The Wage Distribution Across the Life-Cycle and the Role of Unemployment Benefits: France versus the US

SARAH LE DUIGOU *
GAINS-TEPP (Le Mans University)
sarah.le_duigou.etu@univ-lemans.fr
February 1, 2011

The way wage increases along the life-cycle of workers is not homogenous between France and the US. Indeed the discrepancy between the seniors and the young’s wages is wider in France than in the US at the end of the distribution and wider in the US than in France at the bottom of it. Besides, seniors’employment rate is far lower in France than in the US. We propose to explain these facts thanks to a model based on the Burdett and Mortensen (1998) framework with life-cycle. We consider the difference between France and the US in terms of unemployment benefits. Indeed when French workers enjoy rather high and progressive unemployment benefits, the American workers have rather low and flat benefits. We assume therefore that the high and progressive unemployment benefits induce the selection effect responsible for the facts on employment and wage distributions. We first find that the endogenous productivity, the frictions, and the ”on the job search” assumptions are enough to account for the difference between the young’s and the seniors’wages. Then, by introducing high and progressive unemployment benefits consistent with French labor market, we find that this difference is larger especially for the higher wages and that the seniors’employment rate decreases sharply.

1 Introduction

How does wage distribution evolve with life-cycle? How does aging affect the wage trajectory of workers? It is rather intuitive to think of two main effects that aging can have on workers’

*Address: GAINS-TEPP (FR CNRS: 3126), Université du Maine - Avenue Olivier Messiaen, 72085 Le Mans Cedex 9, France.
wage; first the time spent on the labor market increases the probability for workers to have
found in the past higher paying jobs; second, when the worker becomes senior, he has a shorter
working life horizon and therefore can be offered lower wages. Table (1) \(^1\) displays the ratios of
the seniors’ wage distribution deciles and the youth’s wage distribution deciles in France and in
the US. This ratio is computed for the entire population and for the low skilled workers. This
table shows that even when the seniors and the young are evenly productive -we exclude most of
general human capital accumulation by selecting the low skilled workers- seniors still get higher
wages. Therefore empirically the experience on the labor market effect seems to dominate the
horizon effect. Explaining the wage trajectory of workers during their life-cycle has generated
rather few papers. However, recently Menzio and Telyukova designed a matching model with
deterministic aging, general human capital accumulation and directed search from firms. They
reproduce quite well the hump of the wage trajectory observed in the US thanks to the arbitrage
between the horizon effect, the increase in productivity and the search effect.

Table 1: Ratio of Wage Decile: \( \frac{D_{i}(w_{s})}{D_{i}(w_{j})} \)

<table>
<thead>
<tr>
<th>Deciles</th>
<th>All population</th>
<th>low skilled</th>
<th>higher progression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>US</td>
<td>France</td>
<td>US</td>
</tr>
<tr>
<td>1</td>
<td>1.158</td>
<td>1.070</td>
<td>1.136</td>
</tr>
<tr>
<td>2</td>
<td>1.181</td>
<td>1.172</td>
<td>1.138</td>
</tr>
<tr>
<td>3</td>
<td>1.182</td>
<td>1.187</td>
<td>1.163</td>
</tr>
<tr>
<td>4</td>
<td>1.173</td>
<td>1.185</td>
<td>1.143</td>
</tr>
<tr>
<td>5</td>
<td>1.200</td>
<td>1.208</td>
<td>1.122</td>
</tr>
<tr>
<td>6</td>
<td>1.210</td>
<td>1.231</td>
<td>1.143</td>
</tr>
<tr>
<td>7</td>
<td>1.206</td>
<td>1.264</td>
<td>1.146</td>
</tr>
<tr>
<td>8</td>
<td>1.207</td>
<td>1.306</td>
<td>1.154</td>
</tr>
<tr>
<td>9</td>
<td>1.200</td>
<td>1.306</td>
<td>1.125</td>
</tr>
</tbody>
</table>

Yet, the comparison we can draw between France and the US highlights another stylized fact:
this process is not homogenous according to economies and across the distribution. Figure (1)
displays that the hump at the end of the wage trajectory is for instance not present in the
French economy. More precisely, Table 1 shows that whether the workers are skilled or not,
the wage increase is higher at the bottom of the distribution in the US and for the rest of the

\(^1\) The data used for this table and for table 2 as well as for the graph in appendix are the "enquêtes emplois" for France and the Current Population Survey for the US
distribution in France. At first, and considering the trade-off of Menzio and Telyukova, this can appear paradoxical. French seniors have shorter working lives since they on average retire earlier. The firms should therefore be reluctant to offer them high wages. Firms post high wages because they expect to keep the worker for a long time by avoiding poaching. For seniors close to retirement, this strategy becomes useless.

In several articles from 1996 to 2002, Ljunqvist and Sargent notably stressed the major role of generous unemployment benefits (UB) in accounting the differences in the labor market outcomes between France and the US. In particular, generous unemployment benefits achieve to explain the higher unemployment rate of France compared to the US since they induce a high reservation wage which lengthens the unemployment duration especially among seniors. Can UB explain the disparity, at the end but also at the bottom of the distribution, of wage trajectories between France and the US?

<table>
<thead>
<tr>
<th>Table 2: UB in function of wages (in euros)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>$w &lt; 1080$</td>
</tr>
<tr>
<td>$1080 &lt; w &lt; 1170$</td>
</tr>
<tr>
<td>$1170 &lt; w &lt; abt 1370$</td>
</tr>
<tr>
<td>$abt 1370 &lt; w &lt; 1950$</td>
</tr>
<tr>
<td>$1950 &lt; w &lt; 4770$</td>
</tr>
</tbody>
</table>

In contrast with the US, as the table (1) shows it, in France the UB are rather high and progressive with the wage. According to our findings, this difference can explain the slower
increase in wages of low paid young workers in France and the selection effect which accounts for the larger discrepancies between the young and the seniors’ wages at the end of the distribution. To do so, in this paper, we augment the Burdett and Mortensen framework with life-cycle. We introduce two categories of workers, the young and the seniors. We differentiate the seniors from the young by the length of their working life: a young still have many years ahead to work while a senior is soon to be retired. Their productivity is the same.

How do firms deal with workers of different ages and same productivity on the labor market and how does it affect the wage distribution? We suppose that discrimination is banned, therefore firms are necessarily constraint to offer the same wage to everyone based on an expected intertemporal profit. As the youth starts their working life as unemployed, a large proportion of them accepts every job offer. On the opposite the seniors due to the time they spent on the labor market, are very likely to be employed. When the workers face the same wage offers, the youth will earn most of the low wages, since they are less selective on wage, and the seniors more high wages in proportion. Knowing that, as the youth induce the larger intertemporal profit, firms are incited to keep wage rather low to avoid hiring the senior workers. Firms achieve therefore to discriminate seniors even when they cannot identify them. There exists another pervert effect of considering life-cycle in an economy: when firms cannot discriminate, like in the founding model of Burdett and Mortensen, they face an unobservable heterogeneity which gives them a certain market power and reduce average wage. Indeed, as firms post wages without knowing the age of the workers on the labor market, they undergo a marginal cost of labor superior to the labor supply which gives them a certain market power. Then, when a firm wants to attract an extra worker by increasing its wage offer, the cost of this raise is superior to the increase in wage needed to attract this extra worker. Indeed, firms have to pay the same wage to the youth and the seniors whereas the youth has lower a reservation wage and could have been hired anyway.

Protecting seniors by implementing a no age discrimination policy can represent a cost in terms quality of the jobs created. We will see that the youth are the first victims of this behavior.

How generous and progressive UB affect the wage of workers along the life-cycle? As the UB are indexed on the previous wage of the worker, their implementation at first mostly benefit to the unemployed seniors since senior workers who have had a longer experience on the labor market earn higher wages. Unemployed seniors enjoy therefore high reservation wage which explain that they can only earn high wages now. As Ljunqvist and Sargent showed it, this high reservation wage is also the source of the high unemployment rate of seniors. Indeed, the
reservation wage of senior unemployed workers depends on the productivity the worker had in his or her previous job. The time seniors have spent in their previous job made them develop specific human capital, which is difficult to use in another firm. Considering their age, a new firm cannot expect the same productivity from them anymore. These unemployed seniors earn UB generating a reservation wage above their actual expected productivity. Seniors with high reservation wage can either be hired at a high wage or remain unemployed, it's the selection effect which simultaneously account for the particularly high wages and unemployment rate of French senior workers.

Generous and progressive UB also have an impact on the youth’s wage. As UB are progressive, they raise the return for firms to push wages up since now the supply of labor composed of unemployed workers also raises with wage. The mass of unemployed workers who accept any job offer is dramatically reduced. This raise consequently occurs at the bottom of the distribution and entails an increase in the wages offered in the first part of the distribution. The youth who are those who accept the lowest wages are those who eventually benefit the most from this wage raise. This explains why the increasing path of wages is flatter at the beginning of the distribution in France.

Which economy is the most desirable?

We have seen that the UB can reduce the firms’ market power responsible for an inefficient level of wages in the economy. However, as seniors have higher reservation wages thanks to the UB, it is also easier now for the firms to avoid hiring them by posting lower wage.

The global effect of this institution on the level of wage is therefore ambiguous. Our results show that the positive effect more than compensates the negative one and that wages increase. Firms create therefore on average more quality jobs by investing more in human capital at job creation date. The effect of this institution on the global output and on the welfare is not straightforward neither, since this policy also increases significantly the number of unemployed workers who in addition need to be financed. Yet, according to our simulations, the economy with UB has higher an output and better a welfare.

At last, as the market power of firms is strengthened by the fact that age heterogeneity is not observable from firms, this can lead us to question the relevance of age discrimination ban policy. To assess the loss of efficiency that the discrimination ban induces, we can compute the equilibrium wage distributions of the youth and the seniors when firms can discriminate the workers according to their age. When firms can discriminate, simulations show that the
youth get the higher wages on average since seniors are discriminated due to their short working horizon. Yet, the decisions of firms are more efficient and the average level of wages, the quality of the jobs, and the welfare are far higher than in the other economies.

The next step in this work is to introduce the UB in the model with directed search and to assess their effects on the wage distribution of the youth and of the seniors and in terms of welfare.

The second section of this paper is dedicated to the review of the literature about wage distribution. The third section presents the theoretical models to reproduce the US wage distribution and the French wage distribution. The fourth section analyzes these two economies in term of welfare. Eventually, the fifth section concludes.

2 Literature

Explaining the wage distribution generated a large number of papers. According to the recent literature, there are two main approaches to do so. First Butters (1977), Burdett and Judd (1983), Mortensen (1990), and Burdett and Mortensen (1998) showed that there exists wage dispersion even though we assume firms and workers ex-ante homogenous. Indeed, in their seminal article of 1998, Burdett and Mortensen achieve to explain this puzzle by assuming that workers can search for a job while already employed and "undirected search" from firms. These assumptions, in a frictional labor market, lead to pure wage dispersion. Indeed if employees can search "on the job", as they will not accept a job unless its wage is superior to their current one, the labor supply increases with wage. Consequently, as in a frictional labor market, the hiring process is costly in term of time at least, in a non-cooperative game, firms will set different wages. This dispersion is fueled by an arbitrage between high immediate profit, and both quick hiring process and high retention of workers. In this framework, "on the job search" assumption can be regarded as an unobservable heterogeneity of workers which does not affect their productivity and which relies only on the working history of each worker. Workers are ex-ante identical, but ex-post they earn different wages. Notice in this framework that the imperfections which are the frictions and the imperfect information on the status of the workers lead to an inefficient equilibrium. As firms post wages without knowing the status of the workers on the labor market, they undergo a marginal cost of labor superior to the labor supply which gives them a certain market power. Indeed when a firm wants to attract an extra worker by increasing its wage offer, the cost of this raise is superior to the increase in wage needed to attract this extra
worker. Indeed, it is possible that the worker the firm meets and pays this augmented wage is an unemployed worker, who would have accepted any wages above minimum wage. One of the consequences of these imperfections is that firms are incited to keep wages low.

Yet, this only source of dispersion is not sufficient to account for the wage distribution. Indeed using this method only, we face a wage spreading going from once to twice the minimum wage and a high wages over-estimation with a strictly growing wage density. Second, using the same framework, workers and firms’ heterogeneity can fill out the first approach. Mortensen (1990) assumes for instance a discrete heterogeneity in the workers reservation wage. We can also assume that these reservation wages are Pareto distributed. Yet we keep on facing the same problem. It is possible to assume that productivities are Pareto distributed, in that case the theoretical model explains quite well the actual wage distribution trends but underestimate the spreading of wages and sounds a bit artificial.

Mortensen (2000) completes the initial framework of Burdett and Mortensen (1998) with ex-ante homogenous workers and firms by allowing firms to invest in capital on each job: firms are still ex-ante identical but have now ex-post different productivity. He therefore endogenously generates a firms’ productivity distribution. As Acemoglu and Shimer (1999) discussed it, it is that way possible to design simultaneously a wage and a productivity distribution since they are both the results of firms’ optimal decision. Therefore so far, literature achieves to explain wage dispersion to a certain extent even when we assume ex-ante identical workers and firms. This paper totally belongs to this literature.

Workers are ex-ante identical in the way that all the workers are the same type. Yet there exists an heterogeneity in terms of working horizon in the sectional population which change firms’ and workers’ behaviors. Life-cycle has never been introduced in such framework.

3 Theory

There are two age groups in the labor force, the youth $m_j$ and the seniors $m_s$. The retired workers are noted $m_R$. The youth become senior with the probability $p$, the seniors retire with the probability $\delta$, and the retirees die with the probability $\pi$. Each time an employer hires a worker, he or she can invest in human capital to increase the productivity of the match. In this section, firms cannot discriminate workers, we assume undirected search from the firms, i.e. they only choose the wage they post, but cannot decide to hire a young or a senior worker.
In order to maximize the profit of the firm, the employer choose simultaneously the couple of variables wage, capital. We distinguish four exogenous arrival rates of wage offers, $\lambda^j_0$ and $\lambda^s_0$ for respectively the young and senior unemployed, and $\lambda^j$ and $\lambda^s$ for respectively the young and the senior employed. Each job can be destroyed due to an exogenous event which occurs with the probability $s$.

We normalize the total population $(m_j + m_s + m_R = 1)$ and as at steady state, 

$$p.m_j = \delta.m_s = \pi.m_R$$  \hspace{1cm} (1)

we can deduce the population of each category at the equilibrium:

$$m_s = \frac{1}{\left(\frac{\delta}{s} + \frac{\delta}{p} + 1\right)}$$

$$m_j = \frac{1}{\left(\frac{\delta}{s} + \frac{\delta}{p} + 1\right)} \cdot \frac{\delta}{p}$$  \hspace{1cm} (2)

$$m_R = \frac{1}{\left(\frac{\delta}{s} + \frac{\delta}{p} + 1\right)} \cdot \frac{\delta}{\pi}$$

### 3.1 Stylized facts for the US

To reproduce the stylized facts for the US, we assume the UB are so low that they induce no job rejection from unemployed workers considering the minimum wage we chose.

#### 3.1.1 Wage density

Let $G_j(w)$ (resp. $G_s(w)$) be the cumulative distribution function of the wages earned by the youth (resp. the seniors) workers. We denote the young’s unemployment by $u_j$ and the seniors’ unemployment by $u_s$. $(m_j - u_j)G_j(w)$ and $(m_s - u_s)G_s(w)$ represent therefore the number of youth, resp. of seniors employed at a wage less or equal to $w$. These populations depend on the cumulative distribution function of the wages offered by the firms $F(w)$ - which is addressed just as well for the youth as for the seniors - as the following flows equations show it:

$$(s + p + \lambda^j(1 - F(w)))(m_j - u_j)G_j(w) = \lambda^j_0F(w)u_j$$

$$(s + \delta + \lambda^s(1 - F(w)))(m_s - u_s)G_s(w) = \lambda^s_0F(w)u_s + p(m_j - u_j)G_j(w)$$  \hspace{1cm} (3)

On the left side of these equations, there is the flow of young and senior workers leaving the employers offering a wage less or equal to $w$, by experiencing an exogenous shock, changing group of workers - from youth to seniors or from seniors to retirees- or resigning after having been poached by another firm. On the right side there is the flow of workers who become a
young or a senior employed worker with a wage less or equal to \( w \). The first equation suggests that the youth begin their working life as unemployed. At that moment, they accept any wage offer. The second one which includes in the "flows in" a part of young employed workers suggests that part of the seniors are already employed when they become seniors. They therefore have a certain reservation wage which make them reject the lower wages.

Because of "on the job search", senior workers have unambiguously higher reservation wages and higher wages than the young.

### 3.1.2 Unemployed workers’ distribution

By evaluating equations (17) in \( w \), we can deduce the unemployment rate of the youth and the seniors:

\[
\begin{align*}
u_j &= \frac{(p + s)m_j}{\lambda^j + p + s} \\
u_s &= \frac{(\delta + s)m_s - p(m_j - u_j)}{\lambda^s + \delta + s}
\end{align*}
\]  

Let’s assume for the sake of simplicity of the analysis, that the contact probabilities are the same for all the workers and that the youth’s life span is the same as the seniors’ life span. In that case, the unemployment rate of the youth is higher than the unemployment rate of the seniors. This difference comes from the queue phenomenon, i.e. the senior newcomers are partly already employed, while those who are young are all unemployed.

### 3.1.3 Labor Demand

As in Mortensen (2000), each employer commits to both the wage offered and the extent of its match specific human capital investment. Let \( r \) be the real interest rate, the expected present value of the employer’s future flows of quasi-rents once a young and a senior worker is hired at wage \( w \) is respectively:

\[
\begin{align*}
J_j(w, k) &= \frac{y(k) - w + pJ_s(w, k)}{r + p + s + \lambda^j(1 - F(w))} \\
J_s(w, k) &= \frac{y(k) - w}{r + \delta + s + \lambda^s(1 - F(w))}
\end{align*}
\]  

where \( k \) is the match-specific investment per worker that the firm incurs to obtain the productivity \( y(k) \), which is an increasing and concave function of this investment as it follows:

\[
y(k) = y + \left( \frac{q}{\alpha} \right) k^\alpha
\]
y, q and α are strictly positive exogenous parameters. By making the same assumption as in 3.1.2, it is straightforward according to 5 that for a positive p the present value of employing a young worker is higher than the present value of employing a senior. Indeed if the job has not been destroyed before the firm which employs a young who become a senior keeps its job filled, whereas the firm which employs a senior who retire loose its worker. As the employers cannot direct their search, the ex-ante value associated to a job is given by this expected profit:

$$\Pi(w, k) = \max_{w > w, k > 0} \{h_j(w)(J_j(w, k) - k) + h_s(w)(J_s(w, k) - k)\}$$

(7)

where \(h_j\) and \(h_s\) represent the probability that a wage offer \(w\) is respectively accepted by a young and a senior worker. These probabilities are strictly increasing in wage as it follows:

\[
\begin{align*}
  h_j(w) &= \lambda_j^0 u_j + \lambda_j^1 (m_j - u_j)G_j(w) \\
  h_s(w) &= \lambda_s^0 u_s + \lambda_s^1 (m_s - u_s)G_s(w)
\end{align*}
\]

(8)

We consider a minimum wage \(w\) which bounds below the wage distribution and gives the level of iso-profit. The cumulative distribution function of the wages offered by firms \(F(w)\) is such that any wage proposal yields the exact same profit to firms. The arbitrage which fuels this wage offered distribution is that the probability for the employers that an offer is accepted and the retention of the employed workers increase with the wage posted, while the gross margin - the term \((y(k) - w)\) - decreases with it. As workers retention increases with wage, firms offering higher wages also propose higher amount of investment in human capital. This investment can be discounted in a longer run.

The minimum wage allows firms to hire the entire unemployed population. An increase in the wage offer allows firms to speed up their hiring process thanks to the poaching possibility, how fast the hiring is depends on the cumulative distributions of wages.

As the cumulative distribution is composed of fewer high wages and more low wages for the youth than for the seniors, the strategy of wage increase is mostly meant to poach the seniors. Yet, if the firm increases its wage offer, whoever the firm meets - a young or a senior, an employed or an unemployed -, it has to pay the same increased wage. Therefore, the marginal cost of labor is superior to the labor supply which gives the firms a certain market power. The consequence of this market power is that firms are incited to keep wages - and the investment in human capital - low.

Besides, the composition of wage distribution of the youth and the seniors also supposes that the probability to hire a young is higher when the firm offers a low wage. As the profit associated to
hiring a young worker is higher due to the longer working life horizon than the profit associated to hiring a senior, the firms are incited to offer lower wages when life-cycle is taken into account. Firms achieve therefore to discriminate senior workers even when they cannot identify them and, considering the wage density of the young and of the seniors, posting a low wage is a way to select the young workers.

To conclude, the only assumption of the difference in the working life horizon between the youth and the seniors is enough to account for the higher wages of the seniors. Yet, this wage trajectory can be affected by institutions on the labor market in economies such as the French economy. This is the issue tackled in the next section.

### 3.2 Stylized facts for France

To be consistent with the French economy we now introduce the UB like in Chéron and Langot (2009). The unemployment benefits denoted by $b$ are proportional to the past wage of the worker such as:

$$b = \rho w_{-1} + all$$ (9)

where $w_{-1}$ stands for the former wage and $all$ is a lump-sum transfer.

Unemployed workers are now heterogenous in term of income. We denote the young’s unemployment income density and cumulative function by $u_j(R_j(b))$ and $U_j(R_j(b))$ and the seniors’ unemployment income density and cumulative function by $u_s(R_s(b))$ and $U_s(R_s(b))$ with $R_j(b)$ and $R_s(b)$ the reservation wage of the young and senior unemployed workers associated to $b$. Given their unemployment benefits, unemployed workers can therefore accept or refuse wage offers.

#### 3.2.1 Reservation wage

$$rU_j(b(w)) = \left(\frac{b(w) + \tau}{1 - \sigma}\right)^{1-\sigma} + \xi \int_{R_j}^{\infty} (V_j(x) - U_j(b(w))) dF(x) - p(U_j(b(w)) - U_s(b(w)))$$

$$rU_s(b(w)) = \left(\frac{b(w) + \tau}{1 - \sigma}\right)^{1-\sigma} + \lambda \int_{R_s}^{\infty} (V_s(x) - U_s(b(w))) dF(x) - \delta(U_s(b(w)) - V_R(w))$$ (10)

With $\tau$ the tax which finances the unemployment benefits. The value $V_R(w)$ is the value of
being retired with the last wage earned on the labour market $w$. This value is:

$$ V_R(w) = \left( \frac{\gamma w + \tau}{\pi + r} \right)^{1-\sigma} \tag{11} $$

### 3.2.2 Wage density

The distribution of wages in the economy now depends on the unemployment income distribution since now the probability to go out of unemployment with a given wage depends on the number of unemployed workers who have a reservation wage below the given wage:

$$(p + s + \lambda'(1 - F(w)))(m_j - u_j)g_j(w) = \lambda'_j \int_{-\infty}^{w} (F(w) - F(x))u_j(x)dx$$

$$(\delta + s + \lambda'(1 - F(w)))(m_s - u_s)g_s(w) = \lambda'_s \int_{-\infty}^{w} (F(w) - F(x))u_s(x)dx + p(m_j - u_j)g_j(w) \tag{12}$$

On the right side, the flow of workers hired with a wage less or equal to $w$ increase with the wage.

Here, the main difference with the previous section is that the number of unemployed workers that the firm can hire now depend on the level of this wage due to the function $u$. To say it differently, a wage offer to an unemployed worker is not enough to create a filled job, the unemployed worker needs now to agree with that wage offer, i.e. to have a reservation wage below it. The higher the wage offer is, the more likely the unemployed workers agree with it. As a result the number of workers earning high wage increases compared to the case without UB while the number of workers earning low wages decrease compared to the the case without UB.

On average, the wages of the young and of the seniors increase. Yet, we can wonder wether the gap between the seniors’wages and the young’s wage gets wider or not, to answer this question, we need to look closer at the unemployed workers’distribution.

### 3.2.3 Unemployed workers’distribution

The unemployment income distribution depends now on the offered wage density since an unemployed worker is hired only if the wage offered is above his or her reservation wage $R(\rho w + all)$. When $w = \underline{w}$, $u_j(R_j(\rho w + all))$ is such as:

$$ pm_j + s(m_j - u_j)g_j(w) = \lambda'_j u_j(R_j(\rho w + all))(1 - F(R_j(w))) + pu_j(R_j(\rho w + all)) \tag{13} $$

For any other values of $w$, $u_j(R_j(\rho w + all))$ is such as:

$$ s(m_j - u_j)g_j(w) = \lambda'_j u_j(R_j(\rho w + all))(1 - F(R_j(w))) + pu_j(R_j(\rho w + all)) \tag{14} $$
For all values of $w$, $u_s(R_s(\rho w + all))$ is such as:

$$s(m_s - u_s)g_s(w) = \lambda_0 u_s(R_s(\rho w + all))(1 - F(R_s(w))) + \delta u_s(R_s(\rho w + all))$$  \hspace{1cm} (15)

The unemployment rates are the sum of the densities of unemployed workers for all the unemployment incomes. These equations highlight that the unemployed young who begin their working life are uninsured while the unemployed seniors who start their senior working life are entitled to unemployment benefits thanks to the wage they earned in their previous job. Consequently, the senior unemployed workers have on average higher unemployed benefits and therefore higher reservation wages than the young unemployed workers. As a result, $U_s$ is higher for high wages than $U_j$ and the gap between the young’s wages and the seniors’s wages tends to get wider than without unemployment benefit. To say it differently, the time senior workers spent working during the first part of their life necessarily affects positively their unemployment benefits and their reservation wages once they become unemployed, and they will thus accept only rather high wages.

### 3.2.4 Labor demand

The same way as for the unemployment income distribution, the probability that a job with wage offer $w$ is accepted by a young and a senior worker has to take into account the fact an unemployed worker will not accept a wage offer unless it is above his or her reservation wage. These two probabilities are now respectively:

$$h_j(w) = \lambda_0 \int_w^\infty u_j(x)dx + \lambda^j (m_j - u_j)G_j(w)$$

$$h_s(w) = \lambda_0 \int_w^\infty u_s(x)dx + \lambda^s (m_s - u_s)G_s(w)$$  \hspace{1cm} (16)

The probability for firms to hire a worker increase now more with the wage than previously especially at the beginning of the distribution, since the workers - unemployed- who gets a higher reservation wage because of the UB are those who in the US economy accepted any wage offer. The elasticity of labor supply to wage increases for the lowest wages because of the UB and this incites firms to increase their wage offers. As the youth are those who accept the lowest wages, they are those who eventually benefit the most from this wage raise. Next subsection illustrates these results and shows that the UB allow us to explain the main differences between the wage trajectory in France and in the US.
3.3 Numerical results

The simulations we have carried out, are only numerical exercises, therefore no calibration has been really established yet. However, the parameters from first chart are set accordingly to Chéron and Langot (2009) and in order to get reasonable value of unemployment rate for France and the US \(^2\). To simplify the interpretation, we so far consider that the probability contact are the same, and that \(\gamma\) is such that a senior refuses the same level of wage for a given unemployment benefit as a young.

<table>
<thead>
<tr>
<th>Table 3: Simulation parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma)</td>
</tr>
<tr>
<td>0.04</td>
</tr>
<tr>
<td>(\lambda^j_0)</td>
</tr>
<tr>
<td>1.13</td>
</tr>
</tbody>
</table>

Simulations confirm that senior workers get higher wages than the young workers. Indexed UB increase workers reservation wage and the interest for firms to increase their wage offer, especially at the bottom of the distribution. The young workers’ low wages are higher and their progression over the life-cycle is slighter. The higher the UB and the bigger the

\(^2\)Considering values for \(p\), \(\delta\) and \(\pi\), real unemployment rate corresponds to the one calculated here multiplied by \(\frac{3}{2}\)
Table 4: Simulation for French Economy

<table>
<thead>
<tr>
<th>all</th>
<th>$\rho$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.4</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Figure 3: French Case

Table 5: Results

<table>
<thead>
<tr>
<th>Quintiles</th>
<th>France</th>
<th>US</th>
<th>higher progression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0541</td>
<td>1.0969</td>
<td>US 4.28%</td>
</tr>
<tr>
<td>2</td>
<td>1.0513</td>
<td>1.027</td>
<td>France 2.43%</td>
</tr>
<tr>
<td>3</td>
<td>1.075</td>
<td>1.0257</td>
<td>France 4.93%</td>
</tr>
<tr>
<td>4</td>
<td>1.0952</td>
<td>1.0488</td>
<td>France 4.64%</td>
</tr>
<tr>
<td>5</td>
<td>1.1111</td>
<td>1.0698</td>
<td>France 4.13%</td>
</tr>
<tr>
<td>6</td>
<td>1.102</td>
<td>1.0852</td>
<td>France 1.68%</td>
</tr>
<tr>
<td>7</td>
<td>1.1321</td>
<td>1.077</td>
<td>France 5.51%</td>
</tr>
<tr>
<td>8</td>
<td>1.1</td>
<td>1.0517</td>
<td>France 4.83%</td>
</tr>
<tr>
<td>9</td>
<td>1.0746</td>
<td>1.0303</td>
<td>France 4.43%</td>
</tr>
<tr>
<td>UR young</td>
<td>4.14%</td>
<td>3.2%</td>
<td></td>
</tr>
<tr>
<td>UR senior</td>
<td>5.11%</td>
<td>1.99%</td>
<td></td>
</tr>
</tbody>
</table>
loss of specific human capital, the higher the unemployment rate: we observe it mostly among seniors. Table (5) displays the ratio computed thanks to our simulation of French economy wage distribution and the US economy wage distribution. We find as in the stylized facts that the wage increase is higher at the bottom of the distribution in the US and for the rest of the distribution in France.

4 Welfare

We have seen that the UB can reduce the firms’ market power responsible for an inefficiently low level of wages in the economy. However, as seniors have higher reservation wages thanks to the UB, it is also easier now for the firms to avoid hiring them by posting lower wage.

The global effect of this institution on the level of wage is therefore ambiguous. Our results in the last two column of table (6) show that the positive effect more than compensates the negative one and that wages increase. Firms create therefore on average more quality jobs by investing more in human capital at job creation date. The effect of this institution on the global output and on the welfare is not straightforward neither, since this policy also increases significantly the number of unemployed workers who in addition need to be financed. Yet, according to our simulations, the economy with UB has higher an output and better a welfare.

At last, as the market power of firms is strengthened by the fact that age heterogeneity is not observable from firms, this can lead us to question the relevance of age discrimination ban policy. To assess the loss of efficiency that the discrimination ban induces, we can compute the equilibrium wage distributions of the youth and the seniors when firms can discriminate the workers according to their age. The presentation of the model with directed search is on appendix A. The results of this equilibrium is displayed in the first column of table (6). When firms can discriminate, simulations show that the youth get the higher wages on average since seniors are discriminated due to their short working horizon. Yet, the decisions of firms are more efficient and the average level of wages, the quality of the jobs, and the welfare are far higher than in the other economies.

To conclude on this section, we observe that:

1. When firms are allowed to discriminate, they offer **higher wages to young workers and create rather quality jobs**. Yet there is a rather important inequality among workers which **penalizes seniors**.
Table 6: Results of the three economies

<table>
<thead>
<tr>
<th></th>
<th>L-C</th>
<th>UB (France)</th>
<th>Undir.S (US)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Young</td>
<td>Seniors</td>
</tr>
<tr>
<td>Average</td>
<td>3.664</td>
<td>3.942</td>
<td>3.397</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(93%)</td>
<td>(108%)</td>
</tr>
<tr>
<td>Gross</td>
<td>2.252</td>
<td>1.187</td>
<td>1.062</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(48%)</td>
<td>(52%)</td>
</tr>
<tr>
<td>Net</td>
<td>2.103</td>
<td>1.062</td>
<td>1.040</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(48%)</td>
<td>(52%)</td>
</tr>
<tr>
<td>UB Cost</td>
<td>0.010</td>
<td>0.006</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(40%)</td>
<td>(60%)</td>
</tr>
<tr>
<td>MeanWage</td>
<td>1.640</td>
<td>1.725</td>
<td>1.557</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(49%)</td>
<td>(60%)</td>
</tr>
<tr>
<td>Efficiency</td>
<td>-1.014</td>
<td>-1.387</td>
<td>-1.480</td>
</tr>
</tbody>
</table>

2. When discrimination is banned, **senior workers have slightly higher wages** than the young (and than the senior of 1) but the **wages on average are lower** since firms discriminate the seniors by offering lower wages.

3. The UB incite firms to offer **higher wages on average** and to create higher quality jobs. The less paid young and the best paid seniors have higher wages. Yet, the **seniors’ unemployment (and total too) become higher**.

5 Conclusion

The way wage increases along the life-cycle of workers is not homogenous between France and the US. Taking into account the particularly generous and progressive UB implementing in France allows us to explain the main features of these two wage trajectories. Indeed, UB does not affect workers’ wage the same way if the worker is young or senior. Their effects on the wage distribution are also quite complex since more than a mere translation, they change significantly the shape of the distribution of wage of the young and of the senior workers. Introducing an heterogeneity in term of life-cycle also has for a consequence to reduce the average wage due to an increase in the firm’s market power. The UB are often accused of causing pervert effects on the labor market such as the persistent high unemployment rate in France. Here we observe that they also incite firms to offer higher wages and therefore allow them to create higher quality jobs. Eventually the presence of UB improve the welfare in the economy. Yet, let’s note that the welfare can be far higher if we allow firms to discriminate workers. In that case, average wages are higher but senior workers are discriminated and earn on average lower wages than the young workers. The next step in this work is to introduce the UB in the model with directed search and to assess their effects on the wage distribution of the youth and of the seniors and
in terms of welfare.

A Directed search from firms

We consider that the firms can discriminate workers, we assume directed search from the firms. They now choose simultaneously the set of variables wage, population targeted, capital which maximizes its profit.

A.1 Wage density

The cumulative distribution function of the wages of the young and senior workers now depend on two cumulative distribution functions of the wages offered, the one addressed to young workers $F_j(w)$ and the one addressed to senior workers $F_s(w)$, as it follows:

\[
(p + s + \lambda_j(1 - F_j(w)))(m_j - u_j)G_j(w) = \lambda_j F_j(w)u_j
\]

\[
(\delta + s + \lambda_s(1 - F_s(w)))(m_s - u_s)G_s(w) = \lambda_s F_s(w)u_s + p(m_j - u_j)G_j(w)
\]

The youth can only be hired by employers targeting the youth, and the seniors by employers targeting the seniors. Yet a employer which targets only young workers is exposed to employ senior workers since getting older does not induce the breaking-off of the employment contract.

By differentiating equations (17), we can deduce the number of young and senior workers who earn the wage $w$ in function of $f_j(w)$ and $f_s(w)$, the density of wages offered to the youth and the seniors by firms.

\[
(m_j - u_j)g_j(w) = \frac{\lambda_j f_j(w)u_j + \lambda_j(m_j - u_j)G_j(w)f_j(w)}{p + s + \lambda_j(1 - F_j(w))}
\]

\[
(m_s - u_s)g_s(w) = \frac{\lambda_s f_s(w)u_s + \lambda_s(m_s - u_s)G_s(w)f_s(w) + p(m_j - u_j)g_j(w)}{\delta + s + \lambda_s(1 - F_s(w))}
\]

If the density of wages offered were similar like in the undirected search case, the seniors would get higher wages since the low paid jobs of the youth have to high a turnover to survive until the senior part of the working life. This effect still exists with directed search, but now the wage distribution of the youth and the seniors also greatly depends on the firms’ behavior -the wages offered to the youth and to the seniors.

A.2 Labor Demand

The difference here lies mostly on the profit of the firms. As there is two category of firms, there are two different profits, the maximization of which leading to two different wage distributions.
and productivity distributions. The ex-ante value associated to a job offered to a young and to a senior is therefore respectively given by:

\[
\Pi_j(w, k) = \max_{w > \tilde{w}, k > 0} \{ h_j(w)(J_j(w, k) - k) \}
\]

\[
\Pi_s(w, k) = \max_{w > \tilde{w}, k > 0} \{ h_s(w)(J_s(w, k) - k) \}
\]

with,

\[
J_j(w, k) = \frac{y(k) - w + pJ_s(w, k)}{r + p + s + \lambda(1 - F_j(w))}
\]

\[
J_s(w, k) = \frac{y(k) - w}{r + \delta + s + \lambda^s(1 - F_s(w))}
\]

The probability of contact remains unchanged. As young workers start as unemployed when seniors as employed, with on the job search, senior workers have higher reservation wage and firms do not offer them low wages. Therefore, the young workers earn most of low wages. Yet the intertemporal profit of hiring a young is higher than hiring a senior. Firms offer more high wages to young workers because they prevent from poaching and avoiding poaching is more profitable when employing a young than a senior. When it comes to compare the wages of the youth and of the seniors in that configuration, we therefore face two effects: the negative effect for the youth because of their low reservation wage and the positive effect for the youth because the retention is more valuable when employing them. Without numerical exercises, it is difficult to know which effect dominates. It is therefore possible that the seniors get lower wages than the youth because firms discriminate them due to their short working horizon. Note that when firms can observe the age of the workers life-cycle does not induce any inefficiencies anymore. The only remaining inefficiency comes from the founding unobservable heterogeneity between employed workers and unemployed workers which already existed in B and M model.

A.3 Numerical Exercise

Simulations confirm this ambiguity, the young getting the lowest and the highest wage. Yet, on average, table (efficiency) shows that the youth get higher wages than the seniors. Protecting the seniors from discrimination and reducing the inefficiency is possible thanks to the UB.
B Young and seniors wages distribution in France and in the US

Figure 4: Discrimination

Figure 5: Wage density of young (16-55) and senior (55-65) French men in 2000
Figure 6: Wage density of young (16-55) and senior (55-65) American men in 2000

References


• G. Menzio, Telyukova, Directed Search over the Life Cycle, Working paper (version 2010)