and workers invest less in job-search and labor-hoarding activities at the end of the life cycle. This implies that the hiring (firing) rate decreases (increases) with the age of the worker. It thus constitutes the cornerstone in the consideration of older worker employment and policies. Beyond its theoretical interest, we believe that this framework is able to deliver realistic empirical predictions.

A recent OECD report (OECD [2006]) indeed shows that the rate of entry (exit) of 50-64 year-old workers is lower (higher) than that of younger workers (aged between 30-49). Accordingly, Figure 1 first shows that the employment rate of older workers exhibits a sharp fall in major OECD countries. Furthermore, this fall is as sharp when the retirement age gets closer, whatever the country considered and its average employment rate. Two country groups emerge very clearly: those with high employment rates for workers aged 55-59 (Canada, Great Britain, Japan, the United States and Sweden) and those which already experience a huge decrease at these ages, around 25 points with respect to the 50-54 age group (Belgium, France, Italy and the Netherlands). As documented by Gruber and Wise [1999], the second group of countries is indeed characterized by an effective retirement age of
Figure 1: The fall in employment rates from age 30 to 64 for OECD Countries

![Bar chart showing employment rates by age group for different OECD countries.](image)

**Rank of effective retirement age**

Source: OECD data for 1995 (Authors’ calculation). In each country, each bar refers to the employment of age groups: 30 - 49 (first bar on the left), 50 - 54, 55 - 59 and 60 - 64 (last bar on the right).

60 (versus 65 in the first group).

Specific assistance programs just before the early retirement age and the consequent effect on the labor supply are traditionally considered as the primary candidates for explaining the low older worker employment rate\(^1\). The originality of our paper is to put forward the shortened horizon that we consider as the intrinsic feature of both labor supply and labor demand for older workers. In this paper, we show that the workhorse labor market model of Mortensen and Pissarides [1994] naturally delivers an outcome for the older worker employment rate consistent with Figure 1, once a finite horizon, typically determined by retirement, is explicitly taken into account. Firms’ policies of both hiring and firing are detrimental to aged workers when labor

\(^1\)The labor demand for older workers has also been scrutinized and its relative weakness due to ongoing technological progress certainly helps to explain the decrease in the employment rate at the end of working life (see for instance Hellerstein, Neumark and Troske [1999] and Crépon, Deniau and Perez-Duarte [2002]). However, this argument no longer explains the larger drop in older worker employment rates in economies with a low retirement age. There is no reason to believe that these countries are more sensitive to ongoing technological progress.
markets are characterized by job-search and recruiting costs. In this sense, there is strictly speaking no discrimination against aged workers: these latter are objectively less profitable. Adding the unemployed workers’ endogenous job-search effort reinforces the mechanisms at work in a life cycle setting: as retirement gets closer, individuals put less effort into the job-search as the expected value of employment is lower.

In addition, this paper provides a framework well-suited to analyzing the effects of the numerous existing policies designed to favor older workers, such as higher employment protection, specific hiring subsidies, higher unemployment benefits or anti-age discrimination legislation. For instance, in Belgium, Finland, France, or Norway, it is more costly for firms to lay off older workers because of longer notice periods or higher severance pay. We show that these policies reduce the firing rate of older workers, but also decrease (increase) the hiring (firing) probability of younger workers. The introduction of anti-age discrimination policies such as the Age Discrimination in Employment Act (ADEA) in the US, is shown to increase the hiring rate of the older workers at the expense of a reduced hiring rate for other workers. These two policy cases exemplify the general result we get: older worker policies have backward effects on younger worker employment, which must be taken into account in any empirical evaluations.

If our theoretical model is able to generate a drop in the employment rate of workers close to the retirement age, it is important to show that it may explain the actual labor market features in a quantitative way. We calibrate a version of the model on French data. We show that our theoretical framework is able to replicate the main features of the French labor market, in particular the shrinking of the employment rate just before the early retirement age. The distance from retirement appears as the key variable for the origin of this drop, much more than labor market institutions, in particular the great generosity of the unemployment benefits at the end of the working cycle. These empirical results confirm our basic intuition: the retirement age is the central factor to take into consideration in any analysis of older worker employment rates, whereas the employment rate of younger workers is governed on the contrary, and more conventionally, by the labor market institutions (unemployment benefits, employment protection, and so on), since the influence of the retirement age vanishes as the distance from retirement increases. Moreover, it must be emphasized that the firms’ behaviors play a major role in these results, more than that of the workers.

If the low employment rate of older workers can be explained by the short horizon created by the coming retirement, the next issue is then to examine the social optimality of such outcomes. Before engineering any policy devices to prevent firms from discriminating against older workers, it is necessary to
study the social optimality of such behaviors.

We show that, when the job-search is costly, maximizing social welfare implies that older workers, due to their impending retirement, come first (last) in the firing (hiring) process. Without any other distortions than the matching process, it is optimal to discriminate against older workers. We then show that the decentralized equilibrium even coincides with the first best outcome when the Hosios condition holds. What should be the age profile of firing costs and hiring subsidies when the Hosios condition no longer holds or unemployment benefits exist? As the effect of any distortions depends on the expected life-time of the match, we show that age constitutes the cornerstone of any optimal labor market policies, and in a way which is sometimes quite opposed to the sense of current legislations.

The first section presents the benchmark model and the age-dynamics properties of the “laissez-faire” equilibrium. A second section studies the effect of policies designed to protect or stimulate older worker employment. A third section provides some empirical evidence in favor of this approach, based on the French experience. The last section examines the first best allocation and establishes the optimal age-profile of employment policies.

2 How Do Job Creation and Job Destruction Vary over the Life Cycle?

Let us consider an economy à la Mortensen - Pissarides [1994]. Labor market frictions imply that there is a costly delay in the process of filling vacancies, and endogenous job destructions closely interact with job creations. Wages are determined by a specific sharing rule of the rent generated by a job. The latter can be interpreted as the result of a bargaining between workers and employers. At this stage, no other frictions or inefficiencies are introduced.2

Contrary to the large literature following Mortensen - Pissarides [1994], we consider a life cycle setting characterized by a deterministic age at which workers exit the labor market. Firms are free to target their hirings by age: directed-search by age is technologically possible and legally authorized. This means that we are considering at this stage a “laissez-faire” economy.3

2Unemployed workers job-search effort is discarded at this point for clarity of exposition. We present in Appendix A an extended version of the model with endogenous job-search effort for unemployed people. It allows us to assess the robustness of our results.

3If we consider ex-ante undirected search but ex-post match-specific heterogeneity, it adds some complexities but our main results should hold. Typically, the reservation productivity allowing the job to start will increase with a worker’s age, hence reducing the probability of being employed. In our equilibrium the latter result will hold due to the
2.1 Benchmark Model Description

2.1.1 Worker Flows

We consider a discrete time model and assume that at each period the older worker generation retiring from the labor market is replaced by a younger worker generation of the same size (normalized to unity) so that there is no labor force growth in the economy. We denote \( i \) the worker’s age and \( T \) the exogenous age at which workers exit the labor market: they are both perfectly known by employers. There is no other heterogeneity across workers. The economy is at steady-state, and we do not allow for any aggregate uncertainty. We assume that each worker of the new generation enters the labor market as unemployed.

Job creation takes place when a firm and a worker meet. Firms are small and each has one job. The flows of newly created jobs result from a matching function the inputs of which are vacancies and unemployed workers. The destruction flows derive from idiosyncratic productivity shocks that hit the jobs at random. Once a shock arrives, the firm has no choice but either to continue production or to destroy the job. Then, for age \( i \in (2, T-1) \), employed workers are faced with layoffs when their job becomes unprofitable. At the beginning of each period, a job productivity \( \epsilon \) is drawn in the general distribution \( G(\epsilon) \) with \( \epsilon \in [0, 1] \). The firms decide to close down any jobs whose productivity is below an (endogenous) productivity threshold (productivity reservation) denoted \( R_i \).

Let \( u_i \) be the unemployment rate and \( v_i \) the vacancy rate of age \( i \). For any age, we assume that there are matching functions that give the number of hirings as a function of the number of vacancies and the number of unemployed workers, \( M(v_i, u_i) \), where \( M \) is increasing and concave in both its arguments, and with constant returns-to-scale. Let \( \theta_i = v_i/u_i \) denote the tightness of the labor market of age \( i \). It is then straightforward to define the probability of filling a vacancy as \( q(\theta_i) \equiv \frac{M(u_i, v_i)}{v_i} \) and the probability for unemployed workers to meet a vacancy as \( p(\theta_i) \equiv \frac{M(u_i, v_i)}{u_i} \).

At the beginning of their age \( i \), the realization of the productivity level on each job is revealed. Workers hired when they were \( i-1 \) years old (at the end of the period) are now productive. Workers whose productivity is below the reservation productivity \( R_i \) are laid off. For any age \( i \), the flow from employment to unemployment is then equal to \( G(R_i)(1-u_{i-1}) \). The other workers who remain employed \( (1-G(R_i))(1-u_{i-1}) \) can renegotiate their wage. The dynamics by age of unemployment are then given by:

decrease in the number of vacancies targeted at older workers.
\[ u_i = u_{i-1} (1 - p(\theta_{i-1})) + G(R_i)(1 - u_{i-1}) \quad \forall i \in (2, T - 1) \tag{1} \]

for a given initial condition \( u_1 = 1 \). The overall unemployment rate \( u \) is then defined by \( u = \frac{\sum_{i=1}^{T-1} u_i}{T-1} \).

### 2.1.2 The Hiring Decision

Any firm is free to open a job vacancy and engage in hiring. \( c \) denotes the flow cost of recruiting a worker and \( \beta \in [0, 1] \) the discount factor. Let \( V_i \) be the expected value of a vacant job directed to a worker of age \( i \):

\[ V_i = -c + \beta [q(\theta_i)J_{i+1}(1) + (1 - q(\theta_i))V_i] \]

where \( J_i(\epsilon) \) is the expected value of a filled job by a worker of age \( i \) with idiosyncratic productivity \( \epsilon \). Following Mortensen and Pissarides, we assume that new jobs start at the highest productivity level, \( \epsilon = 1 \).

As \( J_T(1) = 0 \), no firms search for workers of age \( T - 1 \), that is \( \theta_{T-1} = 0 \). The zero-profit condition \( V_i = 0 \quad \forall i \in (1, T - 2) \) allows us to determine the labor market tightness for each age \( \theta_i \) from the following condition:

\[ \beta J_{i+1}(1) = \frac{c}{q(\theta_i)} \tag{2} \]

As \( 1/q(\theta_i) \) is the expected duration of a vacancy directed to a worker of age \( i \), the market tightness is such that the expected and discounted job value is equal to the expected cost of hiring a worker of age \( i \).

### 2.1.3 The Firing Decision

For a bargained wage \( w_i(\epsilon) \), the expected value \( J_i(\epsilon) \) of a filled job by a worker of age \( i \) is defined by:

\[ J_i(\epsilon) = \epsilon - w_i(\epsilon) + \beta \int_{R_{i+1}}^{1} J_{i+1}(x)dG(x) + \beta G(R_{i+1}) \max_i \{V_i\} \quad \forall i \in [1, T - 1] \tag{3} \]

A first thing to note is that with probability \( G(R_{i+1}) \) the job is destroyed and the firm can freely choose to direct its vacant job to workers of any age. It is also worth emphasizing that the deterministic exit at age \( T \) leads to an exogenous job destruction, whatever the productivity realization: \( J_T(\epsilon) = 0 \quad \forall \epsilon \).

The (endogenous) job destruction rule \( J_i(\epsilon) < 0 \) leads to a reservation productivity \( R_i \) defined by \( J_i(R_i) = 0 \quad \forall i \in [2, T - 1] \):
\[ R_i = w_i(R_i) - \beta \int_{R_{i+1}}^{1} J_{i+1}(x)dG(x) - \beta G(R_{i+1}) \max_i\{V_i\} \forall i \in [2, T-1] \] (4)

The higher the wage, the higher the reservation productivity, and hence the higher the job destruction flows. On the other hand, the higher the option value of filled jobs (expected gains in the future), the weaker the job destructions. Because the job value vanishes at the end of the working life, labor hoarding of older workers is less profitable. It is again worth determining the terminal age condition: \( R_{T-1} = w_{T-1}(R_{T-1}) \).

### 2.1.4 Wage Bargaining

The rent associated with a job is divided between the employer and the worker according to a wage rule. Following the most common specification, wages are determined by the Nash solution to a bargaining problem.

Values of employed (on a job of productivity \( \epsilon \)) and unemployed workers of any age \( i \), \( \forall i < T \), are respectively given by:

\[ W_i(\epsilon) = w_i(\epsilon) + \beta \left[ \int_{R_{i+1}}^{1} W_{i+1}(x)dG(x) + G(R_{i+1})U_{i+1} \right] \] (5)

\[ U_i = b + \beta [p(\theta_i)W_{i+1}(1) + (1 - p(\theta_i))U_{i+1}] \] (6)

with \( b \geq 0 \) denoting the opportunity cost of employment.\(^4\)

For a given bargaining power of the workers, considered as constant across ages, the global surplus generated by a job, \( S_i \equiv J_i(\epsilon) + W_i(\epsilon) - U_i \), is divided according to the following sharing rule:

\[ W_i(\epsilon) - U_i = \gamma [J_i(\epsilon) + W_i(\epsilon) - U_i] \] (7)

As in MP, a crucial implication of this rule is that the job destruction is optimal not only from the firm’s point of view but also from that of the worker. \( J_i(R_i) = 0 \) indeed entails \( W_i(R_i) = U_i \).

According to this Nash bargaining solution, we then derive the following expression for the wage (see Appendix C for details):

\[ w_i(\epsilon) = (1 - \gamma)b + \gamma (\epsilon + \theta_i) \forall i \in [1, T - 1] \] (8)

This is a traditional wage equation, except that age matters through the market tightness. If this latter diminishes along the life cycle, the age profile

\(^4\)We assume that \( W_T = U_T \) so that the social security provisions do not affect the wage bargaining and the labor market equilibrium.
of wages is decreasing. This could counteract the incentives for firms to fire older workers.\footnote{From an empirical standpoint, such a decrease in wages along the life cycle is a shortcoming. A similar result holds also in MP, where wages are expected to decrease after hiring. As stated in Appendix A, it could be easily overcome by allowing for general human capital accumulation. If the worker’s reservation wage and the productivity grow at the same rate over the life cycle, only the discount factor is changed at the equilibrium (see Appendix A).}

2.2 The “Laissez-Faire” Equilibrium

The main objective of this section is to characterize the life cycle pattern of hirings and firings. For didactic reasons, we first rely exclusively on the firm behavior, without considering wage retroactions. Wages are assumed to be fixed at the reservation wage level $b$. This “wage posting” case could be rationalized by a bargaining power for workers equal to 0 ($\gamma = 0$ in (8)). Then, we will turn to our benchmark labor market equilibrium when it allows for wages adjustments over the life cycle. The introduction of endogenous job-search effort and its implications on equilibrium job creations and job destructions will be also examined. Lastly, we will show that the age profile of employment rates is typically hump-shaped.

2.2.1 The Wage Posting Equilibrium

If wages are equal to $b$, the firing policy, defined by $R_i$, is independent of the hiring one.

**Proposition 1.** If $\gamma = 0$, a labor market equilibrium with wage posting exists and it is characterized by a sequence $\{R_i, \theta_i\}$ solving:

\[
\frac{\partial R_i}{\partial \theta_i} = \beta(1 - R_{i+1}) \quad (JC_{PartialEq})
\]

\[
R_i = b - \beta \int_{R_{i+1}}^{1} [1 - G(x)]dx \quad (JD_{PartialEq})
\]

with terminal conditions $R_{T-1} = b$ and $\theta_{T-1} = 0$.

**Proof.** See Appendix D.1. \qed

It is then possible to derive the age profile of hirings and firings along the life cycle.

**Property 1.** $R_{i+1} \geq R_i$ and $\theta_{i+1} \leq \theta_i, \forall i.$
Proof. See appendix D.2. \qed

Older workers are more vulnerable to idiosyncratic shocks. A shortened horizon relative to younger workers make them more exposed to firings. Otherwise stated, this reflects that labor-hoarding decreases with worker’s age. In turn, it creates a downward pressure on the hirings of older workers. It reinforces the decrease in hirings at the end of the working cycle due to a shortened horizon which makes vacancies unprofitable.

2.2.2 The Equilibrium with Wage Bargaining

If wages are bargained according to the equation (8), the firing policy depends now on the market tightness. The wage decrease over the life cycle is then likely to offset the direct effect of the shortening horizon on firings. This could put into question the decreasing age profile of firings, hence of hirings.

**Proposition 2.** A labor market equilibrium with wage bargaining exists and it is characterized by a sequence \( \{R_i, \theta_i\} \) solving:

\[
\frac{c}{q(\theta_i)} = \beta(1 - \gamma)(1 - R_{i+1}) \quad (JC)
\]

\[
R_i = b + \left(\frac{\gamma c}{1 - \gamma}\right) \left[\frac{\beta(1 - \gamma)}{c}(1 - R_{i+1})\right]^{1-\psi} - \frac{\beta}{2} (1 - R_{i+1})^2 \quad (JD)
\]

with terminal conditions \( R_{T-1} = b \) and \( \theta_{T-1} = 0 \).

**Proof.** See Appendix D.3. \qed

**Corollary 1.** Let be \( M(v, u) = v^\psi u^{1-\psi} \) with \( 0 < \psi < 1 \), and \( G(\epsilon) = \epsilon, \ \forall \epsilon \in [0, 1] \), with \( b \leq 1 \leq 2b/\beta \), the labor market equilibrium with wage bargaining can be summarized by a sequence \( \{R_i\}_{i=2}^{T-1} \) solving:

\[
R_i = b + \left(\frac{\gamma c}{1 - \gamma}\right) \left[\frac{\beta(1 - \gamma)}{c}(1 - R_{i+1})\right]^{1-\psi} - \frac{\beta}{2} (1 - R_{i+1})^2 \quad (9)
\]

with terminal condition \( R_{T-1} = b \).

**Proof.** Straightforward. \qed

\[\text{From (9) it is straightforward to see that } b \leq 1 \leq 2b/\beta \text{ is sufficient for an interior solution to exist (} R_i \geq 0 \ \forall i)\].
The sequence of $R_i$ is no longer necessarily monotonic. If the wage decreases sufficiently at the end of working life because of the weakness of the market tightness, then firms might be less inclined to hire these older workers.\footnote{It is worth emphasizing that this incentive to keep older workers still remains when there is human capital accumulation that allows wages to grow over the life cycle at the same rate as productivity (see Appendix A).} The following property and corollary state restrictions, implying that this indirect effect of age through wages does not dominate the direct impact of age on labor-hoarding and firing.

Property 2.

\[
if \ 1 \geq \begin{cases} 
\frac{\gamma}{1-\psi} \left[ \frac{\beta(1-\gamma)}{\epsilon} \right]^{\psi/(1-\psi)} & \text{for } \psi \geq 1/2 \\
2\gamma \left[ \frac{\beta(1-\gamma)}{\epsilon} \right]^{\psi/(1-\psi)} \left( 1 - b \right)^{\frac{2(1-\gamma)}{1-\psi}} & \text{for } \psi \leq 1/2 
\end{cases}
\]

then the labor market equilibrium verifies $R_{i+1} \geq R_i$ and $\theta_{i+1} \leq \theta_i \ \forall i$.

Proof. See Appendix D.4.

Figure 2 gives the age dynamics of the labor market. This figure shows that if $\frac{dR_i}{dR_{i+1}} \geq 0$ and $b \geq R_i \ \forall i$, the model generates a monotonous increasing sequence of $R_i$. While this result is not ambiguous in the case of an exogenous wage, a parameter restriction is required when the wage derives from a Nash bargaining. The value of recruiting costs ($c$) is central for understanding this result. It determines how age influences the vacancy rate. The higher the recruiting cost, the less sensitive labor market tightness to age, the steeper the age profile of wages. If $c$ is sufficiently high, the wage effect cannot counteract the horizon effect on the reservation productivity: the age profile of firings is still decreasing.

Corollary 2. If $\psi = 1/2$ the condition $c \geq \beta \gamma (1-\gamma) 2$ ensures that the labor market equilibrium verifies $R_{i+1} \geq R_i$ and $\theta_{i+1} \leq \theta_i \ \forall i$.

Proof. Straightforward from Property 2 with $\psi = 1/2$.
2.2.3 The Role of the Endogenous Job-Search Effort

For didactic reasons, until now we have neglected the incidence of the life cycle hypothesis on workers’ job-search effort. Making the latter endogenous would actually reinforce the decrease in the employment rate at the end of working life. As the retirement age gets closer, the return on job-search investments decreases because the horizon (the expected job duration) over which they can recoup their investment is reduced. This point can easily be stated by considering the following unemployed problem to define job-search intensity:\footnote{See the Appendix A for a detailed description of the model with a more general specification of preferences.}

\[
U_i = \max_{\epsilon_i} \left\{ b - \frac{\epsilon_i^2}{2} + \beta [e_i p(\theta_i)] W_{i+1}(1) + (1 - e_i p(\theta_i)) U_{i+1} \right\}
\]

where the labor market tightness is now defined by \( \theta_i \equiv \frac{v_i}{[\bar{\epsilon}_i u_i]} \) with a matching function \( M(v_i, \bar{\epsilon}_i u_i) \) where \( \bar{\epsilon}_i \) is the average job-search effort of workers of age \( i \).

The optimal decision rule shows that it is in the older unemployed workers’ interest to reduce their job-search intensity, since the discounted sum of
surplus related to employment is decreasing with age:
\[ e_i = \beta p(\theta_i) [W_{i+1}(1) - U_{i+1}] \]
This provides an addition to existing forces that lead job creation to decrease with age. This point can be highlighted by examining the equilibrium properties with job-search effort.

**Proposition 3.** A labor market equilibrium with wage bargaining and endogenous job-search effort exists and it is characterized by a sequence \( \{R_i, \theta_i\} \) solving:

\[
\begin{align*}
\frac{c}{\overline{q}(\theta_i)} &= \beta(1 - \gamma)(1 - R_{i+1}) & (JC_{eff}) \\
R_i &= b + \frac{1}{2} \left( \frac{\gamma}{1-\gamma} \theta_i \right)^2 - \beta \int_{R_{i+1}}^{1} [1 - G(x)] dx & (JD_{eff})
\end{align*}
\]

with terminal conditions \( R_{T-1} = b \) and \( \theta_{T-1} = 0 \).

Proof. See Appendix D.5. \( \square \)

**Property 3.** Let \( M(v, eu) = u^\psi(eu)^{1-\psi} \) with \( 0 < \psi < 1 \), and \( G(\epsilon) = \epsilon \), \( \forall \epsilon \in [0, 1] \), if \( 1 \geq \beta \frac{\gamma^2}{1-\psi} \left( \frac{\beta(1-\gamma)}{c} \right)^{\frac{\psi}{1-\psi}} \), then the labor market equilibrium with wage bargaining and endogenous job-search effort verifies \( R_{i+1} \geq R_i \) and \( \theta_{i+1} \leq \theta_i \) \( \forall i \).

Proof. See Appendix D.6. \( \square \)

**Corollary 3.** If \( \psi = 1/2 \) the condition \( c \geq \beta \gamma (1-\gamma) \sqrt{2/\beta} \) ensures that the labor market equilibrium verifies \( R_{i+1} \geq R_i \) and \( \theta_{i+1} \leq \theta_i \) \( \forall i \).

Proof. Straightforward from Property 3 with \( \psi = 1/2 \). \( \square \)

This corollary shows that the restriction on \( c \) to ensure the existence of an equilibrium with \( R_{i+1} \geq R_i \) is weaker than that imposed by corollary 2. This point demonstrates the positive role of job-search effort in explaining the increase (decrease) of firings (hirings) with age.

### 2.2.4 The Age Profile of the Employment Rate

The age profile of hirings and firings has been recursively determined from terminal conditions. On the other hand, the age profile of unemployment \( u_i \) (or employment \( n_i = 1 - u_i \)) depends on the arbitrary initial condition \( u_1 \). This explains why it is ambiguous:

\[ u_i \geq \frac{G(R_{i+1})}{G(R_{i+1}) + p(\theta_i)} \Rightarrow n_{i+1} \geq n_i \forall i \]
Property 4. For $u_1 = 1$, there exists a threshold age $\tilde{T}$ so that $n_i \geq n_{i-1} \forall i \leq \tilde{T}$ and $n_i \leq n_{i-1} \forall i \geq \tilde{T}$.

Proof. See Appendix D.7.

In the case where all the new entrants are unemployed, high vacancy rates and low firing rates at the beginning of the working life cycle make the employment rate increasing with age until the age $\tilde{T}$. Until this threshold age, this increase in employment rate is simply the result of a queue phenomenon. From $\tilde{T}$ on, the employment rate evolution by age mimics the age profile of firings and hirings. The age heterogeneity across workers then leads to a low employment rate for older workers. The overall age-dynamics of employment is thus hump-shaped, as found in OECD data.

2.2.5 The Age Profile of Wages

Since the seminal empirical work of Mincer [1962], it is well-known that the wage increases with age and declines at the end of the life cycle. This stylized fact can be explained in our model simply by including general human capital accumulation as shown in Appendix A. The wage increases with the accumulation of human capital, but the decrease of the labor market tightness
slows down this growth. At the end of the life cycle, the large decrease of the tightness can dominate the dynamic of human capital accumulation.

**Property 5.** If $2\beta(1 - \gamma) < c$, then $w_{i+1}(R_{i+1}) > w_i(R_i), \forall i \leq \tilde{T}$.

*Proof.* See Appendix D.8. □

Moreover, the discrimination against the older workers leads to another phenomenon: at the end of the life cycle, firms hoard their workers less (the reservation productivity increases with the age of the worker). Only the more productive remain at work. The average wage thus increases due to a composition effect. This can explain why Hellerstein and Neumark [2004] on US data or Crépon, Deniau and Perez-Duarte [2004] on French data find that older workers seem to be overpaid. It suggests that this could be a statistical artefact.

### 2.2.6 Distance from Retirement instead of Age

An important parameter of the model is the retirement age. Only the distance between the current age and the retirement age matters according to a horizon effect. On the contrary, the biological age does not matter in itself.

**Property 6.** For two retirement ages, $T$ and $T + N$, we have $R_{T-1-i} = R_{T+N-1-i}$ and $\theta_{T-1-i} = \theta_{T+N-1-i}, \forall i$.

*Proof.* Proposition 2 and Corollary 1 clearly show that for all $T$, we have the same terminal condition: for two retirement ages, $T$ and $T + N$, we have $R_{T-1} = R_{T+N-1} = b$ and $\theta_{T-1} = \theta_{T+N-1} = 0$. Then, using backward induction, the equations $(JC)$ and $(JD)$ show that the $R_{T-1-i} = R_{T+N-1-i}$ and $\theta_{T-1-i} = \theta_{T+N-1-i}, \forall i$. □

This explains why some countries experience a drop in their employment rate as soon as the age of 55 (see Figure 1 in the introduction): the 55-59 years-old workers are close to their retirement age. Finally, the model explains the observed heterogeneity in the employment rates observed in OECD countries by a pure horizon effect due to the heterogeneity in terms of retirement age.
3 Older Worker Policies and Labor Market Equilibrium

It is in the interest of firms to differentiate their hiring and firing decisions by age in a life cycle setting. It potentially\(^9\) provides an original explanation for the observed decrease in the employment rate at the end of the working cycle. Faced with the low employment rate of older workers, governments have put in place different policies to counteract this trend or to compensate for it by specific assistance programs for older workers. These policies are certainly crucial to understanding the different forces at work when considering the end of the working life cycle. Our approach provides a theoretical framework to analyze their consequences by age on firings andhirings.

It is possible to distinguish two types of labor market policies oriented toward older workers.

- The first type is designed to protect them. Some policies for older workers are indeed characterized by (i) higher firing taxes (ii) higher "unemployment" benefits through specific assistance programs.

In some countries, (e.g. Belgium, Finland, France, Japan, Korea, Norway), it is indeed more costly for firms to lay off older workers because of longer notice periods or higher severance pay. Another important feature concerning the older workers is that there exists specific inactivity and disability programs in most European countries. They provide generous substitution incomes for people eligible for these programs until retirement: this leads to an increase in the non-employment incomes. Moreover, these workers must be considered as inactive (the job-search costs are higher than for the younger) as the benefits are not conditional on a job-search activity.

- The second type of policies aims at increasing the likelihood for older workers to find a job. In the UK, firms receive a subsidy if they hire an older worker. In the US, the objective of the anti-age discrimination law is to give the same employment opportunities to all individuals, whatever their age.

3.1 The Equilibrium with Unemployment Benefits, Firing Taxes and Hiring Subsidies

Let \(z_i\) be the age-dependent unemployment benefit, or more generally the non-employment incomes, whose financing is allowed by a non-distortionary

\(^9\)In the next section, we explore the empirical relevance of our theoretical framework.
tax.\textsuperscript{10} \( F_i \) is the tax that the firm must pay when it fires a worker of age \( i \) and \( H_i \) the hiring subsidy that the firm gets when it hires a worker of age \( i \). The equilibrium allocation with wage bargaining and endogenous search effort is now featured by (see Appendix B for details):

**Proposition 4.** For given sequences of policy instruments \( \{H_i, F_i, z_i\} \), a labor market equilibrium exists and it is characterized by a sequence \( \{R_i, \theta_i\} \) solving:

\[
\frac{c}{q(\theta_i)} = \beta (1 - \gamma) (1 - R_{i+1} + H_{i+1} - F_{i+1}) \quad (JC_{pol})
\]

\[
R_i = b + z_i + \frac{1}{2} \left( \frac{\gamma c}{1 - \gamma} \right)^2 - \beta \int_{R_{i+1}}^1 [1 - G(x)] dx - F_{i+1} - F_i \quad (JD_{pol})
\]

with terminal conditions \( R_{T-1} = b + z_{T-1} - F_{T-1} \) and \( \theta_{T-1} = 0 \).

**Proof.** See Appendix B. \( \Box \)

From this system, it can easily be shown that age-specific policies have an impact on the targeted population, but also on younger agents due to the forward-looking behaviors of firms.

### 3.1.1 The impact of age-increasing unemployment benefits

The unemployment benefit \( (z_i) \) is found to exert a conventional upward pressure on wages. This leads to an increase in the productivity threshold \( R_i \): the number of job destructions rises and the hiring rate decreases.

To get more insights into the role of the age profile of unemployment benefits, let us examine the impact of an additional unemployment income for a senior. For simplicity, let us now consider that \( z_{T-1} = z \) and \( z_i = 0 \ \forall i \in [2, T-2] \). Assume also \( H_i = F_i = 0 \ \forall i \) and \( \gamma = 0 \). From proposition 4, it is straightforward to see that:

\[
R_{T-1} = b + z_{T-1} \quad ; \quad \frac{c}{q(\theta_{T-2})} = \beta (1 - b - z_{T-1})
\]

\[
R_{T-2} = b - \beta \int_{R_{T-1}}^1 [1 - G(x)] dx \quad ; \quad \frac{c}{q(\theta_{T-3})} = \beta (1 - R_{T-2})
\]

As could be expected, this first shows that the greater generosity of assistance programs at the end of the working life increases firings and decreases hirings of older workers \((\frac{\partial R_{T-1}}{\partial z_{T-1}}) < 0 \) and \((\frac{\partial R_{T-2}}{\partial z_{T-1}}) > 0 \). But it is also important to

\textsuperscript{10}We assume that unemployed and employed workers face an age-specific lump sum tax, \( X_i \), so that \( X_i = z_i u_i \ \forall i \).
notice that the value of labor-hoarding for younger workers is also reduces
\[
\frac{\partial f_{T-1}}{\partial \gamma} [1 - G(x)] = \frac{\partial f_{T-1}}{\partial \gamma} [1 - G(x)] \times \frac{\partial R_{T-1}}{\partial \gamma} < 0.
\] This leads to higher (lower) firing (hiring) rate for workers who are not yet eligible for the generous unemployment benefit system \((\frac{\partial R_{T-2}}{\partial \gamma} > 0 \text{ and } \frac{\partial R_{T-1}}{\partial \gamma} < 0)\).

Unambiguously, the age-increasing non-employment income implies a rise in the non-employment rate of all workers (see equation (1)). Nevertheless, the firing probability of older workers is higher than that of the younger workers because the expected effects of this policy are smaller than its instantaneous impact.

3.1.2 The impact of the age-increasing firing tax

Whereas \(F_i\) tends, as expected, to push down \(R_i\) by increasing the current cost of firing, \(F_{i+1}\) increases \(R_i\) by reducing the value of labor-hoarding, i.e. the expected future gain associated with the job (see the term in brackets, equation \(JD_{pol}\)). This suggests that job destruction is crucially related to the age profile of the employment protection, \(F_{i+1} - F_i\).

More severe employment legislations protect workers who already have a job, but at the expense of those without a job. This point has already been stated theoretically by Mortensen and Pissarides [2000].

To get further insights into the role of the age profile of firing taxes, let us, for instance, examine the impact of a policy introducing an additional tax when laying off older workers. For simplicity, consider \(F_T > 0\) and \(F_i = 0 \quad \forall i \in [2, T_2 - 2]\). Without loss of generality, we assume again that the bargaining power of the workers is equal to zero \((\gamma = 0)\). Assume also \(H_i = 0\) and \(z_i = z, \forall i\). From proposition 4, it is straightforward to see that:

\[
R_{T-1} = b + z - F_{T-1} ; \quad \frac{c}{q(\theta_{T-2})} = \beta(1 - b - z)
\]

\[
R_{T-2} = b + z + \beta F_{T-1} - \beta \int_{R_{T-1}}^{1} [1 - G(x)] dx ; \quad \frac{c}{q(\theta_{T-3})} = \beta(1 - R_{T-2})
\]

This clearly shows that for workers of age \(T - 1\), the introduction of the tax reduces firings \((\frac{\partial R_{T-1}}{\partial F_{T-1}} < 0)\) while the number of newly-hired workers who enter to employment at age \(T - 1\) is unchanged \((\frac{\partial \theta_{T-2}}{\partial F_{T-1}} = 0)\): unambiguously, the firing tax implies an increase in the employment rate of the oldest workers (see equation (1)). On the contrary, for workers of age \(T - 2\), it appears that the firing probability increases and simultaneously the hiring one decreases \((\frac{\partial \theta_{T-3}}{\partial F_{T-1}} > 0 \text{ and } \frac{\partial \theta_{T-2}}{\partial F_{T-1}} = \frac{\partial \theta_{T-3}}{\partial F_{T-1}} \times \frac{\partial R_{T-2}}{\partial F_{T-1}} < 0)\). The expected firing tax in \(T - 1\) indeed reduces the value of job continuation when the worker is \(T - 2\) years old on the labor market (less labor-hoarding). The associated
increase in productivity thresholds translates into lower expected duration of jobs, hence lower hirings: the employment rate of workers of age \( T - 2 \) falls unambiguously with \( F_{T-1} \).

Overall, our framework illustrates that the aggregate employment impact of age-dependent employment protection is not determined \textit{a priori}.

### 3.1.3 The impact of the age-increasing hiring subsidy

In the equations \((JC_{pol})\) and \((JD_{pol})\), the hiring subsidies have a direct impact only on the job creation rule, unlike the firing taxes which affect both \((JC_{pol})\) and \((JD_{pol})\).

Let us, for instance, examine the impact of a policy introducing an additional subsidy when hiring older workers. For simplicity, consider \( H_{T-1} > 0 \) and \( H_i = 0 \quad \forall i \in [2, T - 2] \). We assume that the bargaining power of the workers is equal to zero \((\gamma = 0)\), and \( F_i = 0 \) and \( z_i = z, \forall i \). From Proposition 4, it is straightforward to see that:

\[
R_{T-1} = b + z \quad ; \quad \frac{c}{q(\theta_{T-2})} = \beta(1 - b - z + H_{T-1})
\]

\[
R_{T-2} = b + z - \beta \int_{R_{T-1}}^{1} [1 - G(x)] dx \quad ; \quad \frac{c}{q(\theta_{T-3})} = \beta(1 - R_{T-2})
\]

It clearly appears that when the bargaining power of the workers is equal to zero, the hiring subsidy targeted for the age \( T - 1 \) workers has an impact only on this age group, through an increase in their hiring rate. This policy unambiguously increases the employment rate of the older workers, without introducing any distortions for the younger workers.

Nevertheless, this result is not general. Indeed, when the bargaining power of the workers is strictly positive, the higher exit rate from unemployment tomorrow pushes up the wage today and then the productivity threshold. This leads to an increase (decrease) of the firing (hiring) rate for the younger workers. This adjustment of the wage could be neglected in the analysis of the unemployment benefit and the firing tax because it amplifies the direct effects of the institutional changes. This is no longer the case for the hiring subsidies.

### 3.2 The Influence of Legislation Prohibiting Age-Discrimination

The second issue of this section is about legislation prohibiting age discrimination. This legislation in the US dates back to the 60’s (Age Discrimination in Employment Act in 1967, and subsequent amendments). In 2000, the
European Union Council Directive also required all 15 EU countries to introduce legislation prohibiting direct and indirect discrimination at work on the grounds of age.

When age discrimination is efficiently prohibited, the job-search is undirected and there is only one labor market tightness, $\theta$. Let consider that search effort is constant, value functions now solve:

$$V = -c + \beta \left[ q(\theta) \sum_{i=1}^{T-2} \frac{u_i}{u} J_{i+1}(1) + (1 - q(\theta))V \right]$$

(10)

$$J_i(\epsilon) = \epsilon - w_i(\epsilon) + \beta \left[ G(R_{i+1}) V + \int_{R_{i+1}}^1 J_{i+1}(x) dG(x) \right]$$

(11)

$$W_i(\epsilon) = w_i(\epsilon) + \beta \left[ G(R_{i+1}) U_{i+1} + \int_{R_{i+1}}^1 W_{i+1}(x) dG(x) \right]$$

(12)

$$U_i = b + z_i + \beta \left[ p(\theta) W_{i+1}(1) + (1 - p(\theta)) U_{i+1} \right]$$

(13)

The free entry condition, $V = 0$, now implies:

$$\frac{c}{q(\theta)} = \beta \sum_{i=1}^{T-2} \frac{u_i}{u} J_{i+1}(1)$$

which means that the average recruiting cost equals the firm’s expected gain related to the match according to the age of the worker it meets. With probability $q(\theta)$ the vacant job is filled, and with probability $u_i/u$ it is filled with a worker of age $i$.

**Proposition 5.** Let $M(v, u) = v^\psi u^{1-\psi}$ with $0 < \psi < 1$, and $G(\epsilon) = \epsilon$, $\forall \epsilon \in [0, 1]$, a labor market equilibrium with wage bargaining and age discrimination legislation exists and it is characterized by a sequence $\{R_i, \theta, u_i\}$ solving:

$$\frac{c}{q(\theta)} = \beta(1 - \gamma) \sum_{i=1}^{T-2} \frac{u_i}{u} (1 - R_{i+1})$$

$$R_i = b + z_i + \gamma p(\theta) \beta (1 - R_{i+1}) - \frac{\beta}{2} (1 - R_{i+1})^2$$

$$u_i = G(R_i)(1 - u_{i-1}) + (1 - p(\theta)) u_{i-1}$$

with boundary conditions $R_{T-1} = b + z_{T-1}$ and $u_T = 1$.

**Proof.** See Appendix C.3 for the derivation of the equilibrium wage equation and then of the equation of the productivity threshold. \[\square\]
If we assume for simplicity that the bargaining power of the workers is equal to zero ($\gamma = 0$), Proposition 5 implies that the dynamic of the productivity threshold is the same as that without an anti-discrimination law. Then, this law has no impact on firing rates by age in this case. Turning to the hiring process, if we assume that $\theta = \theta_{T-2}$, with $\theta_{T-2}$ obtained in the model with discrimination, then $\theta > \theta_{T-1}$ since $\theta_{T-2} > \theta_{T-1}$. Hence, unambiguously, the employment rate of the older workers increases with the anti-discrimination law.

More generally, such a law imposes the same job creation rate whatever the worker’s age, on the basis of the average expected gain. As a consequence, it is favorable to older workers’ recruitment as regards the laissez-faire economy. Labor market tightness for older workers is indeed increased by the age discrimination legislation. But, in turn, since wages are positively related with labor market tightness, equilibrium wages and destruction rates of older workers are also increased. The overall impact on employment is thus theoretically undetermined, but one might expect that the direct impact on firings is less than offset by the indirect impact on firings, that is, this legislation increases the employment rate of older workers.

4 A Quantitative Investigation

The objective of this section is twofold. On the one hand, we aim at showing that the model is able to account for the observed age-dynamics of job creation and job destructions flows. On the other hand, we verify that the feedback effect of the retirement age is at the first rank in the explanation of the decrease in the employment rate of older workers, relative to existing labor market institutions that are traditionally favored in any model aiming at explaining labor market flows. We completely discard the entry-stage process of the labor market and so the younger worker flows, typically focusing on employment age heterogeneity after the age of 30 in our empirical investigations.

We choose to evaluate the model on French data. Firstly, the retirement age is known without any uncertainty. All men exit the labor force when they reach their full pension age and they are not allowed to continue working after drawing their pension. Until recently, more than 90% of men left for retirement at 60. All these features makes France very close to our framework which leaves aside any questions related to heterogeneity and uncertainty about the exit date from the labor force.

France is also characterized by the existence of specific and generous assistance programs for older workers. Workers are eligible for these programs
in the case of layoffs, conditionally on being aged more than 57. They then receive a generous income until retirement. Another important feature of the French labor market is the specific firing tax that firms incur in the case of laying off older workers (“Delalande Tax”).

4.1 Calibration and Quantitative Assessment

The empirical performance of the model is now evaluated on French data by simulating the equilibrium with age-policies \((JC_{pol})-(JD_{pol})\). The calibration is based on the period prior to 1993, that is before the Social Security reform which has introduced more heterogeneity in the retirement age. Similarly, we discard modifications to the layoff tax scheme that have been implemented from 1993 on.

We consider flows between employment and non-employment as older workers are mostly entitled to specific assistance programs which are slightly more generous than unemployment benefits.

Calibration. Specifications of functional forms for the distribution of idiosyncratic shocks and the matching function are first required. As it is usually assumed, we consider an uniform distribution and a Cobb-Douglas matching function. More precisely, we set:

\[
G(\epsilon) = \epsilon \quad \forall \epsilon \in [0, 1] \\
\ln(q(\theta_i)) = \psi \quad 0 < \psi < 1
\]

A first set of parameters \(\Phi_1 = \{\beta, \psi, \gamma, F, z, F(old), z(old)\}\) is then based on external information, where \(F\) (resp. \(z\)) is the firing tax (resp. unemployment benefits) for workers less than 55 (resp. 57) years old, and \(F(old)\) (resp. \(z(old)\)) refers to older worker specific labor market policies for workers more than 55 (resp. 57) years old.

The discount factor \(\beta\) equals 0.96, which yields an annual interest rate of 4%. The elasticity of the matching function is set to the extensively-used value \(\psi = 0.5\) (see, among others, Mortensen and Pissarides [2000]). The bargaining power of the workers is fixed at \(\gamma = 0.3\), which is the mean of admissible values estimated on French microdata (see Abowd and Allain [1996]). Overall firing costs represent in France 15 months of average wage earnings (see Abowd and Kramarz [2003]). No evidence exists for France on the decomposition of these costs between severance transfers to workers and

22
administrative costs. Garibaldi and Violent [2002] provides such a decomposition for Italy, whose employment protection OECD indicator is reported to be very close from France: 1/3 of firing costs are related to administrative costs. Applying this value for the overall firing cost in France implies, we have that the layoff tax represents in our economy 5 months of the average annual wage earnings, which is obtained by setting $F = 0.22$. For workers between 55 and 60, we also include the additional administrative cost targeted on older workers (the “Delalande tax”). This additional component represents 25% of the annual wage earnings (French law until July 1992). This leads to $F_{\text{old}} = 0.385$, which implies that the overall firing tax for older workers is 8 months of the average annual wage earnings. Unemployment benefits are set to reproduce an average net replacement ratio of 55% (Martin [1996]) for workers less than 57 years-old: this implies $z = 0.275$. For workers over 57 we add a premium of 20% which reflects the fact workers are entitled to unemployment insurance for 3 years (until 60), whereas workers aged less than 57 are eligible for two years to the insurance scheme and then fall in assistance programs where their earnings are in average cut by two.

Due to a lack of information on the value of leisure and the flow of recruiting cost, we choose to calibrate a second set of parameters $\Phi_2 = \{b, c\}$ in order to reproduce stylized facts on employment rate in France in 1993. Accordingly, the model will be assessed on its ability to account for job flows in and out of non-employment rather than on the replication of the age-dynamics of the employment stock variable. The value of $b$ and $c$ are set such that the model matches the observed average employment rate for 30-44 and 55-59 years old workers, denoted respectively $n_{(30-44)}$ and $n_{(55-59)}$. This implies $b = 0.19$ and $c = 0.47$.

**Model Assessment.** Our calibration strategy hence implies that by definition the model the employment rate for the two age groups, 30-44 and 55-59. The employment rate of workers aged between 45 and 55 is well replicated. The key issue about the empirical relevance of our model then relies on its ability to account for job creation and job destruction flows. The model could replicate the age-dynamics of the stock variable (employment), but understates or overstates by ten both hirings and firings. Figure 4 shows entry and exit rates for three age groups, 30-44, 45-54 and 55-59, so that our assessment is based on six moments. Interestingly, this figure shows that our model matches quite well the age-pattern of firing and hiring rates over the life cycle, except the constancy of the firing rate over the workers aged between 30 and 55 years old.

However, it must be acknowledged that the levels are not perfectly repro-
Figure 4: The age-dynamics of the labor market

- **Employment Rate by Age Group**

- **Annual Hiring Rate by Age Group**

- **Annual Firing Rate by Age Group**
duced. More particularly, the firing and hiring rates of the age group 45-55 are overestimated. Yet, we consider that this simple model works surprisingly well to generate the age profile of hirings and firings over the life cycle.

4.2 Identifying the mechanisms at work

What are the main driving forces behind the age-dynamics generated by our model? First, it is of primary interest to identify the relative importance of the age policies and the horizon effect in this result. It is often argued that the generosity of unemployment benefits is the primary cause of low employment (see Lunqvist and Sargent [2002]). Accordingly, one could expect that higher UB for workers aged 57 or more in France is the key feature at the origin of the decline in the older worker employment rate. We will show that the intrinsic horizon effect actually is dominant. Secondly, it is crucial to identify the respective contributions of the supply and demand for older workers. Which side of the labor market is the most sensitive to the horizon effect? It will be revealed that the demand side seems dominant in explaining the decrease in the employment rate when the retirement age comes closer.

4.2.1 Disentangling the horizon effect and the age policies

The generosity of assistance programs after 55 could explain the decline in the employment rate for these workers, but also that of the 45-54 age-groups by a feedback effect. Even if this last effect is inherent to our framework, this explanation would be a simple transposition to our life cycle model of a well-identified and classic mechanism.

In order to identify the relative contribution of the horizon effect, we consider along our benchmark calibration the case where the unemployment benefit and the firing tax are constant throughout the working life cycle. Table 1 ("without age policies") gives a quantitative measure of the decrease of employment in this economy (see also figure 5). It appears that there still exists a large decrease in the employment rate after 55 with a constant UB profile. Consistently, the hiring and firing rates are deeply affected by the proximity of the retirement age. Without any age policies, the decrease in the 55-59 employment rate (with resp. to 45-54) is equal to 25 points. These results clearly show that the "horizon effect" largely dominates the dynamics of the employment rate at the end of the life cycle. Indeed, the horizon effect represents roughly two-thirds of the observed decline of the employment rate (see Table 1).

By comparing the first and the second lines, it appears that the increasing profile of UB after 57 is responsible for a decrease of 12.2 points for the 55-
Figure 5: Age-dynamics of the labor market with and without age-specific policies

Table 1: Policies versus Horizon Effect

<table>
<thead>
<tr>
<th>Age</th>
<th>Employment</th>
<th>Hiring</th>
<th>Firing</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-44</td>
<td>30-44</td>
<td>30-44</td>
<td>30-44</td>
</tr>
<tr>
<td>45-54</td>
<td>45-54</td>
<td>45-54</td>
<td>45-54</td>
</tr>
<tr>
<td>55-59</td>
<td>55-59</td>
<td>55-59</td>
<td>55-59</td>
</tr>
<tr>
<td>Without Age Policies</td>
<td>87.1 83.5</td>
<td>24.5 21.6</td>
<td>3.3 5.3</td>
</tr>
<tr>
<td>Without Age Firing Cost</td>
<td>87.0 81.3</td>
<td>46.3 24.0</td>
<td>18.1 5.5</td>
</tr>
<tr>
<td>Benchmark</td>
<td>86.7 77.0</td>
<td>49.0 24.0</td>
<td>17.5 5.8</td>
</tr>
</tbody>
</table>
59 employment rate, but also of 2.2 points for the 45-54 age-groups. Higher firing costs after 55 lead to better employment protection for the 55-59 years-old, who benefit from an increase in the employment rate of 2.7 points due to this legislation. However, this latter negatively affects the employment rate for workers aged 45-54 which decreases by 4.1 points. The net effect on employment rates is thus close to zero (see also figure 5). These results are consistent with a recent econometrical evaluation of the Delalande Tax (Behaghel, Crépon and Sédillot [2005]).

4.2.2 The respective role of the firms and the workers

If the horizon effect seems to play an important role at the end of the working life, both firms and workers are potentially responsible for this outcome. It is possible to identify their respective roles by considering economies with a constant job-search effort for workers. More precisely, we compare two economies, with and without an endogenous job-search effort at each age. In this latter case, the constant job-search effort is set to its average value over the 30-44 age range.

<table>
<thead>
<tr>
<th></th>
<th>Employment</th>
<th>Hiring</th>
<th>Firing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30-44</td>
<td>45-54</td>
<td>55-59</td>
</tr>
<tr>
<td>Benchmark</td>
<td>86.7</td>
<td>77.0</td>
<td>49.0</td>
</tr>
<tr>
<td>Benchmark $e_i = \overline{r}$</td>
<td>86.7</td>
<td>80.3</td>
<td>66.2</td>
</tr>
<tr>
<td>Without Age Policies</td>
<td>87.1</td>
<td>83.5</td>
<td>58.5</td>
</tr>
<tr>
<td>Without Age policies, $e_i = \overline{r}$</td>
<td>87.1</td>
<td>84.8</td>
<td>66.4</td>
</tr>
</tbody>
</table>

We first consider the benchmark economy (with age policies) in these two cases (the first two lines of Table 2). By comparing these two economies, we see that the employment rate of 55-59 years old workers is reduced by 20.5 points (with resp. to 20-44), instead of 37.7 points when search effort is endogenous. This suggests that 55% of the decrease in the employment rate is related to the firms’ behavior (demand side). Moreover, the intrinsic role of the workers’ behavior (supply side) is certainly over-emphasized, as the unemployment benefits are particularly generous at the end of the working life. The relative contribution of the labor demand could be even greater when restricting our attention to the horizon effect. We now consider the two economies without age policies (the last two lines) with and without endogenous search efforts. The contribution of the firms’ behavior goes up to 70%.
5 Efficient Job Creation and Job Destruction over the Life Cycle

Our model precisely highlights economic rationales sustaining age discrimination: there is less time over which to recoup creation costs and temporary negative productivity shocks for older workers. A *laissez-faire* equilibrium is then typically featured by job creation (destruction) rates decreasing (increasing) with age. Hence, there is age “discrimination” against older workers at the equilibrium. In some countries, policies have been implemented to sustain the demand for older workers. If the previous section has questioned their efficiency, their optimality is still an open question. This section is devoted to the analysis of this issue.

5.1 On the Optimality of Age Discrimination

We first wonder to what extent the *laissez-faire* equilibrium is optimal. We precisely show that the Hosios condition leads to equilibrium efficiency. In this particular case, the discrimination against the older workers is optimal. The intuition of the result is simple: a central planner maximising social welfare will allocate fewer resources to the hiring and the labor-hoarding of older workers, as the same phenomenon as in the decentralized equilibrium is at work: a short horizon makes less efficient any investments in older worker employment.

In line with the analysis of Pissarides [2000] with infinite-lived agents, we derive the optimal steady-state allocation by maximizing the sum of discounted output flows net of recruiting costs for a new entrant generation. This is done over the life cycle of workers\(^1\). In addition, since generations are independent of each other, doing this maximization for each one would lead to the same result.

The planner’s problem is stated as:

\[
\max_{\{R_i\}_{i=1}^{T-1}, \{\theta_i, e_i\}_{i=1}^{T-1}} \sum_{i=1}^{T-1} \beta^i \left( y_i + \left[ b - \frac{1}{2} e_i^2 \right] u_i - c\theta_i u_i e_i \right) \quad (14)
\]

under the constraints:

\[
u_{i+1} = G(R_{i+1})(1 - u_i) + u_i (1 - e_ip(\theta_i)) \quad (15)
\]

\[
y_{i+1} = u_ie_ip(\theta_i)1 + (1 - u_i) \int_{R_{i+1}} G(\epsilon) \quad (16)
\]

\(^{11}\)It is possible to show that it is equivalent to maximizing the expected gain of unemployed workers.
where \( y_i \) is the average output.

**Proposition 6.** Let \( \eta(\theta_i) = -\theta_i q'(\theta_i)/q(\theta_i) \) and \( G(\epsilon) = \epsilon \), the efficient allocation exists and it is characterized by a sequence \( \{R_i^*, \theta_i^*\} \) solving:

\[
\frac{c}{q(\theta_i)} = \beta (1 - \eta(\theta_i^*)) (1 - R_{i+1}^*) \quad (JC^*)
\]

\[
R_i^* \ = \ b + \frac{1}{2} \left( \frac{\eta(\theta_i^*) + \theta_i^*}{1 - \eta(\theta_i^*)} \right)^2 - \beta \int_{R_i^*}^1 [1 - G(x)] \, dx \quad (JD^*)
\]

with terminal conditions \( R_{T-1}^* = b \) and \( \theta_{T-1}^* = 0 \).

**Proof.** See Appendix D.9. \( \square \)

**Property 7.** Let \( \eta(\theta_i^*) = 1 - \psi \) and \( G(\epsilon) = \epsilon \), if \( 1 \geq \beta (1 - \psi) \left( \frac{\Delta \psi}{\psi} \right) \frac{1}{1 - \psi} \), then the efficient allocation verifies \( R_{i+1}^* \geq R_i^* \) and \( \theta_{i+1}^* \leq \theta_i^* \) \( \forall i \).

**Proof.** Let substitute \( 1 - \gamma \) by \( \psi \) in proof of property 3, and the proof is straightforward. \( \square \)

These results therefore suggest that it is socially efficient to discriminate against older workers by providing them with a lower probability of hiring and a higher probability of firing.

**Property 8.** Let \( \eta(\theta_i^*) = 1 - \psi \), if \( \gamma = 1 - \psi \) then \( R_i = R_i^* \) et \( \theta_i = \theta_i^* \) \( \forall i \).

**Proof.** The proof is straightforward by substituting \( \psi \) by \( 1 - \gamma \) in Proposition 6 and by comparing with Proposition 2. \( \square \)

As in Mortensen and Pissarides [1994], the equilibrium is in general not optimal. This is only in the case of the Hosios condition (Property 8). It is easy to show that a competitive job-search equilibrium à la Moen [1997] is also able to generate the socially optimal match surplus sharing rule characterized by the Hosios condition (proof available upon request). Our life cycle economy indeed does not introduce any additional source of externalities.

### 5.2 Optimal Age-Dependent Firing Costs and Hiring Subsidies

It is well-known that the existence of unemployment benefits distorts the *laisser-faire* equilibrium and legitimates the implementation of firing cost and hiring subsidy policies (Mortensen and Pissarides (1999)). We show in this section that the first best allocation can also be recovered in our life cycle setting by such policies, but only on condition that they are age-dependent.

Overall, we show that the age profile of hiring subsidies and firing taxes is crucially related to the value of the worker’s bargaining power and the level of unemployment benefits. The intuition of these results is the following:
• Firstly, high unemployment benefits increase the labor cost and then reduce the incentives for firms to hire workers. Nevertheless the reduction of the labor demand is not the same at each age. Indeed, for an older worker, the firm only pays the tax introduced by the UB during a short period. On the other hand, for a younger worker, the firm anticipates paying this tax for a longer period. In this case the comparative advantage of the younger worker becomes a handicap. In order to restore the efficient allocation, firing taxes and hiring subsidies typically must decrease with age.

• Secondly, if the bargaining power of the workers is too low relatively to its optimal value, firms over-invest in the process of hiring the younger workers. Because of congestion effects, the rotation costs are higher than their optimal value. In order to restore the first-best allocation, firing taxes and hiring subsidies typically must increase with age.

By comparing the equilibrium allocation with the first best one, it is possible to feature the first best policies.

**Proposition 7.** Let \( \eta(\theta_i) = -\theta q'(\theta_i)/q(\theta_i) = 1 - \psi \), assume a given sequence for unemployment benefits \( \{z_i\} \), the optimal labor market policy is a sequence \( \{H_i^*, F_i^*\} \) solving:

\[
H_{i+1}^* = F_{i+1}^* + \left[ \frac{\gamma - (1 - \psi)}{(1 - \gamma)\psi} \right] \frac{c}{\beta q(\theta_i^*)} \tag{17}
\]

\[
F_i^* = z_i + \beta F_{i+1}^* + \left[ \left( \frac{\gamma}{1 - \gamma} \right)^2 - \left( \frac{1 - \psi}{\psi} \right)^2 \right] \frac{1}{2} (\theta_i^*)^2 \tag{18}
\]

with boundary conditions \( H^*_{T-1} = F^*_{T-1} = z_{T-1} \), and where \( \theta_i^* \) is given by the solution of the dynamical system \((JC^*)-(JD^*)\).

**Proof.** The proof is straightforward by allowing \( H_i \) and \( F_i \) in \((JC_{pol})\) and \((JD_{pol})\) to be consistent with \((JC^*)\) and \((JD^*)\).

To understand the policy implications of this proposition, we disentangle the role played by each distortion, either related to unemployment benefits or search externalities. The two following corollaries deal successively with these two sources of distortions and their respective implications on policy.

**Corollary 4.** Let \( \eta(\theta_i) = -\theta q'(\theta_i)/q(\theta_i) = 1 - \psi \), assume \( \gamma = 1 - \psi \) and take \( z_i = z \) \( \forall i \) as given, the age dynamics of hiring subsidies and firing taxes is characterized by \( F_i \geq F_{i+1} \geq z \) and \( H_i \geq H_{i+1} \geq z \).
Proof. The proof is straightforward by considering $\gamma = 1 - \psi$ in Proposition 7.

Assuming $\gamma = 1 - \psi$ (job-search externalities are internalized), we are focusing on the policy implications of the distortions related to unemployment compensations.

Why does the firing tax decrease with worker’s age? Let us first consider a job with a worker of age $T - 1$. Correcting for an excessive wage implies $F_{T-1} = z$. With a worker of age $T - 2$, not only $z$ but also $F_{T-1}$ must be internalized: both $z$, by increasing the wage, and $F_{T-1}$ by reducing the value of labor- hoarding are found to increase $R_i$. Accordingly, $F_{T-2} > F_{T-1}$. By backward induction, it thus appears that $F_i^* = z \sum_{j=0}^{T-1} \beta^j$: the firing tax internalizes the sum of discounted unemployment benefits until the exit from the labor market.\(^\text{12}\) Hiring subsidies are then introduced to avoid the distortion induced by termination costs, $H_i = F_i$ \(\forall i\).

If we allow for some exogenous heterogeneity in unemployment benefits, that is $z_i \neq z_i + 1$, we have $F_i^* = z_i + \beta F_{i+1}^*$. Then, if $z_i \geq z_i + 1$, it is straightforward to see that the optimal age profile of firing taxes is still decreasing with age, $F_i^* \geq F_{i+1}^*$. On the contrary, if $z_i \leq z_i + 1$ it can be the case that the optimal employment protection for older workers who benefit from high unemployment compensations is higher than for the younger.

Corollary 5. Assume $z_i = 0$ \(\forall i\) and Proposition 6 is satisfied, the age dependence of hiring subsidies and firing taxes is characterized by:

\[
\begin{align*}
\text{if } \gamma > 1 - \psi, \text{ then } & \ H_i^* > H_{i+1}^* \geq 0 \text{ and } F_i^* > F_{i+1}^* \geq 0. \\
\text{if } \gamma < 1 - \psi, \text{ then } & \ H_i^* < H_{i+1}^* \leq 0 \text{ and } F_i^* < F_{i+1}^* \leq 0.
\end{align*}
\]

Proof. Imposing $\theta_{i+1}^* \leq \theta_i^*$ from Proposition 6 into proposition 7, the proof is straightforward.

If $\gamma > 1 - \psi$, the worker’s bargaining power is higher than its efficient value. This implies that equilibrium wages are higher than required by the optimum. Consequently, there are not enough vacancies at the equilibrium. Hiring subsidies have to be introduced in order to be consistent with $\theta_i = \theta_i^*$. But at the same time, the large value of $\gamma$ together with hiring subsidies are responsible for an excessive rate of job destruction: $\frac{2}{1-\gamma} c\theta_i^* \text{ (from } JD_{pol}) > \frac{1-\psi}{\psi} c\theta_i^* \text{ (from } JD^*)$. This requires a positive tax on firings. Until now, the

\(^\text{12}\)If it was assumed that agents have an infinite life horizon on the labor market, as in Mortensen-Pissarides ($T \to \infty$), it would be straightforward to see that $F_i = z/(1-\beta) \ \forall i$. 

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same results would have been obtained in a Mortensen-Pissarides economy with infinite life horizon.

Our additional point is that the size of the distortions related to $\gamma \neq 1 - \psi$ is decreasing with a worker’s age. This is due to $\theta_i \geq \theta_{i+1}$, which indicates that the wage incidence of $\gamma$ is as smaller the older the worker. Ultimately, even if $\gamma > 1 - \psi$, we have $F_{T-1} = H_{T-1} = 0$ (for $z = 0$). Consistently, when $\gamma > 1 - \psi$, we find at optimal to reduce the size of employment protection and the amount of hiring subsidies as a worker’s age increases.

In turn, when $\gamma < 1 - \psi$, equilibrium wages are not high enough so that it is optimal to tax hirings and simultaneously encourage firings. For the same reason as before, distortions being lower for older workers, hiring taxes and firing subsidies are optimally increasing with a worker’s age.

Overall, the age dynamics of firing taxes and hiring subsidies depend both on the value of unemployment benefits and on worker bargaining power. In particular, even though $\gamma < 1 - \psi$, it can be the case that $F_i \geq F_{i+1}$ if the value of $z$ is high enough to have equilibrium wages higher than their efficient value. In other words, higher unemployment benefits make a decreasing profile of hiring subsidies and firing taxes by age more likely. On the contrary, if $\gamma$ and $z$ are low enough, the dynamics are reversed. Interestingly, this can easily be formally stated by considering the case $\beta \to 1$.

**Corollary 6.** Let $\eta(\theta_i) = -\theta_i q'(\theta_i)/q(\theta_i) = 1 - \psi$, assume $\beta \to 1$, $\gamma < 1 - \psi$, $z_i = z \ \forall i$ and property 7 is satisfied, then

$z \geq \tilde{z}$ is a sufficient condition for $F_i \geq F_{i+1} \geq z$ and $H_i > H_{i+1} \geq z_i$,

$z \leq \tilde{z}$ is a sufficient condition for $F_i \leq F_{i+1} \leq z$ and $H_i < H_{i+1} \leq z_i$.

where $\tilde{z} = \left[\left(\frac{1-\psi}{\psi}\right)^2 - \left(\frac{\gamma}{1-\gamma}\right)^2\right] \frac{c^2}{2} \left[\frac{\psi(1-b)}{c}\right]^{1-\psi}$ and $\tilde{z} = \left[\left(\frac{1-\psi}{\psi}\right)^2 - \left(\frac{\gamma}{1-\gamma}\right)^2\right] \frac{c^2}{2} \left[\frac{\psi}{c}\right]^{1-\psi}$.

**Proof.** See Appendix D.10.14

These results should provide important insights on the optimal age-pattern of employment protection and hiring subsidies in OECD countries. In Anglosaxon countries with low unemployment benefits, the shape of the firing

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13 Lastly, it is worth emphasizing that if we had assigned an efficiency motive to unemployment benefits by removing redistributive considerations, it is straightforward to see from Proposition 7 that for $F_i = 0 \ \forall i$ and $\gamma > 1 - \psi$, $z_i^* \leq z_{i+1}^* \leq 0$, whereas if $\gamma < 1 - \psi$, $z_i^* \geq z_{i+1}^* \geq 0$.

14 If $\gamma < 1 - \psi$ and $z \in [\tilde{z}, \tilde{z}]$, the age dynamics of $H_i$ and $F_i$ are typically non-monotonic (first increasing and then decreasing).
tax should be increasing, whereas the revers hold in Europe countries such as in France.

5.3 The Role of Legislation Prohibiting Age Discrimination

As emphasized above, most OECD countries have adopted a law to prohibit age discrimination. Despite its effectiveness being still controversial, it is of primary interest to examine whether this labor market policy is welfare-improving or not. Interestingly, our answer is again that it depends on the level of unemployment benefits.

Of course, this law cannot be a first best policy\textsuperscript{15}. However, in a second best world, where job-search externalities are not internalized or unemployment benefits are paid to unemployed workers, prohibiting age discrimination can be optimal.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig6.png}
\caption{Welfare gain from age discrimination legislation}
\end{figure}

Calibration: The French economy except that $F = 0$ and $e_i = 1$, $\forall i$.

\textsuperscript{15}This can be easily demonstrated. The program of the social planner when age discrimination is prohibited incorporates a supplementary constraint, $\theta_i = \theta$. 

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ers (the older ones). On the other hand, when employment is too high (too much rotations), the legislation which reduces the expected gain of hirings is now welfare improving. This point is illustrated in figure 6.\textsuperscript{16}

This clearly suggests that it is welfare-degrading to implement age discrimination laws, if the unemployment benefits are high. This result calls into question the adoption of this law in some European economies such as France.

6 Conclusion

The originality of this paper is to incorporate life-cycle features into the job creation - job destruction framework. We show the significance of the retirement date in the the ability of our canonical model to account for the observed fall in the older worker employment rate. This result relies neither on retirement programs nor on declines in productivity. It simply refers to the expected distance from retirement, that determines the expected duration of jobs and, in turn, both firms and workers’ forward-looking strategies at the end of the working life.

We then show that the optimal age-profile of older worker policies sharply differs among countries according to differences in unemployment benefit institutions. While in a US type economy hiring subsidies and firing taxes should be more favorable to older workers, the reverse holds true in European countries with high unemployment compensation. This clearly calls into question the optimality of higher employment protection for older workers and age discrimination legislation adopted in some European countries. Anti-age discrimination legislation could be a proxy of the first best policy in the US.

Overall this paper rehabilitates the life cycle view of the labor market,

\textsuperscript{16}To compare equilibrium welfare with and without directed job-search we use the following definitions, respectively:

\[
W^d = \sum_{i=1}^{T-1} \beta^i \left( y_i^d + bu_i^d - c\theta_i^d u_i^d \right)
\]

\[
W^{nd} = \sum_{i=1}^{T-1} \beta^i \left( y_i^{nd} + bu_i^{nd} - c\theta_i^{nd} u_i^{nd} \right)
\]

where subscripts $d$ and $\text{nd}$ stand for the equilibrium with age discrimination (benchmark) and without age discrimination, respectively. In order to capture only the impact of a constant hiring rate implied by the age discrimination law on the labor market dynamics, we assume that the job-search effort is constant over the life-cycle.
both for understanding supply and demand characteristics, and for implementing welfare-improving policies.

References


A Extended Model with Endogenous Search Effort and Human Capital Accumulation

This section describes an extended version of our benchmark model that allows both for endogenous job-search effort and general human capital accumulation. Our objective is twofold:

- It is first to examine the robustness of our results with respect to the introduction of job-search effort.

- It is then to show that a basic incorporation of human capital accumulation reconciles the model with the hump-shaped profile of wages over the life cycle.

The matching function that gives the number of hirings, \( M(v_i, \tau, u_i) \), now includes the average job-search effort \( \tau \). Accordingly, labor market tightness is defined as \( \theta_i = v_i / [e_i u_i] \), so that the probability for unemployed workers to meet a vacancy is \( e_i p(\theta_i) = e_i \frac{M(v_i, \tau, u_i)}{e_i u_i} \) and the probability for a firm to fill a vacancy is \( q(\theta_i) = \frac{M(v_i, \tau, u_i)}{e_i u_i} \).

Let us consider some general preferences for the opportunity cost of employment and the search effort, \( \phi(b_i, e_i) \), the value of the unemployed position writes as:

\[
U_i = \max_{e_i \geq 0} \left\{ \phi(b_i, e_i) + \beta [e_i p(\theta_i) W_{i+1}(1) + (1 - e_i p(\theta_i)) U_{i+1}] \right\}
\]

with \( \phi'_1 > 0, \phi'_2 < 0 \) and \( \phi''_1 < 0, \phi''_2 > 0 \). The optimal job-search effort decision rule then solves:

\[
-\phi''_2(b_i, e_i) = \beta p(\theta_i) [W_{i+1}(1) - U_{i+1}]
\]

General human capital accumulation over the life cycle is for further simplicity assumed to continuously increase with age, whatever the worker’s status on the labor market (either employed or unemployed). Firms’ value functions are now defined by:

\[17] In the main text we consider a simpler specification, \( \phi(b_i, e_i) = b_i - \frac{e^2}{2} \).

\[18] A useful extension of this model would be to allow for some depreciation in human capital skill with periods of unemployment. Currently, our goal is primarily to show that human capital accumulation provides an opposite force to the age-horizon on the age dynamics of wages.
\[ V_i = -c_i + \beta [q(\theta_i)J_{i+1}(1) + (1 - q(\theta_i))V_i] \]
\[ J_i(\epsilon) = h_i\epsilon - w_i(\epsilon) + \beta \int_{R_{i+1}}^{1} J_{i+1}(x)dG(x) + \beta G(R_{i+1}) \max_i \{V_i\} \quad \forall i \in [1, T - 1] \]

where \( h_i \) reflects upon the productivity impact of human capital for a worker of age \( i \). \( c_i \) is the recruiting cost which eventually depends on the worker’s age due to different levels in human capital levels (see hereafter).

In that context, the wage bargaining (see Appendix C for more details) solves:

\[ w_i(\epsilon) = (1 - \gamma)\phi(b_i, e_i) + \gamma (h_i\epsilon + c_i\theta_i e_i) \quad (20) \]

Before turning to the equilibrium definition, one must provide some assumptions on laws of motion for human capital, recruiting costs and opportunity costs of employment. In a way consistent with an infinite-lived agents matching model with growth (see Pissarides [2000]), we consider a deterministic trend on human capital, and that recruiting costs and workers’ reservation wage are indexed to this trend:

\[ h_{i+1} = (1 + \mu)h_i \]
\[ c_i = ch_i \]
\[ \phi(b_i, e_i) = h_i\phi(b, e_i) \]

with \( \mu \geq 0 \), a given initial condition \( h_1 = 1 \) and some positive parameters \( c, b \geq 0 \).

**Proposition 8.** A labor market equilibrium with wage bargaining, general human capital accumulation and endogenous search effort exists and it is characterized by a sequence \( \{R_i, \theta_i\} \) solving:

\[ \frac{\epsilon}{\phi(\theta_i)} = \beta(1 + \mu)(1 - \gamma)(1 - R_{i+1}) \quad (JC_{eff}) \]
\[ R_i = \phi(b, \Phi(\theta_i)) + \left( \frac{T}{1 - \gamma} \right) c\theta_i e_i - \beta(1 + \mu) \int_{R_{i+1}}^{1} [1 - G(x)]dx \quad (JD_{eff}) \]

with terminal conditions \( \theta_{T-1} = 0 \) and \( R_T = \phi(b, 0) \), and where \( \Phi(\theta_i) \equiv -\phi^{-1}_2 \left( \frac{T}{1 - \gamma} c\theta_i \right) \).
Proof. Similar to the proof of Proposition 2, but by making in addition use of the fact that \( -\phi'_2(b_i, e_i) = \beta p(\theta_i) [W_{i+1}(1) - U_{i+1}] = \frac{\gamma}{1-\gamma} \theta_i \) which implies \( e_i = -\phi'_2^{-1} \left( b_i, \frac{\gamma}{1-\gamma} c \theta_i \right) \equiv \Phi(\theta_i). \) \( \square \)

If \( \mu = 0 \), it is then possible to derive the Corollary 3 in the main text (see appendix D.6). This corollary demonstrates the positive role of the job-search effort in explaining the increase (decrease) of firings (hirings) with age.

Turning to the influence of human capital accumulation on equilibrium wages, let us remove the endogenous search effort. The equilibrium wage bargaining is:

\[
w_i(\epsilon) = h_i [(1 - \gamma)b + \gamma (\epsilon + c \theta_i)]
\]

Since \( h_i \) is increasing with age whereas \( \theta_i \geq \theta_{i+1} \), according to the value of \( \mu \), the age dynamics profile of wages is typically hump-shaped. Simulations of the efficient equilibrium with \( \mu = 0.01 \) illustrate this point.

B Benchmark Model with Labor Market Policy

Let us denote \( H_i \) a lump sum paid to the employer when a new worker of age \( i \) is hired, \( F_i \) the firing cost and \( z_i \) the unemployment benefits. We follow MP by considering that the wage structure that arises as a Nash bargaining solution has two tiers. The first tier wage reflects the fact that hiring subsidy is directly relevant to the decision to accept a match and that the possibility of incurring firing costs in the future affects the value the employer places on the match. In turn, the second tier wage applies when firing costs are directly relevant to a continuation decision.

Let the subscript \( i = 0 \) index the initial wage and the value of a job under the terms of the two-tier contract, firms' value functions solve:

\[
V_i = -c + \beta q(\theta_i) [J_{i+1}^0 + H_{i+1}] + \beta (1 - q(\theta_i)) V_i
\]

\[
J_i^0 = 1 - w_i^0 + \beta \int_{R_{i+1}}^1 J_{i+1}(x) dG(x) + \beta G(R_{i+1}) \left[ \max_i \{V_i\} - F_{i+1} \right]
\]

\[
J_i(\epsilon) = \epsilon - w_i(\epsilon) + \beta \int_{R_{i+1}}^1 J_{i+1}(x) dG(x) + \beta G(R_{i+1}) \left[ \max_i \{V_i\} - F_{i+1} \right]
\]
The optimal firing decision rule now solves:

\[ J_i(R_i) = -F_i \]

Adding the free entry condition, \( V_i = 0 \), it emerges that labor market tightness and productivity threshold are derived from the following two equations:

\[ \frac{c}{q(\theta_i)} = \beta \left[ J_{i+1}^0 + H_{i+1} \right] \tag{21} \]

\[ R_i = w(R_i) - F_i - \beta \left[ \int_{R_{i+1}}^1 J_{i+1}(x) dG(x) - G(R_{i+1})F_{i+1} \right] \tag{22} \]

Let us now examine the derivation of the two-tier wage structure. The latter is characterized by the following two sharing rules (as a result of Nash bargaining):

\[ \mathcal{W}_i(1) - \mathcal{U}_i = \gamma \left[ J_i^0 + H_i + \mathcal{W}_i(1) - \mathcal{U}_i \right] \Rightarrow w_i^0 \tag{23} \]

\[ \mathcal{W}_i(\epsilon) - \mathcal{U}_i = \gamma \left[ J_i(\epsilon) - \left( \max_i \{V_i\} - F_i \right) + \mathcal{W}_i(\epsilon) - \mathcal{U}_i \right] \Rightarrow w_i(\epsilon) \tag{24} \]

so that the equations for the initial and subsequent wage bargaining are (see Appendix C.2 for details):

\[ w_i^0 = (1-\gamma)(b + z_i) + \gamma (1 + c\theta_i + H_i - \beta F_{i+1}) \tag{25} \]

\[ w_i(\epsilon) = (1-\gamma)(b + z_i) + \gamma (\epsilon + c\theta_i + F_i - \beta F_{i+1}) \tag{26} \]

### C Wage Equations Derivations

#### C.1 Wage Bargaining in a “Laissez-Faire” Equilibrium

Let first consider the derivation of the wage equation in a “Laissez-Faire” equilibrium with both endogenous job-search effort and human capital accumulation.

The sharing rule (7) can take the form:

\[ -(1-\gamma)\mathcal{U}_i = \gamma [J_i(\epsilon) + \mathcal{W}_i(\epsilon)] - \mathcal{W}_i(\epsilon) \tag{27} \]
From value functions (3), (5) and (6), it turns out to be that:

\[
\gamma [J_i(\epsilon) + W_i(\epsilon)] - W_i(\epsilon) = \gamma h_i \epsilon - w_i(\epsilon) + \gamma \beta \int_{R_i+1}^1 [J_{i+1}(x) + W_{i+1}(x)] dG(x) \\
- \beta \int_{R_i+1} W_{i+1}(x) dG(x) \\
-(1 - \gamma)\beta G(R_i+1)U_i+1
\]

Similarly,

\[
\gamma \beta \int_{R_i+1}^1 [J_{i+1}(x) + W_{i+1}(x)] dG(x) = \gamma \beta \int_{R_i+1}^1 [J_{i+1}(x) + W_{i+1}(x) - U_{i+1}] dG(x) \\
+ \gamma \beta [1 - G(R_i+1)]U_i+1
\]

\[
\beta \int_{R_i+1} W_{i+1}(x) dG(x) = \beta \int_{R_i+1} [W_{i+1}(x) - U_{i+1}] dG(x) \\
+ \beta [1 - G(R_i+1)]U_i+1
\]

Since (7) holds for each age:

\[
\gamma \beta \int_{R_i+1}^1 [J_{i+1}(x) + W_{i+1}(x) - U_{i+1}] dG(x) = \beta \int_{R_i+1}^1 [W_{i+1}(x) - U_{i+1}] dG(x)
\]

so that (28) can be written as:

\[
\gamma [J_i(\epsilon) + W_i(\epsilon)] - W_i(\epsilon) = \gamma h_i \epsilon - w_i(\epsilon) - (1 - \gamma)\beta U_i+1
\]

In turn, the unemployed value (6) is:

\[
U_i = \phi(b_i, e_i) + \beta [e_i p(\theta_i)(W_{i+1}(1) - U_{i+1}) + U_{i+1}] \\
= \phi(b, e_i) + \beta \left[ e_i p(\theta_i) \frac{\gamma}{1 - \gamma} J_{i+1}(1) + U_{i+1} \right]
\]

From the free entry, it derives:

\[
J_{i+1}(1) = \frac{c_i v_i}{\beta M(\bar{e}_i u_i, v_i)} = \frac{c_i}{q(\theta_i)}
\]

so that since \(p(\theta_i) = M(\bar{e}_i u_i, v_i)/[\bar{e}_i u_i]\) and \(e_i = \bar{e}\) in equilibrium, we have:

\[
(1 - \gamma)U_i = (1 - \gamma)\phi(b_i, e_i) + \gamma c_i e_i \theta_i + (1 - \gamma)\beta U_{i+1}
\]
Let substitute for (29) and (30) in (27), we find:

\[-(1 - \gamma)\phi(b_i, e_i) - \gamma c_i e_i \theta_i - (1 - \gamma)\beta U_{i+1} = \gamma h_i \epsilon - w_i(\epsilon) - \beta(1 - \gamma)U_{i+1}\]

This lastly leads to the following wage equation:

\[w_i(\epsilon) = h_i [(1 - \gamma)\phi(b_i, e_i) + \gamma (\epsilon + ce_i \theta_i)]\]

Abstracting from the endogenous job-search effort and human capital accumulation, it is straightforward to get the equation (8).

C.2 Wage Bargaining with Labor Market Policy

For further simplicity, let us assume \(h_{i+1} = h_i = 1\). Under this assumption, we derive the two-tier wage structure.

The sharing rule (23) first can be written as:

\[-\gamma H_i - (1 - \gamma)U_i = \gamma [J_i^0 + W_i(1)] - W_i(1)\]  
\[(31)\]

Following the same derivation strategy as for the case without policy, we find that:

\[\gamma [J_i^0 + W_i(1)] - W_i(1) = \gamma 1 - w_i^0 - (1 - \gamma)\beta U_{i+1} - \gamma \beta F_{i+1}\]
\[(32)\]

\[(1 - \gamma)U_i = (1 - \gamma)\phi(b, e_i) + \gamma c e_i \theta_i + (1 - \gamma)\beta U_{i+1}\]
\[(33)\]

Substituting out for (32) and (33) in (31), it emerges that the initial wage solves:

\[w_i^0 = (1 - \gamma)\phi(b, e_i) + \gamma (1 + ce_i \theta_i + H_i - \beta F_{i+1})\]

Similarly, the sharing rule (24) can be written as:

\[-\gamma F_i - (1 - \gamma)U_i = \gamma [J_i(\epsilon) + W_i(\epsilon)] - W_i(\epsilon)\]
\[(34)\]

This in turn leads to the following wage equation:

\[w_i(\epsilon) = (1 - \gamma)\phi(b, e_i) + \gamma (\epsilon + ce_i \theta_i + F_i - \beta F_{i+1})\]
C.3 Wage Bargaining with Age-Discrimination Legislation

>From the equation (10)-(11)-(12)-(13) and the free-entry condition, it is possible to show that the total surplus related to the job match, \( S_i(\epsilon) \equiv J_i(\epsilon) - V + W_i(\epsilon) - U_i \) solves

\[
S_i(\epsilon) = \epsilon - b - z_i - \beta p(\theta)(W_{i+1}(1) - U_{i+1}) + \beta \int_{R_{i+1}}^{1} S_{i+1}(x)dG(x)
\]

In addition, since the sharing rule can be stated both as \( \gamma S_i(\epsilon) = W_i(\epsilon) - U_i \) and \( (1 - \gamma)S_i(\epsilon) = J_i(\epsilon) \), it emerges that:

\[
J_i(\epsilon) = (1 - \gamma)[\epsilon - b - z_i - \beta p(\theta)(W_{i+1}(1) - U_{i+1})] + \beta \int_{R_{i+1}}^{1} J_{i+1}(x)dG(x) \quad (35)
\]

This entails \( J_i'(\epsilon) = 1 - \gamma \), hence \( J_i(\epsilon) = (1 - \gamma)(\epsilon - R_i) \) and \( W_i(\epsilon) - U_i = \gamma(\epsilon - R_i) \). It is then straightforward to derive from the sharing rule the following wage equation:

\[
w_i(\epsilon) = \gamma \epsilon + (1 - \gamma)b + \gamma(1 - \gamma)\beta p(\theta)(1 - R_{i+1})
\]

In turn, the job destruction rule solves \( S_i(R_i) = 0 \),

\[
R_i = b + z_i + \beta \gamma p(\theta)(1 - R_{i+1}) - \beta \int_{R_{i+1}}^{1} [x - R_{i+1}]dG(x)
\]

D Proofs of propositions, properties and corollaries

D.1 Proof of proposition 1

For \( \gamma = 0 \), the differentiation of (3) with respect to \( \epsilon \), implies that \( J_i'(\epsilon) = 1 \) \( \forall i \). Since \( J_i(R_i) = 0 \), the value of a filled job verifies \( J_i(\epsilon) = \epsilon - R_i \). The equation (2) can be written as:

\[
\frac{c}{q(\theta_i)} = \beta(1 - R_{i+1})
\]

This gives immediately \( JC_{PartialEq} \). Since by integrating by parts \( \int_{R_{i+1}}^{1} J_{i+1}(x)dG(x) = \int_{R_{i+1}}^{1} J_{i+1}(x)[1-G(x)]dx = \int_{R_{i+1}}^{1} [1-G(x)]dx \) and \( V_i = 0 \) \( \forall i \) (free entry condition), it is straightforward to see that the equation (4) verifies \( JD_{PartialEq} \).
D.2 Proof of property 1

The proof is straightforward. Making use of \((JD_{\text{PartialEq}})\), we indeed obtain:

\[
R_{T-1} = b \\
R_{T-2} = b - \beta \int_{R_{T-1}}^{1} [1 - G(x)]dx \leq R_{T-1} \\
R_{T-3} = b - \beta \int_{R_{T-2}}^{1} [1 - G(x)]dx \leq R_{T-2} \\
\ldots
\]

As can be seen in \((JC_{\text{PartialEq}})\), \(\theta_i\) depends negatively on \(R_{i+1}\), and it turns out that \(\theta_{i+1} \leq \theta_i \forall i\).

D.3 Proof of proposition 2

By differentiating (3) with respect to \(\epsilon\), it emerges that \(J'_i(\epsilon) = 1 - \gamma \forall i\). Since \(J_i(R_i) = 0\), the value of a filled job verifies \(J_i(\epsilon) = (1 - \gamma)(\epsilon - R_i)\). The equation (2) can then be written in order to determine the sequence of \(\theta_i\) as an expression of \(R_i\) (equation \((JC)\)). Moreover, by combining the wage equation (8) and the equation (4), and integrating by parts as in proof of proposition 1, one gets the equation \((JD)\) describing the age profile of \(R_i\).

D.4 Proof of property 2

If \(\frac{dR_i}{dR_{i+1}} \geq 0\) [condition (C1)] and \(R_i \leq b\) [condition (C2)] the solution to the dynamical equation (9) necessarily verifies \(R_{i+1} \geq R_i\). Given the definition of \((JC)\) and \(q'(\theta_i) \leq 0\), it then comes that \(\theta_{i+1} \leq \theta_i\).

Making use of (9), we have:

\[
\frac{dR_i}{dR_{i+1}} \geq 0 \iff 1 \geq \left( \frac{\gamma}{1 - \psi} \right) \left( \frac{\beta(1 - \gamma)}{c} \right)^{\frac{\psi}{\epsilon}} (1 - R_{i+1})^{\frac{2\psi - 1}{1 - \psi}} \quad (36)
\]

\[
R_i - b \leq 0 \iff 1 \geq 2\gamma \left[ \frac{\beta(1 - \gamma)}{c} \right]^{\frac{\psi}{\epsilon}} (1 - R_{i+1})^{\frac{2\psi - 1}{1 - \psi}} \quad (37)
\]

If \(\psi \geq 1/2\), evaluating (36) for \(R_{i+1} = 0\) is sufficient to insure that both conditions (C1) and (C2) hold simultaneously. On the contrary, if \(\psi \leq 1/2\), evaluating (37) for \(\max\{R_i\} = b\) implies that (C1) and (C2) hold.
D.5 Proof of proposition 3

Similar to proof of proposition 2, but by making in addition use of the fact that:

\[ e_i = \beta p(\theta_i) [W_i(1) - U_i] \]
\[ = \frac{\gamma}{1 - \gamma} \delta \theta_i \quad (38) \]

Since the wage solves in that case the equation (20) for \( \phi(b, e_i) = b - \frac{n^2}{2} \), combine this wage equation with (38) and plug into (4) we get \( JD_{eff} \). Proof of \( JD_{eff} \) is straightforward.

D.6 Proof of property 3

Following the same procedure as for the proof of Property 2, we now find that conditions (C1) and (C2) are (from Proposition 3):

\[ \frac{dR_i}{dR_i + 1} \geq 0 \iff 1 \geq \beta \gamma^2 \left( \frac{\beta(1 - \gamma)}{c} \right) \frac{2^\psi}{1 - \psi} (1 - R_i) \frac{2^\psi}{1 - \psi} \quad (39) \]
\[ R_i - b \leq 0 \iff 1 \geq \beta \gamma^2 \left( \frac{\beta(1 - \gamma)}{c} \right) \frac{2^\psi}{1 - \psi} (1 - R_i) \frac{2^\psi}{1 - \psi} \quad (40) \]

It is thus obvious that the condition (39) is more stringent than condition (40) whatever the value of \( \psi \). Evaluating the former condition with \( R_i = 0 \), one gets the sufficient condition reported in the main text.

D.7 Proof of property 4

Let us denote \( \Psi(R_{i+1}, \theta_i) = \frac{G(R_{i+1})}{G(R_{i+1}) + p(\theta_i)} \). By definition, \( \frac{\partial \Psi(R_{i+1}, \theta_i)}{\partial R_{i+1}} > 0 \) and \( \frac{\partial \Psi(R_{i+1}, \theta_i)}{\partial \theta_i} < 0 \). For \( \theta_{i+1} \leq \theta_i \) and \( R_{i+1} \geq R_i \) (property 2), it thus appears that \( \Psi(R_{i+1}, \theta_i) \leq \Psi(R_{i+2}, \theta_{i+1}) \) \( \forall i \).

Let us first reason by contradiction by assuming \( u_1 < \Psi(R_2, \theta_1) \). Since \( \Psi(R_{i+1}, \theta_i) \leq \Psi(R_{i+2}, \theta_{i+1}) \) this necessarily implies \( u_2 < u_1 \). In turn since \( \Psi(R_2, \theta_1) < \Psi(R_3, \theta_2) \), we have \( u_3 < u_2 \), and \( \Psi(R_3, \theta_2) < \Psi(R_4, \theta_3) \). This shows that in that case we would have \( n_{i+1} \leq n_i \) \( \forall i \).

On the contrary, by definition \( u_1 = 1 > \Psi(R_2, \theta_1) \), we have that \( n_2 > n_1 \). \( \Psi(R_{i+1}, \theta_i) < \Psi(R_{i+2}, \theta_{i+1}) \) \( \forall i \) then insures that there exists an age \( \bar{T} \) which verifies \( u_{\bar{T}} = \frac{G(R_{\bar{T}+1})}{G(R_{\bar{T}+1}) + p(\theta_{\bar{T}})} \), and so that \( n_{\bar{T}+1} \leq n_{\bar{T}} \).
D.8 Proof of property 5

The equation (8) gives the wage at age \(i\) for a productivity level \(\epsilon\). Then the wage gap by age, at the level of the productivity reservation by age is given by the difference of the equation (8) for age \(i + 1\) and age \(i\), for the productivity \(R_{i+1}, R_i\):

\[
w_{i+1}(R_{i+1}) - w_i(R_i) = \gamma [(R_{i+1} - R_i) + c(\theta_{i+1} - \theta_i)]
\]

If we assume that \(\psi = 1/2\), we have:

\[
w_{i+1}(R_{i+1}) - w_i(R_i) = \frac{c}{1 - \gamma} \left[ 1 - \frac{c}{2\beta(1 - \gamma)} \right] (\theta_{i+1} - \theta_i)
\]

Then \(w_{i+1}(R_{i+1}) - w_i(R_i) > 0\) if \(1 - \frac{c}{2\beta(1 - \gamma)} < 0\) because \(\theta_{i+1} - \theta_i < 0\).

D.9 Proof of proposition 6

Let us denote \(\lambda_i\) and \(\mu_i\) the Lagrange multiplier associated with constraints (15) and (16), optimal decision rules with respect to \(R_{i+1}, \theta_i\) and \(u_i, y_i\) are respectively given by:

\[
\lambda_i = \mu_i R_{i+1}
\]

\[
\beta^i c = (\mu_i - \lambda_i) p'(\theta_i)
\]

\[
\beta^i (e_i + c\theta_i) = p(\theta_i) (\mu_i - \lambda_i)
\]

\[
\lambda_{i-1} = \beta^i \left( b - \frac{1}{2} e_i^2 - c\theta_i e_i \right) + \lambda_i [1 - e_i p(\theta_i) - G(R_{i+1})] + \mu_i \left[ e_i p(\theta_i) - \int_{R_{i+1}}^1 c dG(\epsilon) \right]
\]

\[
\mu_i = \beta^{i+1}
\]

It is then possible to derive the following expression of the productivity threshold:

\[
R_i = b - \frac{1}{2} e_i^2 - c\theta_i e_i - \beta \int_{R_{i+1}}^1 c dG(\epsilon) + c \frac{e_i p(\theta_i)}{p'(\theta_i)} + \beta R_{i+1} (1 - G(R_{i+1}))
\]

Let us remark that \(c\theta_i + e_i = cp(\theta_i)/p'(\theta_i)\) implies \(e_i = \epsilon \left[ p(\theta_i)/p'(\theta_i) - \theta_i \right] = c\theta_i \frac{\theta_i}{1 - \eta(\theta_i)} (\text{since } p(\theta_i)/p'(\theta_i) = \frac{\theta_i}{1 - \eta(\theta_i)})\) and \(\beta R_{i+1} (1 - G(R_{i+1})) - \beta \int_{R_{i+1}}^1 c dG(\epsilon) = -\beta \int_{R_{i+1}}^1 (\epsilon - R_{i+1}) dG(\epsilon) = \int_{R_{i+1}}^1 [1 - G(\epsilon)] d\epsilon\), one gets \((JD^*)\). The derivation of \((JC^*)\) is straightforward from the optimality conditions of the planner’s problem.
D.10 Proof of corollary 6

From proposition 7, we have that the optimal age profile of firings solve for $\beta \rightarrow 1$:

$$F_i - F_{i+1} = \left[ \left( \frac{\gamma}{1 - \gamma} \right)^2 - \left( \frac{1 - \psi}{\psi} \right)^2 \right] \frac{1}{2} (c\theta_i)^2 - z$$

Accordingly, from $(JC^*)$ since $\theta_i \leq \left[ \frac{\psi}{c} \right]^{\frac{1}{1-\psi}} \forall i$, $z \geq \left[ \left( \frac{1 - \psi}{\psi} \right)^2 - \left( \frac{\gamma}{1 - \gamma} \right)^2 \right] \frac{c^2}{2} \left[ \frac{\psi}{c} \right]^{\frac{2}{1-\psi}}$ insures that $F_i > F_{i+1}$. In turn, since for $i \in [1, T - 2]$, $\theta_i \geq \left[ \frac{\psi(1-b)}{c} \right]^{\frac{1}{1-\psi}}$, it emerges that $z \leq \left[ \left( \frac{1 - \psi}{\psi} \right)^2 - \left( \frac{\gamma}{1 - \gamma} \right)^2 \right] \frac{c^2}{2} \left[ \frac{\psi(1-b)}{c} \right]^{\frac{2}{1-\psi}}$ insures that $F_i < F_{i+1}$. 

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