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Deviation of the modal waves excited by an ultrasonic monochromatic beam in an anisotropic layer

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Abstract. The aim of the Note is to describe the physical phenomenon of the excitation of modal waves, such as Lamb waves, in anisotropic media by a monochromatic incident beam, and to explain the deviation of the modal wave beam so created. Using a stationary phase approach, the modal wave beam is found to be deviated in the direction normal to the modal slowness curve. Due to the anisotropy, this direction is no longer in the sagittal plane. © 2001 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

acoustics / waves / vibrations / ultrasound / Lamb waves / anisotropic / beam

Déviation des ondes modales excitées dans une couche anisotrope par un faisceau ultrasonore monochromatique

Résumé. *Cette Note a pour but de décrire physiquement l'excitation par un faisceau incident monochromatique des ondes modales telles que les ondes de Lamb, dans des milieux anisotropes, et d'expliquer la déviation du faisceau d'ondes modales ainsi créé. L'utilisation d'un argument de phase stationnaire permet de trouver la direction dans laquelle le champ d'ondes modales est dévié. Cette direction, normale à la courbe des lenteurs d'ondes modales, n'appartient plus au plan sagittal en raison de l'anisotropie. © 2001 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS*

acoustique / ondes / vibrations / ultrasons / ondes de Lamb / anisotrope / faisceau

Version française abrégée

Les méthodes de Contrôle Non Destructif font appel à des techniques ultrasonores qui mettent en jeu des faisceaux acoustiques bornés venant insonifier des milieux anisotropes plongés dans un fluide. Selon la configuration de contrôle, des ondes modales [1–3], telles les ondes de Lamb ou de Rayleigh, sont excitées localement dans la structure. Le faisceau incident, de par sa nature bornée, va alors donner naissance à un faisceau d'ondes modales dans la structure, faisceau qui rayonne ensuite dans le milieu environnant. En raison de l'anisotropie du matériau, la direction du faisceau d'ondes modales va subir une déviation par rapport au plan sagittal du faisceau borné incident. Le but de la présente note est de décrire la génération d'un tel faisceau modal et d'en prévoir la déviation angulaire.

Note présentée par Henri CABANNES.

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Le champ acoustique produit à la fréquence ω dans le fluide externe par le transducteur peut s'exprimer comme une somme d'ondes planes sous forme d'une double transformée de Fourier spatiale [4], comme le montre l'équation (1) pour la pression acoustique par exemple, où k_x, k_y sont les composantes du vecteur d'onde courant sur les axes liés au plan $z = 0$ (voir *figure 1*). L'amplitude $\hat{A}^E(k_x, k_y, a; \omega)$ est maximale pour le couple (k_{x_0}, k_{y_0}) correspondant à l'axe acoustique. La pression réfléchie dans le fluide, donnée par l'équation (3), fait intervenir le coefficient de réflexion $\mathcal{R}(k_x, k_y, h, \tilde{\rho}; \omega)$ de chaque onde plane. Ce coefficient présente des singularités proches des solutions de l'équation de dispersion (4) pour les ondes modales de la structure dans le vide. Pour une valeur critique de l'angle d'incidence, le domaine d'intégration significatif Ω pour l'intégrale double (3) contient une partie significative de la courbe de dispersion modale $\mathcal{F}$ (voir *figure 3*), décrite par l'équation (4) [2,5]. A fréquence angulaire ω fixée, cette courbe est le lieu des extrémités du vecteur d'onde modal $\vec{k}_\Lambda$, de composantes (k_x, k_y), en fonction de l'angle azimutal φ. Un exemple de courbes (des lenteurs) modales est donné sur la figure 2, les caractéristiques du matériau pouvant être trouvées dans la référence [3]. En raison de l'anisotropie, ces courbes ne sont pas circulaires.

Lors de l'excitation d'une onde modale principale correspondant au couple (k_{x_0}, k_{y_0}), les ondes modales au voisinage de ce point sur la courbe $\mathcal{F}$, sont également excitées. L'ensemble de ces ondes constitue un faisceau d'ondes modales dans la structure. A la traversée de la courbe $\mathcal{F}$, le coefficient de réflexion $\mathcal{R}$ subit une rapide variation de phase. On peut alors montrer que l'intégrale (3) se ramène, au voisinage de $\mathcal{F}$, à une intégrale simple (6) le long de $\mathcal{F}$ qui représente le faisceau modal et son rayonnement dans le fluide externe. L'intégrale (7) décrit en particulier ce faisceau sur l'interface $z = 0$. L'amplitude $\hat{A}_\Lambda(k_x, a, h; \omega)$ dépend de k_x mais aussi des dimensions caractéristiques de la zone d'impact et de l'épaisseur de la structure. Sous une hypothèse de champ lointain, on peut considérer que les variations de phase liées à « a » et « h » sont négligeables par rapport à celles de la phase $\Phi(k_x, k_y; \omega)$ donnée par l'équation (8). La méthode de la phase stationnaire, exprimée par l'equation (9), permet alors de montrer qu'en champ lointain chaque onde modale, de vecteur d'onde $\vec{k}_\Lambda(k_x, k_y)$, se trouve localement distribuée selon une « direction de groupe » définie par la normale en (k_x, k_y) à la courbe $\mathcal{F}$ (voir *figure 5*). Cette onde modale, localement dominante, est une onde progressive qui se propage avec la vitesse de phase V_{ph} dans la direction du vecteur d'onde modal $\vec{k}_\Lambda$. Il est à noter que pour deux directions de groupe différentes ① et ②, les vecteurs d'onde modaux $\vec{k}_{\Lambda_1}$ et $\vec{k}_{\Lambda_2}$ des ondes locales correspondantes, sont différents (voir *figure 4*). La contribution principale dans l'intégrale (7) venant du point central (k_{x_0}, k_{y_0}) du spectre spatial du faisceau, le faisceau modal a alors pour direction d'énergie maximum la direction de groupe correspondant au point principal (k_{x_0}, k_{y_0}). En raison de l'anisotropie de la structure, la courbe modale n'est pas circulaire et cette direction de groupe est située en dehors du plan sagittal (voir *figure 5*).

Ce faisceau d'ondes modal rerayonne dans le fluide suivant un plan oblique contenant la direction de groupe principale et l'axe acoustique du faisceau réfléchi spéculaire. Les équations de ce plan peuvent être obtenues à partir de la représentation intégrale (6) par application de la méthode la phase stationnaire, équation (10), à la phase exprimée par l'équation (5). Les effets non spéculaires [6–8] seront donc à observer dans ce plan oblique et non plus dans le plan sagittal. Lorsque la fréquence varie, les courbes des lenteurs modales se déforment. L'effet de déviation du faisceau modal dépend donc de la fréquence.

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1. Introduction

Among the different ultrasonic technics used in Non Destructive Testing of plane structures, the generation of modal waves (after the terminology defined by Hayes [1]) is of great interest. Guided waves such as Lamb waves, and surface waves such as Rayleigh waves, are examples of modal waves [2,3]. In practice, the medium is in contact with a fluid in which the emitter transducer is immersed, and bounded acoustic beams are to be considered. At a characteristic pair (angle θ of the acoustic axis, frequency), the

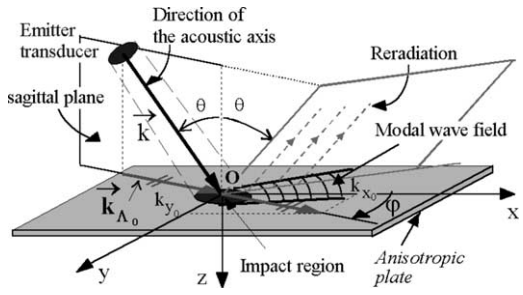

Figure 1. Deviation and reradiation of the modal beam.

Figure 1. *Déviation et rerayonnement du faisceau modal.*

acoustic beam generates locally a modal wave while the bounded nature of the beam rather provokes a modal wave beam in the structure. This modal beam re-radiates in the fluid. In the case of an anisotropic layer, the direction of the modal wave beam is deviated with respect to the sagittal plane, determined by the incident bounded beam (see *figure 1*). As the generation of such modal wave beams seems not to have been frequently mentioned in the litterature, the aim of the present work is to describe physically the phenomenon and to explain the deviation of modal wave beams in the anisotropic case, at a given frequency.

2. General features and description of the phenomenon

In a classical manner, the acoustic field of the incident bounded beam in the external fluid may be expressed in terms of plane waves in the form of a two dimensionnal space Fourier integral [4]. For example, for the acoustical pressure:

$$\widehat{P}_{\mathrm{inc}}(x,y,z,t;\omega) = \iint \hat{A}^E(k_x,k_y,a;\omega)\mathrm{e}^{\mathrm{i}(k_x x + k_y y + k_z z - \omega t)}\,\mathrm{d}k_x\,\mathrm{d}k_y \tag{1}$$

where ω is the angular frequency of the waves, k_x and k_y are the components of the current wave vector on the x and y-axis of the interface plane $z = 0$ (see *figure 1*), and k_z is given by the dispersion relation in the fluid:

$$c_0\sqrt{k_x^2 + k_y^2 + k_z^2} = \omega \tag{2}$$

where c_0 denotes the velocity of sound.

$\hat{A}^E$ is an amplitude function which depends on the particular boundary condition on the emitter. A bounded beam is characterized by a strongly marked maximum of the amplitude function $\hat{A}^E$ for a pair (k_{x_0}, k_{y_0}) which corresponds to the acoustic axis. In the vicinity of the interface, the impact region of the beam can be discerned, and the parameter 'a' denotes its characteristic length. Then, the reflected acoustic field in the fluid may be expressed in the following form:

$$\widehat{P}_{\mathrm{ref}}(x,y,z,t;\omega) = \iint \hat{A}^E(k_x,k_y,a;\omega)\mathcal{R}(k_x,k_y,h,\widetilde{\rho};\omega)\mathrm{e}^{\mathrm{i}(k_x x + k_y y - k_z z - \omega t)}\,\mathrm{d}k_x\,\mathrm{d}k_y \tag{3}$$

where $\mathcal{R}(k_x,k_y,h,\tilde{\rho};\omega)$ is the reflection coefficient of each plane wave. This reflection coefficient depends on the geometrical and physical properties of the layer (and of the external fluid). h denotes the thickness of the layer and the adimensional parameter $\tilde{\rho}$ is representative of the ratio of the voluminal mass of the fluid to that of the layer. Notably, $\mathcal{R}$ contains all the informations on the structure modal waves. In fact, the function $\mathcal{R}$ is strongly related to the modal slowness curve equations such as they are described in the following section. Moreover, the anisotropy of the structure does affect the form of the reflection coefficient $\mathcal{R}$ which is no longer invariant for a rotation about the z-axis, contrary to the case of isotropic media.

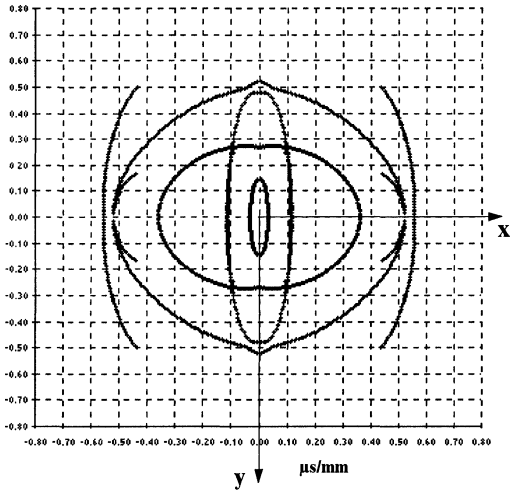

Figure 2. Slowness curves for Lamb modes in unidirectionnel carbon/epoxy layer (direction of the fibers parallel to Ox). $f.h. = 1$ MHz·mm.

Figure 2. *Courbes des lenteurs des ondes de Lamb pour une couche en carbone/époxyde unidirectionnelle (direction des fibres parallèle à Ox). $f.h. = 1$ MHz·mm.*

3. Modal waves and dispersion curves

Generally speaking, modal waves propagate with a phase velocity V_{ph} in the layer plane. When the anisotropic layer is in contact with vacuum, the writing of the boundary conditions leads to a dispersion relation for modal waves:

$$\omega = F(k_x, k_y) \tag{4}$$

For a given propagation direction (angle φ) of the layer plane, V_{ph} can be plotted as a multi-valued function of the angular frequency ω, which gives dispersion curves for the corresponding modal waves (Lamb or Rayleigh-type). For a given frequency ω and a given direction φ, there may be a number of possible values for V_{ph}, corresponding to different families of modal waves. When φ varies, plotting V_{ph} leads to a number of modal curves $\mathcal{F}$, given implicitly by equation (4) in the (k_x, k_y) plane. An example of such curves for Lamb modes is presented on *figure 2* for a Carbon/Epoxy layer, in the more usual form of the so-called 'slowness curves'. Other examples may be found in references [2,5]. It is worthwhile to note that, in the present case, these slowness curves depend on the frequency.

4. Beam effect: specular reflection and modal wave field

We come back here to expression (3) of the reflected field. In the preceeding section, the modal waves have been described for a layer in vacuum. When the external medium is a fluid, these modal waves are slightly modified if the ratio $\tilde{\rho}$ of the voluminal mass of the fluid to that of the solid is small. This is practically the case, even for an interface water-solid, and the term 'generalized modal waves' is usually employed. From a mathematical point of view, this slight modification of the modal waves is related to the fact that singularities of the reflection coefficient $\mathcal{R}(k_x, k_y, h, \tilde{\rho}; \omega)$ are close to the solutions of the modal equation (4) in the complex domain $\mathbb{C}^2$ in (k_x, k_y). More specifically, since the reflection coefficient has its magnitude less than or equal to one, it may be seen that, simultaneously, zeroes of $\mathcal{R}$ are close to the solutions of equation (4). As a consequence, the singular behavior of $\mathcal{R}$ in the vicinity of the modal curve $\mathcal{F}$ in the (k_x, k_y) plane is expressed by an abrupt change of the phase of $\mathcal{R}$ across the curve $\mathcal{F}$.

As explained in Section 2, the beam feature is due to a maximum of the amplitude $\hat{A}^E$ for the main point (k_{x_0}, k_{y_0}) corresponding to the acoustic axis in the (k_x, k_y) plane. Then the relevant integration domain, for integrals (1) and (3), is some region Ω around the axial point (k_{x_0}, k_{y_0}).

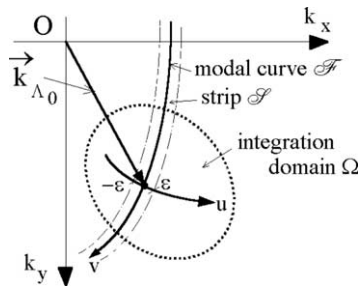

Figure 3. Curvilinear coordinate system associated to the modal curve $\mathcal{F}$ crossing the integration region Ω.

Figure 3. *Système de coordonnées curvilignes associé à la courbe modale $\mathcal{F}$ traversant le domaine d'intégration Ω.*

When the frequency and the incident angle are so that some modal wave in the structure may be excited by the incident beam, the axial point (k_{x_0}, k_{y_0}) belongs to the corresponding modal curve $\mathcal{F}$. In other words, in such circumstances, a branch $\mathcal{F}$ of the modal curve crosses the integration region Ω (see *figure 3*).

Two main parts may then be distinguished in the reflected field described by the integral (3). These two parts may be well described by a far field analysis based on a stationary phase argument. First, looking for the stationary value of the phase (ω and t being constant):

$$\Phi = k_x x + k_y y - k_z z - \omega t \tag{5}$$

with the dispersion condition (2) leads to the usual reflected rays which constitute the specular reflected beam. As this is a classical result, we do not go further in details about it. The outline of the beam, and in particular the reflected axis, is due to the variations of the amplitude $\hat{A}^E$ in terms of (k_x, k_y). In this case, the reflection coefficient $\mathcal{R}$ plays no specific part as long as it is smoothly varying. However, this is no longer true in the vicinity of the modal curve $\mathcal{F}$. Precisely, the second main part of the reflected field is due to the integration of integral (3) in a narrow strip $\mathcal{S}$ of the region Ω along the curve $\mathcal{F}$. If a local curvilinear coordinate system (u, v) is introduced in the strip $\mathcal{S}$, as shown on *figure 3*, a first integration, with respect to the coordinate u across the curve $\mathcal{F}$, will smooth the rapid phase variation of $\mathcal{R}$. The adimensional width ε of the strip $\mathcal{S}$ is of the order of the ratio $\tilde{\rho}$ of the voluminal masses. Assuming that the asymptotic assumption $\tilde{\rho} \ll 1$ is stronger than the far field assumption, the phase (5) may be considered as constant for the u-integration in the small range $[-\varepsilon, +\varepsilon]$. So, the contribution of the strip region $\mathcal{S}$ to the integration of (3) reduces to a single integration in terms of the coordinate v, i.e., along the modal curve $\mathcal{F}$. This 'modal' integral may be given in the following form:

$$\widehat{P}_\Lambda(x, y, z, t; \omega) = \int_{\mathcal{F}} \hat{A}_\Lambda(k_x, a, h; \omega)\, \mathrm{e}^{\mathrm{i}(k_x x + k_y y - k_z z - \omega t)}\, \mathrm{d}k_x \tag{6}$$

where ω is fixed, k_y is a function of k_x via the modal dispersion relation (4) and k_z depends on k_x and k_y through the fluid dispersion equation (2). However, due to the beam feature, in this last integral, the main contribution will remain that of the main point (k_{x_0}, k_{y_0}) which belongs, as an hypothesis, to the modal curve $\mathcal{F}$.

5. Angular deviation of the modal beam

It can be seen, from integral (6), that the modal wave excited by the incident axis is not the only wave present in the reflected field, but also are present the neighbouring modal waves corresponding to the part of the modal curve $\mathcal{F}$, in the vicinity of the main point (k_{x_0}, k_{y_0}), in the region Ω. This superposition of neighbouring modal waves results in the making up of a modal beam in the layer. As we are first interested in the description of this modal beam, let us look at the value of the reflected field on the interface, taking

$z = 0$ in equation (6):

$$\widehat{P}_\Lambda(x, y, 0, t; \omega) = \int_{\mathcal{F}} \hat{A}_\Lambda(k_x, a, h; \omega)\, \mathrm{e}^{\mathrm{i}(k_x x + k_y y - \omega t)}\, \mathrm{d}k_x \tag{7}$$

The direction of this modal beam is now given by a far field assumption, using again a stationary phase argument. Indeed, let us suppose now that the phase term

$$\Phi(k_x, k_y; \omega) = k_x x + k_y y - \omega t \tag{8}$$

varies rapidly, compared to the amplitude function $\hat{A}_\Lambda(k_x, a, h; \omega)$, when the modal wave vector $\vec{k}_\Lambda$, with the components k_x and k_y, follows the modal curve $\mathcal{F}$. Thus, an argument of stationary phase may be introduced. This will be the case if the phase variations involved in the amplitude $\hat{A}_\Lambda(k_x, a, h; \omega)$, due to the presence of the impact characteristic length 'a' and of the thickness of the layer 'h', are negligible in comparison to those of the phase Φ. This circumstance will be encountered if the relevant distance r of the observation point $M(x, y)$ in the acoustic field is much larger than the characteristic dimensions a and h. In other words, the stationary phase procedure is valid here in the far field assumption.

Let $M(x, y)$ be given in the far field of the plane interface (O, x, y). The main plane wave at this point (at a given time t) is given by the stationary phase method, using equation (8):

$$\mathrm{d}\Phi = 0 \quad \Longleftrightarrow \quad x\,\mathrm{d}k_x + y\,\mathrm{d}k_y = 0 \tag{9}$$

which expresses the orthogonality between the position vector $\overrightarrow{OM}(x, y)$ and the differential vector $\mathrm{d}\vec{k}(\mathrm{d}k_x, \mathrm{d}k_y)$. This last vector is tangent to the modal curve $\mathcal{F}$ at a point $\vec{k}_\Lambda$. Following the stationary phase argument, this modal wave vector gives the dominant modal wave at $M(x, y)$. The set of points M such as $\overrightarrow{OM}$ is perpendicular to $\mathrm{d}\vec{k}$ constitutes a 'group direction' along which, locally on the wave-length scale, the modal wave field is governed by the same modal wave vector $\vec{k}_\Lambda$, as schematized on *figure 4* by the hatched circles. This locally dominant modal wave is a progressive wave which propagates with the phase velocity V_{ph} in the direction of $\vec{k}_\Lambda$. It should be noted that, for two different group directions ① and ②, two different modal wave vectors $\vec{k}_{\Lambda_1}$ and $\vec{k}_{\Lambda_2}$ are found.

As above mentioned in Section 4, the main contribution in the integral (7) comes from the central point (k_{x_0}, k_{y_0}) of the beam spectrum, i.e., from the modal wave vector $\vec{k}_{\Lambda_0}$. Thus, the most energetic part of the modal beam will be found along the group direction which corresponds to this vector $\vec{k}_{\Lambda_0}$, i.e., on the direction of the normal to the modal curve $\mathcal{F}$ at the point (k_{x_0}, k_{y_0}). Due to anisotropic effects, the slowness curve is not circular in general, so that this normal direction is different from the radial direction of the propagation vector $\vec{k}_{\Lambda_0}$ (see *figure 5*). The modal beam is thus deviated in the group direction associated to the modal wave vector of the acoustic axis.

6. Reradiation of the modal beam

This modal beam, as it propagates along the anisotropic medium, reradiates in the external fluid. This reradiation phenomenon is centered in an oblique plane corresponding to the reflected direction of the acoustic axis (see *figure 1*). The intersection of this oblique plane with the interface direction is the main group direction of the modal beam. This result could be well obtained again by a stationary phase argument applied to integral (6) where the phase Φ is given by equation (5). Owing to the dispersion relation (2), the stationary condition is now written in the following form:

$$\left(x + z\frac{k_x}{k_z}\right)\mathrm{d}k_x + \left(y + z\frac{k_y}{k_z}\right)\mathrm{d}k_y = 0 \tag{10}$$

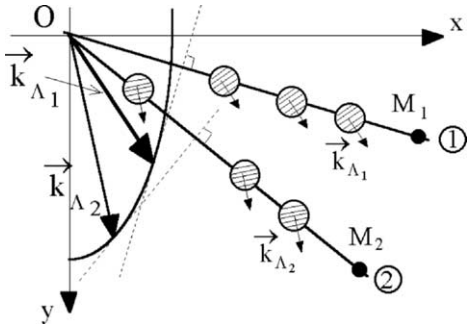

Figure 4. Set of the points M constituting a group direction, for two modal wave vectors. The aligned hatched regions correspond to the same wave vector.

Figure 4. *Ensemble des points M constituant une direction de groupe, pour deux vecteurs d'onde modaux. Les zones hachurées correspondent au même vecteur d'onde.*

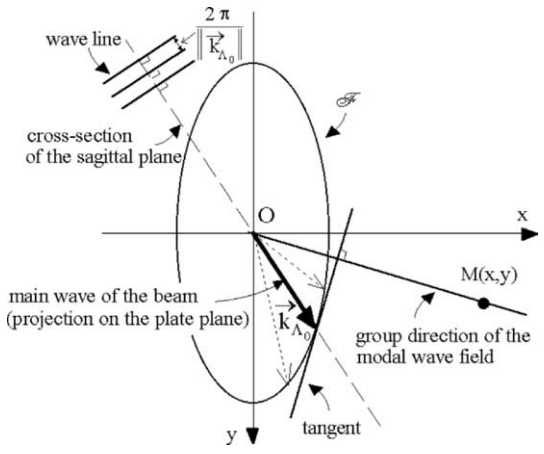

Figure 5. Obtention of the main group direction of the modal wave field, for a particular branch of the modal curve.

Figure 5. *Obtention de la direction de groupe principale du champ d'ondes modales pour une branche particulière de la courbe modale.*

which leads to the equation of the oblique plane under the constraint that the differential vector $(\mathrm{d}k_x, \mathrm{d}k_y)$ be tangent to the modal curve. All the nonspecular effects observed by Neubauer [6] and modelized by several authors, among them those of refs [7,8], are to be searched for in this oblique plane and not in the sagittal plane. In this context, a bi-dimensional modelization of the acoustic beam becomes insufficient.

7. Conclusion

For some ultrasonic testing configurations of an anisotropic structure, the plane wave corresponding to the acoustic axis of the emitter transducer immersed in the external fluid can excite a modal wave in the structure. In this case, the incident beam does not only generate the main modal wave in the direction of the acoustic axis, but also the neighbouring modal waves along the modal slowness curve, so that a modal wave beam is generated in the structure. Due to anisotropic effects, the main axis of the modal beam is no longer contained in the sagittal plane of the incident beam. This deviation effect has been explained by using a stationary phase approach, valid in the far field. Modal slowness curves have been plotted. The more the modal slowness curve branch is strongly elliptic, the more the deviation effect is marked. In the case of modal waves, it should be noted that there is also a frequential dispersion phenomenon in each direction, which leads to the variation of the modal slowness curves with the frequency. As a consequence, the angular deviation effect which is described in the present work, evoluates with the frequency.

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