Observation of nonlinear interaction of acoustic waves in granular materials: demodulation process

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Abstract

Experimental study of the demodulation effect of ultrasonic waves in glass beads are reported. Both impulsive (burst) and continuous regime have been studied for amplitude modulated signals with its carrier frequency of 100 kHz. It is shown that two low frequency acoustical modes corresponding to the modulation frequency (in the range of few kHz) are excited due to the nonlinear processes in the granular medium. The excitation of a first mode propagating mainly through the beads (solid-based mode) and the excitation of a second mode propagating mainly through the air saturating the granular material (fluid-based mode) are observed. Possible physical mechanisms of generation and regimes of propagation in the granular assemblages are discussed. © 2001 Elsevier Science B.V. All rights reserved.

1. Introduction

There is a rising interest to the application of the nonlinear acoustics methods for the nondestructive evaluation of the nonconsolidated (e.g., sand and beads) and consolidated granular materials (including rocks) [1–4]. The principal goal is extracting the information on the material properties from the acoustic signal excited due to frequency-mixing nonlinear processes. The other possible application of the nonlinear effects might be the creation of the parametric transmitters in granular media. The advantages of similar transmitters in underwater applications (parametric antennas) are well known [5]. However, the creation of the parametric antenna for the underground applications, for example, is not a straightforward extension of the existing underwater antennas principles, because of the inhomogeneity of the geological structures. This so-called microinhomogeneity [2] (the mesoscopic character [3]) of rocks and sands introduces specific physical mechanisms of the nonlinear interaction of the acoustic waves (such as Hertzian contact nonlinearity [1,6] and hysteretic nonlinearity [3,4,7]) which are absent in homogeneous substances as water.

The investigation of the excitation of the low-frequency (LF) acoustic waves due to the interaction of two high-frequency (HF) waves in consolidated and nonconsolidated granular materials is just starting [1, 6,8]. Two years ago the observation of the demodulation of the acoustic bursts in sand was reported [1],
where one LF acoustic mode in the granular structure had been detected [1]. In the present Letter we report observations of two LF modes in the glass beads assemblage. In addition to the generation of the solid-based mode, the nonlinear excitation of the fluid-based mode propagating mainly through the air saturating the glass beads assemblage has been documented for the first time.

2. Experimental observations

Registration of demodulation effect has been performed on 1.4 mm glass beads mounted in a cylindrical tube having 60 cm length and 15 cm diameter. A schematic of the used set-up is shown in Fig. 1. An ultrasonic wide-band transducer which is mounted at the bottom of the cylinder can be driven in four regimes at 100 kHz: two continuous (with low frequency amplitude modulation and without) and two burst (modulated in amplitude or with simple rectangular window). Reception of the signals is provided by two PVDF sensors implanted in the beads (each having $1 \times 2$ cm surface) at 17 and 37 cm from the transducer. There is also a high-sensitivity microphone that is placed in air just above the free surface of the beads. Other equipment of the set-up is standard and precised at the caption of Fig. 1.

Nonlinear demodulation effect itself can be demonstrated by simply digging the transducer in the beads (whatever is the container of the beads) and feeding the transducer with a continuous sinusoidal signal at the high frequency well above the hearing limit. As soon as the HF signal is amplitude modulated by a LF audio range sinusoidal component, one can hear a sound tone which corresponds to the modulation frequency. If modulation of HF is then disabled or if the emitting transducer is taken out of the beads, no more sound is heard. An alternative way to observe the effect is to deal with the excitation of the emitting transducer by a burst, which is modulated or not. “Burst” is understood here as a sinusoidal signal with temporal rectangular window. It allows recording the waveforms and performing precise wavespeeds measurements.

There are two different modes propagating in the medium. The first one is so-called solid-based mode with its energy mainly located inside the beads.

Fig. 2(a) presents a characteristic waveform recorded by the PVDF sensors. The transducer excitation signal here is a burst at 100 kHz central frequency which is 100% modulated in amplitude at 3.5 kHz. The duration of the burst is chosen to be equal to 5 periods of the LF modulation. This signal is then low-pass filtered (with a cut-off at 30 kHz) which results in the noise-free waveform of Fig. 2(b). This waveform clearly demonstrates the demodulation effect occurring in the glass beads: there is a well seen burst having a 3.5 kHz central frequency. The unfiltered signal, shown in Fig. 2(a), provides some temporal reference. This signal presents a mixture of the demodulated acoustical signal with unwanted electromagnetic high frequency parasite generated by the power amplifier (Fig. 1, item 4) and picked up by the PVDF sensor acting also as a radio antenna. Amplitude of both signal and parasite are of the same order, but their frequency domains are strongly different. The parasite repeats the waveform of the transducer excitation HF signal, thus having its spectrum centered around 100 kHz. The first peak in Fig. 2(a) corresponds to the
onset of modulation and may be used as a reference to estimate delay time of the front of the acoustical signal in Fig. 2(b). Additionally, one can notice that the first few maxima in Fig. 2(a) are symmetrical corresponding to the shape of the modulated burst, fed to the transducer. The following ones are nonsymmetrical due to the arrival of the LF demodulated component.

The precise wavespeed measurements of the solid-based mode was done with the help of a second PVDF sensor embedded in the beads further from the emitting transducer at 20 cm from the first one. Waveforms from the second PVDF sensor are very similar to the signals previously discussed, and they are shown in Fig. 3. The very beginning of the filtered acoustical signals is then used each time as a reference, enabling to perform accurate time-delay measurements between both PVDF sensors and then to determine the corresponding wavespeed. From these pictures, the measured velocity of the acoustical wavespeed in the glass beads is around 190 m/s. Further measurements of speed variations were also done by mechanically loading the column showing, as expected, that the solid mode is getting faster with load and that hysteretic phenomenon is present (Fig. 4 and next section for further discussion and interpretation). Extra information could also be retrieved from the analysis of the complete shape of the signals: number of peaks, their amplitude, etc. The present study does not go deeper along that direction because the shape of the LF signals is significantly influenced by the geometry and limited size of the beads container. At 3.5 kHz, the LF wavelength is just three times less than the diameter of the cylinder, and accordingly great caution should be taken in order to interpret the detailed shape of the recorded signals.

The second mode is air-based. It has been carefully monitored by mounting a PVDF sensor and a microphone both very close to the upper surface between the beads and air, in this study at 37 cm away from the emitting transducer. The PVDF sensor was placed just below the surface to still be contacting the beads, while the microphone was hung above the beads to avoid any contact with them. For that specific experiment, an unmodulated burst (having 30 periods at 100 kHz) was fed to the emitting transducer in order to get a short LF acoustic pulse in the medium. The demodulation effect in this case occurs over the burst rectangular window. The moment of excitation is depicted in Fig. 5(a) as a large black strip, whose origin is again an electromagnetic parasite. The following delayed signal is the acoustical response, which is also presented more clearly after low-pass filtering in Fig. 5(b). The delay-time between the onset of the parasite and the front of the acoustical pulse corresponds to the propagation time over the distance from the emitting transducer to the receiving PVDF sensor. The air-based mode, recorded by a Brüel–Kjaer microphone, is shown next in Fig. 5(c). In all Figs. 5(a)–(c), the time scale and the absolute delays during waveform recording were kept unchanged, thus allowing...
relative and precise time-delays comparison between them. Accordingly, the wavespeed for the air-based mode is significantly larger than those of the solid-based mode. Its wavespeed is around 280 m/s, a number which is in very good agreement with prediction of the Johnson–Allard “fluid equivalent model” (see further details in the next section). The same glass beads material was previously tested by Allard et al. [9] in order to study the air-based mode only with the background of the linear “equivalent fluid” theory [10,11].

3. Discussion

Though a detailed quantitative theory has still to be developed in parallel with the conduction of the qualitative and quantitative experiments some preliminary discussion can be proposed based on the analysis of the different existing theories and experiments. We start from the semilinear Biot theory for the rigid-framed porous materials [12,13]:

\[
L_{sf}(\vec{u}, \vec{U}) = \rho_{11}\ddot{u}_i + \rho_{12}\ddot{U}_i - C_{ijkl}u_{l,j} - Q_{jj}U_{i,jj} = N_{sf}(\vec{u}, \vec{U}),
\]

\[
L_{fs}(\vec{u}, \vec{U}) = \rho_{12}\ddot{u}_i + \rho_{22}\ddot{U}_i - Q_{jj}u_{i,jj} - RU_{i,il} = N_{fs}(\vec{u}, \vec{U}).
\]

Here \(\vec{u}\) and \(\vec{U}\) represent the displacement fields of the solid phase (\(s\)) and of the fluid phase (\(f\)), where \(\ddot{u}\) and \(\ddot{U}\) denote the derivative over time and the derivative over the spatial coordinate \(x_j\) correspondingly. Linear differential operators \(L_{sf}\) and \(L_{fs}\) in the left-hand side of Eqs. (1) and (2) are traditional ingredients of the linear Biot theory [14,15], \(C_{ijkl}\) represents the 4th rank elasticity tensor of the skeleton, \(Q_{jj}\) is a 2nd rank...
tensor coupling the displacement fields in the frame and in the saturating fluid. The density coefficients are given by

\[ \rho_{12} = -(\alpha_\infty - 1) \phi \rho_f, \]
\[ \rho_{11} = (1 - \phi) \rho_s - \rho_{12}, \]
\[ \rho_{22} = \phi \rho_f - \rho_{12}, \]

where \( \rho_f \) and \( \rho_s \) are the actual densities of the solid and of the fluid, \( \phi \) is the porosity, \( \alpha_\infty \) is the tortuosity. Nonlinear terms \( N_{sf} \) and \( N_{fs} \) in the right-hand side (r.h.s.) of Eqs. (1) and (2) describe the nonlinearity of the deformation of the solid and of the fluid, and the coupling between the strain fields in the solid and in the fluid caused by the nonlinearity of the stress-strain relationship in the porous material.

Note that in the so-called semilinear Biot theory [14, 15] these nonlinearities (which include the Hertzian contact nonlinearity, for example) are assumed to dominate over the kinematic nonlinearity. However, in the present analysis we can consider that kinematic nonlinearity is included in \( N_{sf} \) and \( N_{fs} \).

The equations governing the demodulation process are obtained by the separation of the displacement fields into the LF and the HF parts \( \tilde{u} = \tilde{u}_\Omega + \tilde{u}_\omega, \tilde{U} = \tilde{U}_\Omega + \tilde{U}_\omega \), by the substitution into Eqs. (1) and (2), and by averaging over the high-frequency period \( 2\pi/\omega \):

\[ L_{sf}(\tilde{u}_\Omega, \tilde{U}_\Omega) \approx \langle N_{sf}(\tilde{u}_\omega, \tilde{U}_\omega) \rangle, \]  
\[ L_{fs}(\tilde{u}_\Omega, \tilde{U}_\Omega) \approx \langle N_{fs}(\tilde{u}_\omega, \tilde{U}_\omega) \rangle. \]
The self-action and the interaction of the LF waves are neglected in Eqs. (3) and (4) because it is assumed that the waves excited due to the nonlinear processes are weak [5]. In the absence of the HF pump waves \( \vec{u}_{\omega} \) and \( \vec{U}_{\omega} \), Eqs. (3) and (4) describe the existence of the two uncoupled acoustic modes [12,13]. In the case of weak interaction between the vibrations of the grains and the vibrations of the fluid (this is expected for the case of the glass beads saturated by air at the used high frequencies) the first acoustic mode transports its energy mainly through the beads (the contribution of the fluid motion to this solid-based mode is small). In contrast, the second mode carries its energy mainly through air (fluid-based mode). The fact that in accordance with the Biot theory both modes contain vibrations of both solid frame and of the saturating fluid provides a clear explanation for the excitation of both modes in the nonlinear demodulation process. Indeed, even if we assume, for example, that the only source of the nonlinearity is one of the Hertzian contact in vacuum, that it is not modified by the presence of the air, and that it contributes only to \( N_{sf} \) (because Eq. (3) describes the motion of the granular assemblage when \( \rho_f = 0 \)) and this contribution depends on \( \vec{u}_{\omega} \) only, even then this single nonlinearity in the r.h.s. of Eq. (3) will induce nonzero solutions both for \( \vec{u}_{\Omega} \) and \( \vec{U}_{\Omega} \); and, consequently, it will excite both acoustical modes. Similarly, the nonlinearity of the pressure/density relation in the fluid [5], which definitely contributes to \( N_{f_s} \) and in first approximation depends on \( \vec{U}_{\omega} \) only, also leads to excitation of both modes. This reasoning provides a possible explanation for the excitation of the fluid-based mode in our experiments although the dominant nonlinearity is expected to be that of the grain contacts [1]. However, it is also clear that the quantitative theory should also include other possible mechanisms of the nonlinearity and attention should be paid to the role of the nonlinear terms of the mixed type which depend both on \( \vec{u}_{\omega} \) and \( \vec{U}_{\omega} \) [15].

To get the understanding of the excitation process the distributions of the HF oscillations \( \vec{u}_{\omega} \) and \( \vec{U}_{\omega} \) should be found and substituted into the r.h.s. of Eqs. (3) and (4). This provides an additional independent and very complicated problem because our estimates demonstrate that the effective medium wave equations (1) and (2) cannot be applied for the analysis of the HF waves in our experimental situation. In fact, in Ref. [16] the results of the experiments [17] (where the solid-based mode with a velocity 280 m/s in the air-saturated glass beads of a diameter \( a = 0.5 \) cm had been detected) were analyzed. It was concluded in [16] that the waves at frequencies \( \omega/2\pi = 4 \) kHz in [17] were close to localization. The estimated Ioffe–Regel parameter was \( k\ell^* \approx 0.8 \), where \( k \) is the acoustic wavenumber and \( \ell^* \) is the “renormalized” transport elastic mean free path (\( \ell^* \approx 0.9 \) cm was estimated). This conclusion is also supported by the estimate of the cut-off or natural frequency [18] defined by Einstein via the relation \( k = \pi/a \). For the measured velocities around 200 m/s and \( a = 0.14 \) cm the cut-off frequency is estimated to be \( \approx 70 \) kHz. The situation with the HF fluid-based acoustic mode is expected to be rather similar because its velocity is not strongly different from the velocity of the solid-based mode and for the velocities around 300 m/s the cut-off frequency is estimated to be around 100 kHz. The above estimates of the cut-off frequency is obtained for the localization regime to take place (as it is readily observed experimentally [19]). Strong diffusion is already present when \( \lambda \) is in the range of let say \( 5a \) and the above numbers for the cut off frequency should accordingly be divided by a factor around 5.

In this short Letter we have no intention to describe the shape of the LF demodulated signals \( \vec{u}_{\Omega} \) and \( \vec{U}_{\Omega} \). We concentrate on the analysis of their propagation. For the latter purpose the knowledge of the nonlinear forces in the right-hand side of Eqs. (3) and (4) (and consequently, the information on the precise distribution of the HF waves \( \vec{u}_{\omega} \) and \( \vec{U}_{\omega} \)) is not necessary. It is just important that the excitation of the LF waves (according to the previous paragraph) is localized near the emitter (within a few layers of beads from the emitter). This region is significantly thinner than the propagation length of the demodulated signals. Because of this in our estimates of the propagation distance we may assume that the LF signals are excited at the emitter surface. For the analysis of their propagation the right-hand side of Eqs. (3) and (4) may be neglected. The estimates in our case demonstrate that in the equations \( L_{f_s}(\vec{u}_{\Omega}, \vec{U}_{\Omega}) \cong L_{f_s}(\vec{u}_{\omega}, \vec{U}_{\omega}) \cong 0 \) for the subsequent upward propagation of the demodulated signals, the classical form of the linear operators \( L_{f_s} \) and \( L_{f_s} \) (see Eqs. (1) and (2)) should be significantly modified. The dependence of the effective elastic modulus on the pressure due to gravity (and, consequently, on the dis-
tance from the mechanically free surface) may have a significant influence on the propagation of the solid-based mode [20], and this dependence should be incorporated in Eq. (1). For example, the estimate of the velocity of the solid-based mode using the Hertzian model [1],

\[ v_s = \left( \frac{1}{2} \right)^{1/2} \left[ \frac{n(1 - 2\nu)}{3\pi(1 - \nu)^2} \right]^{1/3} \nu_{\text{glass}}^{2/3}(g H)^{1/6} \]  

(5)

(where \( \tilde{n} \) is the average number of contacts per particle, \( \nu \) is the Poisson ratio for glass, \( g \) is the acceleration of the gravity, \( \nu_{\text{glass}} \) is the velocity of the longitudinal sound waves in glass), provided at the depths \( H \propto 0.1\text{–}0.3 \text{ m} \) from the mechanically free surface the values 280–340 m/s (for \( \tilde{n} = 6 \) characteristic of regular cubic packing of rigid spheres). These values are higher than the experimentally observed (see Fig. 5). The discrepancy indicates that not all possible contacts are loaded and we are rather in the regime of the noncompacted granular assemblage where the variation in the number of the Hertzian contacts with pressure is expected [1,7,17]. In this situation the dependence of the velocity of the solid-based mode on the pressure \( p \) is not \( v_s \propto p^{1/6} \), but rather \( v_s \propto p^{1/4} \) (the latter dependence has been observed experimentally [7,21] and explained theoretically [7]). Note that the regime \( v_s \propto p^{1/4} \) corresponds to the quadratic nonlinearity in the effective stress/strain \( (\sigma/\varepsilon) \) relationship for the medium containing contacts \( \sigma \propto \varepsilon^2 \) [7] (while \( \sigma \propto \varepsilon^{3/2} \) for the case of all possible Hertzian contacts loaded equivalently). Scaling the results of Ref. [20] to our estimated pressures \( (p \propto \rho_f (1 - \phi) g H \propto (1.5\text{–}4.5) \times 10^3 \text{ Pa}) \) we improve the correlation of the estimated velocity \( v_s \propto 230\text{–}310 \text{ m/s} \) with our experimental observations. However, there is still indication that the influence of the vertical walls of the container on the effective pressure at the depth \( H \) [17] is significant.

Summarizing, in the description of the propagation of the solid-based mode the dependence of the wave velocity on the distance from the free surface, caused both by the pressure variation and the variation of the coordination number in the granular skeleton, and the influence of the vertical walls on the effective pressure as a function of depth should be incorporated. Fortunately, these factors do not influence the propagation of the fluid-based mode in first approximation. Moreover, we have verified that the observed velocity of the fluid-based mode is accurately predicted by the Johnson–Allard theory of the “effective fluid” [10,11,13]. In this theoretical model, the general expressions for the effective fluid density \( \rho(\omega) \) and the effective compressibility modulus of the fluid \( K(\omega) \) are given by [13]

\[ \rho(\omega) = \alpha_{\infty} \rho_0 \times \left[ 1 + \frac{\sigma\phi}{j\omega\rho_0\alpha_{\infty}} \left( 1 + \frac{4j\alpha_{\infty}^2 \eta\phi\omega}{\sigma^2\Lambda^2\phi^2} \right) \right]. \]  

(6)

\[ K(\omega) = \left[ \gamma P_0/\gamma - (\gamma - 1) \right] \times \left[ 1 + \frac{\sigma'\phi}{jB^2\omega\rho_0\alpha_{\infty}} \times \left( 1 + \frac{4j\alpha_{\infty}^2 \eta\phi\omega B^2}{\sigma^2\Lambda^2\phi^2} \right) \right]. \]  

(7)

In such expressions \( \sigma' = 8\alpha_{\infty} \eta / \phi \Lambda^2 \), \( j \) stands for \( \sqrt{-1} \), \( \gamma \) is the adiabatic constant, \( P_0 \) the ambient pressure, \( B^2 \) the Prandtl number, \( \phi \) the porosity, \( \alpha_{\infty} \) the tortuosity [22,23], \( \omega = 2\pi f \) the angular frequency, \( \sigma \) the resistivity, \( \rho_0 \) the density of the fluid at rest, \( \eta \) the air viscosity, \( \Lambda \) and \( \Lambda' \) the viscous and thermal characteristic lengths [13,24]. The sound wave speed inside the porous medium is provided by the classical relationship, \( c(\omega) = [K(\omega)/\rho(\omega)]^{1/2} \), with the above effective compressibility \( K(\omega) \) and effective density \( \rho(\omega) \). Because all the model parameters are well known for the tested glass beads [9] the precise dispersion curve can be readily computed and is shown in Fig. 6. This curve predicts the speed of the air-based mode to be 286 m/s at 3.5 kHz, a number that perfectly matches with the measured value which is around 280 m/s.

4. Conclusions

For the first time it has been shown that the two demodulated LF modes propagating in the solid frame and in the saturating fluid could be clearly and unambiguously distinguished, and their wave speed precisely measured. Each mode has some definitive features in terms of dispersion and attenuation that should be further studied. Nonlinear demodulation effects have been demonstrated in several different ways.
Fig. 6. Dispersion curve for the mode propagated in the air saturating the 1.4 mm diameter glass beads calculated from the “fluid equivalent model”. The parameters are the following: $\phi = 0.40$, $\alpha = 1.37$, $\sigma = 6.67 \text{ N m}^{-2} \text{ s}$, $\Lambda = 90 \mu \text{m}$, $\Lambda' = 320 \mu \text{m}$.

Such nonlinear interaction is in fact at the rise of a very dramatic effect, where actually one could hear in real time the glass beads singing at the low frequency amplitude modulation. Due to some resonance effects in the cylindrical duct and expected localization of the high frequency acoustical waves, one observes less obvious characteristic features than the predictions from Zaitsev et al. of “the shape of the detected pulse being close to the third time derivative of the pumping pulse envelope” [1]. The results described in this note are complementary to some recent work performed on the acoustical propagation in unconsolidated granular media such as natural sand [25–29].

Further work may be needed in a free field configuration in a larger tank, as well as theoretical modelling in order to check about the validity of coupled propagation models such as nonlinear Biot theory. The fact that both modes propagated in the solid frame and in the saturating fluid have similar wavespeeds (respectively, 190 m/s in the glass beads and 280 m/s in the saturating air) might indicate that some coupling in the form of synchronism effect occurs in the granular structure over the thin nonlinear generation region. Perhaps this should be taken into account to explain some of the LF nonlinear features.

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