

# $G$ -Expectation and BSDE driven by $G$ -Brownian Motion

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## Abstract

We present recent developments of a notion of sublinear expectation:  $G$ -expectation, the related  $G$ -normal distribution,  $G$ -Brownian motions and the related stochastic calculus, especially the related problem of BSDEs, and potential applications option pricing to measures of risk in finance under volatility uncertainty.

The notion of sublinear expectation (also called upper expectation, coherent prevision in statistics), of which the well-known relation  $\hat{\mathbb{E}}[X] + \hat{\mathbb{E}}[Y] = \hat{\mathbb{E}}[X + Y]$  in classical probability theory becomes the following sublinear one  $\hat{\mathbb{E}}[X] + \hat{\mathbb{E}}[Y] \leq \hat{\mathbb{E}}[X + Y]$ , is proved to be a basic tool in drift and diffusion coefficient (volatility) uncertainty which is crucial in superhedging, superpricing and measures of risk in finance. This is also the start point a new theory of random and stochastic calculus which gives us a new insight to characterize and calculate various kinds of financial risk. A  $G$ -Brownian motion  $B_t$ ,  $t \geq 0$ , is a path-wise continuous stochastic process defined in a sublinear expectation space (in the place of probability space) with stationary and independent increments. Its quadratic variation process  $\langle B \rangle_t$ ,  $t \geq 0$  is also a continuous process with stationary and independent increments.

We present the recent developments of this new field by introducing a crucial notion of  $G$ -normal distributions corresponding to the well-known normal distributions in classical probability theory. This  $G$ -normal distribution will bring us to a new framework of stochastic calculus of Itô's type through the corresponding  $G$ -Brownian motion. The existence and uniqueness other main properties of SDE and BSDE will be presented. We present a probabilistic interpretation of fully nonlinear parabolic equations by BSDEs coupled with a diffusion process driven by a  $G$ -Brownian motion.

### Main References

(other references can be found herein)

Peng, S. (2006)  $G$ -Expectation,  $G$ -Brownian Motion and Related Stochastic Calculus of Itô's type, (pdf-file available in arXiv:math.PR/0601035v2, to appear in *Proceedings of the 2005 Abel Symposium*, Springer.

Peng, S. (2006) Multi-dimensional  $G$ -Brownian motion and related stochastic calculus under  $G$ -expectation, Preprint, in arXiv:math.PR/0601699 v2.

Peng, S. (2006), Lecture Notes:  $G$ -Brownian Motion and Dynamic Risk Measure under Volatility Uncertainty, in arXiv:0711.2834v1 [math.PR] 19 Nov 2007,

Peng, S. (2007) Law of large numbers and central limit theorem under nonlinear expectations, in arXiv:math.PR/0702358v1.

Peng, S.(2008) A New Central Limit Theorem under Sublinear Expectations, in arXiv:0803.2656v1.

Denis, L., Hu, M. and Peng, S. (2008) Function spaces and capacity related to a Sublinear Expectation: application to  $G$ -Brownian Motion Pathes, in arXiv:0802.1240v1 [math.PR] 9 Feb 2008.