Experimental verification of the theory of multilayered Rayleigh waves

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A phenomenon which has been termed "*multilayered Rayleigh modes*" has been presented in previous papers. This study aims to prove experimentally the existence of these waves in anisotropic periodically multilayered media. These modes result from a combination of Floquet waves which propagate in a periodically multilayered medium when all the Floquet waves are inhomogeneous. The experimental verification was done using an acousto-optic technique and a measurement of the reflected field, which was obtained with a hydrophone measurement system, on a carbon/epoxy composite plate. The experimental and calculated dispersion curves of the *multilayered Rayleigh modes* were then drawn. The coincidence of the curves was found quite good, thus confirming our theory. However, two modes were found by the acousto-optic technique not to fit into the theory. One experimentally detected mode was found to correspond to the Lamb mode of the plate and the other was not experimentally detected by the acousto-optic technique. Measurement of the reflected field for this mode, which was obtained with a hydrophone measurement system, and its comparison with the predicted reflected field make it possible to verify the existence of the mode. The combination of both experiments permit a good coincidence to be found. © *1999 American Institute of Physics*. [S0021-8979(99)01314-6]

I. INTRODUCTION

The propagation of surface waves, particularly Rayleigh waves, has become the subject of a large number of studies. Although a complete review of the extensive literature on this subject cannot be undertaken here, the main contributions relevant to this paper are worth mentioning. A Rayleigh wave is a surface wave which propagates at the interface separating a vacuum from an infinite isotropic medium.^{1,2} Registering boundary conditions leads to the cancellation of a determinant which results in Rayleigh wave becomes a leaky Rayleigh wave, and its behavior is related to the pole of the reflection coefficient. The wave numbers corresponding to the pole and to the zero of the reflection coefficient are conjugated complexes.³ The modulus of the reflection coefficient is therefore equal to one, whereas its phase is equal to

Many of the studies on stratified media deal with an infinite substrate medium below the stratified or nonstratified structure. A review of these works is given in Ref. 7 but those of Chimenti *et al.*,⁸ Bogy and Gracewski,⁹ Rokhlin *et al.*¹⁰ and Haskell¹¹ should be mentioned in particular. To our knowledge, as far as infinite periodically stratified media are concerned, there is no complete work on leaky waves that behave as Rayleigh waves in infinite isotropic homogeneous media. In 1990, Nayfeh and Chimenti studied an infinite periodically isotropic structure of copper/steel/....¹² By using both the elastic constants of the corresponding homogenized medium and the coupled mixture equations corresponding to each layer of the medium, they identified a

 $[\]pi$ and presents a point of inflection. Modeling the absorption of infinite isotropic solids leads to a trough of the modulus of the reflection coefficient^{4–6} near the Rayleigh angle. In such solids, Becker and Richardson^{4,5} have proved that the depth of the minimum of the reflected amplitude is the shear wave attenuation per wavelength.

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FIG. 1. Infinite periodically multilayered medium.

dispersive behavior for the medium, and using both analytical and experimental methods they found that the velocity of the Rayleigh wave depends on the frequency.

In 1995, Potel *et al.* showed that, following certain conditions and by analogy with the behavior of the Rayleigh wave in isotropic media, there exist waves that have been termed "*multilayered Rayleigh waves*" (see Refs. 7 and 13–15 and references contained therein). These waves propagate in infinite anisotropic periodically multilayered media (see Fig. 1).

The main properties of *multilayered Rayleigh waves* are that:

- (1) The propagation of a *multilayered Rayleigh wave* is characterized by the cancellation of a (3×3) determinant. This determinant corresponds to the boundary conditions for a vacuum/infinite anisotropic periodically multilayered medium structure.
- (2) A multilayered Rayleigh wave is a linear combination of three inhomogeneous Floquet waves. The Floquet waves are the propagation modes of an infinite periodically multilayered medium (see Refs. 16–18 and references contained therein).
- (3) As the Floquet waves are dispersive, the *multilayered Rayleigh wave* is also dispersive. The propagation of this wave is therefore linked to both an angle and a frequency, which is not the case for the Rayleigh wave.
- (4) A critical attenuation exists, where the depth of the trough of the reflection coefficient is maximum. For this attenuation, the reflection coefficient is equal to zero.
- (5) When the medium is not infinite but is thick enough, it behaves as if it were infinite. The modulus of the reflection coefficient is therefore equal to one when the medium is nonlossy, and presents a trough when the medium is lossy.

All these phenomena, especially the reflection coefficient modulus trough, occur for plane waves. However, the presence of a Rayleigh wave leads to similar phenomena when bounded beams are considered: the reflected beam is split. Bertoni and Tamir¹⁹ have developed an analytical model to describe the nonspecular reflection of ultrasonic beams from a fluid–solid interface for incidence at the Rayleigh angle. In

their results, the specular reflection and the reradiation of Rayleigh waves appear clearly. This was experimentally checked by Breazeale.^{20,21} Bertoni and Tamir's representation for the incident beam was employed by Rousseau and Gatignol^{22,23} who approximated the reflected field integrals for single interfaces and plates, by paraxial high-frequency asymptotics. Ngoc and Mayer²⁴ used the numerical integration approach to extend the Bertoni and Tamir results to arbitrary incidence on fluid-solid interfaces and submerged plates. A finite superposition of inhomogeneous plane waves can also be considered for the beam, which leads to similar results.^{25,26} By defining the incident domain by a Gaussian distribution of velocities along a plane transmitter, Matikas et al.^{27,28} built the incident and reflected beams, alternatively Pott and Harris^{29,30} used the complex source point method. Zeroug et al.^{31,32} also used the second method for expansion of a transducer field in terms of quasi-Gaussian beams. The stratified structure case was considered by Zeroug and Felsen,³³ with emphasis placed on the regime of nonspecular reflection, characterized by strong coupling between the specularly reflected beam and leaky wave supported by the structure. Recently, Rehman et al. have also applied the angular spectrum decomposition method to investigate the nonspecular reflection effects for anisotropic multilayered media.^{34,35}

Initially, ultrasonic nonspecular effects were studied experimentally by Schoch³⁶ who used a Schlieren method: the reflected field was laterally translated according to what the geometrical acoustics predicted. This paper aims to prove the theory developed for plane waves, to *multilayered Rayleigh waves*, by using both an experimental acousto-optic technique linked to bounded beams, and a measurement of the reflected field. The experiments were carried out on a composite of carbon/epoxy layers. After a description of the experimental techniques, the dispersion curves of *multilayered Rayleigh modes* are compared with the experimental curves. The results are then carefully examined, in order to explain some differences.

II. EXPERIMENTAL TECHNIQUES

A. Acousto-optic technique

1. Description of the acousto-optic technique

The acousto-optic technique^{37,38} is used as a nondestructive testing technique for the quality control of layered materials, and is based on the interaction of laser light with ultrasound. Contrary to most of the other existing ultrasonic techniques which focus on the amplitude of the reflected or transmitted ultrasonic wave from the material under investigation, the acousto-optic technique focuses on the phase difference between the incident and the reflected ultrasound. This phase information and the critical angle seem to be extremely sensitive to the specific characteristics of layered materials even for low frequency ultrasound. Laser light can be used to find this phase information quite easily and to detect the critical angle. This new nondestructive testing technique has the advantage over other high-resolution techniques in that it uses low-frequency ultrasound and low power laser light. It is also extremely sensitive to very thin



FIG. 2. Experimental setup to make Schlieren images.

layers and small defects at or near the surface of a material. Even the hardness of flat material or the surface roughness of a plate can be measured in this way.³⁹ Figure 2 shows a scheme of the experimental setup of the acousto-optic technique. One 10 mW He-Ne laser was used with a high power ultrasonic transducer of 3.5 MHz. The ultrasonic transducer is powered by a sine wave generator (type SG4511 pulse/ function generator from IWATSU) and is mounted on a motorized rotation table having a resolution of 0.001°. The angle rotation is carried out automatically (Newport Motion Controller MM3000). Each specimen was placed in a water tank on an aluminum platform perpendicular to the incident plane. The laser beam was incident perpendicular to and diffracted by the overlap of the incident and the reflected ultrasound waves. The diffraction pattern can be separated into two regions: the optical near field and the optical far field. In order to determine the critical angles of the investigated material, the incident as well as the reflected ultrasound waves were visualized in the optical far field using the Schlieren image technique.⁴⁰ For this, a slit was used in order to eliminate diffraction orders 0, -1, -2, etc. and to make an interference with the other orders, in order to get the Schlieren image, as illustrated in Fig. 2. It should be noted that in the experimental setup, the specimen can be moved in an x-yplane which makes it possible to focus on different positions of the investigated material.

2. Experiments on a carbon/epoxy composite

The medium used was made up of stacked identical hexagonal layers of carbon/epoxy, each at 45° to the previous one $(0^{\circ}/45^{\circ}/90^{\circ}/135^{\circ}$ medium). This composite plate consisted of six superlayers (24 layers), each layer being 0.12 mm thick. It was supplied by Aerospatiale (France). As the aim of this research was to determine different critical angles for the specified composite, Schlieren images were made for changing incident angles and different ultrasonic frequencies. Figure 3 is a photograph of a Schlieren image in which the ultrasonic frequency is 3 MHz and for which the incident angle is a critical angle (20°) of the composite. The critical angle differs from a noncritical incident angle (16°, Fig. 4) by the presence in the reflected field of a "null" region, a region in which there is no ultrasound at all, between two regions of ultrasound. It is the difference between



FIG. 3. Schlieren image of an ultrasonic wave (3 MHz) incident in a critical angle (20°) of the composite.

the reflected fields, as illustrated in Figs. 3 and 4, which gives the angle and the frequency for which a critical angle occurs. Starting from three different frequencies, 2.8, 3, and 4 MHz, and focusing on one point on the composite, the critical angles between 0° and 45° were detected. Once a critical angle was found for a certain frequency, the same critical angle was sought by changing the ultrasonic frequency. Frequency step sizes were increased by increments of 0.1 MHz while the angle rotation step size was 0.5° . Due to the limitation in bandwidth of the ultrasonic transducer, only frequencies between 2.5 and 4.7 MHz were used. It should be noted that until now, no exact procedure has been found to detect accurately the perpendicular incidence (0°) . Due to this the experimental results might have an uncertainty of $\pm 1^{\circ}$.

B. Measure of the reflected field with a hydrophone measurement system

As explained in Sec. I, the presence of a Rayleigh wave leads to a split of the reflected beam. In order to complement



FIG. 4. Schlieren image of an ultrasonic wave (3 MHz) incident in an angle (16°) different from the critical angles of the composite.



FIG. 5. Experimental setup of the measurement of the reflected field with a hydrophone measurement system.

the experiments using the acousto-optic technique described above, the reflected field was measured for the $0^{\circ}/45^{\circ}/90^{\circ}/135^{\circ}$ plate described in Sec. II A 2.

The plate was excited by a 19 mm transducer, with central frequency equal to 2.25 MHz. The transducer was driven with 20 cycles of a 3 MHz sine wave. The incident angle was equal to 25.8°, which corresponds to the propagation of a *multilayered Rayleigh wave* at f=3 MHz, as it will be seen in Sec. III G. The distance between the transmitting transducer and the plate was 235 mm in the far field. The reflected field was measured in the (A,x,y) plane with a Hydrophone Measurement System (see Fig. 5). The size of the hydrophone element was 0.5 mm. Figure 6 presents a section of the experimental reflected field (thick line).

III. COMPARISON OF MODELED WITH EXPERIMENTAL DISPERSION CURVES

A. Background on multilayered Rayleigh waves

The results presented in this part of the article were carried out in Refs. 7,15,18, and 41.

An infinite periodically multilayered medium is the reproduction of an infinite number of "periods," each one made by the stacking of distinct anisotropic media (see Fig. 1). An oblique incident wave propagates in a semiinfinite medium 0 and is contained in the plane x_1Ox_3 , defined in Fig. 1. The writing of the boundary conditions at each interface separating two successive layers leads to the transfer matrix of one superlayer. This matrix is defined in



FIG. 6. $0^{\circ}/45^{\circ}/90^{\circ}/135^{\circ}$ medium consisting of six periods of f=3 MHz and $\theta=25.8^{\circ}$. Diameter of the transmitting transducer: 2a=19 mm. Section of the reflected profiles: (thick line) experiments, (thin line) calculus.

Ref. 18. It allows the displacement amplitudes of the plane waves in the first layer of a period to be expressed as a function of those in the first layer of the previous period. Due to the fact that, locally, the acoustical state is characterized by six quantities, this matrix is of the sixth order, and the waves which correspond to the eigenvectors of this matrix are the Floquet waves. If the general solution is decomposed on the Floquet basis, the transfer matrix becomes a diagonal matrix. These six Floquet waves are the propagation modes of an infinite periodically multilayered medium. They are linear combinations of the classical waves propagating in each layer of the multilayered medium. As a consequence, the displacement and stress vectors can be expressed in the Floquet wave basis, which permit important numerical problems to be solved when the stratified medium is finite.⁴¹ Moreover, Floquet waves propagating in a multilayered medium can be used as classical waves propagating in a homogeneous medium, which is physically more transparent. The major difference results from the dispersive character of the Floquet waves.

When a finite medium is thick enough, according to the frequency and to the incident angle, it can behave as if it were infinite. This is the case for the composite plate described in Sec. II A 2. From experiments on this plate, submerged in water, it was shown⁷ that the presence of a trough of the experimental reflection coefficient results from the propagation of what we termed a *multilayered Rayleigh wave*, by analogy with the propagation of Raleigh waves in isotropic media. Indeed, this trough was not found on the modeled coefficient when the medium is considered as nonlossy (i.e., the elastic constants are real), whereas it is found on the modeled coefficient when the medium is considered lossy. When the medium is thin, the beginning of a multilayered Lamb mode can be observed on the reflection coefficient, even if the medium is considered nonlossy.

In Sec. I, it was seen that the propagation of a *multilay-ered Rayleigh wave* is characterized by the cancellation of a (3×3) determinant that corresponds to the boundary conditions for a vacuum/infinite anisotropic periodically multilayered medium structure. This determinant depends on k_1 , the projection on the x_1 axis of the wave number vector. k_1 is real. However, it is more convenient to represent this determinant as a function of a fictitious angle of incidence θ , which can be defined in relation to a medium of reference. If the medium of reference is water k_1 is given by

$$k_1 = \frac{\omega}{V_{\text{water}}} \sin \theta \,, \tag{1}$$

with $V_{water} = 1480$ m/s. The value of k_1 for which the (3×3) determinant is equal to zero corresponds to the propagation of a *multilayered Rayleigh wave*, if all the Floquet waves which propagate in the structure are inhomogeneous. Dispersion curves for the *multilayered Rayleigh modes* can therefore be drawn by searching both in angle and in frequency for the cancellation point of the (3×3) determinant when all the Floquet waves are inhomogeneous. If the medium of reference is water, the *multilayered Rayleigh wave* speed is given by

TABLE I. Elastic constants in GPa for a carbon/epoxy medium from Ref. 38 if the sixth-order symmetry A_6 axis is parallel to the (Ox_3) axis.

c_{11}	c_{12}	<i>c</i> ₁₃	c 33	c_{44}	$ ho(kg/m^3)$
13.7+0.13 <i>j</i>	$7.1 \pm 0.04j$	$6.7 \pm 0.04j$	126 + 0.73j	5.8+0.1 <i>j</i>	1577

$$V_R = \frac{V_{\text{water}}}{\sin \theta_R},\tag{2}$$

where θ_R is the multilayered Rayleigh angle.

B. Theoretical dispersion curves for *multilayered Rayleigh waves*

The viscoelastic constants used in the model are those determined by Hosten and Castaings:^{42,43} these constants are complex, in other words the medium is a lossy one (see Table I). The volumetric mass of each layer is equal to 1577 kg/m³. By using the above described procedure, the dispersion curves for *multilayered Rayleigh modes* for a non-lossy 0°/45°/90°/135° medium (see Fig. 7) can be drawn. It should be noted here that representing the incident angle as a function of the frequency is equivalent to representing the velocity of the *multilayered Rayleigh wave* as a function of the frequency.

The experimentally measured critical angles by the acousto-optic technique together with the theoretically predicted critical angles are presented in Fig. 7.

As the precision for the experiments is $\pm 1^{\circ}$, the coincidence between the modeled curves and the experimental curves is quite good for modes (2), (3), (4), (5), and (6), except for some zones which will be carefully examined subsequently. The case of mode (1) will be discussed in Sec. III G. For each mode, the phenomenon will be illustrated by only one example, in order to avoid unnecessary complications, with the help of Figs. 8, 9 and 10. Figures 8–10 allow us to check, for different frequencies, that the experimentally detected wave has or does not have the properties of a *multilayered Rayleigh wave*: modulus of the reflection coefficient equal to one (a), Floquet waves all inhomogeneous (c), and determinant corresponding to the boundary conditions equal to zero (d).



FIG. 7. Experimental measured (full lines) as well as theoretical predicted (dotted lines) critical angles for the $0^{\circ}/45^{\circ}/90^{\circ}/135^{\circ}$ composite.



FIG. 8. $0^{\circ}/45^{\circ}/90^{\circ}/135^{\circ}$ medium consisting of six periods at f=2.7 MHz modulus (a) and phase (b) of the reflection coefficient in water: (thin line) nonlossy medium, (thick line) lossy medium. Number of the Floquet waves which are inhomogeneous in the nonlossy medium (c) and zoom of the modulus of the (3×3) determinant (d) corresponding to the boundary conditions for a vacuum/infinite nonlossy multilayered medium structure, as a function of the incident angle.

C. Study of mode (5)

From 4.2 MHz, experimental and modeled curves coincide quite well. Let us see now, for frequencies inferior to 4.2 MHz, why a mode not predicted by the theory can be found experimentally. It can be seen in Fig. 8 at the frequency f=2.7 MHz, as well in Fig. 9 at f=3 MHz, that although the modulus of the (3×3) determinant corresponding to the boundary conditions is almost equal to zero, only two Floquet waves are inhomogeneous, in other words one Floquet wave is propagative. This mode is not a *multilayered* Rayleigh mode, because in order to be so, all Floquet waves would have to be inhomogeneous. Moreover, the modulus of the reflection coefficient for the nonlossy 24 layer plate is not equal to one, and presents a trough. We can thus conclude that at these frequencies, the mode which is experimentally detected is not a *multilayered Rayleigh mode*, but is a Lamb mode for the plate. Indeed, the acousto-optic technique described in Sec. II cannot distinguish the difference between a Lamb mode and a Rayleigh mode.

At f=4.3 MHz, for example, the experimental and modeled curves coincide. It can be seen in Fig. 10 that the modulus of the determinant is almost equal to zero, that all the Floquet waves are inhomogeneous and that the modulus of the reflection coefficient for the nonlossy plate is equal to one. For this frequency, the plate is thick enough to behave



FIG. 9. Extracted from Ref. 16: $0^{\circ}/45^{\circ}/90^{\circ}/135^{\circ}$ medium consisting of six periods at f=3 MHz modulus (a) and phase (b) of the reflection coefficient in water: (thin line) nonlossy medium, (thick line) lossy medium. Number of the Floquet waves which are inhomogeneous in the nonlossy medium (c) and zoom of the modulus of the (3×3) determinant (d) corresponding to the boundary conditions for a vacuum/infinite nonlossy multilayered medium structure, as a function of the incident angle.

like an infinite medium. The mode experimentally detected is thus a *multilayered Rayleigh mode*.

D. Study of mode 6

For the same reasons, it can be seen in Fig. 10, at f=4.3 MHz, that the mode experimentally detected is a *multilayered Rayleigh mode*: here, all the Floquet waves are inhomogeneous, the modulus of the reflection coefficient for the nonlossy plate is equal to one and the modulus of the (3×3) is almost equal to zero.

E. Study of modes (3) and (4)

The experimentally detected modes are multilayered Rayleigh modes as can be confirmed by examining Fig. 8 at f=2.7 MHz for both modes or Fig. 9 at f=3 MHz for mode (3), all the properties of *multilayered Rayleigh modes* are checked, although the modulus of the reflection coefficient for the plate (see Fig. 9) is not quite equal to one for the mode (4). However, the calculation has been done for other frequencies, and this modulus was found to be equal to one, which confirms that the plate behaves as if it were infinite.

F. Study of mode 2

The experimentally detected mode is not a *multilayered Rayleigh mode* but a Lamb mode, there is a small trough of



FIG. 10. $0^{\circ}/45^{\circ}/90^{\circ}/135^{\circ}$ medium consisting of six periods at f=4.3 MHz modulus (a) and phase (b) of the reflection coefficient in water: (thin line) nonlossy medium, (thick line) lossy medium. Number of the Floquet waves which are inhomogeneous in the nonlossy medium (c) and zoom of the modulus of the (3×3) determinant (d) corresponding to the boundary conditions for a vacuum/infinite nonlossy multilayered medium structure, as a function of the incident angle.

the modulus of the reflection coefficient for the nonlossy plate (see Fig. 8 at f=2.7 MHz) and one Floquet wave is propagative. For other frequencies, it appears that this theoretically detected mode is a surface mode.

G. Study of mode (1)

Theoretically, this mode is a "perfect" multilayered Rayleigh mode, as can be seen in Figs. 8, 9, and 10; the determinant corresponding to the boundary conditions for a vacuum/infinite multilavered medium structure is almost equal to zero. Here, all the Floquet waves are inhomogeneous and the modulus of the reflection coefficient for the nonlossy plate is equal to one, whereas that for the lossy plate presents a trough. Nevertheless, this mode was not experimentally detected by the acousto-optic technique; the nominal frequency and the width of the ultrasonic beam would have to be changed. However, it was detected by the measurement of the reflected field. In fact, in Sec. II A 1, it was seen that the acousto-optic technique is based on the interaction of laser light with ultrasound. Thus, as the luminous intensity is not proportional to the displacement amplitudes of the field, this technique has a saturation level. All the amplitudes which are greater than this level are therefore represented by the same luminous intensity. Consequently, depending on this saturation level, a trough of the reflected field which is not deep enough will not be detected. It will be shown that this is the case for mode (1). Indeed, the modelized reflected field (see explanation below) presents a trough which is not as deep as for other *multilayered Rayleigh modes*. This is why it was not detected by the acousto-optic technique.

1. Prediction of the reflected field

When finite-sized transducers are employed, the bounds of the beam have to be considered so the angular spectrum decomposition method^{19,24,44} was used. The general model for periodically anisotropic multilayered media, first developed for plane waves,^{7,15,18} was thus extended to take into account bounded beams.^{34,35} The predicted reflected field corresponding to the experimental conditions at f=3 MHz and $\theta = 25.8^{\circ}$ is shown in Fig. 6 (thin line). The differences between experimental and predicted results are due to the viscoelastic constants used. In fact, these constants used in the model are those determined by Hosten and Castaings^{42,43} for a lossy medium (see Table I), and do not exactly correspond to those of the sample. In addition, although the modelization of the incident beam by a Tukey window is a good approximation, it does not represent the experimental incident beam correctly. In any case, this comparison aims simply to highlight the fact that the trough of the reflected field does not reach zero. This confirms that the saturation level in the acousto-optic technique was undoubtly too high to be able to detect this trough. In conclusion, we can affirm that mode (1) is a multilayered Rayleigh mode.

IV. CONCLUSION

The aim of this paper was to verify experimentally a theory developed for plane waves using bounded beams, by analogy with the propagation of Rayleigh waves in isotropic media: in periodically anisotropic multilayered media, a surface wave that we termed multilayered Rayleigh wave can propagate if it satisfies several conditions. Within these conditions, the cancellation of a (3×3) determinant corresponding to the boundary conditions of a vacuum/infinite multilayered medium structure can be noted. Moreover, the modulus of the reflection coefficient for a nonlossy medium must be equal to one, whereas that of a lossy medium presents a trough. Another important property is that all the Floquet waves (modes of the infinite periodically structure) must be inhomogeneous. For the experiments, the medium was a 0°/45°/90°/135° carbon/epoxy plate, made up of stacked identical hexagonal layers of carbon/epoxy, each being 45° to the previous one. The measurement of the reflected field by a hydrophone measurement system showed a split of the reflected beam when a multilayered Rayleigh wave propagates. Using angular and frequential scanning, the dispersion curves of multilayered Rayleigh modes for an infinite $0^{\circ}/45^{\circ}/90^{\circ}/135^{\circ}$ medium were drawn. These curves were then compared with those obtained using an experimental acousto-optic technique. According to the angle precision, the coincidence between the two curves was quite good: some modes which were experimentally detected are in fact Lamb modes for the plate. However, all of the predicted modes were detected, except for one mode this technique did not detect. The measurement of the reflected field and the comparison with a three dimensional model have shown that the trough of the reflected field is not deep enough to be detected by the acousto-optic technique. We can conclude that these experiments prove our theory on *multilayered Rayleigh waves* in anisotropic periodically multilayered media.

The possible applications of the propagation of *multilay*ered Rayleigh waves are first of all the classical applications of surface waves: detection of an emerging defect near the surface, even residual stresses. Moreover, as a surface wave propagates along the surface, the presence of a defect will disturb this propagation along a certain distance. This is also the case for Lamb waves or for any surface wave. Thus, it will be possible to test a plate by moving a transducer on a line and not on a surface, which permits an important decrease in the testing time. It has been shown¹³ that the propagation of a multilayered Rayleigh wave depends on the stacking of the layers. A change in the stacking of the first layers could thus be detected, as well as the orientation of the first layer. Following the chosen mode, only some layers are concerned with the *multilayered Rayleigh waves*. As a consequence, these layers can be tested following the incident angle and the frequency.

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