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Characterization of composite materials by ultrasonic methods: modelization and application to impact damage

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This paper presents some ultrasonic methods to detect and to characterize defects, possibly obtained after damage caused in composite materials. Firstly, a two-dimensional ultrasonic cartography, performed section by section, at different positions from the impact point, allows the participation of the delamination mechanisms which took part through the thickness of a pultruded glass fiber-reinforced plastic composite beam, to be analyzed. A very good agreement has been found with destructive testings. Furthermore, some examples are given on the use of an ultrasonic propagation model which has been developed. This model permits optimum experimental configurations to be determined, by the use of transmission and reflection coefficients or of Lamb waves. In addition, experimental and modelized time signals have been compared. © 1998 Elsevier Science Limited. All rights reserved.

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INTRODUCTION

Ultrasounds are one of the classical ways used to examine and characterize materials. As far as composite materials are concerned, the acoustic propagation through anisotropic multilayered media has become the subject of intensive study because of their application to nondestructive evaluation, geophysics, etc. Generally speaking, multilayered media are made by stacking distinct anisotropic media. These multilayered media are now studied through the use of the propagator matrix formalism which was first developed by Thomson[1], then furthered by Haskell[2] and afterwards by Gilbert and Backus[3]. By writing boundary conditions at each interface separating two successive layers, a transfer matrix of the whole medium can be obtained[4,5]. This matrix relates the stresses and displacements at the last interface to those at the first. A very interesting case is the one of anisotropic periodically multilayered media. Such media consist of the reproduction of P anisotropic multilayered medium cells, termed a 'superlayer'. These media have been extensively studied in previous works (see Refs [6–12] and references contained therein). The study of such media leads to Floquet waves that are linear combinations of the classical plane waves propagating in each layer of the multilayered medium[13–18]. They correspond to the propagation modes of the infinite periodically multilayered medium, considered to be an equivalent material. The long wavelength approximation allows equivalent constants for a multilayered medium to be obtained. In 1984, Helbig[19] used this approximation and presented dispersive curves and slowness surfaces for transversally isotropic media. During the same year Shoenberg[20], while studying alternating fluid/solid layers, specified that the homogenized medium which models the behavior of the periodically multilayered medium in the long-wavelength domain must have the same slowness surfaces as those of the multilayered medium. In 1995, a Floquet polarization vector was defined at different interfaces[9,10]. This vector tends towards a limit which is the polarization of the classical plane wave in the homogenized medium. The validity domain of homogenization for multilayered Lamb waves and for what has been termed 'multilayered Rayleigh waves' has also been studied[12].

As far as the characterization of composite materials is concerned, two types of indications can be extracted form ultrasonic studies: information on defects (delamination, porosity, etc.) and information on mechanical characteristics (anisotropic elastic constants), possibly obtained after damaging in these materials. In particular, the problem of fiber composite structures impact performance is important because of the possibility of accidental damage during service. That is why the impact behavior of composite materials has been extensively studied over the past few years[21–30], with different scientific approaches and on

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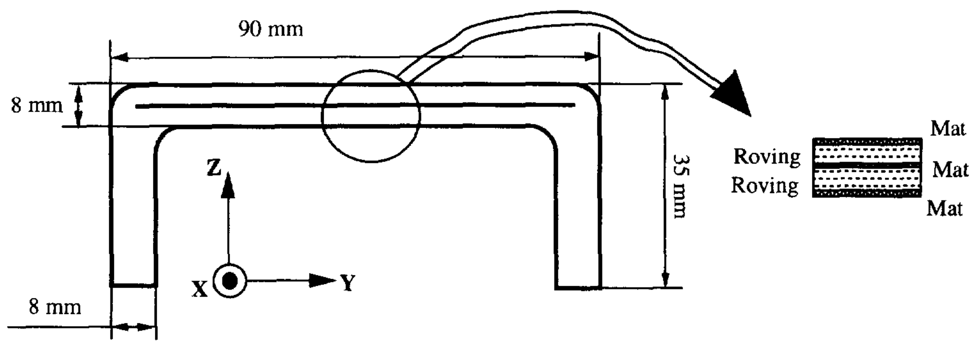

Figure 1 Geometry of the tested pultruded beam

different materials. Recently, the influence of impact parameters on the response of composite laminates has been studied[31–33]. These parameters are numerous and hard to discriminate. An example of some parameters are the impact velocity[34] or the impact energy[35,28,29]. Other studies[36,37] have focused on the analysis and on the numerical modeling of the internal damage due to impact load. Most of these works have been carried out on laminated structures or laminated plates, made up of high performance composite materials such as carbon/epoxy, glass/epoxy, etc. However, pultruded glass fiber-reinforced plastic composites are being increasingly used in structural applications, because of their favorable mechanical properties for a given weight, corrosion resistance and continuous production capability. Pultrusion, which consists in pulling the pre-impregnated required reinforcement through a heated metal die, is an easier process than other fiber-reinforced composite material working processes. Some recent studies[38,39] started to investigate the impact performances of pultruded beams (E-glass/polyester) used in safety roadside barriers. These studies have focused on the understanding of the impact behavior of pultruded composite structures when they are subjected to low velocity impact, such as a car collision into a fixed road object. More recently, another paper[33] has dealt with the influence of impact parameters such as impactor velocity, impactor mass and impactor nose dimensions, on the impact behavior of pultruded beams used in building and construction. The impact behavior under low velocity and low energy dynamic solicitations of these profiles was examined (speed ranging from 4 to 6 m s^{-1}, and impact energy ranging from 110 to 180 J). A good experimental correlation was found between time characteristics and damage characteristics measured by nondestructive techniques.

Different ultrasonic methods can be used in order to obtain information on defects and on mechanical characteristics: cartography in transmission mode or in pulse-echo mode to extract different parameters (such as transmission or reflection coefficients, flight time, etc.), measure of velocity and attenuation for different incident angles, and generation and detection of Lamb waves. Many studies have used this last method. Generally speaking, Lamb waves are guided waves in a plate within a vacuum[40,41]. The registering of boundary conditions leads to the cancellation of a determinant which yields to Lamb modes. When the vacuum is replaced by a fluid, the Lamb waves become leaky Lamb waves, due to the fact that their energy leaks into the fluid. The important investigations of Chimenti and Nayfeh have led to a better understanding of the ultrasonic leaky Lamb wave propagation in fiber-reinforced unidirectional and further nonunidirectional composites when the medium consists of a composite material instead of a homogeneous material[42–48]. The great complexity of the propagation phenomena of the elastic waves is due to the very nature of the composite materials. Consequently, the observed results are sometimes difficult to interpret. This led us to develop a propagation model which can take into account the stacking of any number of layers, each one being arbitrarily oriented.

The aim of this paper is to present some methods which allow a defect or a damage to be detected, by utilizing both mechanical testing and nondestructive ultrasonic investigation. The matching of these methods is illustrated here on an impacted pultruded composite beam. Its structure and the impact testing procedures are thus first exposed. The internal damage extent and the influence of some parameters on the chronology of damage mechanisms are then evaluated with the help of both destructive testing and ultrasonic cartography: notably, a three-dimensional ultrasonic representation is performed. Finally, the developed ultrasonic propagation in anisotropic multilayered media is presented. This model allows some observed results to be interpreted and the limits of the validity of the homogenization hypothesis to be precisely studied. Moreover, the echographic and transmitted time signal, obtained in a real testing configuration can also be reconstructed.

MECHANICAL TESTING AND MATERIALS

As mentioned in the introduction, few works have dealt with the impact behavior of pultruded glass fiber reinforced plastic composites. Due to the inhomogeneous and anisotropic character of these materials, their structure is complex: they are made of by E-glass continuous fibers and polyester unsaturated isophtalic resin. The volume fraction is 0.478 ± 0.020 in fibers. The glass fiber-reinforcement consists of continuous strand Mat and unidirectional direct Roving. The considered profile stacking sequence is an alternating of five

Table 1 Mechanical characteristics for homogeneized material

Young's modulus (GPa)	Poisson's ratio	Coulomb modulus (GPa)
$E_{xx} = (35.1 \pm 1.63)$	$\nu_{xz} = (0.308 \pm 0.008)$	$G_{xz} = (4.30 \pm 0.15)$
$E_{yy} = (9.22 \pm 0.71)$	$\nu_{xz} = (0.087 \pm 0.002)$	$G_{xz} = (4.15 \pm 0.21)$
$E_{zz} = (9.43 \pm 0.71)$	$\nu_{xz} = (0.263 \pm 0.009)$	G_{xz} = to be determined

Table 2 Mechanical characteristics for continuous strand Mat

Young's modulus (GPa)	Poisson's ratio	Coulomb modulus (GPa)
$E_{xx} = 9.2$	$\nu_{xy} = 0.333$	$G_{xy} = 3.67$
$E_{yy} = 9.28$	$\nu_{xz} = 0.407$	$G_{xz} = 1.43$
$E_{zz} = 5.37$	$\nu_{yz} = 0.407$	$G_{yz} = 1.43$

Table 3 Mechanical characteristics for unidirectional roving

Young's modulus (GPa)	Poisson's ratio	Coulomb modulus (GPa)
$E_{xx} = 38$	$\nu_{xy} = 0.21$	$G_{xy} = 3.88$
$E_{yy} = 9.9$	$\nu_{xz} = 0.32$	$G_{xz} = 3.88$
$E_{zz} = 8.5$	$\nu_{yz} = 0.45$	$G_{yz} = 3.45$

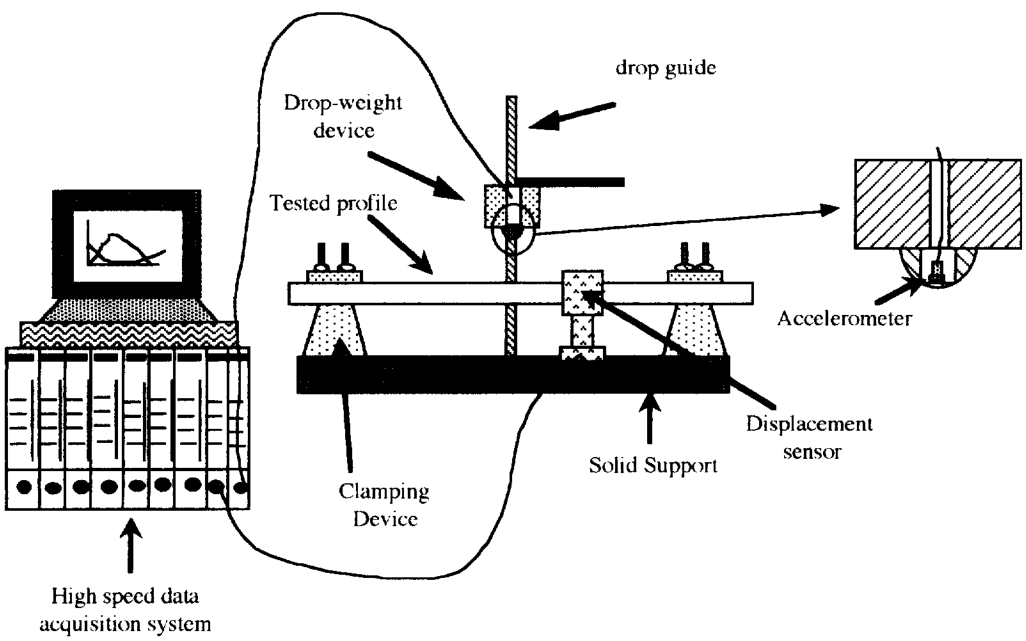

Figure 2 Experimental setup

continuous Mat and Roving layers, with Mat on the outer surfaces. This stacking configuration is shown in *Figure 1*. The average composite material thickness is 8 mm.

Static mechanical characterization has been carried out before impact testing, in order to evaluate the mechanical constants both for the homogenized sequence and for each layer of the composite. *Tables 1–3* present the results of this investigation. It can be seen in *Table 2* that although the fibers are randomly distributed, the Mat layer has isotropic properties in plane.

The impact set-up is displayed in *Figure 2*. The impact testings are conducted with a vertical drop-weight testing machine developed in the laboratory. Affixed to the drop-weight device, an accelerometer, localized in the impactor nose, yields the complete acceleration *versus* time history of impact event, while a displacement sensor gives the values of the drop-weight device displacement. The value of the impact velocity is determined through the displacement data by a simple derivation. The output from the accelerometer and from the displacement sensor are collected by a computer coupled with a data acquisition system. The data acquisition system is triggered by a threshold on the displacement data previously determined. Each specimen is tested clamped on devices with a clamped length equal to 900 mm. The diameter of the hemispherical impactor nose is equal to 50 mm. All the specimens are struck at the center of the span. Concurrently, some impact testings have been filmed by a 4500 frames s^{-1} Kodak high-speed camera, as can be seen in *Figure 3*.

In order to separate the velocity and the mass effect on

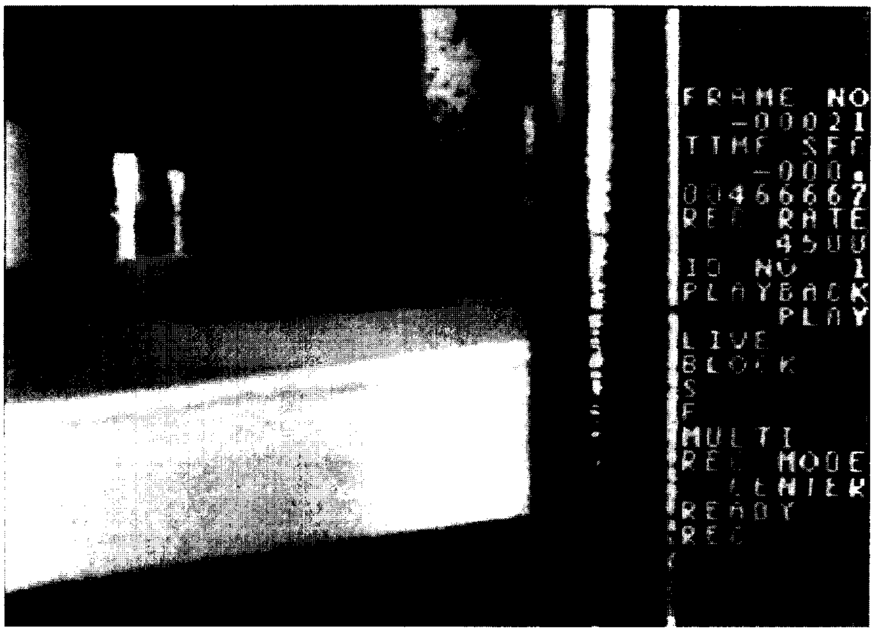

Figure 3 Part of an impact sequence, filmed at 4500 frame s^{-1}

internal damage, two different types of testings have been performed: iso energy testings and iso-velocity testings. The present study is illustrated by two samples tested at 50 and 80 J. For these two impact energy levels, the considered velocity of the hemispherical impactor is equal to 4 m s^{-1} and its diameter nose to 50 mm.

Let us see now how the damage characterization can be evaluated by destructive and nondestructive testings.

A THREE-DIMENSIONAL ULTRASONIC REPRESENTATION FOR DAMAGE CHARACTERIZATION

Nondestructive evaluation: ultrasonic cartography, performed section by section

As mentioned in the introduction, information on defects and on mechanical characteristics can be obtained by various methods. One of them is an ultrasonic cartography, in transmission, or in pulse-echo mode, and it is particularly useful in indentifying parameters such as transmission or reflection coefficients, flight time, etc. The principle of ultrasonic echography is the following: a transducer emits an ultrasonic wave which is then reflected by the interfaces of the material and by defects, see *Figure 4*. At each reflection, an echo is thus observed, the position in the material being a function of time. Selecting a time window is therefore equivalent to selecting a slice of the medium. Hence it is possible to observe the material, section by section, by selecting the appropriate time window.

A two-dimensional ultrasonic cartography (C-scan), performed section by section, in pulse-echo mode, and at different positions relative to the impact point is presented in *Figure 5*. Each section taken was numbered (0–11), and is approximately 0.66 mm thick. This cartography allows several types of damage, such as delamination or cracking, to be identified and localized with a good accuracy[49]. Classical results can be observed: for instance, below the impact point (on the rear surface), the zone of damage is larger than that on the surface of impact. This nondestructive evaluation leads to a better knowledge of the influence of the parameters mentioned in the introduction, and of the chronology of the damage mechanisms, which occur during the impact of these structures. Indeed, this representation presents a clear visualization of the importance of delamination and of the fissuration produced during impact.

From these bi-dimensional visualizations, a three-dimensional representation can be produced. This is shown in *Figure 6*. This three-dimensional cartography allows analysis of the delamination mechanism (present throughout the specimen). This analysis is very important as it allows the chronology of damage spread in the specimen to be determined during the duration of impact.

Destructive evaluation

The ultrasonic cartographies have been correlated with destructive testings: in order to better determine the shape and the extent of internal damage, a post-mortem analysis was carried out. Several transverse cross-sections were cut in the specimen at different distances from the impact point, see *Figure 7*. The fissuration was then visualized with dye, and different sections were cut in order to see the evolution of the damage (*Figure 8*). Here a very good correspondence between this representation and the ultrasonic cartographies that we have seen before. For example, in the transverse section situated 60 mm from the impact point, a delamination under the Mat layer in the middle of the thickness is found, and this coincides with a large ultrasonic echo.

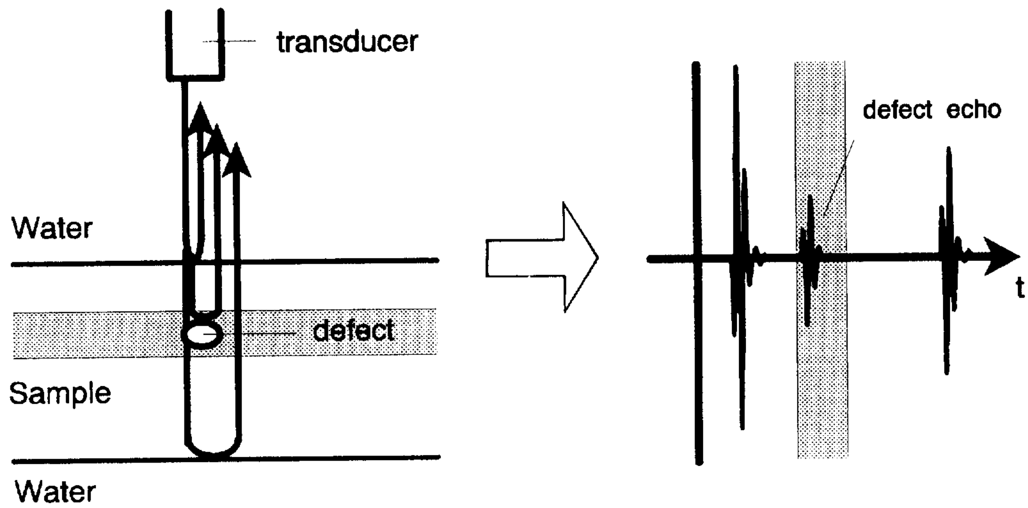

Figure 4 Ultrasonic echography principle

ULTRASONIC PROPAGATION IN ANISOTROPIC MULTILAYERED MEDIA

In order to better interpret some experimental results, an ultrasonic propagation model was developed. As this study has been carried out in previous works (see Refs [6–12]), a quick summary of the main results will be outlined and examples on its utilization will then be given.

Anisotropic periodically multilayered media

A periodically multilayered medium is the reproduction of P 'superlayers', each one made by the stacking of Q distinct anisotropic media (see *Figure 9*). Each layer may have any thickness. A 'superlayer' is, therefore, the period of the medium. When $P = 1$, the medium is not, of course, a periodically medium. Media 0 and $N + 1$ above and below the periodically multilayered medium are semi-infinite. The model is based on the study of the interaction of a monochromatic oblique incident wave propagating through the medium 0 and contained in the plane x_1Ox_3 defined in *Figure 9*, with the structure. By writing the boundary conditions at each interface separating two successive layers, the transfer matrix $[\Phi]$ of one superlayer can be found and thus the transfer matrix of the whole periodically multilayered medium, which is $[\Phi]^P$, where P is the number of superlayers. If the general solution is decomposed on the Floquet wave basis, the transfer matrix becomes a diagonal matrix. These six Floquet waves are the propagation modes of an infinite periodically multilayered medium. They are linear combinations of the classical waves, propagating in each layer of the multilayered medium. The linear combination differs simply according to the layer. One of the reasons for using Floquet analysis is that the displacement and stress vectors are expressed in the Floquet wave basis. Subsequently, Floquet waves propagating in a multilayered medium can be thought of as classical waves propagating in a homogeneous medium, which is physically more transparent. The major difference results from the dispersive character of the Floquet waves. The writing of the boundary conditions at the first and last interfaces of the medium leads to reflection and transmission coefficients, which can be drawn as a function of the incident angle or of frequency. For example, *Figure 10* presents the modulus of the reflection and transmission coefficients in water for a periodically multilayered medium 0°/90° made from layers of carbon/epoxy material. Each layer is 0.13 mm thick and in total the medium is 13 mm thick. The frequency intervals for which the reflection coefficient is equal to one and the transmission coefficient is equal to zero are called stopping bands. Such a result gives an indication for nondestructive evaluation. Indeed, the experimental configuration must not use such an angle and a frequency, otherwise the ultrasonic waves do not penetrate through to the end of the medium. Thus, in order to detect a defect, it is preferable to use a natural frequency of the incident wave equal to 2 MHz rather to 5 MHz.

Comparison between modelized and experimental echographic time signals

This model permits the echographic and transmitted time signal to be reconstructed. As a result, understanding of the composite material is improved. The calculation principle is the following: the frequency spectrum of the transducer is multiplied by the reflection or the transmission modelized coefficient. The inverse fast Fourier transform then gives the reflected or transmitted time signals. Let us take, for example, some results of the pultruded composite beam that has been studied before. *Figure 11* shows the modelized echographic signal when the medium is considered as nonabsorbant (*Figure 11*a), compared with the experimental echographic signal (*Figure 11*c). As the model can also simulate porosity or absorption, *Figure 11*b presents the modelized echographic signal when the medium is considered as an attenuating medium. The model takes into account this effect by considering the elastic constants of the material constituting each layer to be complex. A 0.5 in transducer was used during the experiments and its nominal frequency is equal to 2.25 MHz. Here, a good similarity of these two signals is observed. The first echo is the front face one, whereas the other two correspond to the Mat layers. However, these layers are not thick enough to separate the echoes.

Use of Lamb waves

Generally speaking, Lamb waves are free waves in a plate. When the plate is made up by a stratified medium, the Lamb waves are termed 'multilayered Lamb waves'[12]. When the plate is submerged in a fluid, a Lamb mode can be generated: its generation in the plate depends only on the incident angle and on the frequency of the incident wave propagating through the fluid. For example, the experimental configuration shown in *Figure 12* allows a Lamb wave to propagate through the plate and to be detected by a receiver transducer. If there is a defect in the plate, the propagation of the Lamb wave is disturbed and the defect can thus be detected. The model developed can thus also be employed to determine which configuration (incident angle, incident frequency) will permit defects to be detected in a composite plate, by using multilayered Lamb waves.

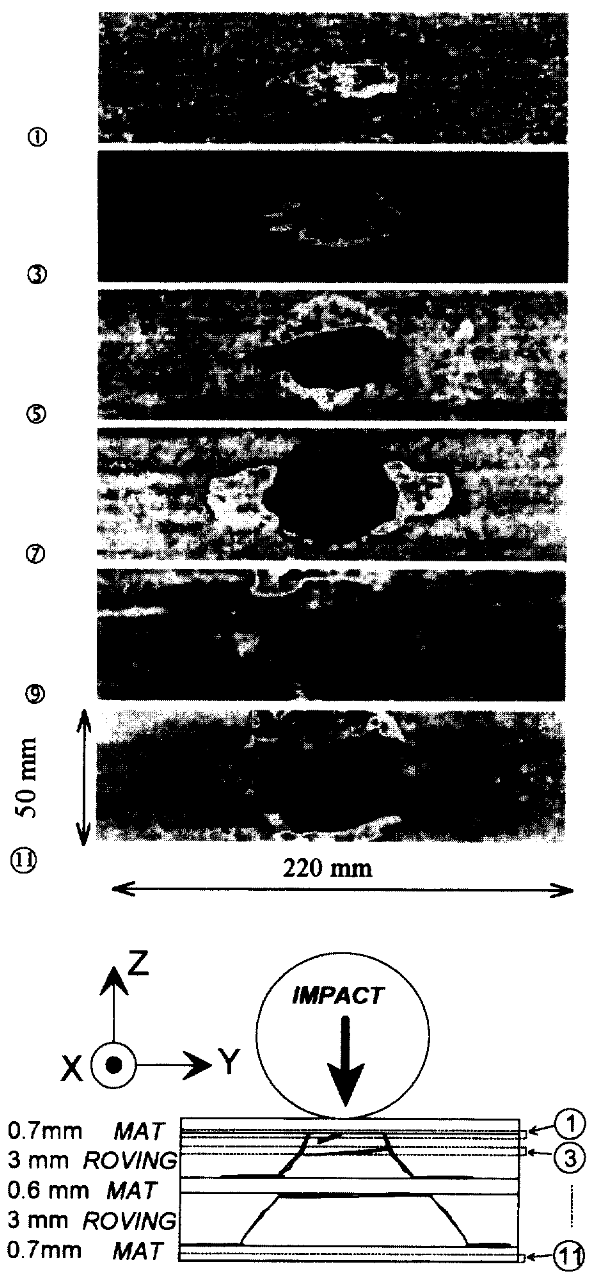

Figure 5 Two-dimensional ultrasonic cartography C-scan on an impacted pultruded beam

Let us see now how dispersion curves for Lamb modes can be drawn. Generally speaking, the presence of Lamb waves in a plate is indicated by a sharp minima in the magnitude and rapid reversals in the phase of the reflection coefficient of the fluid[42]. This approximation is valid, so long as the ratio of acoustic impedances of the fluid and plate is small[43,46,47,50]. The reflection coefficient is thus calculated in such a fluid. For example, the modulus of the reflection coefficient presented in *Figure 13*a is calculated as a function of the incident angle, for a fixed frequency f_1: this is angular scanning. In this figure, three minima were detected for three different incident angles: i_1, i_2 and i_3. As each minimum of the modulus of the reflection coefficient corresponds to the propagation of a Lamb mode, the plotting of these incident angles for different frequencies will permit the dispersion curves for Lamb modes to be drawn (see *Figure 13*b). Note that drawing the incident angle α in a fluid of reference as a function of the frequency is equivalent to drawing the Lamb waves' velocity V_L as a function of the frequency:

$$V_L = \frac{V_{fluid}}{\sin\alpha} \quad (1)$$

An example of these curves is given in *Figure 14* for a $[0°/90°]_{2s}$ carbon/epoxy plate which consists of eight 0.15 mm thick layers. The fluid of reference is the water. It can be seen that for a frequency equal to 1 MHz, there is a multilayered Lamb mode for an incident angle of about 10°. Then, a (25 × 8 mm) Teflon layer was inserted between the third and the fourth layer to simulate a delamination. This was experimentally detected as shown in *Figure 15* where the incident angle equal to 11.2° and a 1 MHz transducer was used. The decrease in the amplitude of the signal corresponds to the defect. At each emission, a distance of about 50 mm exists between the emitter and receiver transducers, thus permitting 50 mm of the material to be tested by each emission. This thus permits an important decrease in the testing time.

CONCLUSION

The use of ultrasonic methods provides a lot of information on composite materials, including information on defects and on mechanical characteristics (possibly obtained after the material has been damaged). The close correlation between destructive and nondestructive evaluations has been illustrated on a pultruded glass

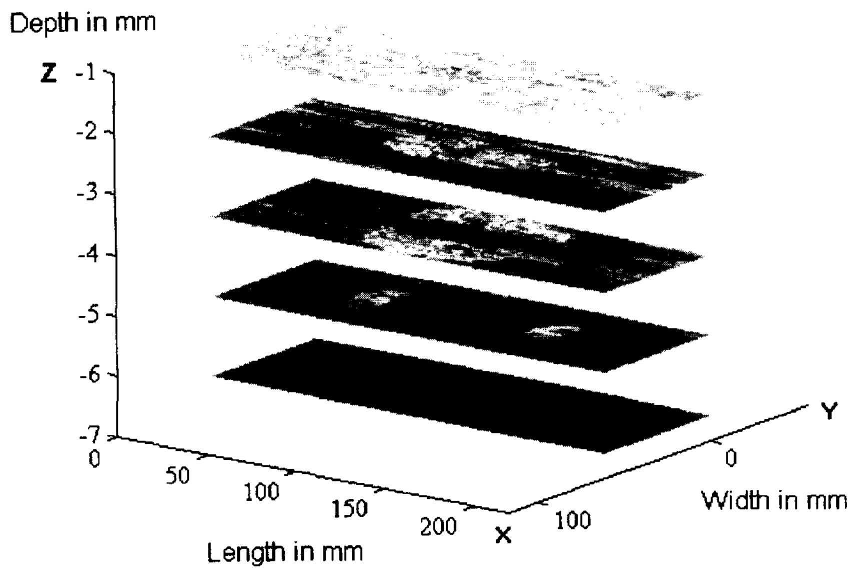

Figure 6 Three-dimensional ultrasonic cartography C-scan on an impacted pultruded beam

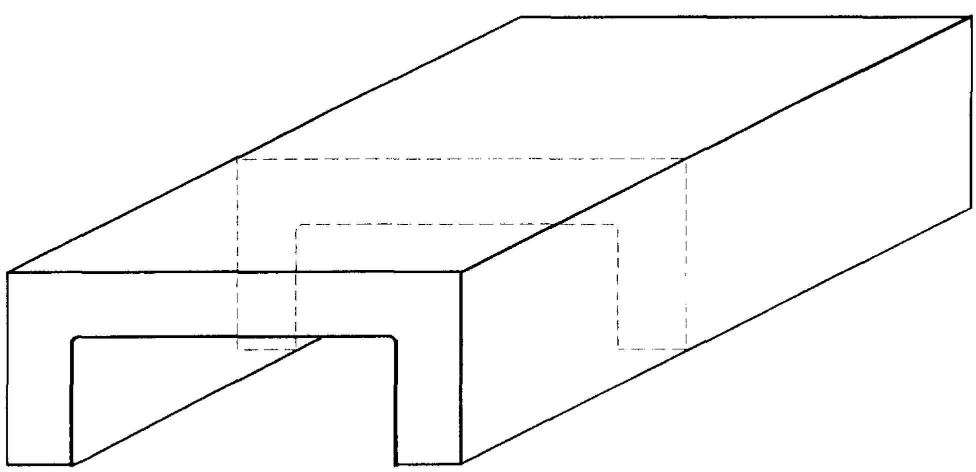

Figure 7 Transverse cross-sections for destructive visualization of the fissuration

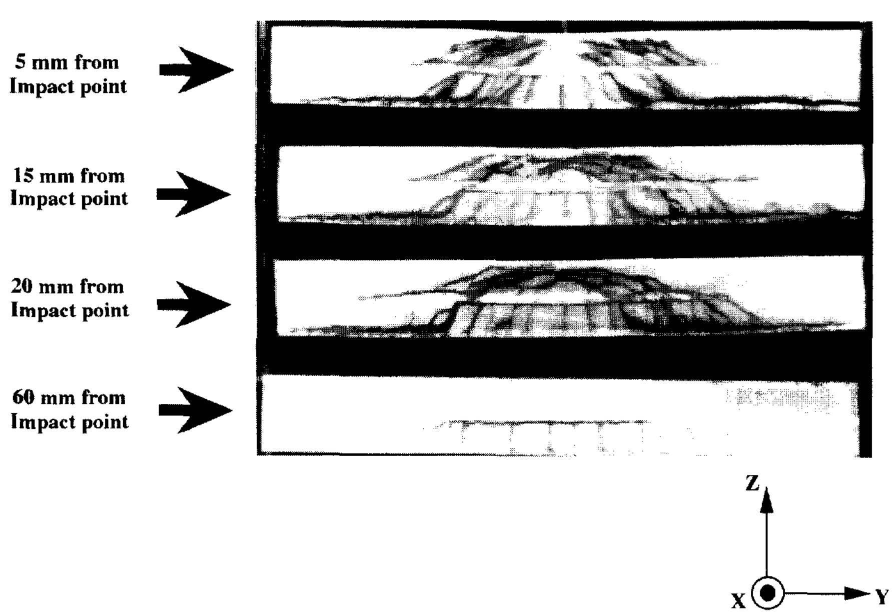

Figure 8 Visualization of the fissuration in transverse cross-sections, by using dye

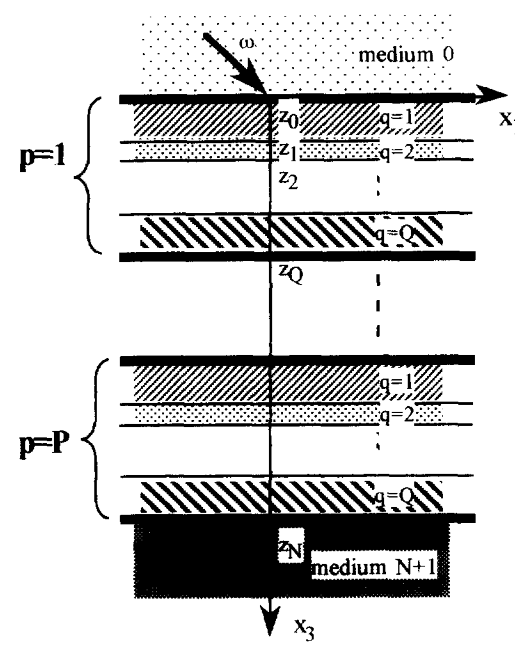

Figure 9 Geometry of a periodically multilayered medium

fiber-reinforced plastic composite beam, and particularly for damages caused by impact loading. A two-dimensional ultrasonic cartography (C-scan), performed section by section, in pulse-echo mode, at different positions from the impact point, has shown to give a good visualization of the delamination participation and of the fissuration produced by the impact. Thus, this leads to a better knowledge of the influence of certain parameters, on the chronology of damage mechanisms which occur in these structures, for example impact velocity or weight of the striker. Moreover, a three-dimensional cartography has been presented. This representation allows the participation of the delamination mechanisms that occured through the thickness of the specimen to be analyzed. A very close agreement was obtained between the C-scan damage localization and visual observations performed (with the help of destructive testing). Indeed, the fissuration in several transverse cross-sections, cut at different positions from the impact point, was visualized with dye: the evolution of the damage in each section corresponds to that in C-scan representation.

In order to better interpret some of the experimental results, an ultrasonic propagation model which takes into account the stacking of any number of layers, each one being arbitrarily oriented, has been developed. This model is based on the study of the interaction of a plane wave propagating in an outer medium (where the composite structure is immersed) with a stratified medium. Three examples of the use of this model are given. Firstly, modelized reflection and transmission coefficients can give an indication for nondestructive evaluation: the experimental configuration chosen must not use the incident angles and frequencies belonging to stopping bands. In these intervals, a defect will not be detected, because the ultrasonic waves do not penetrate through to the end of the medium. The second example concerns the reconstruction of the echographic and transmitted time signals: a good similarity of experimental and modelized signals, obtained from the pultruded beam, was observed. The last example deals with the use of Lamb waves. These waves are free waves propagating in plates, their propagation is disturbed by the presence of a defect. The model permits (only by simulation) the geometric and frequential optimum configuration (incidence and frequency) to be determined. This simulation has been matched with experiment on carbon/epoxy composite plates.

To conclude, it can be said that the ultrasounds can give many indications on defects in composite materials, and that the propagation model we developed permits the

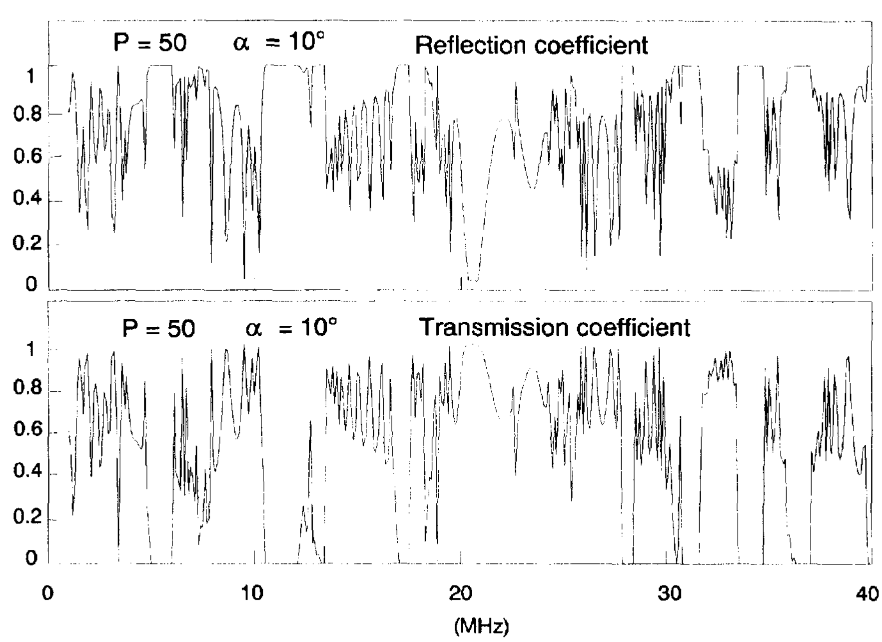

Figure 10 Modulus of the reflection and transmission coefficients in water for a periodically multilayered medium 0°/90° made from layers of carbon/epoxy, as a function of the frequency. $P = 50$; $\alpha = 10°$

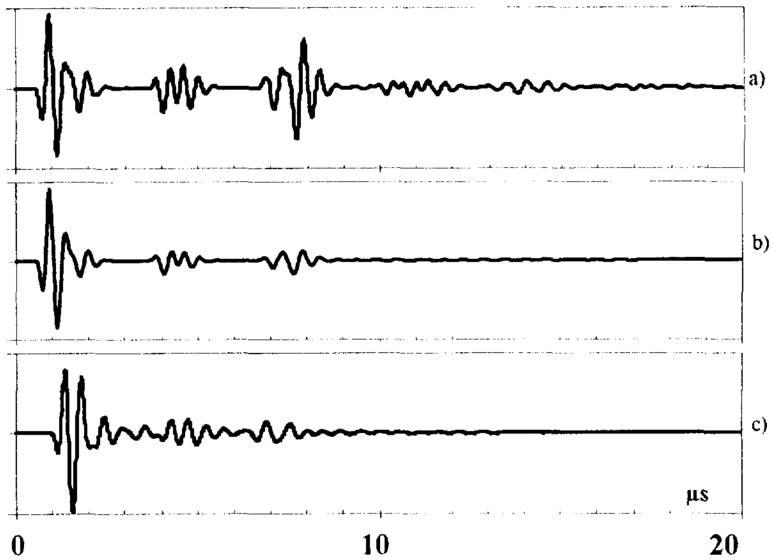

Figure 11 (a) Modelized echographic signal when the medium is nonlossy; (b) modelised echographic signal when the medium is lossy; (c) experimental echographic signal

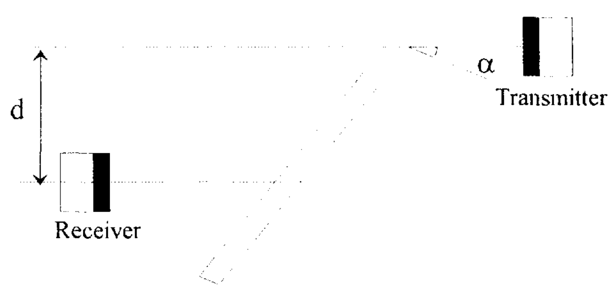

Figure 12 Experimental configuration for the detection of a Lamb mode

experimental results to be understood and optimal configurations for each case to be determined.

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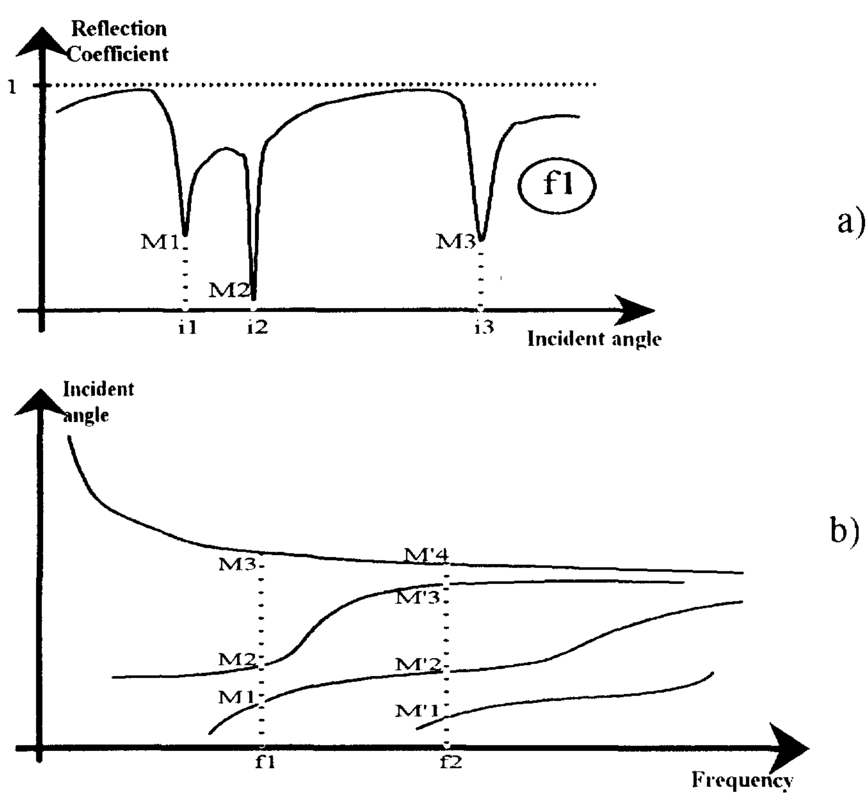

Figure 13 Determination of the dispersion curves for Lamb modes: (a) modulus of the reflection coefficient; (b) dispersion curves for Lamb modes

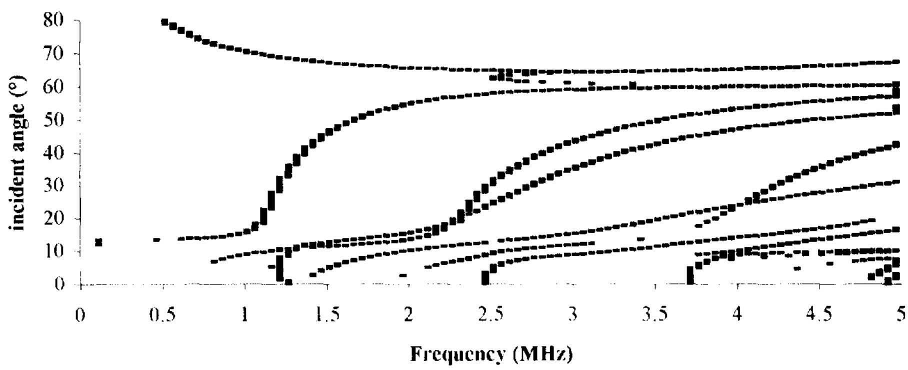

Figure 14 Modelized dispersion curves for Lamb modes for a $[0°/90°]_{2s}$ carbon/epoxy medium

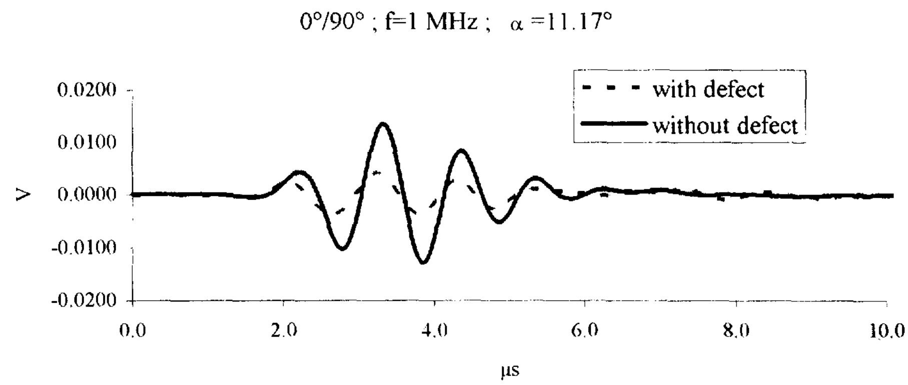

Figure 15 Experimental signals detected in a $[0°/90°]_{2s}$ carbon/epoxy plate when there is no defect in the plate (full line) and when teflon layer is introduced between the third and the fourth layer (dotted line)

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