On the possible applications of the non-linear acoustic phenomena for the evaluation of the granular materials

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Granular materials are widely used in different branches of industry from the building construction to pharmaceutical and production of the nanostructured materials. Classical acoustical methods for their diagnostics are well known to be very useful. By the present communication we would like to attract the attention to the possible applications of the emerging nonlinear acoustical methods for the non-destructive evaluation of the granular materials. Three potential applications are discussed. First, due to high nonlinearity of the granular materials the parametric emitting antenna can operate with sufficient efficiency in these materials. Thus, similar to the advantages achieved in underwater applications of the parametric sonar, compact sources of highly directive low-frequency sound waves can be created for the acoustic spectroscopy, tomography and depth-profiling of the granular piles and columns. Second, due to high sensitivity of the nonlinear phenomena to the state of the inter-grain contacts, such processes involving polarized shear waves are sensitive to the force chains network through its anisotropy when a uniaxial static stress is applied on the medium. Finally, few results related to a recently developed model describing the parametric antenna operation in a uni-dimensional chain of identical elastic beads are discussed. Discrete feature of the lattice and evanescent modes are taken into account.

1 INTRODUCTION

Acoustic waves are useful when they penetrate deeper than other classical or non-classical waves in some media to be tested. This seems to be the case for dense granular materials, from the scale of several grains to large distances. Many physical phenomena are known to play a role in the propagation of an elastic perturbation in a granular material, such as absorption, scattering (due to randomness of the grain positions and inhomogeneity of contacts between the grains), velocity dispersion (a manifestation of the micro-structure), and also nonlinearity, mainly due to the contacts. The equation of state describing the behavior of the contact between two spherical beads is known as the Hertz relation and is fundamentally nonlinear.

A lot of studies have been performed in idealized media for modelling or experiments (typically 1-D chains of beads), but also in the more realistic 3-D granular materials (sandstones, geological media, unconsolidated assemblages of beads, ...) (7). For regularly assembled beads of the same dimension, the equivalent quadratic nonlinear parameter, parameter which characterize the importance of the nonlinear effects of the first order on the acoustic propagation, was evaluated (7). The obtained values were between $10^2$ and $10^4$, to be compared to the classical nonlinear parameters of homogeneous media: water 3, air 1.2 or glass 1.4. As a consequence, it is easy in such media to generate nonlinear effects, even with moderate amplitudes of excitation, which is not the case in classical ho-
The parametric antenna emits low-frequency (LF) acoustic signals due to the demodulation of amplitude-modulated high-frequency (HF) waves. In Fig. 1, regions of existence of HF primary waves and LF demodulated waves are schematically represented.

Figure 1: Qualitative scheme of the parametric antenna

2 ADVANTAGES AND CAPABILITIES OF THE PARAMETRIC EMITTING ANTENNA METHOD

The main interests of applying the parametric antenna method are coming from several inherent features already observed in underwater acoustics such as low frequency radiation with a high directivity (B.K. Novikov et al. 1987). The HF waves (wavelength comparable to several bead diameters) are sensitive to the micro-structure through the velocity dispersion or the scattering, but are usually attenuated before arising to the receiver. In contrast, LF waves (with very large wavelengths compared to the beads diameter) are less dispersed or attenuated and can be detected far from the emitter. It has been shown that the LF waves, radiated due to the demodulation of initially modulated HF waves, contain information on the HF waves propagation (V.Y. Zaitsev et al. 1999a; V.Y. Zaitsev et al. 1999b).

The high directivity of the radiated LF waves allows to perform experiments in samples that have smaller lateral dimensions than the LF wavelength and to avoid at the same time reflexions of these waves on the sample lateral boundaries.

A theoretical model on the parametric antenna operation in granular materials has been recently developed, including velocity dispersion, HF absorption and scattering (V. Tournat et al. 2002). We have shown that velocity dispersion influence on the demodulated LF profile can lead to an important change of its shape (like integration or differentiation depending on the ...). But also, the transition from propagative to diffusive HF primary waves manifests itself by an integration of the demodulated temporal profile, under some established conditions. Detailed considerations can be found in (?), but for instance, in the case of HF waves propagation, the time of arrival $\tau$ of the demodulated signal is related to the HF attenuation time $\tau_a$, the HF waves group velocity $c_g$, the LF waves phase velocity $c_\phi$ and the distance of propagation $d$, by the following relation:

$$\tau = \frac{d + \tau_a (c_\phi - c_g)}{c_\phi} \quad (1)$$

Qualitative explanations of experimental results related to this model have been performed in (V. Tournat et al. 2003). In Fig. 2, are shown experimental LF demodulated pulses obtained for different frequencies of HF waves but with the same Gaussian modulation function.

Electrical signals corresponds respectively to the HF electrical signals radiated first and to the LF accelerations received by an piezo-electric transducer immersed in the medium. Two successive derivations are first observed for the demodulated LF signal with increasing frequency of HF waves, and then an integration. A cross-talked of several different physical phenomena, influencing the process of demodulation at the same time is certainly responsible here to this trend. For example, derivation of the LF temporal profile can occur when the LF wave is diffracted.

In order to take into account this different phenomena at the same time, it was necessary to built a numerical code (in collaboration with V. Aleshin (?)). First encouraging results were obtained, and the work is in progress in order to cover a large bunch of parameters values.
3 PROBING FORCE CHAINS AND INTER-GRAINS CONTACTS BEHAVIOUR WITH NON-LINEAR SHEAR WAVES

In contrast to longitudinal waves that have a cylindrical symmetry along their propagation axis (for axisymmetric transducers), shear waves are polarized along an orthogonal direction. Consequently, along the same propagation direction, it is possible to rotate the polarization direction, and to feel the eventual anisotropy of the medium.

In non-periodic or anisotropic granular materials, a mode conversion between shear and longitudinal waves takes place. This phenomenon makes the measurement of shear waves velocities difficult. Since shear waves are slower than the longitudinal waves, shear signals should arise later than longitudinal ones. However, due to mode conversion, no pure shear wave can be received, and the leading edge of the shear first-radiated signal, traditionally used to determine shear wave velocity is not well evaluated due to the presence of a longitudinal contribution.

A lot of recent studies have been performed on the properties of the force chains network in 2-D or 3-D granular assemblages (M. Manciu et al. 1999). One of the conclusions, concerning an uniaxial externally applied static stress, is that the force chains are preferentially oriented along the direction of this applied stress. We will show that when anisotropy of the chain force network is realized, it influences the acoustical properties of the material due to the differences in the contact prestrains.

Velocity is sensitive to this preferentially orientated chain forces through its dependence on the static applied stress. This stress is applied vertically with a piston adapted at the top of a 0.41m diameter rigid cylindrical container, filled with 2mm diameter glass beads. As a consequence, chain forces should be orientated along the vertical direction. We performed experiments to determine longitudinal wave velocities dependences on the applied static stress for two different directions of wave propagation: vertical and horizontal. When fitting the results with the following power law $v(t) \propto P^{\alpha}$ for the vertical direction of propagation and $v(t) \propto P^{1/9}$ for the horizontal one. The first result was already several times observed experimentally and explained analytically (M.J. Buckingham 1997; Buckingham 2000). This behavior is attributed to the Hertzian contact law between beads and to the creation of new contacts during the increase of the static stress (for low values of stress). The $\alpha \simeq 1/6$ power law has also been observed for high static stresses, when there is no more creation of contacts between the beads in the medium. Our observed $\alpha \simeq 1/9$ behavior can be interpreted as a manifestation of the anisotropy of the applied stress transmission in the medium. The increasing externally applied stress is mainly supported by vertically oriented contacts, in contrast, horizontally oriented contacts are not affected so much by this uniaxial static stress. This acoustical experiment confirms the observed and computed results of anisotropy of force chains in granular materials (\).

It is well known that non-linear Hertzian contacts exhibit a lower nonlinearity if they are strongly than slightly pre-stressed (\). Moreover, when the dynamic elastic strain is higher than the contact pre-strain (which can happen when the contact is slightly pre-stressed), clapping of the contact occurs (\). This type of nonlinear behavior is fundamentally different to the Hertzian behavior, and manifests itself in the acoustical nonlinear effects. As a consequence, non-linear phenomena represent competitive candidates for the evaluation of internal static stress of granular assemblages.

Several experiments have been performed on the shear waves self-demodulation effect. A strong HF carrier wave of 80kHz, modulated in amplitude at 6kHz, is radiated by a shear wave transducer. Due to mode conversion and nonlinearity, this wave is self-demodulated into a 6 kHz longitudinal wave (this feature has been checked precisely using different polarization experiments and different membrane orientations between the HF emitter and the LF receiver).

In order to compare, for the same configuration, this results with previous ones (V.Y. Zaitsev et al. 1999a; V.Y. Zaitsev et al. 1999b), we also emitted HF waves with a longitudinal transducer.

A longitudinal transducer is thus used for the detection, after 16cm of propagation in the unconsolidated assemblage of 2mm diameter glass beads. The usual configuration for the transducers dis-
position geometry is shown in Fig. 3. As for the previous velocity experiment, in order to enlight the anisotropy of the force chains network in the medium, we performed this experiment with horizontal propagation. The vertical externally applied stress is thus orthogonal to the plane of the Fig. 3. As a consequence, one shear emitter (denoted C) is polarized along the force chains and the other (denoted A) orthogonally to these force chains.

Amplitudes of the received LF demodulated longitudinal signals are plotted as a function of the amplitudes of the emitted HF shear waves on Fig. 4. Different features are noticable. First, the signal demodulated from the horizontally polarized shear waves is up to 15dB stronger than the signal demodulated from the vertically polarized shear waves. Second, the two signals are growing quadratically for low amplitudes of excitation and then obey to a 3/2 power law for higher amplitudes of excitation. This transition from a quadratic to a 3/2 power law behavior, occurs around -5dB for the horizontal polarisation and around +8dB for the vertical polarization.

The same experiment has also been performed for a vertical direction of propagation. Amplitudes difference between the two demodulated signals was less than 2dB, ensuring that real efficiencies of the transducers are very close.

The static strain in the medium, corresponding to the applied static stress, can be estimated using the following stress-strain relation (2):

\[ \sigma = \tilde{n}(1 - \alpha)E\epsilon^{3/2} / (3\pi(1 - \nu^2)) \]

with \( \sigma \) the applied stress, \( \epsilon \) the strain, \( \tilde{n} \) the average number of contacts per bead (3-5 for random packing), \( E \) the Young modulus and \( \nu \) the Poisson ratio of the bead’s material, and \( \alpha \) the porosity of the medium. Using the known values for these parameters, we estimate the static strain around \( 10^{-4} \) for our experimental configuration. At the same time, calibration of the shear transducers using a vibrometer, provide an estimation of the dynamic applied strain in the medium at full input electrical amplitude around the same strain value \( 10^{-4} \). Obviously, there exists a little dispersion of the static strain values, from one contact to another, around the estimated mean value.

When the dynamic strain is higher (or at least of the same order) than the static one, the contact has the opportunity to “open” and to close again. This is usually called the regime of clapping. Detailed considerations about this phenomenon and its consequences can be found in (V. Y. Zaitsev et al. 1999a; ?). In fact, it manifests itself by the observed 3/2 power law behavior. Then, the quadratic behavior is associated to the classical pre-stressed Hertzian contacts, and the 3/2 behavior to the clapping of the contacts. Consequently, at the transition between these two behaviors, the amplitude of the dynamic strain is roughly equal to the amplitude of the static strain of the contacts solicitated in the propagation of the dynamic perturbation.

Difference in the amplitude of transition from quadratic to 3/2 power law behavior observed on Fig. 4 is thus associated with the difference of the contacts pre-strain. Shear waves polarized (vertically) along the force chains (vertical) feel an assembly of contacts with a higher pre-strain than the shear waves polarized (horizontally) orthogonally to the force chains. This difference of pre-strain can be estimated using the difference of dynamic ones at the transitions, i.e. around 13dB, which characterize the level of the anisotropy of the force chains network in the granular material.

4 ANALYTICAL RESULT ON SELF-DEMODULATION OF EVANESCENT PRIMARY WAVES

We have developped a model that describes the nonlinear self-demodulation process in a one-dimensional chain of identical elastic beads. Linear (but also nonlinear) elastic propagation in such
media has been actively investigated (C. Coste and B. Gilles 1999). Also, a well-known dispersive feature of periodic lattices, is the existence of a cut-off frequency $\omega_c$ for the elastic waves, above which modes of propagation become evanescent (L. Brillouin and M. Parodi 1956). This cut-off frequency $\omega_c$ is related to the bead radius, the bead’s material properties and the longitudinal static applied stress on the chain (or others sets of parameters). As a result, the cut-off frequency is a source of information on the properties of the granular chain.

When considering characteristic times for the elastic waves that are much higher than the elastic wave travel time along a bead diameter (i.e. less than $10^{-6}$s for 2mm diameter glass beads), the chain is usually modelized by masses linked with non-linear springs as seen on Fig.5. To begin with, the potential energy of the chain $U_p$ is expanded up to the cubic term in bead displacement $U(n)$ (where $n$ is the bead number):

$$E_p = E_{p0} + \frac{\alpha}{2} \sum_n [U(n) - U(n + 1)]^2$$

$$+ \frac{\beta}{6} \sum_n [U(n) - U(n + 1)]^3 + \ldots$$

where the cubic non-linear term corresponds to the quadratic non-linearity for the stress-strain relation, and $\beta$ denotes the quadratic non-linear parameter. Then, the following discrete non-linear equation of motion is derived for each mass $m$:

$$m \frac{\partial^2 U(n)}{\partial t^2} - \alpha [U(n + 1) - 2U(n) + U(n - 1)] \simeq$$

$$\simeq - \frac{\beta m}{2 \alpha} \frac{\partial^2 U(n)}{\partial t^2} [U(n + 1) - U(n - 1)]$$

Using the successive approximation method to solve this equation in the context of the self-demodulation process, i.e. strong modulated HF waves and weak modulated LF wave, we obtain the solution Eq.5 for the following boundary conditions:

$$U_{HF}(n = 0) = A_0 \cos(\omega_1 t) + A_0 \cos(\omega_2 t)$$

$$U_{LF}(n = 0) = 0$$

$$U_{LF}(n) =$$

$$\exists \left\{ \frac{\beta m}{2 \alpha} \left[ \omega_c^2 \sin(k(\omega_1)a) - \omega_c^2 \sin(k^* (\omega_2)a) \right] \right\}$$

$$A_{b1}^{(1)}(0)A_{b2}^{(2)}(0) \frac{e^{-ik(\Omega)n} - e^{i\Delta k an}}{4\alpha \sin^2(\Delta ka/2) - m\Omega^2 e^{i\Omega n}}$$

In this solution $k^*$ denotes the complex conjugate of the wave number $k$, $\Delta k = k^*(\omega_2) - k(\omega_1)$ and $\Omega = \omega_2 - \omega_1$. In order to derive this solution, we used the total dispersion relation, i.e. for both propagative and evanescent modes:

$$k(\omega) = \begin{cases} \frac{2}{k_c} \arcsin(\omega/\omega_c) & 0 \leq \omega \leq \omega_c \\ k_c - i \frac{2}{k_c} \text{arccosh}(\omega/\omega_c) & \omega_c \leq \omega < +\infty \end{cases}$$

where $k_c = \pi/a$ is the maximum (real) wave number.

The Fig.6 represents the demodulated signal amplitude $U_{LF}(n = 2500)$ as a function of the frequency of primary wave for different absorptions,
Figure 7: Demodulated signal amplitude as a function of the frequency of primary wave for different absorptions. From the dark line to the bright, the constant $C$ in the absorption coefficient $\alpha(\omega) = C\omega_n$ is respectively equal to 230, 91, 36, 14, 6, 0.9, 0.36. The normalized demodulated frequency $\Omega_n = \Omega/\omega_c$ is equal to 8.3$ \times 10^{-4}$.

i.e. different additional imaginary parts $\alpha(\omega)$ for the wave numbers. For the highest absorptions (darkest lines), dynamics of the amplitude as a function of pump frequency exhibits a minimum around the normalized frequency $u_n = \omega/\omega_c = \omega_c/\sqrt{2}$. This is an effect of dispersion of nonlinearity, the non-linear quadratic term (or equivalently the cubic term of the potential energy) of the equation of motion has a minimum at this frequency. For the lowest absorptions (brightest lines of Fig. 6), the amplitudes have more and more pronounced minima at some fixed frequencies. This oscillations are due to the velocity dispersion effects between HF waves (sources) and LF wave, being sometimes in phase (maxima) and sometimes out of phase (minima). It can be interpreted as a beating phenomenon between HF sources (the second exponential in brackets of Eq.5) and LF (the first exponential). This phenomenon is also a cause of saturation for the demodulated amplitude.

For the same conditions but a different demodulated frequency $\Omega_n = 8.3 \times 10^{-4}$, the results are plotted on Fig. 7. The feature occurring for the frequency $\omega_n = \omega_c/\sqrt{2}$ is still present and is independent of $\Omega_n$. However, as could be anticipated, the phenomenon of beating depends on $\Omega$ because minima and maxima observed for low absorption (bright lines) are shifted compared to Fig. 6.

In each case, the transition from propagative ($\omega_n < \omega_c$) to evanescent primary waves ($\omega_n > \omega_c$) manifests itself by a strong decrease in the self-demodulation process efficiency. Obviously, this feature is of high interest for experimental purposes because it allows to detect the cut-off frequency of the medium.


5 CONCLUSIONS
Several nonlinear methods have been applied to characterize granular materials through their acoustic wave absorption, scattering, velocity dispersion (section 2), through the anisotropy of force chains network and contacts behavior (section 3), or through the cut-off frequency and various dispersive effects (section 4).

REFERENCES


