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# Models Comparison for the scattering of an acoustic wave on immersed targets

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Abstract. Ultrasonic telemetry techniques consist in locating various immersed structures (for instance, components in the main vessel of fast breeder reactors). The interactions beam-targets give rise to different scattering phenomena: tip diffraction of boundaries and edges of the different parts, specular reflection, and corner effect. In order to conceive and design such imaging techniques, simulation tool needs to account for these effects. Classical methods have been studied for such problems. The diffraction coefficients based on the Geometrical Theory of Diffraction (GTD) fail in the transition regions adjacent to shadow and reflection boundaries. The uniform diffraction theory provides continuous solutions in these regions, but with more sophisticated formulation. Another simple approximation based on the integral equation, widely used for scattering problems, is the so-called Kirchhoff approximation. The Kirchhoff approximation has good performance in the specular reflection zone but fails at predicting amplitude of diffracted waves by edges. A refinement of the Kirchhoff approximation which is based on the Physical Theory of Diffraction (PTD) and combines GTD and Kirchhoff edge diffraction coefficients has been studied. This refined Kirchhoff approximation provides a simple formulation and correct results for all scattered directions, which will be illustrated in the case of a rigid halfplane or wedge.

## 1. Introduction

Monitoring and inspection of nuclear reactor are stringent requirements from operator and safety authorities. The sodium-cooled fast reactor (SFR) is one of the perspectives chosen for the 4<sup>th</sup> generation reactor. The characteristics exhibited by sodium, such as its opacity, have led the designers to devise specific monitoring and inspection techniques. Consequently ultrasonic techniques are seen as suitable candidates. Two approaches are being followed: the core monitoring where transducers are directly immersed in sodium near the reactor's core and the outside inspection with transducers located along the wall of the main vessel (outside sodium medium).

Ultrasonic telemetry is one of the core monitoring techniques that allows checking the position of the various objects contained inside the main vessel and the possible detection of defects inside these

objects. The distance between the transducer and the immersed targets can be determined by measuring the time of flight of backscattering acoustic waves generated by the transducer installed inside. While in-service the flow of sodium creates turbulence that leads to temperature inhomogeneities, which convert into ultrasonic velocity inhomogeneities. These variations of velocity could impact directly upon the location tolerance by introducing times of flight variations. Different scattering phenomena can also be produced while the interaction between the acoustic beam radiated by the probe and the immersed targets: specular reflection, tip diffraction from boundaries and edges of the different parts and corner effect. Thus various variables will influence this technique behavior. In order to optimize the probes parameters that are being developed and to predict the probes performances, a simulation tool is necessary to assist the design of each element of the ultrasonic telemetry.

A wave propagation model has been developed in a previous work to calculate the ultrasonic field radiated in an inhomogeneous medium [1]. Calculations have shown that the inhomogeneous characteristic of the fluid doesn't impact much the time of flight and amplitude of the acoustic wave propagating but leads to beam deviations. In this paper, we will consider the field scattered from the immersed targets; different models describing the scattering phenomenon have been studied and compared. Firstly the scattered acoustic field can be modeled using the high-frequency asymptotics known as the Geometrical Acoustics (GA) and Geometrical Theory of Diffraction (GTD) [2], all based on the ray theory. The former describe incident and reflected waves and the latter, wave diffracted by obstacle edges (the so-called edge waves). The regions that support different kinds of waves are classified as either geometrical regions (illuminated region and shadow region) or transition zones that are the boundaries between an illuminated region and a shadow region. The sum of GA and GTD gives a perfectly adequate description of geometrical regions but fails inside transition ones. A more sophisticated uniform GTD is required to complete the description.

In some NDE applications, another approach, the so-called Kirchhoff approximation (KA) [3], is widely used in high-frequency scattering problems, particularly when dealing with obstacles of a complicated shape. The fundamental principle of this method is the use of Green's function representing in a given region  $\Sigma$  the solution to the Helmholtz equation. The KA provides a correct description of the reflected wave and the fields inside the transition region. The integral formulation of the KA solution enables description of the field in more intricate regions, such as focusing areas, shadow boundaries of edge waves, where the known GTD procedures are no longer applicable. This approximation leads to qualitatively correct description of edge waves, but with incorrect amplitudes. To eliminate the deficiencies of KA and GTD and combine their advantages, a model called here the refinement of KA is proposed to modify KA by employing GTD diffraction coefficients: this approach is based on the Physical Theory of Diffraction (PTD) [4].

The targets to be inspected by telemetry are steel structures immersed in liquid sodium, thus, the assumption of perfect rigid boundary condition should be not suitable here. In order to take into account the real boundary condition of our case, a modified GTD model applied to nearly-rigid wedge has been implemented. The reflection coefficient of this nearly-rigid interface can also be taken into account in the KA integral creating the rigorous KA formalism valid for arbitrary boundary condition.

#### 2. Geometrical Theory of Diffraction

2.1. Non uniform asymptotic solution for scattered waves

Consider a simple example: a plane wave scattered by a two-dimensional halfplane as shown in Figure 1. Presence of this halfplane in a plane incident wave field (with  $\theta_0$ : incident angle) gives rise to a shadow region of incident wave, a reflected wave and the related shadow region. Thus two light-shadow boundaries can be identified which are function of the incident angle:  $\theta_1 = \pi - \theta_0$ ,  $\theta_2 = \pi + \theta_0$ . The incident and reflected wave are given by Geometrical Acoustics and these solutions are discontinued on light-shadow boundaries jumping to zero in the shadow region. These discontinuities

of the field will be eliminated by adding the diffracted fields. Therefore the scattered field can be written as



Figure 1: scattering of a plane acoustic wave by a halfplane.

The geometrical acoustic fields can be written as

$$U^{\text{GA}} = U^{\text{Inc}} + U^{\text{Ref}} = e^{-ikr\cos(\theta - \theta_0)} \pm e^{-ikr\cos(\theta + \theta_0)}, \qquad (2)$$

the plus sign is taken for the Neumann boundary condition (hard case) and the minus sign for the Dirichlet boundary condition (soft case). We'll treat in the following only the Neumann's case (perfect rigid target).

In the problem under consideration the diffraction field is a cylindrical wave generated by the obstacle edge. According to the Geometrical Theory of Diffraction [2], the "main-order" term with respect to large kr in the diffraction field has the form

$$U^{\text{GTD}} \simeq \frac{e^{ikr}}{\sqrt{kr}} D^{\text{GTD}}(\theta, \theta_0), \qquad (3)$$

where  $\theta_0$  is the incidence angle of the plane wave, *r* and  $\theta$  are the polar coordinates of the observation point, *k* is the wavenumber and  $D^{\text{GTD}}(\theta, \theta_0)$  is the diffraction coefficient [2] given by

$$D^{\text{GTD}}(\theta,\theta_0) = -\frac{e^{i\pi/4}}{2\sqrt{2\pi}} \left(\sec\frac{\theta-\theta_0}{2} + \sec\frac{\theta+\theta_0}{2}\right),\tag{4}$$

where the angles  $\theta$  and  $\theta_0$  are measured with respected to the illuminated surface of the halfplane (S<sup>+</sup>) as shown in Figure 1. From eqn.(4) it follows that the diffraction coefficient grows without bound if the observation point is at a light-shadow boundary ( $\theta = \pi - \theta_0$  or  $\theta = \pi + \theta_0$ ) as shown in Figure 2. Clearly this diffraction coefficient is inapplicable in the vicinity of light-shadow boundaries where their poles are located. Hence GTD fails on the light-shadow boundaries.



Figure 2: diffraction coefficient for Neumann boundary condition and incidence angle  $\varphi_0=50^\circ$ 

The situation would be essentially the same when the obstacle is an infinite wedge. The only difference between the wedge and halfplane would manifest itself only in the angular dependence of the edge-wave diffraction pattern, i.e. in the dependence of the diffraction coefficient D on the wedge angle  $\Phi$ . Let us now consider a wedge of angle  $\Phi$  (see Figure 3) with the polar coordinates  $(r, \theta)$ , this wedge occupies the region  $\{(r, \theta): 2\pi - \Phi \le \theta \le 2\pi\}$ .



Figure 3: Scattering of a plane wave by a wedge with the wedge angle  $\Phi$ .

The scattered field from the wedge has the same form as that for the halfplane (eqn.(1)) with diffracted field written as

$$U_{\Phi}^{\text{GTD}} = \frac{e^{ikr}}{\sqrt{kr}} D_{\Phi}^{\text{GTD}}(\theta, \theta_0),$$
(5)

with the diffraction coefficient for a wedge of angle  $\Phi$  [5] defined by

$$D_{\Phi}^{\text{GTD}}(\theta,\theta_0) = -\frac{e^{i\pi/4}}{2\sqrt{2\pi}} \frac{\pi}{\Phi} \Big[ H(\theta+\pi,\theta_0,\Phi) - H(\theta-\pi,\theta_0,\Phi) \Big],$$
  
$$H(\alpha,\beta,\Phi) = \Bigg[ \operatorname{ctg} \frac{\pi}{2\Phi} (\alpha-\beta) + \operatorname{ctg} \frac{\pi}{2\Phi} (\alpha+\beta) \Bigg].$$
 (6)

This diffraction coefficient has also two poles (corresponding to zeros of ctg's arguments) at  $\theta = \pi \pm \theta_0$ , i.e. on the light-shadow boundaries of the incident and reflected waves.

#### 2.2. Uniform asymptotic theory (UAT) for scattered waves

Several uniform theories derived from GTD exist. One of the two most studied methods, the so-called uniform asymptotic theory of diffraction (UAT), involves the application of Fresnel integral near the light-shadow boundaries [5, 6] in order to smooth the abrupt field shift through the boundaries. Let us take the example of halfplane and there are two light-shadow boundaries:  $\theta = \pi + \theta_0$  for the incident wave and  $\theta = \pi - \theta_0$  for the reflected wave. The uniform solution  $U^{\text{UAT}}$  of eqn.(1) is written as

$$U^{\text{UAT}} = U^{\text{GA}} F\{\sqrt{k(s_e - s_{i,r})}\},\tag{7}$$

where F is the Fresnel integral. For a plane incident wave its argument can be given as follows:

$$\sqrt{k(s_e - s_i)} = \sqrt{2kr}\cos\frac{\theta - \theta_0}{2}, \quad \sqrt{k(s_e - s_r)} = \sqrt{2kr}\cos\frac{\theta + \theta_0}{2}, \tag{8}$$

where  $s_i$ ,  $s_e$  and  $s_r$  are respectively the eikonals of incident wave, the edge wave and the reflected wave. In the case of wedge, the eqn.(7) becomes:

$$U^{\text{UAT}} = U^{\text{GA}} F\{\sqrt{k(s_e - s_{i,r})}\} + \frac{e^{ikr}}{\sqrt{r}} \sum_{n=0}^{\infty} (i/k)^{n+1/2} C_n(\theta, \theta_0),$$
(8)

The expansion associated to  $C_n$  corresponds to the differences in the boundary conditions between the halfplane and the wedge, and the zero order of expansion  $C_n$  is given as follow [9]:

$$C_0(\theta, \theta_0) = \left[ D_{\Phi}^{\text{GTD}}(\theta, \theta_0) + \left(2\sqrt{2\pi}\right)^{-1} \sec\frac{\theta - \theta_0}{2} \pm \left(2\sqrt{2\pi}\right)^{-1} \sec\frac{\theta + \theta_0}{2} \right], \tag{9}$$

where  $D_{\Phi}^{\text{GTD}}(\theta, \theta_0)$  is given by eqn. (6). Such a zero-th order approximation defines the primary term of UAT.

Another method proposes a modification of the diffraction coefficient and consists in suppressing the coefficient poles by multiplying it with a transition function having zeros at the poles. This procedure known as the uniform geometrical theory of diffraction (UTD) was described in [8].

#### 2.3. Asymptotic solution for scattered waves from a finite impedance obstacle

To deal with the solid targets immersed in a fluid medium, we have to take into account non perfectly rigid boundary condition at the target surface. Thus the real acoustic impedances of the propagation medium and the obstacle have to be taken into account

$$Z = \rho c \tag{10}$$

with  $\rho$  the density and c the acoustic velocity in the corresponding medium. In our case, the acoustic wave propagates in a liquid sodium and is reflected on a steel target; we can here define the admittance of such fluid-solid interface as

$$\beta = Z_f / Z_s, \tag{11}$$

where  $Z_f$  and  $Z_s$  represent the acoustic impedance of sodium (fluid) and steel (solid) respectively. The boundary condition of the wave equation on this fluid-solid interface can be given by

$$\frac{\partial U(\mathbf{x})}{\partial \mathbf{x}} - ik\beta U(\mathbf{x}) = 0, \tag{12}$$

where  $U(\mathbf{x})$  is the acoustic potential field and  $\mathbf{x}$  denote the chosen Cartesian coordinate system. The mathematical formulations for the diffraction of an acoustic plane wave by a finite impedance wedge have been given by Willams [10] and Pierce *et al.* [11]. In our case, we consider that the interface sodium/steel is a nearly rigid interface, since  $Z_f (200^{\circ}\text{C}) = 2.23 \times 10^6 \text{ kg/ (m}^2\text{.s)}$  and for a typical steel alloy  $\rho = 7700 \text{ kg/m}^3$ , with a longitudinal velocity  $c_1$  of 6000 m/s,  $Z_s = 46.2 \times 10^6 \text{ kg/ (m}^2\text{s})$ , thus,  $\beta = 0.048 \ll 1$ . The diffraction coefficient for a nearly rigid wedge [11] is given by

$$D_{\Phi}^{*}(\theta,\theta_{0},\beta) = D_{\Phi}^{\text{GID}} \times [1 + S_{\Phi}(\theta,\theta_{0}) \cdot \beta].$$
(13)

This is a modification of the GTD diffraction coefficient given previously by multiplying a term containing a function  $S_{\Phi}(\theta, \theta_0)$  and the admittance  $\beta$  of this interface. This asymptotic solution for the scattered-field is an extension of GTD for nearly-rigid wedge. In order to find the function  $S_{\Phi}(\theta, \theta_0)$ , let us define firstly the function

$$M_{\nu}(\theta) = \frac{\cos(\nu\pi) - \cos(\nu\theta)}{\nu \sin(\nu\pi)},\tag{14}$$

with  $v=\pi/\Phi$  and the function

$$Q_{\Phi}(\theta) = -\nu \sin(\nu \pi) \sum_{n=1}^{1/2(p-1)} \frac{1}{\sin\left[\nu\left(\theta + 2n\pi\right)\right] \sin\left\{\nu\left[\theta + \pi\left(2n-1\right)\right]\right\}} - \sum_{m=0}^{q-1} \frac{\sin\left(\theta - 2m\Phi\right) + \sin\left[\theta - \beta(2m+1)\right]}{\sin\left(\theta - 2m\Phi\right) \sin\left[\theta - \beta(2m+1)\right]},$$
(15)

in which the wedge angle  $\Phi$  should take the form  $p\pi/2q$  (i.e. for instance for a right-angled wedge  $\Phi=3\pi/2$ , p=3 and q=1). Finally the function  $S_{\Phi}(\theta, \theta_0)$  is given as follows:

$$S_{\Phi}(\theta,\theta_0) = 2 \left[ M_{\nu}(\theta+\theta_0) + M_{\nu}(\theta-\theta_0) \right]^{-1} - Q_{\Phi}(-\theta) - Q_{\Phi}(-\theta_0).$$
(16)

With the modified diffraction coefficient  $D^*_{\Phi}(\theta, \theta_0, \beta)$ , the non-uniform solution of scattered field from a nearly rigid wedge can be written as

$$U_{\Phi}^{\text{Nearly-rigid}} = U^{\text{Inc}} - \frac{1 - \sin\theta_0/\beta}{1 + \sin\theta_0/\beta} U^{\text{Ref}} + \frac{e^{ikr}}{\sqrt{r}} D_{\Phi}^*.$$
(17)

The second term of (17) refers to the reflected field, where the reflection coefficient R is

$$R = -\frac{1 - \sin\theta_0/\beta}{1 + \sin\theta_0/\beta}.$$
(18)

In the extreme case where the obstacle is perfectly rigid  $Z_s \rightarrow \infty$ , so  $\beta \rightarrow 0$  we have R = 1 or where it is a perfect acoustic absorbent surface  $Z_s \rightarrow 0$ ,  $\beta \rightarrow \infty$ , we get R = -1. In our case, the module and the phase of reflection coefficient on the sodium/steel interface are given below with  $\varphi = \pi - \theta_0$ .



**Figure 4:** Reflection coefficient on the interface sodium/steel versus the complementary angle of incidence ( $\gamma = \pi/2 - \theta_0$ ): modulus (solid curve) and phase (dash curve).

#### 3. Kirchhoff approximation

The geometrical theory of diffraction provides short-wave asymptotic solutions accurate to the given orders of k for some typical model problems. Unfortunately, many model problems of practical interest have neither rigorous solutions to extract short-wave asymptotics nor appropriate asymptotics. Under these circumstances, one has to resort to approximate methods. A widely used method employed for large flaw size compared to the wavelength is the Kirchhoff approximation (KA) [3]. For any geometry, however complicated, the solution of KA is formulated as an integral of the field over the illuminated side of the reflector.

Consider how the KA method formulates a solution to the scattering field from a halfplane (see Figure 1). Let us introduce the associated Cartesian coordinate system  $\mathbf{x}$  ( $x_1$ ,  $x_2$ ), so that we have

$$x_1 = r\cos\theta$$
 and  $x_2 = r\sin\theta$ . (19)

Let us consider this acoustic problem with the acoustic potential field  $U(\mathbf{x})$  satisfying the Helmholtz equation. Fundamental of this method is the use of Green's function *G* to obtain, by superposition of elementary fields, an expression in the form of integral equation for a given boundary S ( $x_1, x_2=0$ )

$$U^{\text{Scat}}(\mathbf{x}) = \int_{S} \left[ G(\mathbf{x}, \mathbf{x}') \frac{\partial U(\mathbf{x}')}{\partial \mathbf{n}} - U(\mathbf{x}') \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial \mathbf{n}} \right] d\mathbf{s}(\mathbf{x}').$$
(20)

Here,  $U(\mathbf{x})$  refers to the field on the surface S,  $\partial/\partial \mathbf{n}$  implies differentiation along the inward-directed normal to S,  $\mathbf{x}'$  denote one point on the surface S and the Green function  $G(\mathbf{x}, \mathbf{x}')$  for the two-dimensional problem studied here takes the form :

$$G(\mathbf{x}, \mathbf{x}') = (\mathbf{i}/4) H_0^{(1)}(\mathbf{k} |\mathbf{x} - \mathbf{x}'|)$$
(21)

with  $H_0^{(1)}$  the Hankel function of first kind.

3.1. Approximation solutions for a perfect boundary condition

The Kirchhoff approximation is based on the assumption that for  $\lambda = (2\pi/k) \ll L$  (*L* is a typical size of the reflector), i.e. for  $kL \gg 1$ , one can use the approximation of geometrical acoustics for the total field  $U(\mathbf{x}')$  near the surface. In the shadow region, we can set

$$U(x') = \frac{\partial U(x')}{\partial \mathbf{n}} = 0.$$
(22)

When evaluating  $U(\mathbf{x})$  and  $\partial U(\mathbf{x})/\partial \mathbf{n}$  in the illuminated region, the total field is equal to the sum of incident and specularly reflected fields for the Neumann condition or to a difference of these fields for the Dirichlet condition. Therefore, on S:

$$U(\mathbf{x}') = 2U^{\text{Inc}}(\mathbf{x}') \text{ and } \frac{\partial U(\mathbf{x}')}{\partial \mathbf{n}} = 0 \quad \text{for the Neumann condition };$$
 (23-a)

$$U(\mathbf{x}')=0 \text{ and } \frac{\partial U(\mathbf{x}')}{\partial \mathbf{n}} = 2 \frac{\partial U^{\text{lnc}}(\mathbf{x}')}{\partial \mathbf{n}} \text{ for the Dirichlet condition.}$$
 (23-b)

Thus, in the KA, the scattered field from a perfectly rigid surface (Neumann condition) can be written as

$$U^{\mathrm{KA}}(\mathbf{x}) = -2 \int_{0}^{+\infty} U^{\mathrm{Inc}}(\mathbf{x}') \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial \mathbf{n}} ds(\mathbf{x}'), \qquad (24)$$

where s is the surface element along the illuminated surface of halfplane. In the wedge case, this integration must be carried out on each wedge face when the two faces are illuminated by the incidence wave.

#### 3.2. Approximated solutions for an impedance boundary condition

The eqn.(20) can also be calculated for a given arbitrary boundary condition whose corresponding reflection coefficient is R. In this condition, the assumption (22) can remain but the approximations given by eqn. (23) should be rewritten [12] as follows:

$$U(\mathbf{x}') = [1+R] U^{\text{Inc}}(\mathbf{x}'), \quad \frac{\partial U(\mathbf{x}')}{\partial \mathbf{n}} = [1-R] \frac{\partial U^{\text{Inc}}(\mathbf{x}')}{\partial \mathbf{n}}.$$
 (25)

R = 1 corresponds to the Neumann condition and R = -1 gives the Dirichlet condition. Therefore the Kirchhoff integral for a given impedance interface is given by

$$U^{*KA}(\mathbf{x}) = \int_{S} \left[ G(\mathbf{x}, \mathbf{x}') \frac{\partial U^{\text{lnc}}(\mathbf{x}')}{\partial \mathbf{n}} (1 - R) - U^{\text{lnc}}(\mathbf{x}') (1 + R) \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial \mathbf{n}} \right] ds(\mathbf{x}').$$
(26)

### 4. Refinement of the Kirchhoff approximation

The Kirchhoff approximation has some limitations; the most important one is the incorrect prediction of the diffraction wave amplitudes. To overcome this limitation we are going to correct the Kirchhoff approximation by employing GTD diffraction coefficient. As we can see the GTD diffraction coefficient can be computed in an efficient manner using algorithm given by eqn.(4), this refinement of the Kirchhoff approximation should be quite fast.

In order to correct the Kirchhoff approximation, we should identify different parts inside the Kirchhoff integral (eqn.(24) for a Neumann boundary condition for example). Using the stationary phase method, we find that the integral (24) involves two critical points, a stationary point corresponding to the geometrical field  $U^{GA}$  and the lower limit contribution where  $\mathbf{x}'=0$  corresponding to the diffraction field. This diffraction field contribution has the same form as the  $U^{GTD}$  with a different diffraction coefficient:

$$U^{\mathrm{KA(Diff)}} = \frac{e^{ikr}}{\sqrt{kr}} D^{\mathrm{KA}}(\theta, \theta_0).$$
(27)

U

Using [5] to find the asymptotic contribution of integration domain boundary, the Kirchhoff diffraction coefficient, for a Neumann boundary condition (plus sign) and a Dirichlet boundary condition (minus sign), turns to be

$$D^{\mathrm{KA}}(\theta,\theta_0) = -\frac{e^{i\pi/4}}{2\sqrt{2\pi}} \left( \tan\frac{\theta-\theta_0}{2} \pm \tan\frac{\theta+\theta_0}{2} \right).$$
(28)

This means that outside the penumbrae areas, the non-uniform asymptotics of the Kirchhoff integral are

$$U_{\text{non-uniform}}^{\text{KA}}(\mathbf{x}) = U^{\text{GA}}(\mathbf{x}) + U^{\text{KA}(\text{Diff})}(\mathbf{x}).$$
(29)

Thus the KA integral has been decomposed in two parts. The refinement of the Kirchhoff approximation is to correct the diffraction field amplitudes by employing the GTD

$${}^{\text{RKA}}(\mathbf{x}) = U^{\text{KA}}(\mathbf{x}) + U^{\text{GTD}}(\mathbf{x}) - U^{\text{KA(Diff)}}(\mathbf{x})$$
$$= U^{\text{KA}}(\mathbf{x}) + \frac{e^{ikr}}{\sqrt{kr}} \left( D^{\text{GTD}}(\mathbf{x}) - D^{\text{KA}}(\mathbf{x}) \right).$$
(30)

Finally the refinement of the Kirchhoff (RKA) consists in correcting, thanks to GTD, the KA contribution corresponding to the field scattered by the edge. This correction leads to add a corrective term to the KA field which is the difference of wave amplitudes diffracted by the edge given by GTD and KA. The diffraction coefficients for KA diffraction contribution and GTD have the same singularities at  $\theta = \pi + \theta_0$  and  $\theta = \pi - \theta_0$  (see Figure 5); when we make the difference of the two coefficients, their singularities cancel each other.



**Figure 5:** Diffraction coefficient for GTD and KA for  $\theta_0 = 50^\circ$ .

#### 5. Models comparisons and discussion

## 5.1. Perfectly rigid halfplane

The scattering of a plane wave by a halfplane is a canonical problem and it has an exact solution which allows us to compare with the GTD non-uniform eqn.(1) and uniform solutions eqn.(7). Those results are represented by their radiation pattern (containing the maximum power) and shown in Figure 6 and Figure 7. The incidence angle  $\theta_0$  is taken at 50°; the observation points are located around the edge for two distances from the edge  $r = \lambda$  and  $r = 5\lambda$  where  $\lambda$  is the wave length. According to the incidence  $\theta_0$  the light-shadow boundaries are  $\theta_1 = 130^\circ$  and  $\theta_2 = 230^\circ$  on which we find the singularities of the non-uniform GTD solution (black dash-dot curve). However the uniform solutions (green dash curve) coincide quite well with the exact solution (red solid curve).





**Figure 6:** Radiation pattern comparison of the scattered field by a perfectly rigid halfplane given by exact solution, GTD and UAT.

**Figure 7:** Radiation pattern comparison of the scattered field by a perfectly rigid halfplane given by exact solution, GTD and UAT.

Applying Kirchhoff approximation to the perfectly rigid halfplane in the same configuration as in Figure 6 and Figure 7, we obtain the results in Figure 8 and Figure 9. Kirchhoff approximation provides a qualitatively correct description of the scattered field. Errors can be found near the boundaries and in the shadow region where the edge diffraction wave dominate. When the observation is done far from the boundaries ( $r = 5\lambda$ ) and it is usually the case in non destructive evaluation, the KA field coincide quite well with the exact solution (Figure 9).

In a two dimensional space, the distribution of KA's normalized errors around the halfplane can be represented in Figure 10. The errors located near the edge are caused by the approximations we made in eqn.(23) on the boundary condition. The errors away from the edge are indeed due to the incorrect prediction of the diffracted wave amplitudes.

To correct the errors produced by the Kirchhoff approximation, we apply the refinement of the Kirchhoff approximation by employing GTD diffraction coefficients. The configurations of Figure 8 and Figure 9 have been recalculated using the refined Kirchhoff approximation. The results are given in Figure 11 and Figure 12. The results after the refinement coincide quite well with the exact solutions. The errors are greatly reduced Figure 13 (from 20% max to 1.5% max).



**Figure 8:** Radiation pattern comparison of scattered field from a rigid halfplane given by exact solution and Kirchhoff approximation.



**Figure 9:** Radiation pattern comparison of scattered field from a rigid halfplane given by exact solution and Kirchhoff approximation.



**Figure 10**: Distribution of KA errors to the exact solution with the  $\theta_0 = 50^\circ$ .







**Figure 11:** Radiation pattern comparison of scattered field from a rigid halfplane given by exact solution and refined KA.

**Figure 12:** Radiation pattern comparison of scattered field from a rigid halfplane given by exact solution and refined KA

**Figure 13**: Distribution of refined KA's errors to the exact solution with  $\theta_0 = 50^\circ$ .



**Figure 14:** Radiation pattern comparison of the scattered field from a perfectly rigid right-angle wedge given by GTD and exact solution.



**Figure 15:** Radiation pattern Comparison of the scattered field from a perfectly rigid right-angle wedge given by KA and the refined KA.

# 5.2. Perfectly rigid wedge

The GTD extension for a wedge (6) has been applied to a rigid right-angled wedge (Figure 14). For an incidence of 120°, there is no shadow region outside the wedge and the incident wave is reflected on both surfaces in the direction of 60° and 240°. Two singularities are found in the these two specular directions, but outside the singularity zones, the GTD model gives a quite good prediction. The Kirchhoff approximation can be also used to calculate the scattered field by a wedge and if two surfaces are illuminated, the integral (24) should be calculated on both surfaces. The refinement procedure of Kirchhoff approximation can be extended for a rigid wedge. The radiation pattern of KA and refined KA are illustrated in Figure 15 and we see the refined KA leads to a slight improvement.

## 5.3. Nearly rigid wedge

The scattered fields from a nearly rigid wedge have been calculated for two different incidences:  $\theta_0 = 50^\circ$  (before the abrupt phase shift of the reflection coefficient, see Figure 16) and  $\theta_0 = 20^\circ$  (after the abrupt phase shift, see Figure 17). The result, given by eqn.(17) (GTD <sub>nearly-rigid</sub>: red solid curve), is a non-uniform asymptotic solution (GTD extension) and considered as an almost exact solution outside

the penumbrae areas. Then we compare this GTD result (outside the penumbrae areas) to those given by the Kirchhoff approximation with the Neumann boundary condition [eqn. (24)] (KA <sub>rigid</sub>: green dot curve). For the incidence  $\theta_0 = 50^\circ$ , the amplitude of KA <sub>rigid</sub> is little high than that of GTD <sub>nearly-rigid</sub> which can be explained by the modulus of the reflection coefficient |  $R(\theta_0 = 50^\circ)$  |  $\approx 0.85$  in the nearly rigid case and |  $R(\theta_0 = 50^\circ)$  | = 1 in the perfectly rigid case. At this incidence the pattern lobes of the two results have the same orientations, since there is no phase-shift yet, and for the incidence  $\theta_0 = 20^\circ$ , there's no obvious amplitude difference, but the pattern lobes have different orientations which is due to the phase shift of R. We add in the Figure 16 and Figure 17 the results given by eqn.(26) (KA<sub>impe</sub>: black dash curve): the Kirchhoff approximation solution for a finite impedance boundary condition. We find a quite good coincidence between GTD<sub>nearly-rigid</sub> and KA<sub>impe</sub> outside the penumbrae areas.



**Figure 16:** Radiation pattern comparison of the scattered field from a nearly-rigid wedge

**Figure 17:** Radiation pattern comparison of the scattered field from a nearly-rigid wedge

#### 6. Conclusion

Two classic wave scattering models: Geometrical Theory of Diffraction (GTD) and the Kirchhoff approximation have been studied for a rigid halfplane model and a rigid wedge. The results have been compared with exact solutions. The asymptotic GTD formalism can be computed in an efficient manner and gives a perfectly adequate description of geometrical regions but is inapplicable inside transition regions. A more sophisticated uniform GTD is required to complete the description. The Green's-function-based KA formalism's results are uniform with respect to the observation point. KA correctly describes the geometrical field but leads to qualitatively correct description of diffraction wave but with incorrect amplitudes. In order to eliminate the deficiencies of GTD and KA and combine their advantages, the refinement of KA has been developed which consists in correcting, thanks to GTD, the KA contribution corresponding to the field scattered by the edge. This correction leads to add a corrective term to the KA field which is the difference of wave amplitudes diffracted by the edge given by GTD and KA. The refined KA gives accurate results compared to the exact solutions and with its simple formalism it can deal with obstacles of a complicated shape.

As to the wedge with finite impedance which is representative of the solid targets immersed in fluid which are inspected by telemetry, a non-uniform asymptotic model has been implemented. It is a modification of the classic GTD model by account of the real admittance of the fluid-solid interface. This model gives accurate description of the scattered field from a finite impedance surface outside the transition regions. The reflection coefficient of the fluid-solid interface can be taken into the KA integral and this KA extended to impedance wedges yields scattered fields from a nearly-rigid wedge coinciding with the modified GTD fields.

By coupling the impedance Kirchhoff model to the wave propagation model in inhomogeneous medium, it will be possible to simulate the complete telemetry technique and predict the possibility to

locate or detect objects contained in the reactor main vessel, which will help the development of visualization tools under sodium for the new generation nuclear reactor.

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