Frequency up-conversion and frequency down-conversion of acoustic waves in damaged materials

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Abstract

Experimental results have been obtained on thermally damaged thick glass plates. A study of nonlinear phenomena with different amounts of damages from the virgin (undamaged) case to a strongly cracked configuration has been performed. The cascade process of harmonic excitation, interaction of a high-frequency wave with a low-frequency one (nonlinear parametric receiving antenna) and demodulation of the amplitude modulated high-frequency wave (nonlinear parametric emitting antenna) have been implemented. One observes a dramatic increase in generation of the second harmonic as well as the demodulation signals versus the pump wave amplitude. A clear correlation exists between these nonlinear signatures and the amount of damages. Preliminary determinations of the coefficient of quadratic nonlinearity have been achieved. The results in the configuration of parametric emitting antenna are believed to be the very first on cracked materials.

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1. Introduction

During the last 10 years a true revival of the nonlinear acoustics in solids has been observed. A new class of solids, the so-called “mesoscopic materials” \cite{1} having macroscopic inhomogeneities, is concerned. Applications dealing with the detection of cracks \cite{2–6}, the monitoring of materials fatigue \cite{7}, the characterization of materials used in microelectronics \cite{8}, have been recently described. Emerging new tools and innovative methods have been proposed in order to decipher the intricacies of nonlinear effects related to acoustical wave propagation in mesoscopic and damaged materials \cite{9–14}.

Even for a single crack, early detection with nonlinear acoustics tools is sought, because the nonlinear propagation of acoustic waves is highly sensitive to defects. Consequently the coefficient of nonlinearity of a cracked material should be quite high, in the range of $10^2$–$10^3$ or higher. Recent advances are reported towards the characterization of micro-cracked materials. For instance, Zaitsev et al. have proposed \cite{14} to use for crack diagnostics the so-called “Luxembourg–Gorky effect”, which consists in the transfer of low-frequency modulation from high-
amplitude (pump) acoustic wave to low-amplitude initially unmodulated (probe) acoustic wave. However, the classical method to probe material nonlinearity by detecting the excitation of the harmonics (2ω, 3ω, ...) of a fundamental wave at frequency ω [15] is still very useful. This method was applied in the last few years to the diagnostics of mesoscopic materials (such as rocks [16], microcrystalline metals [17] and ceramics [18]) and to the detection of cracks [3,6] and related phenomena. Another effective method for crack detection patented more than 25 years ago [19] is based on the mixing of high frequency (ω) acoustic wave with low-frequency (Ω) vibration, which leads to the excitation of side lobes at frequency ω ± Ω. Generally speaking this is the same physical principle as for the ultrasonic parametric receiving antenna in underwater applications [20]. Recently this method has been extensively applied for different materials in a variety of geometrical configurations [2, 5, 10, 12].

In the research described in the present Letter we apply the method of harmonics excitation (frequency up-conversion) of acoustic waves for the diagnostics of cracks produced by thermal shock in glass. We also demonstrate efficient frequency mixing in cracked glass. By applying modulated pump wave (with spectral components at ω, ω − Ω and ω + Ω) we were able to observe the processes of frequency down-conversion leading to the excitation of acoustic waves at frequencies Ω and 2Ω, i.e., (ω + Ω) − (ω − Ω) ⇒ 2Ω; ω − (ω − Ω) ⇒ Ω; (ω + Ω) − ω ⇒ Ω. It was possible to follow both frequency up-conversion and frequency down-conversion processes simultaneously using wide-frequency band detection system. The configuration using modulated pump wave is well-known in underwater acoustics as a parametric emitting antenna [20]. It was applied for the investigation of nonlinear seismic effects [21, 22]. The rectification effect was also demonstrated experimentally on an interface between two solids (glass–glass contact interface) in pulsed regime [23, 24]. Very recently parametric antenna has been used to characterize the nonlinear demodulation process in unconsolidated granular materials [25, 26]. However, to the best of our knowledge the experiments described in the present Letter are the first where the principle of parametric emitting antenna is applied to the diagnostics of cracks in solids.

2. Experimental set-up and measurements

The experimental set-up, which has been used in this work, is described on Fig. 1. It is the similar configuration of a parametric antenna, with a pump wave generated by a piezoelectric transducer at 100 kHz with appropriate power electronics. This pump wave can be amplitude modulated by a low-frequency wave. The transmitted and reverberated signal is then probed with another piezoelectric transducer that is properly mounted on the opposite edge of the plate. Rectangular 18 mm thick glass plates have been used, as shown on Fig. 2 for virgin and damaged samples. The plates were damaged through a thermal process. After heating them in an oven, they were immersed in cool water in order to produce macroscopic cracks inside the volume. By modifying the parameters of the thermal process (maximum temperature, duration of the treatment, conditions of the water cooling), different spatial configurations of cracks and various degree of damages were achieved [27].

The received acoustical signal was displayed at the same time on a LeCroy oscilloscope and on a vector signal analyzer. Some recorded LeCroy spectra are shown on Fig. 3a–d, for the case of harmonic 100 kHz transducer excitation. Two levels of transducer driving voltage ($U_0$ and $U_0 + 30$ dB) were used in order to demonstrate the great differences between damaged and virgin samples in spectral domain. On the selected spectra the first ones (Fig. 3a and b) show up a significant increase of the harmonic rays (frequency-up conversion) when increasing the driving amplitude on the damaged material. For the undamaged plate (Fig. 3c and d), the amplitudes of the harmonics are almost unchanged when increasing the driving amplitude level.

A dramatic increase of the higher harmonics observed on the damaged plate is a direct indication of higher nonlinearity of the plate compared to the virgin one. It is important that such increase of higher harmonics takes place at relatively weak mechanical strain induced by the ultrasonic wave. It was possible to determine the strain from the movement of the transducer surface (coupled to glass) by means of laser...
Fig. 3. FFT spectra corresponding to strongly damaged plate #1 and virgin plate #9 as obtained on the LeCroy oscilloscope. For each plate, the nonmodulated (a)-(d) and modulated (e)-(h) signal spectrum are presented at two different transducer driving level, i.e., at $U_0$ and at $U_0 + 30$ dB. $U_0$ represents a reference level for the settings of the power electronics which corresponds to transducer driving voltage of 0.5 V p-p. The dB vertical scales are expressed in arbitrary units (u.a.), depending upon the settings of the power amplifier and the used amplitude scale of the oscilloscope.
velocimetry. The maximum speed of surface oscillation was $v_{\text{max}} = 25 \text{ mm s}^{-1}$. Consequently, the largest relative strain $\varepsilon_{\text{max}}$ that we were able to achieve with the used transducer at the maximum driving voltage was $\varepsilon_{\text{max}} = v_{\text{max}}/C_0 \times 10^{-6}$ (here $C_0 = 5800 \text{ m s}^{-1}$ is the sound velocity). To obtain the observed strong manifestation of the harmonic generation at that level of ultrasonic excitation, one should expect the nonlinear parameter to be extremely high, in the range of $10^2$–$10^3$, when there is a sufficient influence of the nonlinear term in the stress/strain relationship (see additional explanations in the next section).

The emitting piezoelectric transducer used in these experiments is specially designed to provide very low level of higher harmonic radiation. However, some harmonics are still seen at $U_0$ on the spectre of the virgin sample being generated mainly by the transducer. They are absent (near noise level) on damaged sample because of much stronger attenuation of cracked media and different spectral scale. The last one was changed in function of sensitivity set-ups of the reception circuit in order to obtain better visualisation of the spectra. On these qualitative spectra, the absolute values of harmonics are not indeed crucial as we are interested to show the relative changes in harmonics amplitude compared to the fundamental frequency amplitude.

For detailed spectral analysis, the absolute amplitudes of the harmonics were measured with the help of the FFT vector signal analyser providing a full 100 dB dynamic range. The data useful for further explanation are given on Table 1. One can observe that the amplitude of the fundamental frequency $A_1$ is directly proportional to the transducer driving voltage. For plate #8, for example (strongly damaged), the 10 dB voltage increments corresponding to the voltage levels $U_0 + 20$, $U_0 + 30$ and $U_0 + 40 \text{ dB}$ result in about 10 dB steps in amplitude for $A_1$, while in about 20 dB steps for the second harmonic amplitude $A_2$. Our measurements indicate that the second harmonic amplitude $A_2$ can be approximated as being proportional to the square of the fundamental wave amplitude $A_1$ in the sample. Consequently, the increase of the second harmonic amplitude as compared to the amplitude of the fundamental wave provides an opportunity to determine $\Gamma$, the coefficient of quadratic nonlinearity. Details on the analytical treatment and on the data analysis are provided below. It should be emphasized that these measurements are certainly the very first determination of the coefficient of nonlinearity for cracked glass material.

3. Determination of the coefficient of quadratic nonlinearity

In the phenomenological description of nonlinear acoustic behaviour of homogeneous solids, wave interaction is caused by nonlinear dependence of stress tensor $(\sigma)$ on the components of the displacement gradient tensor $(\partial u_i/\partial x_j)$, where $u_i$ are the components of mechanical displacement. This nonlinear dependence is due both to nonlinearity of the stress/strain relationship $(\sigma = M_0 \varepsilon + M_1 \varepsilon^2 + \cdots$ (where $\varepsilon$ is the mechanical strain and $M_0, M_1$ are the second and third order elastic moduli, respectively) and to nonlinearity of the relationship between the tensors $\varepsilon$ and $\partial u_i/\partial x_j$ [15]. The latter (so-called kinematic) nonlinearity provides important contribution to total nonlinearity of homogeneous materials but can be neglected in microinhomogeneous materials where the ratio $(M_1/M_0)$ characterizing the nonlinearity of the stress/strain relationship is about $10^2$–$10^3$ times higher than in homogeneous media [1,4,7,10,28]. Consequently the quadratic nonlinearity of microinhomogeneous materials can be characterized by a parameter $\Gamma = M_1/M_0$. However, it should be mentioned that in contrast to homogeneous materials, both real and imaginary parts of $M_1$ can be important for wave interactions in microinhomogeneous materials [11–13]. In other words both reactive (elastic) and dissipative (inelastic) nonlinearities of the stress/strain relationship should be taken into account in microinhomogeneous materials in general.

The formal relationship between the amplitude $A_1$ of the fundamental wave (at frequency $\omega$) and $A_2$ of its second harmonic (at $2\omega$), which follows from our experiment can be presented as:

$$A_2 = \alpha(2\omega) \Gamma A_1^2,$$

where $\alpha(2\omega)$ is a calibration factor depending mainly on the attenuation at frequency $2\omega$ of the second harmonic. This equation enables to calculate the relative coefficient of nonlinearity by comparing two different objects, i.e., a damaged glass plate (noted D) and a virgin one (noted V), or two damaged plates having
different degree of damage. We consider the fundamental and first harmonic amplitudes in damaged plate as \( A_D \) and \( A_Y \) and the same quantities in a virgin plate as \( A_D^V \) and \( A_Y^V \). When these amplitudes are expressed on a logarithmic scale (for instance, in dB), Eq. (1) yields:

\[
20 \log \left( \frac{A_D}{A_Y} \right) = (A_D^V - 2A_Y^V) - 20 \log \left( \frac{A_D^V}{A_Y^V} \right).
\]

The ratio \( \frac{A_D}{A_Y} \) is different in general at second harmonic \( 2(\omega) \) and at the fundamental frequency \( \omega \). However, it is well-known that in a wide class of microinhomogeneous materials with soft mechanical elements such as rocks and sands [29,30] the experimentally observed attenuation is proportional to frequency in a very wide spectrum band. That is why for our present estimates we assume that \( \frac{A_D(2\omega)}{A_Y(2\omega)} \approx \frac{A_D^V(\omega)}{A_Y^V(\omega)} \). The latter ratio is estimated from the measured amplitudes of the fundamental wave in different samples but for the same settings of the power electronics, and finally can be written out as

\[
20 \log \left( \frac{A_D}{A_Y} \right) \approx A_Y^V - A_D^V.
\]

It has been checked that all frequencies of interest in the experiment were sufficiently far from sharp resonances of the plates by recording the resonance spectra for each plate. As a consequence, the designed experimental system can be considered as being nonresonant and the contribution of the changes in harmonic amplitudes due to crack-induced variations in linear elasticity can be neglected. Accordingly, from Eq. (1), one finally obtain the estimation of the coefficient of quadratic non linearity in the form:

\[
20 \log \left( \frac{A_D}{A_Y} \right) = (A_D^V - A_D^V) - (A_Y^V - A_Y^V),
\]

where all the above spectral amplitudes (both on fundamental and second harmonic waves) are expressed in dB. Consequently, due to the differential nature of the technique, one gains access to the coefficient of nonlinearity of the damaged plate compared to the virgin one. The same procedure is evidently available to determine the ratio of the coefficients of nonlinearity for two plates having different level of damages, i.e., to rank their degree of damages.

Eq. (3) has been used to evaluate three glass plates (described on Table 1) having different amount of damages. With the data gathered on Table 1, one obtains the ratio of the coefficient of nonlinearity between any two of the three plates. For example, for the virgin and strongly damaged plates, 20\( \log \left( \frac{A_D^V}{A_Y^V} \right) = 50 \) dB \( \Rightarrow \frac{A_D^V}{A_Y^V} = 300 \). The slightly damaged plate exhibits a \( \Gamma \) coefficient that is 6 times smaller than \( \Gamma \) of the strongly damaged one. The evaluation of log\( \left( \frac{A_D^V}{A_Y^V} \right) \) was almost independent on the driving voltage applied to the pump transducer within 20 dB range from \( U_0 + 20 \) to \( U_0 + 40 \) dB. The values of \( \Gamma \) for glass being differently damaged did show up a clear correlation between the coefficient of nonlinearity and the amount of experienced damage. More damage leads to higher nonlinearity.

For virgin raw glass, \( \Gamma \) is around 1–2 (see Refs. [15,30–32] and the references therein), one deduce that the most damaged glass plate tested here has at least \( \Gamma_D \approx 300–600 \). This value actually indicates the lower limit of the coefficient of nonlinearity. Our analysis is based on the relative differences in second harmonic amplitudes for which a contribution to the second harmonic generation from transducer nonlinearity is strongly compensated. We do not take it into account in our estimates. Neglecting the contribution from transducer’s second harmonic

<table>
<thead>
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<th>Level</th>
<th>( U_0 + 40 ) dB</th>
<th>( U_0 + 30 ) dB</th>
<th>( U_0 + 20 ) dB</th>
<th>( A_1 ) (plate #8)</th>
<th>( A_2 ) (plate #8)</th>
<th>( A_1 ) (plate #9)</th>
<th>( A_2 ) (plate #9)</th>
<th>( \Gamma(8)/\Gamma(9) )</th>
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<tr>
<td></td>
<td>(-38.6') dB</td>
<td>(-48.4') dB</td>
<td>(-59.2') dB</td>
<td>(-70.5') dB</td>
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<td>(-111.5') dB</td>
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<td></td>
<td>(-4.0') dB</td>
<td>(-14.4') dB</td>
<td>(-38.6') dB</td>
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leads, in fact, to some underestimation of the absolute value of the parameter $I^{D} = 300–600$. The derived value is actually indicating the lower limit of the coefficient of nonlinearity. The prime goal of the present estimation has been to demonstrate the drastic increase of nonlinearity when the material is cracked and link nonlinearity to the amount of damage.

4. Frequency conversion due to high nonlinearity

In the media with such high level of nonlinearity one can expect to have strong manifestation of nonlinear phenomena even at relatively weak amplitudes of the acoustical excitation. Let us illustrate this point by the processes of frequency conversion which are attributed to a highly nonlinear media.

Fig. 3e–h represent spectra for the plates pumped by amplitude modulated signals (100 ± 5 kHz). Complex frequency-up conversion process is well seen on Fig. 3f compared with Fig. 3h and e where several groups of higher harmonics appear around 200, 300 and 400 kHz like sets of side lobes with 5 kHz intervals between them. No such remarkable changes are observed on Fig. 3h relative to Fig. 3g as on Fig. 3f relative to Fig. 3e. The last spectra demonstrate earlier emergence of frequency-up conversion phenomena on damaged sample at a given level of ultrasonic excitation amplitude.

Frequency-down conversion takes place in the form of a demodulation process. Strong LF signal at 5 kHz (and also at 10 kHz) having an amplitude comparable to those of the pump signal is observed on damaged plate (Fig. 3f). Temporal profiles of the signals corresponding to the configurations on Fig. 3e and f are shown on Fig. 4a.b.

For three plates with different amount of cracks (strongly damaged, medium damaged and virgin) the emergence of demodulated LF signal is shown on Fig. 5 as a function of driving voltage applied to the pump transducer. A strong demodulation occurs on the most damaged plate. Demodulation is less efficient when the sample is slightly damaged. When the driving voltage is set at its maximum, the LF amplitude on damaged plates is much larger than the LF signal created by the transducer non linearity (as can be observed on the curve corresponding to the virgin plate).

It has been shown for the first time that parametric LF emission was achieved in damaged solid material. Based on this result, the authors have implemented the same parametric interaction to produce an audio range emitter made of the cracked glass plate. Using previous configuration, the pump transducer was driven by a 100 kHz carrier frequency ultrasonic signal, being amplitude modulated by a musical record (e.g., “tango”). As a result of pump wave demodulation which occurred within the cracks, the music was heard from the damaged plate in real time while no sound was heard from the virgin plate under the same conditions.

The opposite multiwave process (modulation) was also obtained on the damaged plate. Two continuous harmonic signals of high and low frequencies from two different sources were mixed onto the plate. The nonlinear mixing of both signals resulted into the creation of new spectral lines within the plate at $\omega \pm \Omega$. 

![Temporal signals for the case of plate #1 corresponding to the configuration of Fig. 3h (a) and Fig. 3f (b).](image)
Fig. 5. Amplitude of the demodulation spectral peak (normalized to noise) in function of transducer driving voltage $U$ (normalized to reference level $U_0$) studied for three glass plates with different damages, (●) virgin undamaged glass plate (plate #9); (△) glass plate with medium damage (plate #6); (■) strongly damaged glass plate (plate #1).

(96, 104 kHz) which are demonstrated on Fig. 6a,b. In the temporal domain it means amplitude modulation of HF wave by LF wave. During this experiment, LF audiosignal was launched at 4 kHz by a loudspeaker mounted a few centimeters away from the plate and coupled to it via air. The LF signal transmitted into the plate in this way appears as a spectral line at 4 kHz. The HF ultrasonic nonmodulated signal was launched as before from the same pump transducer working at 100 kHz. Nonlinear frequency mixing shown here can also be considered as a LF parametric reception.

The described experiments on frequency conversion due to anomalous nonlinearity of damaged glass represent the first successful demonstration of parametric emitting antenna in solid media with defects. The authors here refer to the similarities with well-known parametric antennas used in underwater acoustics in terms of parametric LF emission (Fig. 3f) and parametric LF reception (Fig. 6b). In unconsolidated granular media, working prototype of such antenna was recently described by the authors [23].

5. Discussion

The huge quadratic nonlinearity of the cracked glass plates is clearly of nonclassical nature. We remind here that classical quadratic nonlinearity of solids is associated with weak nonlinearity of the stress/strain relationship (due to anharmonicity of interatomic potential) and with kinematic nonlinearity (due to nonlinearity of strain tensor in terms of displacement gradient). It should be also pointed out here that quadratic hysteretic nonlinearity (to which many nonlinear phenomena in mesoscopic materials such as rocks, microcrystalline metals and ceramics are attributed [11,17,33–35] is not effective either in harmonic (2ω) excitation [35,36] or in the demodulation of quasi-harmonic low-frequency signal [37]. The hysteretic quadratic nonlinearity is one
of odd type, resulting in the excitation of uneven harmonics \( (3\omega, 5\omega, \ldots) \) \[35,36\]. For the parametric antenna operation in media with hysteretic quadratic nonlinearity, the strong pump wave should have a wide frequency spectrum \[37\]. For example, in the simplest situation of a pump wave containing strong \( \omega \) and strong \( 2\omega \) simultaneously modulated at low frequency \( \Omega \), this pump wave contains \( \omega, \omega \pm \Omega \) and \( 2\omega, 2\omega \pm \Omega \). Then the frequency mixing processes involving one phonon from the second group and two phonons from the first group can cause demodulation (rectification) of the pump wave in hysteretic material. An example of such a process is \( (2\omega \pm \Omega) - \omega - \omega = \pm \Omega \). However, in the absence of the pump components near \( 2\omega \) (as it is in our experiment here) quadratic hysteretic nonlinearity does not contribute to emission at low frequencies.

The plausible candidate providing huge nonlinearity at the cracked glass is the nonlinearity of soft contacts, existing between the lips of the cracks. It is well-documented that such soft Hertzian contacts provide enhanced nonlinearity of unconsolidated granular materials \[38,39\]. However, at the current stage of our experiments it is difficult to discriminate between elastic nonlinearity of soft contacts and their contribution to nonlinear dissipation \[12–14\], as well as to distinguish between different possible regimes of contact nonlinearity \[38–41\]. In particular, we do not know currently if there are any clapping (popping) contacts \[36\] inside the cracks at the applied level of excitation, or if there are any periodic stick-depinning (bond forming–bond breaking) processes \[42\]. One of the opportunities to answer these questions is to include in the analysis higher harmonics \( (3\omega, 4\omega, \ldots) \), which are also accessible by our wide-band detection system (see Fig. 3), and to develop theoretical models for nonlinear dynamics of clapping contacts. It is also highly desirable to extend the range of available excitation amplitudes in order to study in bulk acoustic waves such phenomena as hysteresis and transition to chaotic oscillations, reported just recently in experiments with surface acoustic waves and surface-breaking cracks \[6\].

6. Conclusions

Cracked glass plates have been studied with intense ultrasonic fields. Great difference between damaged and intact samples has been demonstrated in the spectral domain. Dramatic increase in higher harmonics generation clearly indicates huge nonlinearity of damaged glass. Anomalous nonlinearity found in this damaged material was confirmed by pronounced manifestation of nonlinear frequency conversion phenomena. LF reception from an external loudspeaker has been demonstrated by the effect of modulation of HF. Parametric audio emission in damaged solid material was achieved for the first time.

A spectral analysis of the second and fundamental harmonics amplitudes showed that the second harmonic amplitude can be approximated as being proportional to the square of the fundamental wave amplitude. Based on this relationship, a method to determine the relative coefficient of quadratic nonlinearity is proposed. It links together damage and nonlinearity, and provides a practical way to rank the amount of damage via the coefficient of quadratic nonlinearity: more damage leads to higher nonlinearity. The method also allows unique estimation of the absolute value of the nonlinear parameter for strongly damaged glass as being at least 300.

The nonclassical nature of huge quadratic nonlinearity of a damaged material has been put in evidence. Further studies are needed to understand better the physics and the mesoscopic mechanisms at the origin of such anomalous nonlinearity. Knowledge in this field will find many applications, particularly in nondestructive testing of the onset of microcracks at the very early stage. Nonlinear frequency conversion phenomena seem to be a promising solution to the problem.

References