Damage detection by nonlinear interaction of ultrasonic waves

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RESUME

Experimental results have been obtained on thermally damaged thick glass plates. A study of nonlinear acoustic phenomena (from the undamaged to a strongly cracked configuration) has been performed. The cascade process of harmonic excitation, interaction of a high frequency wave with a low frequency one and demodulation of the amplitude modulated ultrasonic wave have been implemented. A clear correlation exists between these nonlinear phenomena and the amount of damage. Preliminary determinations of the coefficient of quadratic nonlinearity for cracked samples have been achieved. Reported realisation of a parametric emitting antenna on damaged material is believed to be the very first.

I. INTRODUCTION

During the last 10 years a true revival of the nonlinear acoustics in solids has happened. A new class of solids, the so-called "mesoscopic materials" [1] having macroscopic inhomogeneities, is concerned. Applications are dealing with the detection of cracks, the monitoring of materials fatigue, the characterization of materials used in micro-electronics. Emerging new tools and innovative methods have been proposed in order to decipher the intricacies of nonlinear effects related to acoustical wave propagation in mesoscopic (such as rocks, microcrystalline metals, ceramics) and damaged materials. Even for a single crack, early detection with nonlinear acoustics tools is sought, because the nonlinear propagation of acoustic waves is highly sensitive to defects. Consequently the coefficient of nonlinearity of a cracked material should be quite high.

In the research described in the present publication we apply the method of harmonics excitation (frequency up-conversion) of acoustic waves for the diagnostics of cracks produced by thermal shock in glass. We also demonstrate efficiency of the frequency mixing. By applying modulated pump wave (with spectral components at $\omega$, $\omega-\Omega$ and $\omega+\Omega$) we were able to observe the processes of frequency down-conversion leading to the excitation of acoustic waves at frequencies $\Omega$ and $2\Omega$. It was possible to follow both frequency up-conversion and frequency down-conversion processes simultaneously using wide-frequency detection system. The configuration using modulated pump wave is well-known in underwater acoustics as a parametric emitting antenna [2]. Very recently parametric antenna has been used to characterize the nonlinear demodulation process in unconsolidated granular materials [3]. To the best of our knowledge the experiments described in the present publication are the first where the principle of parametric emitting antenna is applied to the diagnostics of cracks in solids.

II. DISCUSSION

II.1. Experimental set-up and measurements

The experimental set-up is described on Figure 1. It is the configuration similar to a parametric antenna, where a pump wave is generated by a piezoelectric transducer at 100 kHz with appropriate power electronics. This pump wave can be amplitude modulated by a low frequency wave. The transmitted and reverberated signal is then probed with a wide-band piezoelectric transducer that is properly mounted on the opposite edge of the plate. Rectangular 18 mm thick glass plates have been used for virgin and damaged samples. The plates were damaged through a thermal shock process. After heating them in an oven, they were immersed in cool water in order to produce macroscopic cracks inside the volume. By modifying the parameters of the thermal process (maximum temperature, duration of the treatment, conditions of the water cooling), different spatial configurations of cracks and various degree of damages were achieved.

The received acoustical signal was displayed at the same time on a LeCroy oscilloscope and on a vector signal analyzer. Some recorded LeCroy spectra are shown on Figure 2a-d, for the case of harmonic 100 kHz...
transducer excitation. Two levels of transducer driving voltage were used in order to demonstrate the great differences between damaged and virgin samples in spectral domain. On the selected spectra the first ones (Fig. 2a and Fig. 2b) show up a significant increase of the harmonic rays (frequency-up conversion) when increasing the driving amplitude on the damaged material. For the undamaged plate the amplitudes of the harmonics were almost unchanged when increasing the driving level.

A dramatic increase of the higher harmonics observed on the damaged plate is a direct indication of higher nonlinearity of the plate compared to the virgin one. It is important that such increase of higher harmonics takes place at relatively weak mechanical strain induced by the ultrasonic wave. The strain was determined from the movement of the transducer surface (coupled to glass) by means of laser velocimetry. The maximum speed of surface oscillation was \( v_{\text{max}} = 25 \text{ mm/s} \). Consequently, the largest relative strain \( \varepsilon_{\text{max}} \) that we were able to achieve with the used transducer at the maximum driving voltage was \( \varepsilon_{\text{max}} = v_{\text{max}}/C_0 \approx 4 \times 10^{-6} \) (here \( C_0 = 5800 \text{ m/s} \) is the sound velocity). To obtain the observed strong manifestation of the harmonic generation at that level of ultrasonic excitation, one should expect the nonlinear parameter to be extremely high, in the range of \( 10^2 - 10^3 \), when there is a sufficient influence of the nonlinear term in the stress/strain relationship (see additional explanations in the next section).

For detailed spectral analysis, the absolute amplitudes of the harmonics were measured with the help of the FFT vector signal analyser providing a full 130 dB dynamic range. The data useful for further explanation are given on Table 1. One can observe that the amplitude of the fundamental frequency \( A_1 \) is directly proportional to the transducer driving voltage. For plate \# 8 for example (strongly damaged), the 10 dB voltage increments corresponding to the voltage levels \( U_0 + 20 \text{ dB}, U_0 + 30 \text{ dB} \) and \( U_0 + 40 \text{ dB} \) result in about 10 dB steps in amplitude for \( A_1 \), while in about 20 dB steps for the second harmonic amplitude \( A_2 \). Our measurements indicate that the second harmonic amplitude \( A_2 \) can be approximated as being proportional to the square of the fundamental wave amplitude \( A_1 \) in the sample. Consequently, the increase of the second harmonic amplitude as compared to the amplitude of the fundamental wave provides an opportunity to determine \( \Gamma \), the coefficient of quadratic nonlinearity. It should be emphasized that these measurements are certainly the very first determination of the coefficient of nonlinearity for cracked glass material.

**II.2. Determination of the coefficient of quadratic nonlinearity**

In the phenomenological description of nonlinear acoustic behaviour of homogeneous solids, wave interaction is caused by nonlinear dependence of stress tensor \( \sigma \) on the components of the displacement gradient tensor \( \partial u / \partial x \), where \( u_i \) are the components of mechanical displacement. This nonlinear dependence is due both to nonlinearity of the stress/strain relationship

\[
\sigma = M_0 \varepsilon + M_1 \varepsilon^2 + \ldots
\]

(where \( \varepsilon \) is the mechanical strain and \( M_0 \), \( M_1 \) are the second and third order elastic moduli, respectively) and to nonlinearity of the relationship between the tensors \( \varepsilon \) and \( \partial u / \partial x \), [4]. The latter (so-called kinematic) nonlinearity provides important contribution to total nonlinearity of homogeneous materials but can be neglected in microinhomogeneous materials where the ratio \( M_1/M_0 \) characterizing the nonlinearity of the stress/strain relationship is about \( 10^2 - 10^3 \) times higher than in homogeneous media [1, 5, 6]. Consequently the quadratic nonlinearity of microinhomogeneous materials can be characterized by a parameter \( \Gamma = M_1/M_0 \). However it should be mentioned that in contrast to homogeneous materials, both real and imaginary parts of \( M_1 \) can be important for wave interactions in microinhomogeneous materials. In other words both reactive (elastic) and dissipative (inelastic) nonlinearities of the stress/strain relationship should be taken into account in microinhomogeneous materials in general.

**Figure 2.** FFT spectra corresponding to strongly damaged plate. The non-modulated (a-b) and modulated (c-d) signal spectrum are presented at two different transducer driving level, i.e. at \( U_0 \) and at \( U_0 + 30 \text{ dB} \). \( U_0 \) represents a reference level for the settings of the power electronics which corresponds to transducer driving voltage of 0.5 V p-p. The dB vertical scales are expressed in arbitrary units (u.a.), depending upon the settings of the power amplifier and the used amplitude scale of the oscilloscope.

The formal relationship between the amplitude \( A_1 \) of the fundamental wave (at frequency \( \omega \)) and \( A_2 \) of its second harmonic (at \( 2\omega \)), which follows from our experiment, can be presented as:

\[
A_2 = a(2\omega) \Gamma A_1^2,
\]

where \( a(2\omega) \) is a calibration factor depending mainly on the attenuation at frequency \( 2\omega \) of the second harmonic. This equation enables to calculate the relative coefficient
of nonlinearity by comparing two different objects, i.e., a damaged glass plate (noted D) and a virgin one (noted V), or two damaged plates having different degree of damage. We consider the fundamental and first harmonic amplitudes in damaged plate as \((A_e^D, A_e^D)\) and the same quantities in a virgin plate as \((A_e^V, A_e^V)\). When these amplitudes are expressed on a logarithmic scale (for instance in dB), equation (1) yields:

\[
20 \log \left( \frac{\Gamma^D}{\Gamma^V} \right) = (A_e^D - 2A_e^D) - (A_e^V - 2A_e^V) - 20 \log \left( \frac{\alpha_2^D}{\alpha_2^V} \right). \tag{2}
\]

The ratio \((\alpha_e^D/\alpha_e^V)\) is different in general at second harmonic \((2\omega)\) and at the fundamental frequency \((\omega)\). However it is well-known that in a wide-class of microinhomogeneous materials with soft mechanical elements such as rocks and sands [7,8] the experimentally observed attenuation is proportional to frequency in a very wide spectrum band. That is why for our present estimates we assume that

\[
\frac{\alpha_2^D}{\alpha_2^V} \equiv \frac{\alpha_2^D}{\alpha_2^V}.
\]

The latter ratio is estimated from the measured amplitudes of the fundamental wave in different samples but for the same settings of the power electronics, and finally can be written out as

\[
20 \log \left( \frac{\alpha_2^D}{\alpha_2^V} \right) \equiv A_e^V - A_e^D.
\]

It has been checked that all frequencies of interest in the experiment were sufficiently far from sharp resonances of the plates by recording the resonance spectra for each plate. As a consequence, the designed experimental system can be considered as being non resonant and the contribution of the changes in harmonic amplitudes due to crack-induced variations in linear elasticity can be neglected. Accordingly from equation (1), one finally obtain the estimation of the coefficient of quadratic non linearity in the form:

\[
20 \log \left( \frac{\Gamma^D}{\Gamma^V} \right) = (A_e^D - 2A_e^D) - (A_e^V - 2A_e^V).
\]

where all the above spectral amplitudes (both on fundamental and second harmonic waves) are expressed in dB. Consequently, due to the differential nature of the technique, one gains access to the coefficient of nonlinearity of the damaged plate compared to the virgin one. The same procedure is evidently available to determine the ratio of the coefficients of nonlinearity for two plates having different level of damages, i.e. to rank their degree of damages.

Equation (3) has been used to evaluate three glass plates described on Table 1. With these data one obtains the ratio of the coefficient of nonlinearity between any two of the three plates. For example, for the virgin (No.9) and strongly damaged (No.8) plates,

\[
20 \log \left( \frac{\Gamma^V}{\Gamma^D} \right) = 50 \text{ dB} \Rightarrow \Gamma^V = 300 \Gamma^D
\]

The third slightly damaged plate exhibits a \(\Gamma\) coefficient that is 50 times greater than \(\Gamma\) of the undamaged one. The evaluation of \(\log (\Gamma^V/\Gamma^D)\) was almost independent on the driving voltage applied to the pump transducer within 20 dB range from \(U_o + 20 \text{ dB} \) to \(U_o + 40 \text{ dB}\). The values of \(\Gamma\) for glass being differently damaged did show up a clear correlation between the coefficient of nonlinearity and the amount of experienced damage. More damage leads to higher nonlinearity.

<table>
<thead>
<tr>
<th>Level</th>
<th>(A_1) (pl.8)</th>
<th>(A_2) (pl.8)</th>
<th>(A_1) (pl.9)</th>
<th>(A_2) (pl.9)</th>
<th>(\Gamma^V/\Gamma^D)</th>
</tr>
</thead>
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<tr>
<td>(U_0 +40)</td>
<td>-38,6</td>
<td>-70,5</td>
<td>-4,0</td>
<td>-88,2</td>
<td>+52,3</td>
</tr>
<tr>
<td>(U_0 +30)</td>
<td>-48,4</td>
<td>-92,2</td>
<td>-14,4</td>
<td>-108,0</td>
<td>+49,8</td>
</tr>
<tr>
<td>(U_0 +20)</td>
<td>-59,2</td>
<td>-111,5</td>
<td>-38,6</td>
<td>-107,1</td>
<td>+16,2</td>
</tr>
</tbody>
</table>

Table 1. Data for fundamental \(A_1\) and second harmonic \(A_2\) amplitudes in dB of three plates (# 8, # 3, # 9) at different levels of ultrasonic excitation. Evaluation of the relative coefficient of non linearity \(\Gamma\) (in dB) between these plates having different level of damages. The quantity \(U_0\) represents a reference level.

For virgin raw glass, \(\Gamma\) is around 1-2 (see references [4,8]), one deduce that the most damaged glass plate tested here has at least \(\Gamma^D = (300-600)\). Our analysis is based on the relative differences in second harmonic amplitudes for which a contribution to the second harmonic generation from transducer nonlinearity is strongly compensated. Neglecting the contribution from transducer's second harmonic leads, in fact, to some underestimation of the absolute value of the parameter \(\Gamma^D=300-600\). The derived value is actually indicating the lower limit of the coefficient of nonlinearity. The prime goal of the present estimation has been to demonstrate the drastic increase of nonlinearity when the material is cracked and link nonlinearity to the amount of damage.

II.3. Frequency conversion due to high nonlinearity

In the media with such high level of nonlinearity one can expect to have strong manifestation of nonlinear phenomena even at relatively weak amplitudes of the acoustical excitation. Let us illustrate this point by the processes of frequency conversion which are attributed to a highly nonlinear media. Fig. 2 c-d represent spectra for the plates pumped by amplitude modulated signals (100 kHz ± 5 kHz). Complex frequency-up conversion process is well seen on Fig. 2d where several groups of higher harmonics appear around 200, 300 and 400 kHz like sets of side lobes with 5 kHz intervals between them.

Frequency-down conversion takes place in the form of a demodulation process. Strong LF signal at 5 kHz (and also at 10 kHz) having an amplitude comparable to those of the pump signal is observed on damaged plate
(Fig. 2d). The last result demonstrates that parametric LF emission is possible in damaged solid material. Based on this result, the authors have implemented the same parametric interaction to produce an audio range emitter made of the cracked glass plate. Using previous configuration, the pump transducer was driven by a 100 kHz carrier frequency ultrasonic signal, being amplitude modulated by a musical record (e.g. "tango"). As a result of pump wave demodulation which occurred within the cracks, the music was heard from the damaged plate in real time while no sound was heard from the virgin plate under the same conditions.

The opposite multiwave process (modulation) was also obtained on the damaged plate. Two continuous harmonic signals of high and low frequencies from two different sources were mixed onto the plate. The nonlinear interaction of both signals resulted into the creation of new frequencies of high and low frequencies from two independent sources inside the plate with medium damage. HF pump transducer is always active when the LF loudspeaker is either turned off (a) or turned on (b).

The non-classical nature of huge quadratic nonlinearity of a damaged material has been put in evidence. Further studies are needed to understand better the physics and the mesoscopic mechanisms at the origin of such anomalous nonlinearity. This field will find many applications, particularly in nondestructive testing of the micro-cracks at the very early stage, for which nonlinear frequency conversion phenomena seem to be appropriate.

**REFERENCES**


