Low frequency in situ metrology of absorption and dispersion of sound absorbing porous materials based on high power ultrasonic non-linearly demodulated waves

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Abstract

The present work is related to acoustic in situ free-field measurements of sound absorption in porous materials, such as cellular plastic foams, glass–wool or recycled felt materials. The emphasis is given towards fine metrology of absorption in view of potential industrial applications. A powerful ultrasonic array working at 40 kHz is used. It enables to measure absorption acoustical data down to 100 Hz due to the exploitation of the non-linear ultrasonic demodulation phenomenon in air. Fine measurements of acoustic absorption are compared to numerical predictions based on the “equivalent-fluid” model (when the squeleton frame is motionless), and to some measurements performed on a Bruel and Kjaer impedance tube. Dispersion curves are also measured and compared to the numerical predictions for some automotive felt materials which are compressed at various ratios. Data obtained with a dedicated portable instrument are also discussed for the same type of materials and configurations.

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1. Introduction and position of the problem

In order to obtain the acoustic absorption coefficient of porous absorbing materials, one generally uses a Kundt or impedance tube. This procedure can be quite time consuming because one needs to cut the tested samples, especially in the automotive industry which needs to know rapidly the absorption characteristics of the used panels at various compression ratios. Furthermore, in many cases the mounting of the sample inside the tube modifies the acoustical properties by adding some stiffness due to the mechanical boundary conditions on the sides [1]. Moreover, for inhomogeneous or compressed panels, it is difficult to describe the spatial variations of the acoustical properties, as measurements are solely done on the cut and tested samples. Consequently, in situ free-field methods are sought even if this problem is not trivial to solve. For instance, a simple method working with two microphones (the so-called microphone doublet method) was designed more than 20 years ago (e.g. [2–4]). It is simple and correctly working at low frequencies but it needs to have a sound source (i.e. a loudspeaker) mounted at a sufficient distance, to generate approximately plane waves nearby the sample. Consequently, this technique needs to use an anechoic chamber, which is a strong limitation for routine on-site measurements. Other experimental methods were proposed during the 1990s, as extensions of the two microphones method, with their positions being scanned over a straight line [5,6]. By using some reconstruction algorithms based on Fourier–Hankel dual temporal and spatial transform methods (i.e. holographic inversion procedures), the surface
impedance (or alternatively the absorption coefficient) can be retrieved versus frequency. By using some stretched pulses, one can achieve these measurements, but again some anechoic treatment of the room where the measurements are performed is needed (e.g. see [6]). The automotive industry, has evidently implemented some alternative measuring methods. For instance one can measure the absorption coefficient from the transmission loss when the sample is mounted between two small rooms (or cabins). One can also use the so-called alpha cabin method which somehow measures the Sabine absorption coefficient of diffuse fields, i.e. by integrating for many incidence angles covering the full solid angle produced by the acoustical source. However, such methods cannot provide accurate measurements which could be truly compared to theoretical or numerical predictions based on the current fundamental models. More recently, several researchers were proposing various new methods to measure acoustical properties in the free-field configurations, or even on-site or in situ. For instance, the use of the microflown sensors [7] seems to be efficient for such purposes, as was reported very recently [8].

In the present work, we describe a completely different technique to obtain essential acoustical data [9]. This method is based on an amplitude auto-demodulation process taking place in air (and also inside the air-saturated porous media), the so-called parametric array technique. The physical principle of a parametric array is apparently quite simple. One starts by using a powerful ultrasonic source (the so-called “pump” transducer working at some high frequency \(\omega\), which is amplitude modulated, at a low frequency \(\Omega\) (with \(\omega \gg \Omega\)). Consequently, the produced spectrum of the emitted signal basically contains three frequencies, which are the pump frequency \(\omega\) and the lateral shifted components \(\omega + \Omega\) and \(\omega - \Omega\). At that step only linear response of the system is observed and the three spectral components are all high frequency (HF), while the demodulation process (or rectification) deals with a nonlinear behaviour of air which produces an energy transfer towards low frequency (LF), allowing to observe and monitor the demodulated component itself at the low frequency \(\Omega\). The use of powerful ultrasonic demodulated waves in air has proved recently to be a very attractive tool to characterize the audio range acoustical properties of absorbing porous materials, which are described with the background of the “equivalent fluid”, i.e. rigid frame, model [10,11]. The very unique feature of that technique, mainly due to its high directivity producing plane waves, lies on the fact it can work in air in the free field, allowing simple and straightforward “in situ” measurements, at least if a proper procedure is used. The definitive advantages of using parametric methods of the nonlinear acoustics are the following: (1) the technique is very simple as only one emitting ultrasonic transducer is needed, and because a single metrology microphone is necessary for detection, and afterwards for further treatment. Fig. 1 provides a basic sketch of the experimental set-up when working in the reflection configuration, or when transmission through the porous plate is studied. (2) it allows to perform measurements at various distances, depending on the physical characteristics of the pump transducer (e.g. from less than one meter towards a few meters). This provides to test remote areas. (3) The high directivity of the ultrasonic beam permits with proper point-like acoustic detection to focus on small areas of a given material, enabling to search for defects, inhomogeneity, lack of adhesion, of a porous material glued onto a support, and so on. Obviously, some drawbacks of the parametric technique exist as well: (1) the aperture of the sonic beam which controls its directivity is always around 10° (at –6 dB) or wider. (2) the demodulation length (or the so-called length of the parametric array) which is somewhat related to the length of nonlinear interaction in air and to the HF ultrasound absorption length. This parameter is evidently linked to the LF acoustic wavelength as there is a minimum distance between the pump transducer and the microphone before some demodulation process significantly arises. (3) the efficiency of the array in terms of the demodulation process, which is inversely proportional to the modulation frequency, that is at 100 Hz, the demodulation is less efficient than at 1 kHz by a factor around 10.

Advances in parametric arrays technology in air have transformed a curiosity in the field of science, discovered 50 years ago in the field of underwater acoustics [12,13], into a proper tool having potentials for fine metrology [14,15] and routine measurements of acoustical properties of air-saturated porous and poroelastic materials. Any ultrasonic transducer is working in the parametric regime, and some preliminary results were described in 2004 in the field of the characterization of porous materials in the reflection configuration as well as in transmission [16].
The advent of powerful LF systems allows to work at lower modulation frequencies, and various measurements and numerical simulations were described with a standard commercially available device manufactured in the USA (from Ultrasonics Technology, San Diego) [17]. Unfortunately, such device when driven at very low frequencies (below 200 Hz) was not working properly, as seen in the results of that publication. In order to measure acoustical absorption for industrial applications one needs to cover the 100 Hz to 6 kHz bandwidth, which was not available with that American ultrasonic array, based on thin PVDF film technology. In order to attain the very low frequencies, alternative designs, for instance in the format of numerous PZT individual transducers are needed. In the present work, an original apparatus manufactured by the European company Sennheiser has been used, because it is very powerful at low frequencies down to 100 Hz, or even better in some cases. The present work reviews some of the unique features of such device for the operation and diagnostics of porous materials at very low frequencies, with a special emphasis on the fine metrology of the acoustical absorption coefficient. Part of the technical information related to that device has also been described at various international conferences, e.g. [18–20], and we wish to produce here some comprehensive and systematic description of the latest results obtained with it.

2. Acoustical propagation in the frame of the “equivalent-fluid” model

When the frame is motionless, the theoretical “equivalent-fluid” model is a very effective approach [10,11]. It necessitates full knowledge of a set of five or six physical parameters (depending on the assumptions), namely the porosity \( \phi \), the tortuosity \( z_{\infty} \), the viscous permeability \( k_0 \), the thermal permeability \( k_0' \), the viscous characteristic length \( A \), and the thermal characteristic length \( A' \) [10,11]. Some of these parameters (i.e. tortuosity and characteristic lengths) are efficiently determined with ultrasonics, while others can readily be measured by other means. For instance, porosity and permeability (or alternatively resistivity) are generally obtained by using static experimental set-ups.

To describe the propagation of acoustical waves in the porous slab within the “equivalent-fluid” model, one needs to start with the Euler (or motion) Eq. (1) and the conservation of mass (or continuity) Eq. (2) linking the acoustic pressure \( p \) to the fluid velocity \( \vec{v} \):

\[
\rho_0 \frac{\partial \vec{v}}{\partial t} = -\nabla p, \tag{1}
\]

\[
\frac{\beta(\omega)}{K_0} \frac{\partial \vec{v}}{\partial t} = -\nabla \vec{v}, \tag{2}
\]

In these two equations, the functions \( \alpha(\omega) \) and \( \beta(\omega) \) are provided by some fundamental expressions taken from the so-called “equivalent-fluid” model, which describe respectively the modification of the mass density (replaced by \( \rho(\omega) = \alpha(\omega) \rho_0 \) in the porous material from \( \rho_0 \) in the free air) and the change of compressibility of the fluid (modified from \( K_0 \) in the free air towards \( K(\omega) = K_0/\beta(\omega) \) inside the porous material). There are several physical parameters related to the properties of the fluid, i.e. the viscosity \( \eta \), (which relates the viscous permeability to the resistivity \( \sigma \) by the relation \( k_0 \sigma = \eta \), the specific heat ratio \( \gamma \), the Prandtl number \( Pr \), and the static pressure of the fluid at rest \( P_0 = K_0/\gamma \). There are also the five (or six by including the thermal permeability) macroscopic parameters describing the porous network which have been outlined above. The angular frequency \( \omega = 2 \pi f \) is additionally a central variable for these two functions \( \alpha(\omega) \) and \( \beta(\omega) \) which are finally written [10,11]:

\[
\alpha(\omega) = z_{\infty} \left( 1 + \frac{\eta \phi}{j0z_{\infty} \rho_0 k_0} \sqrt{1 + j \frac{4 \omega^2 k_0^2 \rho_0 \omega}{\eta \phi^2 A^2}} \right), \tag{3}
\]

\[
\beta(\omega) = \gamma - (\gamma - 1) \left( 1 + \frac{\eta \phi}{j0 \rho_0 k_0 Pr} \sqrt{1 + j \frac{4 \omega^2 k_0^2 \rho_0 \omega Pr}{\eta \phi^2 A^2}} \right)^{-1}. \tag{4}
\]

As explained above, the acoustical fields for the studied 1D problem can be written with plane waves (this has sense to do so with highly directive sound projectors, such as used in this work). Substituting a plane wave solution \( p = p_0 e^{j(\omega t-k_0 z)} \) and \( v = \frac{p_0}{\rho_0} e^{j(\omega t-k_0 z)} \) in Eqs. (1) and (2), directly yields \( k(\omega) = \rho(\omega) c(\omega) \) and \( Z(\omega) = \rho(\omega) c(\omega) \), with,

\[
c(\omega) = \frac{K(\omega)}{\rho(\omega)} = \frac{c_0}{\sqrt{\alpha(\omega) \beta(\omega)}}, \tag{5}
\]

the complex phase velocity inside the porous material \( c_0 = \frac{c_0}{\sqrt{\rho_0}} \) is the wavespeed in air). Hence knowing \( k(\omega) \) and \( Z(\omega) \) – the wavenumber and characteristic impedance – from the two ingredients \( \alpha(\omega) \) and \( \beta(\omega) \), straightforward calculations yield the (normal incidence) reflection coefficient of a layer of material, of thickness \( h \), mounted on a rigid plate which is perfectly reflecting the acoustical waves (see e.g. [21]):

\[
R(\omega) = \frac{z \cos kh - j \phi \sin kh}{z \cos kh + j \phi \sin kh}, \tag{6}
\]

where \( z = \frac{Z(\omega)}{\rho_0 c_0} \) denotes the characteristic impedance ratio. A factor of porosity \( \phi \) explicitly appears because of the continuity of normal flux at the material surface, which is \( \nu \) in air and \( \phi \nu \) in the material. (This factor could be removed by redefining the characteristic impedance ratio as \( z = \frac{z_{\infty}}{z_{\infty}}, \) making the algebra completely similar to that in [21].)

By definition, the absorption coefficient \( A(\omega) \) is the fraction of the incident energy flux which is entering the material. It directly comes from the conservation law of the acoustical energy, which is written:

\[
A(\omega) = 1 - |R(\omega)|^2, \tag{7}
\]

when the material is mounted onto a rigid plate, Eq. (6) applies. This expression (6) has been systematically used, in
the following sections, for the numerical simulations of the absorption coefficient (7).

3. Experimental results and procedures

Fig. 1 provides a basic sketch of the experimental bench, used either in the reflection configuration (in order to obtain the absorption coefficient versus frequency), or in the transmission configuration when the dispersion curves are sought (i.e. the phase velocity versus frequency). The electronic features of the instruments are the following. An Agilent HP 33120A wave generator is used to produce a single wavepacket with proper amplitude. Specifically, the duration of this wavepacket is provided by the temporal period of the low frequency (LF) modulation $T = 1/\Omega$, while it contains a given number $N_p = \omega_0/\Omega = T/\tau$ of the high frequency (HF) temporal period of the pump ultrasonic wave $\tau = 1/\omega_0$ (e.g., a 4 kHz modulation amplitude will contain 10 periods of the 40 kHz HF pump ultrasonic signal within one single wavepacket). A Sennheiser audio sound projector is then fed with a LF signal provided by an Agilent wave generator (see Fig. 2a which presents some raw signals, before some filtering numerical treatment, which clearly show both LF and HF components). This device, which is simply mounted in front of the tested porous material, is perfect to achieve very strong ultrasonic fields in air at 40 kHz (in the range of 110 dB at 2 m along the geometrical axis of the projector), with proper modulation of amplitude. During propagation, the ultrasonic wave produced by the Sennheiser device is partially demodulated due to the non-linearity of air [14,15] and produces LF audio signal (at an approximate 70 dB level at 1 kHz). One should emphasize that after demodulation the propagation is mainly done in the linear regime, and accordingly no special care was done in the present work in order to incorporate any non-linear effects in the description of the observed phenomena, even if such non-linear behavior might also be present in some cases.

After propagation, the acoustical signal is received onto a non-capacitive audio microphone which is mounted between the Sennheiser audio projector and the material, and is further captured on a LeCroy oscilloscope, and then processed by a computer with IEEE488 interface. An example of some obtained temporal pulse signals with a modulation frequency fixed at 2 kHz is provided (Fig. 2), showing the influence of a simple temporal average on the raw data. Because the modulation signal is not electronically synchronized with the HF pump signal, the temporal averaging process tends to reduce the presence of the HF component and consequently will enhance the LF/noise ratio demodulation signal. This is exactly what is seen (Fig. 2), when increasing the number $N$ of temporal averages from $N=1$ (Fig. 2a) (no temporal average) to $N=10$ (Fig. 2b), and then to $N=100$ (Fig. 2c). The averaging process corresponds to some low-pass filtering procedure. For the given example, $N=100$ is amply sufficient to remove most of the ultrasonic field and to strengthen the demodulated LF audio component. In some previous work [17], we have used a very efficient electronic filtering unit (a Butterworth/Bessel multichannel filter from KROHN-
HITE Corporation), but here this step was unnecessary. The incident and reflected wave packets, as clearly visible (Fig. 2), are then separately processed in the Fourier domain. In order to compute the coefficient of reflection $R(\omega)$ versus frequency, one then needs to perform the division of the two power spectra over their bandwidth. The computation of the absorption coefficient versus frequency $A(\omega)$ is then done by using Eq. (7).

Some demodulated temporal waveforms, as a function of the modulation frequency, are shown (Fig. 3), from $\Omega = 1 \text{ kHz}$ to $\Omega = 3 \text{ kHz}$. Evidently, each wavepacket, due to its very short duration is in fact covering a much larger spectral bandwidth as illustrated (Fig. 4b), where it is shown that a wavepacket having a $\Omega = 1.5 \text{ kHz}$ central modulation frequency (as seen in Fig. 4a), in fact covers the 700 Hz to 2.5 kHz bandwidth (at – 6 dB). On the temporal waveforms of the signals of Fig. 3 it is evident to note that the relative amplitude of the reflection coefficient, ratio between the reflected wavepacket onto the incident one diminishes when increasing the modulation frequency. For instance at $\Omega = 1 \text{ kHz}$ the two amplitudes for the incident and the reflected waves are almost identical, while at 3 kHz, the reflected wave is only half the incident one. This trend is also directly seen (Fig. 4b), as the two spectral amplitudes are identical at DC (i.e. at $\Omega = 0 \text{ Hz}$), while they diverge when increasing frequency. Consequently, because of the above relationship linking the absorption to the reflection coefficient, one understands easily that absorption increases with frequency. The transient nature of the used impulses greatly facilitates the measurement of absorption coefficient in a very fast way (almost real time measurements).

In order to perform accurate measurements of the absorption coefficient, one needs to implement an automatic signal processing procedure. This is done directly on the computer by dividing the two amplitude spectra over the attainable bandwidth, and then the absorption coefficient is simply computed with Eq. (7). Some results of such a treatment are shown (Fig. 5) for a 30 mm hard-backed thick plastic foam. On the same graph are collected

![Fig. 3: Temporal waveforms recorded at various modulation frequencies for a given number of signal averaging ($N = 100$). This results demonstrates the decrease in terms of the coefficient of reflection versus frequency. (a) $f_{\text{mod}} = 1 \text{ kHz}$; (b) $f_{\text{mod}} = 1.5 \text{ kHz}$; (c) $f_{\text{mod}} = 2 \text{ kHz}$; (d) $f_{\text{mod}} = 3 \text{ kHz}$.](image-url)
various curves as explained in the caption. Measurements with a Kundt resonance tube are also provided, as well as numerical predictions with the “equivalent-fluid” model. It should be emphasized that these predictions were obtained without fitting the physical parameters of the model, i.e. porosity, resistivity, tortuosity and the characteristic lengths which were all independently measured. The other curves collected in Fig. 5 are measurements performed with the Sennheiser sound projector with numerous modulation frequencies, respectively at 1, 1.5, 2, 2.5 and 3 kHz. Most of these curves are almost superimposed meaning that the measuring procedure is reproducible. Moreover, there is a good agreement between the various measurements and numerical predictions, as well as with the Kundt tube data (even if these are a bit high). In order to tackle very low frequency measurements, we have further concentrated on the lowest modulation frequencies, by improving the shape of the incident impulse. Some optimum measurements are collected onto Fig. 6 for two modulation frequencies, this time at 1.5 kHz, and at 2 kHz. Because the data acquisition system provides data over the necessary audio range bandwidth, very low frequencies measurements are plotted this time down to 40 Hz for the very first data point. It is obvious that such very low frequency data are normally outside the bandwidth of the recorded signals meaning in turn that the achieved measurements below some frequencies are indeed done on noise instead of carrying real acoustic information. As a matter of fact, Fig. 6a demonstrates that the very low frequency data are incorrect as the coefficient of absorption does not tend towards zero at $f = 0$ Hz. There is a simple way to avoid such discrepancy, just by forcing that the two amplitude spectra of Fig. 4b superimpose at $f = 0$ Hz, meaning in turn that the absorption coefficient will be zero (as shown on Fig. 6b). This strong constraint of a zero absorption coefficient at $f = 0$ Hz is really relevant, and it shapes the curve. It is interesting to note by comparing Fig. 6a and b, that the changes due to that normalization occur mainly at low frequencies (where it is already known that some discrepancy occurs) but it is not affecting very much the higher frequency range.

4. Numerical modelling on dispersion and further discussions

As it was shown in a recent paper published in the journal *Ultrasonics* [17], presenting the method and its potential applications to the ultrasound community, it is possible to retrieve information concerning the absorption coefficient at audio frequencies (e.g. below 6 kHz) with the use of parametric sound projectors based on the Kramers–Krönig causal formalism [22–25]. One starts measuring the dispersion curve of a given material in the transmission configuration (cf. Fig. 1b), i.e. the phase velocity versus frequency. Then by fitting this dispersion curve to a simplified power law equation derived from the O’Donnel model [25], one gain access to the absorption coefficient versus frequency over the same spectral bandwidth. Some examples were
provided for a series of four automotive felts, and the obtained results were satisfactory, while not perfect [17]. We would like here to extend further this work and comment on it. Fig. 7b presents experimental data for the dispersion curves of four automotive felts (in this case the data were obtained with the previous system working with PVDF thin films), while Fig. 7a shows up some numerical predictions, obtained in the frame of the “equivalent-fluid” model [17], by using physical parameters provided in Table 1. The experimental dispersion curves where obtained by using the original Sachse and Pao phase spectrum unwrapping algorithm [26], which was slightly modified because the tested porous sample does not contact either the emitting or receiving transducers [27] as it was the case in [26] for direct contact ultrasonic testing.

When dealing with very low frequencies, one needs to write down the asymptotic limit of the effective density and the effective compressibility functions $\lim_{\omega \to 0} \rho(\omega) = \frac{\rho_0}{\rho_0 + \omega^2 / \phi}$; $\lim_{\omega \to 0} \beta(\omega) = \gamma$, which allows to obtain the LF asymptotic behavior of the complex phase velocity versus frequency in the form $c(\omega) \approx \sqrt{\frac{\omega}{\phi}}$. Accordingly, there is a square root dependance on frequency, and this point should be checked. Consequently, the dispersion curves have been plotted again in the format of $c(\omega)$ versus $\omega^{1/2}$ (Fig. 8). Normally, one should expect at low frequencies four straight lines crossing all together at the origin of the graph, with a slope proportional to $\sigma \phi^{1/2}$. This is what is seen on Fig. 8a because in this case, one deals with
numerical predictions based on the equivalent-fluid model. It is even possible to find out the upper frequency limit where the linear approximation is valid (for instance around 570 Hz for the 20 mm porous plate), and to show that such frequency limit depends on the particular plate under study. By contrast, the treatment of the experimental data along these ideas is disappointing as it is not really valid at low frequencies. There are some discontinuities occurring around 300 Hz on one hand and nearby 2.7 kHz on the upper frequency range. These limits were already visible onto the plain dispersion curves, constituting the boundaries of the attainable bandwidth (see [17] for further comments). The meaning of this analysis is very simple: when frequency is below 300 Hz, it should be hazardous to use dispersion curves data in order to recover information on absorption. Small errors in phase velocity measurements become drastic at very low frequencies where the absolute values decrease towards zero (in other words a 3 m/s error for the phase velocity in free air, that is 340 m/s, only represents 1%, but when dealing with a measured phase velocity around 30 m/s, the inaccuracy shifts up to more than 10%). Despite this limitation, we have made extra attempts to use dispersion curves in order to predict fundamental complex acoustical properties (e.g. impedance, complex reflection coefficient, propagation constant) through the causal Kramers–Kronig formalism. One could also find a similar approach recently developed by N. Sebaa in her Ph.D work done at KU Leuven to determine the resistivity from measurements performed in a very long flexible duct [28]. Some interesting work has also been done by Z.E.A. Fellah et al. with more sophisticated procedures to solve several direct and inverse problems related to the propagation of acoustical waves inside porous materials in order to retrieve some physical parameters of interest [29,30].

5. Measurements done with a portable system on some compressed porous mats

For in situ measurements, it is convenient to design and use a portable system. Such device was briefly described at the Euronoise 2006 conference [19] and also at the SAPEM 2005 conference in Lyon [20], France. We describe here that information in a more usable form. Fig. 9 provides a schematic view of the experimental set-up. The audio projector is mounted vertically approximately 1 m above the tested porous plate, in order to produce powerful plane wave orthogonal to the surface of the tested hard-backed material. When using the Sennheiser projector, the effective illuminated area of the sample (at 2 kHz and at –6 dB) is approximately a 30 cm by 40 cm rectangle, i.e. it is almost identical to the size of the projector, as the beam is collimated at least during the first 2 or 3 m during propagation. This size enlarges when the audio modulation frequency decreases, as the directivity becomes poorer. A non-capacitive audio microphone is mounted a few cm above the sample. During a preliminary calibration procedure, which

<table>
<thead>
<tr>
<th>Thickness (mm)</th>
<th>Porosity</th>
<th>Resistivity (N m$^{-4}$ s)</th>
<th>Tortuosity</th>
<th>Viscous length ($\mu$m)</th>
<th>Thermal length ($\mu$m)</th>
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<tr>
<td>20</td>
<td>0.92</td>
<td>24,000</td>
<td>1.04</td>
<td>82</td>
<td>164</td>
</tr>
<tr>
<td>15</td>
<td>0.90</td>
<td>33,000</td>
<td>1.06</td>
<td>63</td>
<td>126</td>
</tr>
<tr>
<td>10</td>
<td>0.86</td>
<td>54,000</td>
<td>1.15</td>
<td>46</td>
<td>92</td>
</tr>
<tr>
<td>5</td>
<td>0.77</td>
<td>125,000</td>
<td>1.30</td>
<td>24</td>
<td>48</td>
</tr>
</tbody>
</table>

Fig. 8. The same dispersion curves as in Fig. 7, but drawn versus the square root of frequency, the horizontal scale being given in arbitrary units, with (a) numerical predictions and (b) experimental data of the dispersion curves over the 0–3000 Hz bandwidth for four different compressed automotive felts, manufactured by Rieter. In the LF regime one expects a linear dependance of $c(\omega)$ versus $\omega^{1/2}$, as it is approximately verified.
should be done from time to time (due to slight changes of the performances of the array during some temperature increase of the audio projector when intensively used), the sample holder (in the form of a “wheel-table”) is firstly removed in order to capture the incident wave field. Next, the table is mounted back in its prior location with a rigid reflector mounted onto it. During that second step one records simultaneously the incident and the reflected field, both having the same amplitude. After this two steps procedure is done, one can perform real absorption measurements just by placing the tested porous plate onto the table (hard-backing configuration). The detected signals are recorded onto a portable computer connected to a low-cost audio card working at a 96 kHz sampling frequency. This is amply sufficient to cover the audio range, between a few 100 Hz and 6 kHz, with proper temporal resolution. The processing of the signals, in order to compute the acoustical coefficient of reflection, is done by using the LabVIEW version 7.1 software. The very same system, with appropriate calibration procedure can work either in the reflection configuration as it was described above, or in transmission. In such case, the dispersion curves can be measured, as it was outlined in the previous section.

Some measurements of the absorption coefficient versus frequency are described next. They can be done over the 100 Hz to 6 kHz bandwidth. The lowest frequency which is available is somehow related to the size of the sample compared to the wavelength, because the measurements are done in the free-field configuration. Accordingly, at 300 Hz, one should use 1 m square sample, or at least 80 cm × 80 cm plates, as the wavelength at such frequency is 115 cm. Another limitation concerning low frequency is related to the distance between the plane where the sample is mounted (i.e. the top, or hard-backing, of the supporting table) and the ground. For instance, with a regular 1 m high table, the lowest frequency is 150 Hz, because otherwise the direct and reflected wave packets at such frequency are superimposing even when using ultra-short signals (one single wavelength duration only as was shown in Figs. 2 and 3). After proper calibration procedure which consists in measuring individual contributions of the incident wave, the reflected wave onto a perfectly rigid reflector, and the same reflected wave after interaction with the tested porous material, one can obtain the absorption coefficient \( A(\omega) \) by using Eq. (7). Some significant examples are provided below, both in reflection configuration (absorption coefficient measurements), and in transmission (dispersion curves).

In order to describe the changes occurring during compression without systematically measuring all the physical parameters, one introduces some simple prediction laws. The changes of the five physical parameters, occurring during a 1D compression of a porous mat can be documented as follows. One starts from the definition of some of these parameters as given by the relationships:

\[
\frac{1}{V} \int_{\Omega} u^2 \, dV = 2 \int_{\Gamma} u^2 \, dS = 2 \frac{V}{S}.
\]

(8)

In these expressions, the volume and surface integrals are calculated over an “average” pore distribution inside an homogenization domain, containing several pores, having a volume \( V \). The microscopic velocity of the air particles is noted by \( u \). The predictions for the changes occurring during compression of some of these parameters (tortuosity, viscous characteristic length) is not trivial because they include in their definition the microscopic fluid velocities \( u \). Such dependance is eventually much easier to predict for the “geometrical” parameters (porosity, thermal characteristic length). Some measurements, done with ultrasonic techniques [31,32], did show that during compression the tortuosity slightly increases, following a simple law of variation, as noted in the following equation (see Appendix for further justification):

\[
\chi^{(n)} = 1 - n(1 - \chi^{(0)}),
\]

(9)

where \( n \) denotes the ratio of thickness change (or compression ratio) of the compressed material, i.e. \( n = h_0/h_n \), where \( h_n \) is the thickness after compression and \( h_0 \) is the reference initial thickness. In Appendix, some calculations are given for an expansion (instead of a compression), with \( n = h_n/h_0 = nh/h_0 \), but the reasoning is somewhat identical.

An analogous equation exists for the porosity, that could be easily derived by some simple calculations [31], in the case of an unidirectional compression:

\[
\phi^{(n)} = 1 - n(1 - \phi^{(0)}),
\]

(10)

in such case porosity evidently decreases during compression.

Moreover, one could also derive with some simplified assumptions the following relationships for the characteristic lengths and the resistivity [32,33] (also see Appendix for further details):

![Fig. 9. Schematic of the used set-up. 1: audio sound projector; 2: mounting frame; 3: audio microphone; 4: poroelastic plate; 5: removable wheel-table; 6: connection towards audio acquisition card and portable computer.](image-url)
\[ A'(\alpha) = \frac{A_0'}{n} \quad \text{and} \quad A(\rho) = \frac{A_0(\rho)}{n} \quad \text{and} \quad \sigma(\rho) = n\sigma_0, \] (11)

The result for the thermal characteristic length \( A' \) can be approximately derived (cf. [31] for some simplified geometrical configurations and distributions of the fibres). The formula dealing with the resistivity \( \sigma \) is obtained by extending to 1D compression some theoretical calculations made by Tarnow [34,35]. It should be emphasized that some incorrect law of variations for the characteristic lengths were published in *Applied Acoustics* by the first author [31], with an inverse square root dependence versus compression ratio instead than the inverse linear law of Eq. (11), cf. Appendix for further discussions and justifications.

This last trend is simply related to the change of size pore which is indeed inversely proportional to the 1D compression ratio, as can be seen from the change in average distance between neighbouring fibres at least along the axis of compression. Obviously, these trends should rigorously apply only when the porosity of the porous mats remain close to one. In the opposite case, for instance with compressed granular media having a small porosity, such behavior, as described through Eqs. (9)–(11), should not rigorously apply [36].

With that portable system using the Sennheiser audio projector, original data are presented as well in the reflection and in the transmission configurations (cf. Fig. 1). Absorption coefficient measurements are shown (Fig. 10) on a 30 mm hard-backed thick plastic foam material manufactured by Tramico. The experimental data are compared to numerical predictions made with the standard “equivalent-fluid” model. For this case, the five physical parameters (porosity, resistivity, tortuosity, viscous and thermal characteristic lengths) were measured independently with dedicated benches, and no numerical fit was needed between the experimental data and the numerical predictions. Similar experimental data are shown (Fig. 11) for a 15 mm thick felt manufactured by Rieter. The numerical predictions are done in the same way, but here the physical parameters were not measured, and instead we have simply used the best numerical fit in order to obtain the physical parameters collected in the caption.

Figs. 12 and 13 deals with a complete set of some glass–wool materials, which have been compressed by keeping unchanged their basic (or surface) weight. There are five different thicknesses from 25 to 5 mm, with 5 mm steps. For the most compressed material (i.e. the 5 mm thick plate), the experimental measurements are tricky to obtain because the absorption coefficient becomes quite small (around or below 0.5 at a few kHz). This might explain some of the observed oscillations in Fig. 13a which are mainly due to diffraction effects on the sides of the sample. The tested plates were 1 m \( \times \) 1 m squares, and edges waves on their sides could produce interferences with the direct incident wave. Fig. 13b shows the numerical predictions for the very same plates. The only change in the numerical treatment (beyond the thickness change) deals with the modifications of the five physical parameters. Starting from the best fit, as done with the 25 mm glass–wool, they are calculated with the help of Eqs. (9)–(11), and they are collected in Table 2 (or alternatively, in some cases they are numerically estimated in order to provide the very best fit with the experimental data). The comparison between experimental data and predicted values of the absorption coefficient, is generally fair, becoming excellent in some cases. Some additional data obtained with a Bruël and Kjaer resonance tube were documented on the very same materials and published separately [19], showing again very
good agreement (also see Fig. 12 for these Kundt tube data given here on two thicknesses only).

Next, we have measured the phase velocity versus frequency for various materials, by using the phase spectrum unwrapping method [26]. Fig. 14 shows the dispersion curve over the 0–2500 Hz bandwidth for a Tramico 20 mm thick plastic foam. The triangles are numerical predictions, while the other symbols are different measurements done on the same plate by slightly changing the electronic settings of the instrumentation. In other words, this attempt constitutes some reproducibility tests done on the experimental set-up. On these curves, one can notice that the very first data is obtained at a very low frequency. In this example it is around 40 Hz, but in some cases, such measurements were achieved down to 20 Hz or even below. Again, as it was outlined in [17], the phase velocity at such very low frequency is very small, here at 40 Hz in the range of 15 m/s, a value which is 20 times smaller than the speed of sound in the free air. Theoretically, the speed of sound is zero at \( f = 0 \) Hz. We have obviously a similar trend with the coefficient of absorption, and the Kramers–Kronig relationships should somehow predict this result. Next, on Fig. 15, the dispersion curves are plotted for a complete set of felt materials manufactured by Rieter automotive, starting with a 20 mm thick plate, down to 5 mm thickness (i.e. a compression ratio at 4, and by keeping the initial basic weight unchanged). Fig. 15a shows the experimental data, while Fig. 15b plots the numerical predictions obtained with the “solid frame” assumption (i.e. using the equivalent-fluid model). Table 3 contains the values of the physical parameters, computed with the help of Eqs. (9)–(11), which have been used for such simulations. The agreement between the two series of plots is excellent showing that the rigid frame model is amply sufficient for such case contrarily to the claim of some authors which argued that the full Biot model should be used instead, as
recently published in *Applied Acoustics* [37] (that work which extends to the Biot theory is quite interesting on the principle however). This very last result should be compared to Figs. 7 a and b, on the same materials, published previously [17], but with a different audio projector, that is with the HSS device (from Ultrasound Technology) providing a 500 Hz to 2.5 kHz bandwidth for a 1.5 kHz central frequency of the used pulse [17]. As a matter of fact, the dispersion curves obtained with that system did show (see Fig. 7 and comments in [17]) some discontinuities which were related to the drop of efficiency of the demodulation process of the HSS audio projector at these frequencies and beyond. With the new Sennheiser system, these accidents are avoided and instead we get smooth dispersion curves. However, the 5 mm compressed felt material has a complicated behaviour, and there still exist some slope changes around 3.5 kHz and beyond, a frequency limit where the efficiency of the system starts to significantly decrease. For the 5 mm plate, it is not obvious to use the compression ratio transformation laws of Eqs. (9)–(11) because the compression ratio is already high (4 in this case, the reference thickness being 20 mm), and also we know that in such case, the compressed material is denser on both surfaces compared to the central core. Accordingly, due to the manufacturing process, the sample should somewhat be considered as a three-layer structure instead of a single homogeneous material.

### 6. Conclusion and perspectives

The described results constitute the very first report on parametric demodulation of ultrasound in air with the objective to achieve very fine measurements of the absorp-

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**Table 2**

<table>
<thead>
<tr>
<th>Thickness (mm)</th>
<th>Porosity</th>
<th>Resistivity (N m$^{-4}$s)</th>
<th>Tortuosity</th>
<th>Viscous length (µm)</th>
<th>Thermal length (µm)</th>
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<td>100,000</td>
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<tr>
<td>5</td>
<td>0.90</td>
<td>200,000*</td>
<td>1.25</td>
<td>20</td>
<td>40</td>
</tr>
</tbody>
</table>

* The resistivity for the 5 mm plate should be larger than the predicted value, as explained in the text.
tion coefficient and dispersion curves in poroelastic materials in the audio range. The covered bandwidth is very large extending between 100 Hz and 6 kHz. The only strong limitation of the technique when working with a 40 kHz pump transducer is the demodulation distance which becomes large, in the order of one meter. The achieved results reported in the present work are very promising. They demonstrate that fine metrology of the coefficient of absorption is possible for free-field configurations in porous media by using parametric demodulation of strong ultrasonic fields. These measurements are difficult to perform with standard audio techniques involving standard loudspeakers and microphones, unless advanced acoustic instrumentation is used. The main advantage of the described technique deals with the simplicity of the experimental set-up which uses a single ultrasonic pump transducer, and one unique metrology audio microphone.

Applications of the present work are possible in many different directions, for “in situ” and “on-line” measurements. Even if only feasibility work so far has been initiated along these directions, this approach will provide ample opportunities for monitoring the quality of cellular and fibrous porous materials in real-time directly over the production lines. No other tool is presently available for that purpose. Additionally, the detection of defects, lack of adhesion (of the porous medium mounted onto a hard-backing plate), inhomogeneity and void detection, acoustic leak detection and other ancillary difficult problems could be eventually tackled by the emerging technique when it will be sufficiently adapted for routine measurements.

Appendix. Invariance for some ultrasonic indicators and compression laws for tortuosity, viscous and thermal and characteristic lengths

It is easy to demonstrate the relationship (A1) between the temporal delay $\Delta t = t - t_0$ due to the presence of the tested sample and the phase velocity $c(\omega)$, of the sound waves inside it. In such expression, $t_0$ represents the time of flight of the pulse when no sample is present, while $t$ is the same quantity when the sample of thickness $h$ is mounted between the emitting and the receiving transducers. Accordingly, these temporal quantities are simply given by the following expressions, $t_0 = \frac{L_0}{c_0}$, and $t = \frac{L_0 - h}{c_0} + \frac{h}{c(\omega)}$, where $L_0$ is the fixed distance between transducers, $c_0$ the speed of sound in the free air (when no sample is mounted). From these equations and with the definition of the time delay $\Delta t$, it is easy to obtain $c(\omega)$ in the form:

$$c(\omega) = \frac{c_0}{1 + \frac{\Delta t}{\pi c_0}}. \tag{A1}$$

Because $\Delta t$ is in fact depending on frequency, $c(\omega)$ is also a function of frequency (the so-called dispersion curve). When the temporal delay is zero, one retrieve the speed of sound in the free air. Consequently, and because in air-saturated porous media the phase velocity $c(\omega)$ is always smaller than $c_0$, the time delay $\Delta t$ is a positive quantity corresponding to a true delay (and not to an advanced time).

Next, tortuosity is defined by its asymptotic high frequency limit $\kappa = \lim_{\omega \to \infty} \left[ \frac{c_0}{c(\omega)} \right]^2$. By the way, this is because of this definition that high frequency methods (i.e. ultrasonics) are used to determine tortuosity. This limit is evidently larger than one because $c_0 > c(\omega)$. Consequently, one can write $\kappa = 1 + \varepsilon \simeq 1 + \frac{\Delta t}{\pi c_0}$, for a porous layer of thickness $h$, as in most cases $\varepsilon = \frac{\Delta t}{\pi c_0} \ll 1$ (or say in other terms, the tortuosity is usually slightly larger than one, as it is seen for all the materials tested in the present study). If we now imagine the same material, which is here expanded (and not compressed) from the initial thickness $h$ towards $nh$ with $n > 1$, one can use a similar expression in order to write $\kappa = 1 + \frac{\Delta t}{\pi c_0}$. By comparison of such expression with the one already written for $\kappa = 1 + \frac{\Delta t}{\pi c_0}$, one easily derive the following equation:

$$\kappa_n = 1 + \frac{1}{n} (\kappa_n - 1). \tag{A2}$$

This result is identical to Eq. (9) when a compression is considered instead of an expansion, by just changing $n$ by $1/n$. In the above reasoning, we made explicit that the time delay $\Delta t$ is invariant during compression (or during expansion). By the way this is exactly what is experimentally observed, as long as the basic weight of the compressed plates is unchanged, or say differently as long as the same average number of fibres or pores is crossed by the sound field. Evidently, as was emphasized above, $\varepsilon$ should be small, and this relies on a moderate compression (or expansion ratio).

Now, let us consider transmission coefficient invariance during compression. The physical argument for such invariance is basically identical to the temporal delay invariance, that is the same quantity of pores (or fibres) interact with the sound waves (in the form of ultrasound here because we deal with the high frequency asymptotic behaviour). As a matter of fact, the HF limit of the coeffi-

<table>
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<th>Viscous length (µm)</th>
<th>Thermal length (µm)</th>
</tr>
</thead>
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<td>100</td>
</tr>
<tr>
<td>5</td>
<td>0.80</td>
<td>140,000</td>
<td>1.30</td>
<td>25</td>
<td>50</td>
</tr>
</tbody>
</table>
cient of transmission is given by the following expression [38]:

\[
\lim_{\omega \to \infty} \ln T(\omega) = \frac{\alpha}{\epsilon_0} \sqrt{\frac{2}{\pi \rho c}} \sqrt{\frac{\delta}{2} + \frac{\gamma - 1}{\sqrt{\rho c}} h},
\]

(A3)

where \( \delta = \sqrt{\frac{2 \rho c}{\rho_\infty}} \) is the viscous skin depth. Because \( \lambda' = 2 \) or \( 3 \lambda' \), and because tortuosity change is small as long as the compression (or expansion) is limited (see above discussion), then the invariance of the coefficient of transmission induces that \( h/\lambda' \) should be constant, that is:

\[
\frac{h}{\lambda_b} = \frac{n h}{\lambda_{bh}} \Rightarrow \frac{\lambda_{bh}}{\lambda_b} = n \lambda_{bh},
\]

(A4)

Again by considering a compression instead of an expansion, Eq. (11) for the characteristic lengths is obtained. As explained in the text, these relationships are formally approximate, because in the definition of these quantities (tortuosity and viscous characteristic length) there is the influence of the particle velocity \( u \). Accordingly on a more theoretically basis, these transformation laws should be approximate. Nevertheless, as long as the time delay invariance applies, then Eq. (A1) is valid, and as long as the coefficient of transmission invariance is verified, then Eq. (A4) is meaningful. A last point is related to fine measurements of the viscous characteristic length with standard ultrasonic techniques [39,40]. Eq. (A4) has been systematically observed on experimental data for many fibrous materials (glass–wools, recycled felts, etc.) in our lab during the last 10 years (e.g. see [31–33]). A significant example is provided here with the felt materials manufactured by Rieter as shown on Table 1, where the Eq. (A4) is approximately verified, even if the plates were different as felt materials are generally quite inhomogeneous (in the range of a few 10 % for these physical parameters).

References


