I. INTRODUCTION

The propagation of sound in acoustic materials, i.e., mainly various foams and felts which are air saturated, is under active study at numerous institutions. A recent reference book on the subject reviews some of the state-of-the-art theoretical models which are currently used. An important distinction has to be made depending on the nature of the displacement field in the solid frame. In the case of a rigid, homogeneous, and isotropic frame, two frequency-dependent response factors, $\sigma(\omega)$ and $\beta(\omega)$, must be evaluated in order to model sound propagation. Inertial and viscous effects are expressed in the effective density $\rho_0 \tilde{\sigma}(\omega)$ such that, for harmonic motion,

$$\rho_0 \frac{\partial v}{\partial t} = -\nabla p,$$

where $\rho_0$ is the ambient fluid density, $v$ is the pore-volume average of the fluid velocity, and $-\nabla p$ is the macroscopic pressure gradient. Thermal effects are expressed in the dynamic compressibility $(1/K_a)\beta(\omega)$ such that

$$\frac{1}{K_a} \frac{\partial p}{\partial t} = -\nabla \cdot v,$$

where $K_a$ is the adiabatic bulk modulus of the fluid. Johnson et al. analyzed the general properties of the response factor $\tilde{\sigma}(\omega)$ and demonstrated that it is conveniently reproduced for a wide range of geometries by means of a simple scaling function. This scaling function presents the exact asymptotic behavior at high and low frequencies. In some cases, however, when the frame is not rigid, the full Biot theory has to be considered. Other applications of the Biot theory exist in the field of geophysics.

II. ANISOTROPY OF ACOUSTICAL MATERIALS

Porous materials can be, to some extent, anisotropic. The propagation of acoustic waves in these materials is quite intricate. An indirect way to characterize such anisotropy was recently achieved by measuring the coefficient of reflection versus incidence angle for reticulated plastic foams. Whereas noticeable discrepancies exist between the experimental curves of the reflection coefficient versus incidence angle and the theoretical description given by the isotropic Johnson-Allard theory, a good agreement between experimental and theoretical reflection curves has been obtained when the anisotropy of the relevant parameters was taken into account. In this paper, other interesting features are reported for the wave speed and attenuation measurements.

The amount of anisotropy observed in reticulated plastic foams is slight, in the order of 10%-20%. In fact, these materials are also heterogeneous, and the acoustic properties depend on the location which is probed. The observed variations of the phase wave speed and transmission coefficient at normal incidence are again in the range of 5%-20%.

Along any principal axis of the material we may use the simple equations (1) and (2). The propagation will be characterized by the complex refractive index $n = \sqrt{\tilde{\sigma}(\omega) \beta(\omega)}$. Experimentally, a simple phase wave speed measurement gives the propagation index $n = \text{Re}(n)$

$$n_r = \frac{C_0}{C_s(\omega)} = \text{Re}(\sqrt{\tilde{\sigma}(\omega) \beta(\omega)}),$$

where $C_0$ is the adiabatic speed of the sound in the fluid and $C_s(\omega)$ is the frequency dependent phase wave speed in the same fluid when it saturates the material. This feature is well documented for water-saturated media such as rocks or ceramics. These authors made use of a routine based on the phase spectrum. The influence of dispersion can be drastic in some cases with water-saturated samples. Air-saturated porous materials, however, exhibit slight dispersion.

Measurements were done by probing the propagation index versus angle. A simple numerical routine has been recently reported by J. F. Allard, B. Castagnide, M. Henry and W. Lauriks [Rev. Sci. Instrum. 65, 754 (1994)]. Further experimental work related to the anisotropic nature of these materials has been done by measuring the coefficient of reflection versus incidence angle for reticulated plastic foams. Air-saturated porous materials are also heterogeneous, and the acoustic properties depend on the location which is probed. The observed variations of the phase wave speed and transmission coefficient at normal incidence are again in the range of 5%-20%.

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The
TABLE I. Values of $n^2$, the squared propagation index, for the Tramico plastic foam measured independently along two principal directions (in-plane and out-of-plane) at various frequencies. Tortuosity, $\alpha$, was calculated from these values of $n^2$ using the Johnson-Allard theory. These measurements, and those presented in Figs. 2 and 3, were all made on the same sample.

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>$n^2_{\text{out-of-plane}}$</th>
<th>$n^2_{\text{in-plane}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>1.23</td>
<td>1.33</td>
</tr>
<tr>
<td>75</td>
<td>1.21</td>
<td>1.32</td>
</tr>
<tr>
<td>180</td>
<td>1.20</td>
<td>1.31</td>
</tr>
<tr>
<td>Tortuosity, $\alpha$</td>
<td>1.17</td>
<td>1.30</td>
</tr>
</tbody>
</table>

The variation of the functions $\tilde{\alpha}(\omega)$ and $\tilde{\beta}(\omega)$ with frequency is generally small at ultrasonic frequencies and $n_\varepsilon$ assumes the form

$$n_\varepsilon = \sqrt{\alpha_c (1 + \epsilon)}$$

where $\epsilon \ll 1$, $\epsilon$ being proportional to $1/\omega$. An expression for $\epsilon$ can be obtained from the Johnson and Allard scaling functions in terms of the ratios between the viscous or thermal skin depth and the characteristic pore-geometry length $\Lambda$ or $\Lambda'$. The tortuosity $\alpha_c$, defined as the limit as frequency tends to infinity of the dynamic tortuosity $\tilde{\alpha}(\omega)$, is also the limiting value of the squared index $n^2_\varepsilon$ because $\tilde{\beta}(\omega)$ tends to one in this limit (adiabatic processes). In the present work, the important parameter $\alpha_c$ has been easily obtained by a regression method from several measurements of $n_\varepsilon$. Some characteristic results obtained from a reticulated plastic foam are given in Table I. Values of $n^2_\varepsilon$ are reported along different directions, the out-of-plane direction 3 normal to the surface of the sample, and the in-plane direction 1 which lies in the plane of the sample. The material is assumed to be transversely isotropic, and, as seen later, direction 3 practically coincides with the principal distinguishable axis $Z$, whereas direction 1 practically coincides with one ($X$) of the freely rotatable axes. The results along the two "principal" axes 1 and 3 are different. For the material tested here, the squared propagation index $n^2_\varepsilon$ at 40 kHz, is within 5% of the tortuosity $\alpha_c$ in the corresponding direction.

For a given direction $\theta$ of the unit wave normal $\mathbf{s}$ ($s_\parallel = \sin \theta$, $s_\perp = \cos \theta$) it can be shown that the propagation index $n_\varepsilon$ is given by the relation (analogous to Fresnel's relations in optics)

$$\frac{1}{n^2_\varepsilon} = \left( \frac{C_\varepsilon(\omega)}{C_0} \right)^2 = \frac{\cos^2 \theta}{n^2_{tr}} + \frac{\sin^2 \theta}{n^2_{sr}}$$

where $n^2_{tr}$ and $n^2_{sr}$ are the principal propagation indices. This relation is justified in Appendix A.

A very simple, but efficient, inversion scheme has been implemented which makes use of a linear regression routine in order to determine the principal propagation indices from measurements of the propagation index performed along numerous non-principal propagation directions. The recovered values can then be directly compared to those contained in Table I.

### III. EXPERIMENTAL RESULTS

Experiments were carried out on a large panel (0.3 m x 0.4 m) of a 50-mm-thick layer of a reticulated plastic foam manufactured by Tramico S.A. (France). The measurements were performed by using an ultrasonic reflecto-refractometer recently designed at LAUM. This is a versatile system which enables measurements to be made in the reflection configuration as well as in the transmission mode. Hence, the coefficients of reflection, transmission, attenuation, and the propagation index versus incidence angle can be measured. We are concerned here with values obtained from the latter. The configuration of the instrument which has been used in the present work is outlined in Fig. 1. Because of the very weak signals obtained after transmission, a Panametrics 5058PR pulser-receiver which provides 900 V peak-to-peak pulses and a 90 dB dynamic range was used. The signals captured by a LeCroy 9310 digitizing oscilloscope were processed by a Macintosh Quadra 950 computer running LabVIEW 3.0. A pair of 40 kHz narrow-band piezoelectric transducers insonified the sample. The phase wave speed measurements were performed in the Fourier domain by using a standard intercorrelation routine. A precision in the order of 0.1% was achieved. Due to the spatial shift that occurs when the ultrasonic beam crosses the sample at nonzero incidence, Snell–Descartes' law of refraction was used in order to position the receiving transducer.

A polar diagram of the squared propagation index $n^2_\varepsilon$ of a Tramico air-saturated sample is shown in Fig. 2. The agreement between theoretical and experimental data is quite acceptable. The recovered principal values, i.e., $n^2_{tr} = 1.22 \pm 0.01$ and $n^2_{sr} = 1.34 \pm 0.01$, are within 1% of measurements taken at normal incidence on samples cut along in-plane and out-of-plane principal directions, as given in Table I. It should be pointed out that the small discrepancy between the two sets of values could be due to the fact that, although the same sample was used, the measurements were not taken from exactly the same areas of the sample.

An interesting finding is that the principal acoustic axes of the sample ($Z,X$) do not coincide with the geometric ones (3,1) (i.e., normal to and in the plane of the sample). The inversion routine applied to the tortuosity predicts a 4° difference between these axes for tortuosity, as shown in Fig. 2. Such asymmetry is also observed in the measurements of the amplitude of the transmitted signal as shown in Fig. 3. Its
Fig. 2. Polar plot of the squared propagation index $n^2_z$ as a function of the angle for a reticulated plastic foam. Experimental data (□) with theoretical regression curve (solid line). Axes 1 and 3 are the in-plane and out-of-plane geometric axes of the sample, respectively. The principal acoustic axes of the sample, X and Z, which are almost aligned with axes 1 and 3, are shown with solid lines. There is a 4° difference between the two sets of axes. The recovered principal values are $n_{xz}^{\text{exp}} = 1.22 \pm 0.01$ and $n_{yz}^{\text{exp}} = 1.34 \pm 0.01$. A measurement (□) done at normal incidence along the in-plane axis 1 (see Table I) shows good agreement with the regression curve ($n_{xz}^{\text{exp}} = 1.33 \pm 0.01$).

maximum being shifted by 9°. Consequently, this technique enables the orientation of the principal acoustic axes of the sample to be determined. Obviously, there might exist additional Euler angles (up to 2) when principal axes are oriented in totally arbitrary directions. In such a case, one needs to probe various nonprincipal planes of propagation in order to recover the orientations of the principal axes.13 Such inversion routines have not been implemented so far. Interestingly, we have observed other results where the angular parallax is more pronounced (in the range of 10° to 20°).

Some porous materials have an almost isotropic propagation index. An example is provided for a fibrous glass wool in Fig. 4, where the polar plot of the squared propagation index as a function of the angle of incidence is a circle. The recovered principal values are $n_{xz}^{1.4 \text{kHz}} = n_{xy}^{1.4 \text{kHz}} = 1.05 \pm 0.01$. This result is also a general check of the implementation of the signal processing and data analysis schemes.

IV. CONCLUDING REMARKS

Experimental results concerning the angular dependence of the propagation index have been reported. From the measured values of this index, the use of a regression routine enables the determination of the components of the tortuosity tensor. These measurements use standard ultrasonic procedures and can be performed by any laboratory having the proper instrumentation. Due to the very strong damping of ultrasound in the foam, in the range of several 10 dB/cm, the characterization of thick samples is possible only when high voltage pulses, here 900 V, are used. The anisotropy of the tortuosity is small but discernible. Other parameters of the frequency-dependent tortuosity tensor model will exhibit some anisotropy as well. Slight deviations of the principal acoustic axes from the geometric axes have been observed for the tortuosity and the transmission coefficient. These misalignments are related to the manufacturing process of the porous panels. Ultrasonics may prove to be an invaluable tool to monitor the anisotropy of acoustic properties in air-saturated porous materials.

ACKNOWLEDGMENT

The authors would like to acknowledge Laurent Guéry from TRAMICO S. A. (Brionne, France) for supplying the samples of reticulated plastic foams.

APPENDIX A

When considering a homogeneous anisotropic material, Eq. (1) must be written in tensorial form

$$\rho_0 \tilde{a}_{ij}(\omega) \frac{\partial v_j}{\partial t} = - \nabla p,$$

(A1)

whereas Eq. (2) is unaffected. Neglecting viscous losses, $\tilde{a}_{ij}$ is a real symmetric tensor. This important symmetry property
can be shown by using energy considerations completely analogous to those used in electrodynamics for demonstrating the symmetrical character of the dielectric tensor $\varepsilon_{ij}$ of a medium. Thus, the tortuosity tensor diagonalizes in a system of three orthogonal axes—the so-called principal axes. In the general case where viscous losses are present, the symmetry of the complex tensor $\tilde{\alpha}_{ij}(\omega)$ is maintained and can be viewed as an example of the general Onsager’s reciprocity rules. However, the real and imaginary part of the tensor $\tilde{\alpha}_{ij}(\omega)$ need not diagonalize in the same system of principal axes. Moreover, the principal axes themselves may be frequency dependent. Such dispersion of the principal axes does not occur, a priori, for transversally isotropic materials such as the ones studied in this paper. In addition, ultrasonic measurements lie in the high-frequency regime where the imaginary part of the tensors $\tilde{\alpha}_{ij}(\omega)$ need not diagonalize in the same system of principal axes.

Let us consider a monochromatic plane wave propagated along the direction of the unit wave normal $s$, i.e., described by the factor $\exp\{i \omega [(n/c) \cdot s \cdot r - t]\}$. Substituting this dependence in (A1) and (2) we obtain

$$\rho_0 \tilde{\alpha}_{ij} \nu_j = -\frac{n}{c} s_i \nu_j; \quad \tilde{\beta} \rho = n \nu_i. \quad \text{(A2)}$$

Eliminating $p$ in the two equations (A2) and making use of coordinate axes coincident with the principal axes of the tensor $\tilde{\alpha}_{ij}(\omega)$ gives the three equations

$$\nu_k = \frac{n^2}{\tilde{\beta} \tilde{\alpha}_k} s_k s \cdot \nu \quad (k = x, y, z). \quad \text{(A3)}$$

Multiplying (A3) by $s_k$ and adding the three resulting equations we obtain

$$\frac{1}{n^2} = \frac{s_x^2}{n_{rx}^2} + \frac{s_y^2}{n_{ry}^2} + \frac{s_z^2}{n_{rz}^2}, \quad \text{(A4)}$$

where $n^2_k (k = x, y, z)$ have been defined by analogy with the isotropic case, such that $n^2 = \tilde{\alpha} \tilde{\beta}$. Let us now consider the above complex index which assumes the form $n = n_r (1 + i \epsilon)$, where $\epsilon \ll 1$, and similar expressions for each of the $n_r$. In effect, based on the Johnson et al. and Champoux-Allard high frequency limits for $\tilde{\alpha}$ and $\tilde{\beta}$, typical values for our materials are $\epsilon \approx 0.05$ at 40 kHz. Thus from (A4) and the definition of the phase velocity $C_r(\omega)$ we get

$$\frac{1}{n^2_r} = \left( \frac{C_r}{C_0} \right)^2 = \frac{s_x^2}{n_{rr}^2} + \frac{s_y^2}{n_{ry}^2} + \frac{s_z^2}{n_{rz}^2} + o(\epsilon^2). \quad \text{(A5)}$$

14. D. Lafarge, a forthcoming paper will discuss this point in more details.