The Luxemburg-Gorky Effect Revived for Elastic Waves: a Mechanism and Experimental Evidence

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Abstract. For many nonlinear effects typical in optics and plasma physics direct acoustic analogies are known. In particular, such effects have been observed in bubble-containing liquids, whose strong acoustic nonlinearity is due to the coupling to soft oscillators, the bubbles. However, no acoustic analogies are known for the Luxemburg-Gorky (LG) effect, which represents one of the pioneering observations in nonlinear wave interactions [Tellegen, B.,"Interaction of radio waves?", Nature, No 6, 840 (1933).]. It consists of the transfer of the amplitude modulation from the radio-wave of a powerful station (originally, Luxemburg and Gorky-city stations) to another carrier wave. The stronger wave perturbs the dissipation in the ionosphere plasma thus causing pronounced amplitude-modulation of the weaker wave, whereas the role the complementary perturbations in the weaker wave velocity is of secondary importance [1]. A majority of the later nonlinear research for waves of different nature focused, however, on the effects of reactive, rather than dissipative nonlinearities. Below a new mechanism is proposed for the linear and amplitude-dependent dissipation due to elastic wave-crack interaction. We have observed one of its strong manifestations in a direct elastic-wave analogue of the L-G effect in a crack-containing glass sample. The counterpart acoustic mechanism implies, first, a drastic enhancement of the thermoelastic coupling at high-compliance microdefects. Second, the high stress-sensitivity of the defects leads to a strong stress-dependence of the resultant dissipation.

INTRODUCTION

It is well known that for homogeneous solids the intrinsic elastic nonlinearity due to the anharmonicity of the interatomic potential is normally very weak, however, during last few years, relatively low-amplitude nonlinear-elastic effects have been intensively studied, such effects being readily observed at strains $\varepsilon \sim 10^{-6} - 10^{-5}$ in rocks, fatigue-damaged metals and in other microstructured materials [3]. For these solids, the presence of defects with highly increased compliance often results in drastic increase in nonlinear elasticity, whereas linear elastic parameters remain only slightly perturbed, which may be understood by means of instructive models describing the interplay between the strong strain-concentration at the high-compliant defects and their small density [3]. Recently, for such solids, some observations were reported [4-6] on pronounced variations in dissipation of a weak elastic wave (at strains down to $\varepsilon \sim 10^{-8} - 10^{-10}$) induced by another moderate-amplitude ($\varepsilon \sim 10^{-5} - 10^{-6}$) elastic wave. This effect can be explained neither by reactive nor by hysteretic nonlinearities and requires assuming the existence of another, non-hysteretic and non-frictional, nonlinear-dissipative mechanism.

STRESS-SENSITIVE DISSIPATION IN SOLIDS WITH CRACKS

The proposed idea is based on a few reliably established microstructural features of cracks. First, strong influence of cracks on material elasticity is due to their high compliance that is characterized by the ratio of crack opening d to its characteristic diameter L, $d/L \ll 1$. Average material strain $\varepsilon \sim d/L$ is enough to completely close a crack. However, normally this strain is significantly larger than typical strains $\varepsilon \sim 10^{-5} - 10^{-6}$, for which the dissipation stress-sensitivity is already pronounced. Second, numerous direct microscopic images of cracks indicate their complex, wavy or zigzag interface shapes. The cracks often have inner strip-like contacts, as schematically shown in Fig. 1. At these regions, local separation (or inter-penetration) $\tilde{d}$ of crack interfaces is much smaller than average separation d. Due to such a geometry, the contacts are strongly perturbed even by average strains, which can be orders of magnitude smaller (roughly $d/\tilde{d} \gg 1$ times) than typical strains $\varepsilon \sim d/L \sim 10^{-3} - 10^{-4}$ required to close the whole crack.

Another crucial fact is that cracks are regions of very effective dissipation for elastic waves. Conventionally, this dissipation is attributed to friction at crack interfaces or adhesion hysteresis, which requires that mutual interface displacement should exceed the atomic size a, as is recently well documented in direct nanoscale experiments. For a crack with diameter L, the average compressional or shear strain ε can produce maximal lateral or normal interfacial displacement [5] $D \sim \varepsilon L$. On the other hand, requirement $D > a$ determines threshold strain $\varepsilon_{th} > a/L$, below which the interfacial displacement is of sub-atomic scale. For a typical atomic size $a \sim 3 \cdot 10^{-10}$ m and crack-size $L \sim 10^{-3}$ m, this yields $\varepsilon_{th} \sim 0.3 \cdot 10^{-6}$, which should be exceeded in order to activate frictional or adhesional hysteretic losses. However, even at much smaller strains, the defects can efficiently dissipate elastic energy due to locally enhanced thermoelastic coupling, which is not threshold. For the whole crack, the following asymptotic expressions for the losses per cycle can be derived (the details will be published elsewhere):

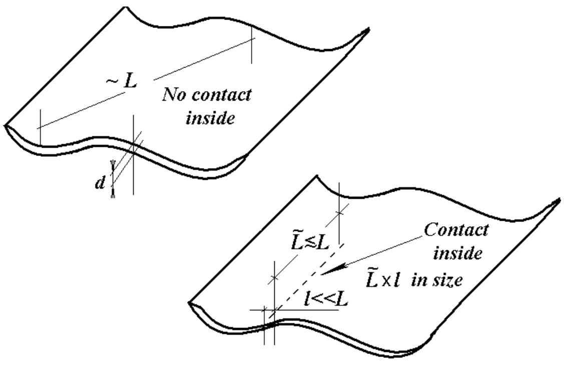

FIGURE. 1. A crack without and with an inner contact is shown here schematically. At $\tilde{L} \to l$ a strip-like contact reduces to a point-like contact.

$$W_{LF}^{dis} \approx 2\pi\omega T(\alpha^2 K^2/\kappa)L^5\varepsilon^2, \qquad \text{at } \omega << \omega_L \approx \kappa/(\rho C L^2) \qquad (1)$$

$$W_{HF}^{dis} \approx 2\pi T(\alpha K/\rho C)^2[1/(\kappa\rho C\omega)]^{1/2} L^2\varepsilon^2, \quad \text{at} \quad \omega >> \omega_L \qquad (2)$$

$$W_{crack}^{\max} \approx 2\pi T(\alpha^2 K^2/\rho C)L^3\varepsilon^2, \qquad \text{at } \omega = \omega_L, \qquad (3)$$

where ω is the wave cyclic frequency, T is the temperature, α is the temperature expansion coefficient of the solid, K is the bulk elastic modulus; ρ is the density; C is the specific heat, ε is the average strain, κ is the thermal conductivity, and ω_L is the relaxation frequency for defect scale L. For example, for $L\sim10^{-3}$ m frequency ω_L falls between 10^{-1}-1 cycle/s for most rocks and metals, and thus the "global" absorption at cracks normally is not so efficient in acoustic/ultrasonic band.

Analogous estimates for inner contact of width $l << L$ and length $\tilde{L} \leq L$ yield:

$$W_{LF}^{dis} = 2\pi\omega T(\alpha^2 K^2/\kappa)l^2\tilde{L}L^2\varepsilon^2, \qquad \text{at } \omega << \omega_l \approx \kappa/(\rho C l^2) \qquad (4)$$

$$W_{HF}^{dis} = (2\pi/\omega)\kappa T(\alpha K/C\rho)^2\tilde{L}(L/l)^2\varepsilon^2, \qquad \text{at} \quad \omega >> \omega_l \qquad (5)$$

$$W_{cont}^{\max} = 2\pi T(\alpha^2 K^2/\rho C)\tilde{L}L^2\varepsilon^2, \qquad \text{at } \omega = \omega_l, \qquad (6)$$

Comparison of Eqs. (3) and (6) indicates the striking result that, for strip-like contacts with $\tilde{L} \sim L$, the maximum losses at the whole crack and at the small inner contact have the same magnitude, whereas the relaxation frequency for narrow, $l << L$, contacts can be 4-6 orders of magnitude higher and reaches the kHz or even MHz band. Next, it is essential that quite moderate average strain, say $\varepsilon \sim 10^{-5} - 10^{-6}$, which is too small to perturb the whole crack, can strongly perturb sizes l and $\tilde{L}$ of soft inner contacts. This can produce strong effect on the dissipation of a weaker probe wave, for which neither adhesion-hysteresic, nor frictional losses are yet important. The complementary variation in material elastic moduli may remain very small, since the stiffness of such contacts is very low. Thus in crack-containing solids, favorable conditions should occur for the direct elastic-wave analogue of the LG-effect, since perturbation of the inner crack contacts by a moderate-amplitude wave via the considered mechanism can noticeably affect dissipation for a weaker wave, just like in the case of the radiowaves in the ionosphere [1].

EXPERIMENTAL DEMONSTRATION

We observed the acoustic counterpart of the LG-effect [7] in the form of the cross-modulation of two longitudinal modes in a glass rod containing three corrugated thermally-produced cracks 2-3 mm in size. In a reference rod without cracks, the modulation sidelobes (existing due to residual parasitic nonlinearities) were 25-40 dB lower than shown in Fig. 2 (a). Resonance curves for the probe wave [Fig. 2 (b)] demonstrate that primarily the dissipation, not the elasticity, is affected by the stronger wave. Magnitudes and frequencies, at which the observed amplitude-dependent variations in dissipation were observed, agree well with estimates based on equations (4)-(6). As argued above, for small enough strains $\varepsilon \sim 10^{-8}$, estimated displacements εL of adjacent crack interfaces are subatomic in scale, so that neither hysteretic, nor

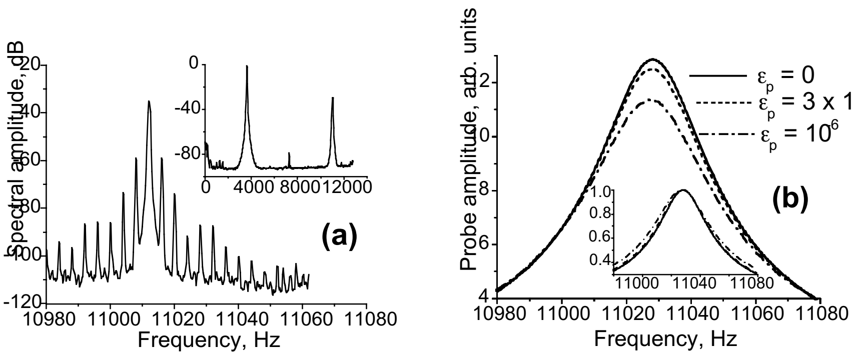

FIGURE. 2. Experimental observation of the elastic wave LG-effect. (a) – Modulation spectrum of weak second mode near 11 kHz with $\varepsilon \sim 10^{-8}$ by a stronger ($\varepsilon_p \sim 10^{-6}$) first mode wave with carrier frequency near 3.6 kHz and slow amplitude modulation at 3 Hz. The inset shows the relative levels of the stronger and the weaker waves. (b) – Resonance curves for the probe wave at different stronger-wave levels, clearly illustrating a greater than 10% variation in the probe mode quality-factor. In contrast, the resonance frequency shift is hardly noticeable. The inset shows the same curves in normalized form.

frictional effects can be important for the probe wave dissipation. These cracks are the only defects present in the transparent sample, and there is no doubt that only their presence is responsible for the observed effects absent in the reference rod.

Since the described defects occur in a vast class of solids, the proposed mechanism of strong enhancement of coupling of thermal phonons and elastic waves is expected to operate widely, in particular, both for the dilatation strain responsible for conventional thermoelastic dissipation and for shear modes. The corresponding effects, including the LG-modulation, should find diagnostic applications in basic solid-state studies, in seismics and in non-destructive testing.

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