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Raising students' interest and deepening their training in acoustics through dedicated exercises[a)]

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ABSTRACT:
The aim of this paper is to briefly present ways to improve the interest of students who begin to attend physical acoustic courses and give examples of exercises (among many others) dedicated to the deepening of their training. Regarding the interest of students who are just beginning, two particular questions arise: how is the attention of these students gained? and how would the students be properly trained when starting out? An attempt to answer these questions is given at the beginning of the paper. Then, the paper continues with a brief presentation of some problems, which are extracted from a three-volume textbook that was published recently, precisely posed, and solved in detail analytically in an exact or approximate manner, even numerically, according to the three levels (bachelor, master, and Ph.D.) while offering an opportunity to deal extensively with the modeling of important topics in acoustics, which are treated as having several ways to be addressed.
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I. INTRODUCTION

During the last decades, acoustic teaching activities have become increasingly concerned with various aspects of academic education in acoustics and, in parallel, student enrollment has been increasing. Accordingly, there was a growing involvement of lecturers in acoustics with attempts at educational innovation in higher education. Courses at the bachelor and master levels in acoustics have been of great interest and present a student with his/her opportunity to find a job in a company (as a high-level technician or engineer, respectively) or prepare for a Ph.D. in acoustics. This is how universities or engineering high schools offer training for jobs in areas ranging from theoretical acoustics to practical acoustics, involving a wide range of applications. Fundamental physical acoustics and basic analytical and numerical modeling are requirements in many such areas. It can even be said that to support the teaching of numerical methods and increasing use of commercial software for numerical simulations, which is a good practice in the mathematical formulation of the acoustic problems and analytical methods of their resolution (even semi-analytical before solving the problem numerically), serve students majoring in engineering technology, environmental, and medical (and so on) fields.

Today, many textbooks[1–9] provide support for the courses in university curricula. They are intended for a very large audience. Most often, these textbooks contain exercises which are briefly posed and whose solutions are limited to very short reasoning or even a simple result. Actually, these short exercises are useful complements to the coursework. But, on one hand, they cannot totally meet the expectations of motivated students who would like to practice on problems when they need to deepen a lesson, desire to fill a gap in their knowledge, or to catch up, or just want to practice completely through exercises. On the other hand, problems precisely posed and solved in detail analytically in an exact or approximate manner, even numerically, which moreover offer an opportunity to deal extensively with the modeling of important topics in acoustics, according to the three levels (bachelor, master, Ph.D.), are requested by the teachers in charge of the tutorials for the students.

One of the topics proposed for this special issue focuses on "Innovative exercises for students to work out as homework problems and possible exam questions that might be given in courses related to acoustics." It is the purpose of this paper to present parts of such exercises extracted from a three-volume textbook (published recently in French[10–12]), highlighting the authors' pedagogical aims, which are deeply marked by their scientific backgrounds, namely physical acoustics, theoretical physics, fundamental mechanics, and applied mathematics. Note that this three-volume textbook (not yet translated into English) is intended for three audiences of increasing levels from the first year of bachelor's degree to the master's degree, starting with basic exercises on acoustic propagation in an ideal fluid (volume 1), then

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deepening and extending these exercises (volume 2), and, finally, completing with more advanced topics (volume 3).

The paper is mainly divided in two sections: after the preliminary comments entitled "how to get the attention of the new students and how to properly train them at the early beginning?" (Sec. II), the main topic of the paper is presented under the title "Innovative exercises to help students deepen subjects" (Sec. III). Section III is divided in four subsections (Secs. III A–III D), which are related to four topics, respectively, the one-dimensional (1-D) modeling of impedance discontinuity in the waveguide (asymptotic approach), transient behavior in the finite-length tube with dissipative fluid, Doppler effect, and radiation of a surface in a plane screen. Note that in each subsection, three exercises, dealing with the subject considered, are presented, two very shortly and the third exercise with more details, showing several ways to address each topic.

II. PRELIMINARY COMMENTS: HOW TO GET THE ATTENTION OF THE NEW STUDENTS AND HOW TO PROPERLY TRAIN THEM AT THE EARLY BEGINNING?

Acoustics is usually taught from the third year of the bachelor's degree or the first year of the master's degree, when students possess enough skills and knowledge in mathematics, mechanics, and physics. However, when dealing with the first and second year of the bachelor's degree, teachers must use different approaches to (i) arouse interest, (ii) simplify the physical complexity (idealized, uniform at rest fluid, and/or 1-D propagation, for instance) without letting students believe that things are so simple, and (iii) address basic academic exercises while maintaining student interest.

The aim of this section is to present some technics/materials/ideas used for the second year students of the bachelor's degree, who have a scientific high school diploma and are often musicians, users of mixing decks, loudspeakers, and so on, and who want to understand the physical phenomena hidden behind their musical experiences.

A. General presentation of the world of acoustics

1. Lindsay's wheel of acoustics and a little history of acoustics

Beginner students usually think that acoustics is related to audible acoustics (music, audio, and room acoustics) and do not really know about the existence of psychoacoustics, underwater acoustics, ultrasonic nondestructive testing of solids, and so on. To arouse their interest, Lindsay's wheel of acoustics, combined with well-chosen examples for each specialized area, is one of the best tools (see Fig. 1 for an updated illustrated version of Lindsay's wheel of acoustics.[13]) A little history of acoustics, from prehistoric reindeer

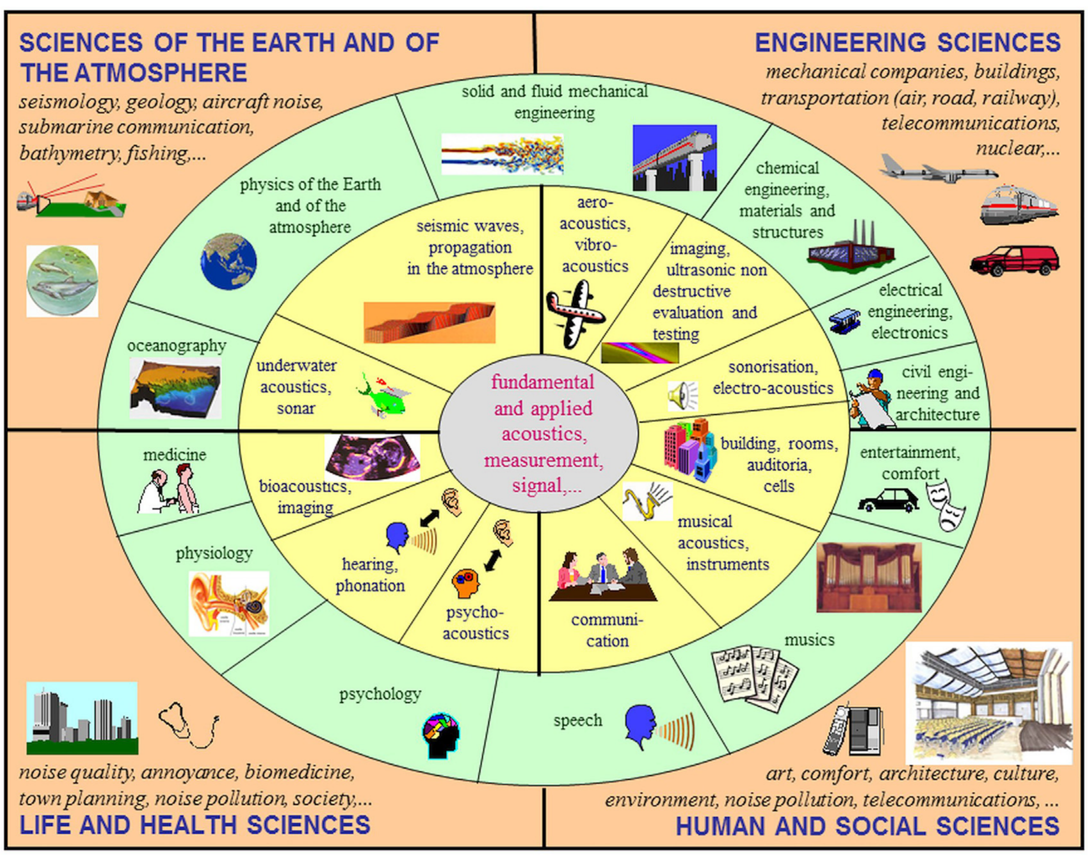

FIG. 1. (Color online) The synoptic view of acoustic skills from the periphery to center depicting the four main fields of activity, engineering domains, and specialized areas. Adapted from Ref. 13.

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whistling phalanxes to Lord Rayleigh by way of Vitruvius, Marin Mersenne, Helmholtz, and Koening, usually leads to many questions and permits the introduction of some vocabulary.

2. Fundamentals of standard acoustics: "Well-posed" problem

Even if the students do not know, yet, what gradient, divergence, and Laplacian operators are, the physical meaning of each fundamental equation of acoustics in fluid media (namely, the Euler equation, mass conservation law, and relation between acoustic pressure and density variations for adiabatic transformations) can be physically explained by considering that fluids have mass density and volume elasticity (the two requisites of wave motion); thus, they have many of the characteristics of a chain of masses and springs:

(i) the first requisite of wave motion (i.e., the vibration of a fluid element) is the mass of the element, which is accounted for by making use of the Newton's second law of motion applied to the particle, leading to the Euler equation (relating acoustic pressure and particle velocity);

(ii) the second requisite of wave motion is the volume elasticity, similar to the spring stiffness, accounted for by the compressibility of the fluid, which relates the change in volume of the compressed fluid to the change in density (conservation of mass equation);

(iii) in addition, one has to specify whether the compressibility is isothermal (near the walls, for example) or adiabatic (in the bulk of the medium), even complex. The current property should be specified by the appropriate differential relation between the density and pressure (obtained as a first approach in assuming, respectively, that the motion is isothermal or adiabatic), leading to the well-known acoustic law $p = c_0^2\rho$ for any motion, which relates the acoustic pressure, density variation, and adiabatic or isothermal propagation velocity c_0.

Finally, eliminating two of the three variables involved in these equations, namely, the pressure or density variations or particle velocity, produces the propagation equation (the process is depicted in Fig. 2).

At this stage, it is worthwhile to introduce the acoustic sources associated with each above mentioned equation [see (i), (ii), and (iii)], respectively, the fluctuating force per unit of mass, volume velocity, and heat flux per unit of mass. As a display to the students during the lesson, the last equation can be very easily highlighted with a compact thermoacoustic source: a quite loud sound whose frequency is related to the length of the tube is generated as soon as the temperature of the lower part of the porous element (see Fig. 3) reaches the necessary threshold.

Finally, the initial and boundary conditions have to be introduced and discussed.

Once an acoustic problem is presented, such as described previously and in Fig. 2, it is easy to start from what we call a "well-posed problem" in many exercises, to familiarize the students with specifying the mathematical expression of the problem before attempting to solve it.

B. From academic exercises to real situations: From the 1-D waveguide of finite length to room acoustics and wind musical instruments

The first academic exercises (closed and open tubes, for example) in idealized fluid media can be easily considered as boring by students, but presenting a closed tube as a 1-D room (corridor), even a 1-D cathedral (!) in which an organ

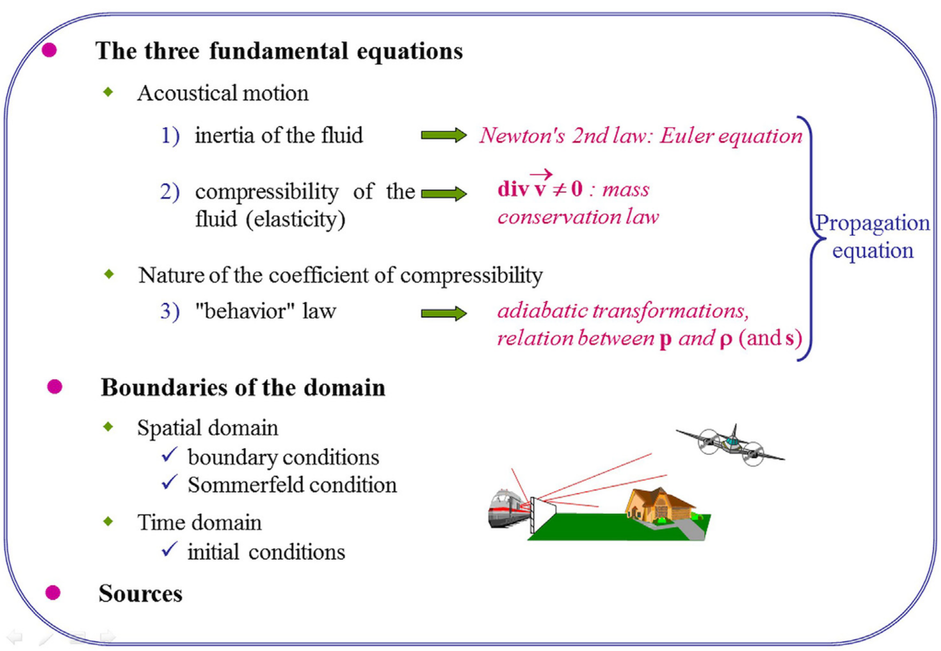

FIG. 2. (Color online) The summary of a finally "well-posed" problem.

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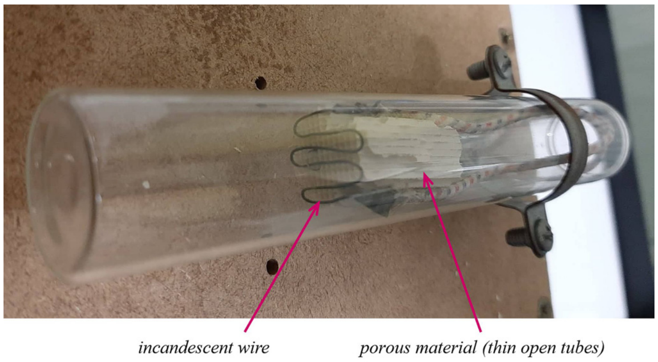

FIG. 3. (Color online) The thermoacoustic source (porous material and heating element in a tube) is put into operation during the lesson to illustrate that the heat flux can create acoustic energy (heat source).

concert is performed, permits to move an eigenvalue problem to a practical room acoustic event. Such an approach can be presented as follows: "*You are attending an organ concert in a cathedral. At the end, the organist plays the final chord and raises suddenly his/her fingers. Thus there is no more acoustic source, and yet you keep hearing a sound dying out in time (this is called 'reverberation'). Which notes/frequencies do you hear? Are they those of the final chord of the source (organ)?*." Then, we must explain that (i) the resulting field in the "cavity" consists of a series of standing waves, each oscillating with its own natural frequency and its own damping out in time, (ii) the natural frequencies present during the reverberation time are those close to the frequencies present in the final chord. This approach permits us to address many notions concerning the room acoustics and allows us to learn more about the problem with or without sources. From the experience of the authors, the students keep on asking questions because they are deeply interested! Note that the starting transient is another problem.

The deep interest of the students arises also when a closed/open tube is considered as a wind instrument (in a very first approach) with the proviso that a remarkable feature of all wind instruments be noted: due to the fact that the radiated pressure amplitude is a tiny fraction of the internal pressure amplitude, the end condition of the closed/open tube is nearly a Dirichlet condition, i.e., the acoustic pressure vanishes (pressure node) at the end of the tube. Here, again, the students willingly accept working on basic exercises, provided, of course, that the teacher can go a little further, for example, by explaining why the effective pressure node mentioned above is displaced beyond the geometrical end of the tube by a distance called the "end correction" (which some of them have already heard about!).

III. INNOVATIVE EXERCISES TO HELP STUDENTS DEEPEN SUBJECTS

The educational goal mentioned here corresponds to treating a subject as having several possible ways to be addressed. Pedagogically speaking, using several approaches to model a given physical phenomenon is intrinsically helpful and physically appealing. The different aspects of the subject treated, even the different levels of the implemented models to treat them, can be covered by several exercises more or less dedicated to the phenomenon considered. To illustrate this objective, below, we present four acoustical phenomena and the way that was used here to model them, extracted from the textbook cited above.[10–12] These overviews take the following form: after recalling briefly the fundamental laws on which the used reasoning is based, three exercises dealing directly or indirectly with the subject considered are presented, either just mentioned or treated as a whole, yet, in a drastically reduced version.

A. Tubes with small admittance discontinuity of wall properties: 1-D approximation

The study of a harmonic motion [$\exp(i\omega t)$] in the waveguide consisting of the assembling of two semi-infinite tubes of circumference L and cross section S (Fig. 4), the first tube (labelled $\ell = 1,\ z \leq 0$) being one with rigid walls and the second tube (labelled $\ell = 2,\ z \geq 0$) consisting of walls of a small admittance $\hat{\zeta}$ [$|\hat{\zeta}|/\zeta_0 \ll 1,\ \zeta_0 = 1/(\rho_0 c_0)$, which is the characteristic acoustical admittance of the medium, and the hat sign indicates that the parameter is complex], can be simplified by assuming that the acoustic field can be assimilated to a plane wave field because the working frequency is lower than the cut-off frequency of the rigid-walled guide $\ell = 1$ and the admittance $\hat{\zeta}$ of the walls of the second guide $\ell = 2$ is relatively small. Moreover, neglecting the evanescent modal waves diffusing in the two media on either side of the interface, permits us to consider that the monochromatic incident field in the guide $\ell = 1$ generates a reflected plane wave and a transmitted plane wave at the interface $z = 0$. Thus, 1-D propagation can be considered. Actually, it is essential to always list all of the hypotheses and assumptions made and, therefore, the requirements for applying the model to let the students know that they must start from the beginning when dealing with a more general case.

One of the things to rectify in the minds of students is that the wave number $\hat{k}$ is not always equal to $k_0 = \omega/c_0$; this exercise is well-suited for demonstrating that. Noting that $(+\hat{\zeta}\hat{p})$ is the velocity normal to the wall (outwardly directed) in guide $\ell = 2$, the fractional change in the volume

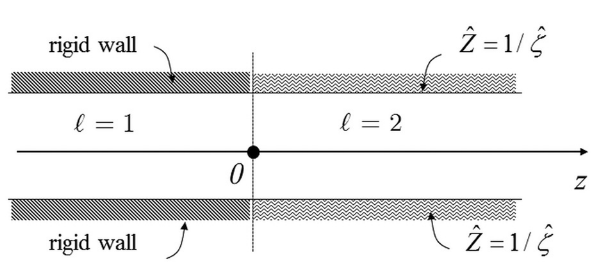

FIG. 4. The infinite tube with the small admittance discontinuity of wall properties for the rigid wall for tube $\ell = 1$ and the small admittance $\hat{\zeta}$ for tube $\ell = 2$.

$\hat{q}$ entering a slice dz of the fluid at the interface (area $L\,\mathrm{d}z$) with the wall of admittance $\hat{\zeta}$ can be written as[6,7]

$$\hat{q} = \frac{(-\hat{\zeta}\,\hat{p})L\,\mathrm{d}z}{S\,\mathrm{d}z} = -\frac{L}{S}\hat{\zeta}\hat{p}. \tag{1}$$

Assuming 1-D motion, the wave equation in guide $\ell = 2$ can be written as

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\hat{p}(z;t) = -\rho_0\frac{\partial \hat{q}(z;t)}{\partial t}. \tag{2}$$

Hence, we get

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} + \hat{k}^2\right)\hat{P}(z)\exp(\mathrm{i}\omega t) = 0, \quad \forall z \geq 0, \quad \forall t, \tag{3}$$

where

$$\hat{k}^2 = k_0^2 - \mathrm{i}k_0\rho_0 c_0\hat{\zeta}L/S, \tag{4}$$

where, under physical considerations, $\mathrm{Re}(1/\hat{\zeta}) > 0$ and $\mathrm{Im}(\hat{k}) < 0$. Finally, writing the problem in each guide with the appropriate discontinuity conditions at $z = 0$ leads to the reflection and transmission coefficients.

Concerning the same subject, slightly more advanced exercises consist in assessing whether the equation of motion (3) is relevant whenever the admittance $\hat{\zeta}$ is very small and, then, under what conditions it is so. One considers, first, a three-dimensional (3-D) acoustic movement governed by the 3-D Helmholtz equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k_0^2\right)\hat{P}(x,y,z) = 0, \tag{5}$$

and one assumes the quasi-plane wave approximation by replacing each term by its average over the area S of the section of the waveguide, which is defined for the acoustic pressure as

$$\langle\hat{P}\rangle_S = \frac{1}{S}\int\int_S \hat{P}\,dS. \tag{6}$$

Using the divergence theorem and assuming that the fluctuation of the field along the edge of the section is negligible, this leads to Eq. (3).

Finally, one considers a more advanced 3-D modeling, which provides the asymptotic underlying conditions that must be fulfilled when using the 1-D approach [Eq. (3)]. The quasi-plane wave is written as

$$\hat{P}(x,y,z) = \hat{P}_0(z) + \varepsilon\hat{P}_1(x,y,z), \tag{7}$$

where ε is a small parameter, such as $\varepsilon|\hat{P}_1| \ll |\hat{P}_0|$, which depends on the ratio of the small specific admittance $\hat{\zeta}/\zeta_0$. Then, two small parameters appear in the problem: the ratio of the admittance of the wall to that of the fluid $|\hat{\zeta}|/\zeta_0$ and the ratio of the average diameter of the duct to the wavelength, in other words, the nondimensional frequency $k_0\sqrt{S}$. After some algebra and assuming the least degeneracy principle, it appears that the (1-D) approximation requires the following conditions:

$$\frac{\varepsilon}{k_0\sqrt{S}} \approx \frac{|\hat{\zeta}|}{\zeta_0} \ll k_0\sqrt{S} \ll 1. \tag{8}$$

B. Transient signal in a finite-length tube with attenuation

In the textbook, several exercises deal directly or indirectly with transient 1-D acoustic movement, starting transient, or transient decay. The discussions of these problems involve the loss of energy of the system considered, caused by thermo-viscous effects (inside or outside the boundary layers)[4–6] and/or energy transfer through the boundaries of the system (radiation). Moreover, considering harmonic motion in the tubular devices when the wavelength is large compared with the transverse dimensions of the tube and when uniform yielding of the tube walls can be considered to be contributing slightly to the movement, the procedures presented in Sec. III A are used to demonstrate that the fluid motion is predominantly parallel to the axis of the tube: the two-dimensional (2-D; or 3-D) problem is reduced to a 1-D problem. Here, we would like to introduce two of these kinds of problems, which were treated in the textbook. The first problem, mentioned only briefly, treats the modeling of the starting transient acoustic signal in a finite-length tube by using, in particular, the Laplace transform method, and the second problem, presented with more details than the first one, deals with the challenge of an acoustic signal dying out in time in a finite-length tube (accounting for the thermo-viscous boundary layer attenuation) while referring to one of the essential features of the functioning of the clarinet.

As mentioned above, the first exercise presented here deals with the analysis of the starting transient signal which leads, after a long time (theoretically infinite), to a monochromatic steady state because the 1-D acoustic movement is assumed to be driven by a causal sinusoidal excitation (starting from a given time), where the source is located at the entrance of a finite-length tube (length L) filled with a first fluid. The end of the tube is either connected to a semi-infinite tube with the same section, filled with a second fluid whose acoustic impedance is greater than that of the first fluid, or closed by a rigid wall. The method used to solve the problem is based on both the Laplace transform and an iterative process,[14–16] which enable us to point out several behaviors of the starting transient, depending on several parameters, namely, the frequency domain considered with respect to the modal behavior of the pipe, the reaction and absorption at the end of the tube, and so on. This modeling involves the calculus of the inverse Laplace transform in the complex plane and its comparison with a direct method available in particular cases. Two results are depicted in Fig. 5.

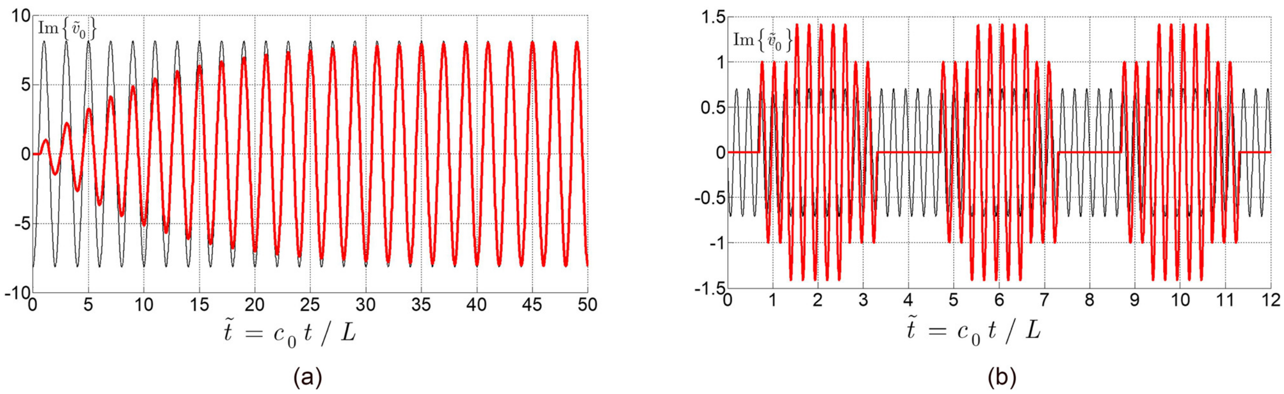

FIG. 5. (Color online) The amplitude of the acoustic pressure as a function of the normalized time $\tilde{t} = c_0 t/L$ for the (a) semi-infinite termination and (b) closed termination (nonresonant case). Extracted from Refs. 10–12.

They represent the amplitude of the acoustic pressure as a function of the normalized time $\tilde{t} = c_0 t/L$ (from $\tilde{t} = 0$) at a given position inside the tube. Figure 5(a) corresponds to the semi-infinite termination, for which part of the acoustic energy is radiated to infinity. The steady state is reached quickly, here, at low frequency (the thin black line represents the steady stage, extended up to $\tilde{t} = 0$). Figure 5(b) corresponds to a closed termination in a nonresonant case (the frequency of the source is different from the eigenfrequencies of the finite-length tube). Then, successive reflections take place without attenuation, and the process never converges to the steady state, which would occur for a sinusoidal excitation (thin black line in Fig. 5).

The second exercise treated below as a whole, yet, in a drastically reduced version, relies on the approximate transient behaviors (starting transient and transient decay) of the quarter-wavelength resonator of the clarinet, accounting for the dissipative effects of the thermo-viscous boundary layers, which lead to an acoustic signal dying out in time when the source is shut off.[17,18] Such a transient problem becomes well-nigh intractable analytically for the realistic thermo-viscous boundary conditions at the walls because their actual behavior in the time domain involves a half-order derivative with respect to the time. Such difficulties are left out of consideration in what follows, in which the aim is merely to explain one of the essential features of the functioning of the clarinet. In fact, the model used is simplified as much as possible while retaining the essential characteristics of the system being studied. Then, based on a number of simplifying hypotheses, the relevant mathematics reduces to the solution of an ordinary differential equation with respect to the time. The model can also be used to gain a deep insight into the main physical behavior of the drastically simplified system considered: the clarinet.

Several exercises, which precede this one, pave the way for modeling the dissipative and reactive effects of the thermo-viscous boundary layers and lead to the following approximate propagation equation for the velocity potential in the frequency domain (which shows a complex wave number):

$$\frac{d^2\hat{\varphi}}{dx^2} + \left(1 + \eta\frac{e^{-i\pi/4}}{\sqrt{\omega}}\right)\frac{\omega^2}{c_0^2}\hat{\varphi} = \hat{v}_0(\omega)\delta(x), \quad x \in (0, L), \tag{9}$$

where $\hat{\varphi}$ is the velocity potential, c_0 is the adiabatic speed of sound, ω is the angular frequency, η is a thermo-viscous real parameter, which depends on the coefficients of the viscosity and conductivity and specific heat of the gas ($\eta \cong 1$ for air under standard conditions), $\hat{v}_0(\omega)$ is the Fourier transform of the velocity $v_0(t)$ of the source set at the entrance $x = 0$ of the tube, L is the length of the tube, $\delta(x)$ is the Dirac delta function, and $\hat{v}_0(\omega)\,\delta(x)$ is the volume velocity of the source. The well-posed problem can then be written as

$$\left\{\frac{d^2}{dx^2} - i\omega\varepsilon_\omega + (1+\sigma_\omega)\frac{\omega^2}{c_0^2}\right\}\hat{\varphi} = \hat{v}_0(\omega)\delta(x), \tag{10}$$

$$\frac{d\hat{p}}{dx} = 0, \quad x = 0, \tag{11}$$

$$\left\{\frac{d}{dx} + i\frac{\omega/c_0}{\hat{\zeta}(\omega)}\right\}\hat{\varphi} = 0, \quad x = L, \tag{12}$$

with

$$\varepsilon_\omega = \frac{\eta\sqrt{\omega}}{\sqrt{2}c_0^2}, \quad \sigma_\omega = \frac{\eta}{\sqrt{2}\sqrt{\omega}}, \tag{13}$$

and where $\hat{\zeta}(\omega)$ is a specific acoustic impedance (here, the specific radiation impedance). Note that the effect of the localized source could have been taken into account in the form of the boundary condition: $d\hat{p}/dx = -i\omega\rho_0\hat{v}_0(\omega)$, $x = 0$, where $\hat{p} = -i\omega\rho_0\hat{\varphi}$ is the pressure variation.

Moreover, the exercise is introduced by several comments, which are needed to support the approximations retained in the following. These comments include (i) the pitch range of the B-flat clarinet from the sounding pitch D_3 (164.8 Hz) to F_6 (1568 Hz) and even to B_7 flat (1865 Hz), attainable by advance players, which defines the frequency range to be considered; (ii) the halftone, defined as the relative frequency difference $(\Delta f/f)_{1/2} = 2^{1/12} - 1 = 0.06$; (iii) the separation between the successive resonances, given approximately by $(\Delta f/f)_{1/2} \cong 0.06$, the maximum relative width of the mth resonance f_m, which is given by $[(\Delta f)_{-3\,\mathrm{dB}}/f_m]_{\max} \cong 0.03$.

To simplify the model as much as possible while conserving the essential characteristics of the resonator of the clarinet, the following approximations can be retained.

(a) A first approximation is that no sound is radiated at all from the instrument (to the musician, this must be a shocking simplification!) because the acoustic pressure amplitude in the sound wave radiated is remarkably small in comparison with the acoustic amplitude in the interior of the resonator. This approximation provides a good basis for modeling the quarter-wavelength resonator of the instrument as the energy absorption at the wall is much greater than the energy radiated from the end of the resonator. Then, the real part of the radiation impedance $\hat{\zeta}(\omega)$, proportional to $(ka)^2$, is neglected in comparison with its imaginary part $8ka/(3\pi)$,[4] assuming that $ka \ll 1$ (k is the wavenumber and a is the radius of the resonator). This approximate expression of the imaginary part is such that the parameter $(\omega/c_0)/\hat{\zeta}(\omega)$ in Eq. (12) is independent of the frequency and it implies that the effective pressure node, set at the geometrical end of the tube when we assume a Dirichlet boundary condition, is displaced beyond by a distance of $8a/(3\pi)$.

(b) A pressure change in the mouthpiece (considered as the entrance of the resonator) modifies the rate at which air flow passes through the slit between the reed and the mouthpiece, which, in turn, changes the pressure in the mouthpiece. A second approximation is that in this valve-effect of the vibrating reed coupled strongly with the resonator, the behavior of the reed, damped by the lower lip and coupled with the mouth cavity of the player, can be simplified as much as possible while retaining the fundamental operating principle of the coupling system (valve-effect).

(c) The parameters ε_ω and σ_ω are smoothly varying functions of ω: their relative variation over the relative width $[(\Delta f)_{-3\,\mathrm{dB}}/f_m]_{\max} \cong 0.03$ of any resonance peak (which are separated from each other) is given readily by $\Delta\varepsilon_\omega/\varepsilon_\omega \cong \Delta\sigma_\omega/\sigma_\omega \cong 0.015$ so that they can be assumed to be constant in these intervals $(\Delta\omega_m)_{-3\,\mathrm{dB}}$. Then, a third approximation is that these parameters depend only on the discretized eigenfrequencies of the resonator,

$$\varepsilon_\omega = \frac{\eta\sqrt{\omega}}{\sqrt{2}c_0^2} \cong \frac{\eta\sqrt{\omega_m}}{\sqrt{2}c_0^2} = \varepsilon_m,$$
$$\sigma_\omega = \frac{\eta}{\sqrt{2}\sqrt{\omega}} \cong \frac{\eta}{\sqrt{2}\sqrt{\omega_m}} = \sigma_m. \quad (14)$$

The construction of the solution in terms of the eigenmodes of the resonator makes use of the modal wave functions satisfying the boundary conditions at both ends of the resonator [Eqs. (11) and (12)]. They are orthogonal because the radiation parameter $(\omega/c_0)/\hat{\zeta}(\omega)$ is an imaginary number independent of the frequency as mentioned above.

The velocity potential in the Fourier domain is then expressed by the sum

$$\hat{\varphi}(x,\omega) = \sum_{m=0}^{\infty} \hat{\alpha}_m(\omega)\Phi_m(x). \quad (15)$$

Substitution into the propagation equation (10) and accounting for Eq. (14) indicates (after some algebra) that the inverse Fourier transform of the coefficients of the expansion $\hat{\alpha}_m(\omega)$ obey the equation

$$\left\{\frac{\partial^2}{\partial t^2} + 2r_m\frac{\partial}{\partial t} + \Omega_m^2\right\}\alpha_m(t) = -\frac{2}{L}c_0^2\, v_0(t), \quad (16)$$

where

$$\Omega_m^2 = \omega_m^2(1-\varsigma-\sigma_m),\;\; 2r_m = c_0^2\varepsilon_m \;\;\text{with}\;\; \varsigma = \frac{16\,a}{3\pi L}. \quad (17)$$

Note that the Fourier transform involves all values of ω but, when considering the functioning of the clarinet, the lower frequency range (say, the first ten modes) is the only frequency range which is involved. The general solution for $\alpha_m(t)$ is given by the convolution integral of the source term and the Green's function associated with Eq. (16),[4–7] assuming that $r_m \ll \Omega_m$,

$$\alpha_m(t) = -\frac{2}{L}\frac{c_0^2}{\Omega_m}\int_{-\infty}^{+\infty} H(t-\tau)e^{-r_m(t-\tau)}\sin[\Omega_m(t-\tau)]v_0(\tau)d\tau, \quad (18)$$

where the Heaviside function $H(t-\tau)$ reflects the causality.

The last part of the exercise furthers this general solution in three particular cases, depending on the velocity profile of the source: (1) a time-periodic source leading to the well-known input impedance of the resonator of the clarinet; (2) an instantaneous unit impulse source applied at $t = 0$ (Dirac delta function), highlighting the forward and backward pulses travelling inside the tube; and (3) the valve-effect of the reed when the player chooses a lip setting which favors the reed vibration profile sustained by the feedback from the resonator (over several periods).

When considering this last "realistic" situation in the frame of the simplified model mentioned above, the shape of the opening of the mouthpiece is rectangular (source term). Therefore, the time dependence of the normalized pressure variation, relatively easy to express, is represented by a rectangular function as shown in Fig. 6 (length of the tube is 0.40 m, 51 eigenmodes), which is characteristic of the clarinet behavior. The starting transients of the signal

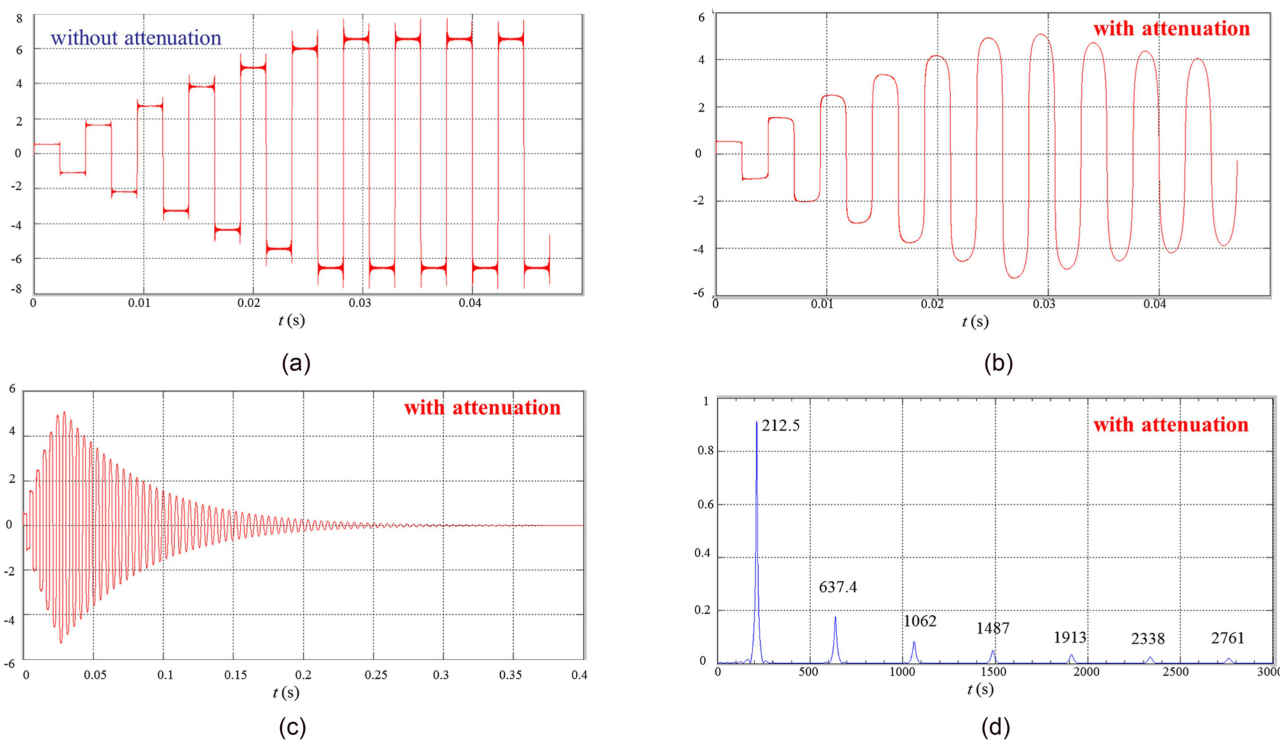

FIG. 6. (Color online) The starting transient of the normalized acoustic pressure (for the rectangular profile of the reed vibrations sustained over six periods) as a function of the time at the entrance of the resonator $x = 0$, respectively, without (a) and with (b) the thermo-viscous dissipation. (c) The starting transient and transient decay (signal dying out in time) and (d) Fourier transform of the signal (frequency spectrum) are depicted. Extracted from Refs. 10–12.

are shown in Fig. 6(a) without the thermo-viscous dissipation and in Fig. 6(b) with the thermo-viscous dissipation (the time range is 0–50 ms), Fig. 6(c) emphasizes the starting transient and the signal dying out in time (the time range is 0–400 ms), and Fig. 6(d) shows the Fourier transform of the signal (frequency spectrum). Note that the Gibbs effect disappears when the dissipation effects are taken into account.

C. Doppler effect

The chapter of the textbook[10–12] which mainly deals with the Doppler effect begins with comments on the monopole point sources, the notion of instantaneous frequency, and the assumptions based on the physical considerations, whereas the Doppler effect is a purely kinematic phenomenon. These assumptions reveal, in particular, that the geometry of the pattern of the wave is invariant and the phase of the wave, orientation of the wave vector, and wavelength are the same for all of the observers whether they are moving with respect to the medium or not. Three exercises dealing directly or indirectly with the subject are then presented in the textbook. Below, we, first, briefly introduce two of the exercises and then we summarize the model proposed in the third exercise.

The first exercise that we would like to mention briefly deals with the acoustic propagation in a cylindrical waveguide with uniform flow,[19] where the acoustic field is generated by a harmonic point source. The natural solution is a Green's function expressed as a 2-D modal expansion, with the coefficients of the expansion depending on the axial coordinate of the guide. By carefully analyzing each parameter involved in this last function, the basic Doppler law is easily obtained.

The second exercise that we would to mention briefly deals with a point source moving uniformly along a straight line and a point of observation moving uniformly parallel to the path of the emission point, where the medium infinite in extent is at rest (this last assumption can be made without loss of generality because only the relative motion between the source and medium and between the observation point and medium occurs). The method used to solve the propagation equation is based on a coordinate system, which enables us to reduce the problem to that of the radiation from a stationary source. Then, the derivative with respect to the time of the phase of the solution leads readily to the frequency shift due to the Doppler effect. Several configurations are finally discussed.

A third presentation of the Doppler effect is based on an asymptotic argument. This is an opportunity to emphasize that a major concern in our educational approach in teaching physical acoustics is to train students to master the dimensional analysis of a problem, identify the relevant dimensionless numbers, and implement asymptotic methods adapted to the specific physical conditions.

With regard to the Doppler effect, the asymptotic argument arises from the assumption that the characteristic time of the signal is much shorter than that of the motion. Consequently, a "two-timing" asymptotic process can be implemented. At the first order, a simple differential calculation leads to the relation between the signal characteristic time of the source and that of the receiver.

In a precise manner, let $\vec{r}_1(t)$ and $\vec{r}_2(t)$ be the (vectorial) time laws for the displacement of the source and receiver, where each is in any movement in a fluid at rest (with the sound speed c_0), and t_1 and t_2 are some associated transmission and reception times. Let

$$r(t_1, t_2) = \| \vec{r}_2(t_2) - \vec{r}_1(t_1) \| \tag{19}$$

be the distance covered by the signal between these times. Then, the characteristic times $\tau_1 = \mathrm{d}t_1$ and $\tau_2 = \mathrm{d}t_2$ of the signal when emitted at the time t_1 and received at the time t_2, respectively, are linked by the relation

$$\left[1 + \frac{1}{c_0}\frac{\partial r(t_1,t_2)}{\partial t_1}\right]\tau_1 = \left[1 - \frac{1}{c_0}\frac{\partial r(t_1,t_2)}{\partial t_2}\right]\tau_2. \tag{20}$$

In the case of a periodic signal, the relation between the emitted and received frequencies (ν_1 and ν_2, respectively) is deduced by using the relation

$$\nu_1 \tau_1 = \nu_2 \tau_2. \tag{21}$$

It is worth noting that in using the analytic procedure given by Eqs. (19)–(21), it is easy to consider several situations analytically, which fall into the framework of the assumptions retained, such as the uniform rectilinear movement of the emitting and/or receiving points, uniform circular movement of one of them, accelerated movement of both on the same axis etc.

It should be mentioned that several other exercises presented in the textbook[10–12] involve asymptotic reasoning. The saddle point method or that of the stationary phase[20] for the high frequency evaluation of Fourier-Laplace integrals leads, in particular, to the approximation of geometric acoustics (see Sec. III D). Similarly, a high frequency asymptotic reasoning, which is applied to various scattering problems, leads to the justification for the diffracted field, the geometric theory of diffraction or GTD introduced by J. B. Keller.[21]

D. Radiation from a vibrating surface in a plane screen

There is a broad class of quite simple acoustic radiation problems in which the acoustic source can be idealized as a piston mounted in a rigid infinite planar baffle and vibrating with velocities that may be either sinusoidal or non-sinusoidal pulse motions. This section deals with such problems. Two standard analytic procedures whereby one expresses the steady-state and transient radiation characteristics are the subject of the exercises in the textbook.[10–12] In the first problem, briefly mentioned below, the acoustic field is expressed as a Green's function integral and in the second problem, presented with more details than the first one, it is expressed as a spatial Fourier integral with respect to a component of the wave number (plane wave representation) making use of the stationary phase method to calculate it. The displayed examples are for the circular piston or rectangular piston (which can be idealized in the form of the 1-D source in a 2-D space).

As mentioned above, the first exercise presented here deals with the analysis of the field in the half-space above the baffled piston, which is expressed as a Green's function integral (over the surface of the piston) solution for the time-dependent pressure.[22] The approach is applicable to the analysis of the acoustic pressure at any spatial point in the half-space for any shape of the piston radiator.

In the time domain, the velocity potential ϕ may be expressed as a convolution integral

$$\phi(\vec{r}, t) = v(t) * h(\vec{r}, t), \tag{22}$$

where the impulse response function $h(\vec{r}, t)$ of the piston to the spatial point of interest is defined as

$$h(\vec{r}, t) = \iint_{S_0} \frac{\delta(t - |\vec{r} - \vec{r}_0|/c_0)}{2\pi |\vec{r} - \vec{r}_0|} dS_0, \tag{23}$$

which is easily seen as the time-dependent velocity potential at $\vec{r}$, resulting from an impulsive velocity of the piston (of area S_0). Applications to the sinusoidal and impulse excitation of the plane circular and rectangular pistons are presented. Two results are depicted in Fig. 7(a) for an impulse velocity at any receiving point on the axis of a circular piston and in Fig. 7(b) for a sinusoidal velocity starting at $t = 0$ (null before $t = 0$) at a given location at the receiving point. The effect of the edge of the source appears in Figs. 7(a) and 7(b), where a is the radius of the piston and b is the distance between the receiving point and the plane of the baffle.

The radiation of a plane emitter is one of the standard examples for which several resolution methods can be proposed. The method that has just been presented in this first exercise consists in describing the acoustic field as that of a distribution of point sources located on the emitter. This field then appears as a Green-Rayleigh integral. In the second exercise, the method, limited to the monochromatic case, uses spatial Fourier integrals. It, then, represents the acoustic field as a superposition of plane waves; see Ref. 23.

The duality between the Green representations (as the distribution of sources on the emitter) and the Fourier representations [as the superposition (integral) of the plane waves], is illustrated by the following representation formula of a field emitted by a point source:

$$\frac{\mathrm{e}^{-\mathrm{i}kr}}{r} = \frac{1}{2\pi\mathrm{i}} \iint_{(k_x,k_y)} \frac{\mathrm{e}^{-\mathrm{i}\left(k_x x + k_y y + \hat{k}_z z\right)}}{\hat{k}_z} \mathrm{d}k_x\, \mathrm{d}k_y \quad (z \geq 0), \tag{24}$$

where $\hat{k}_z = \sqrt{k^2 - (k_x^2 + k_y^2)}$ with the sign conditions [for the temporal convention $\exp(\mathrm{i}\omega t)$]

$$\mathrm{Re}\left\{\hat{k}_z(k_x, k_y)\right\} \geq 0 \quad \text{or} \quad \mathrm{Im}\left\{\hat{k}_z(k_x, k_y)\right\} \leq 0. \tag{25}$$

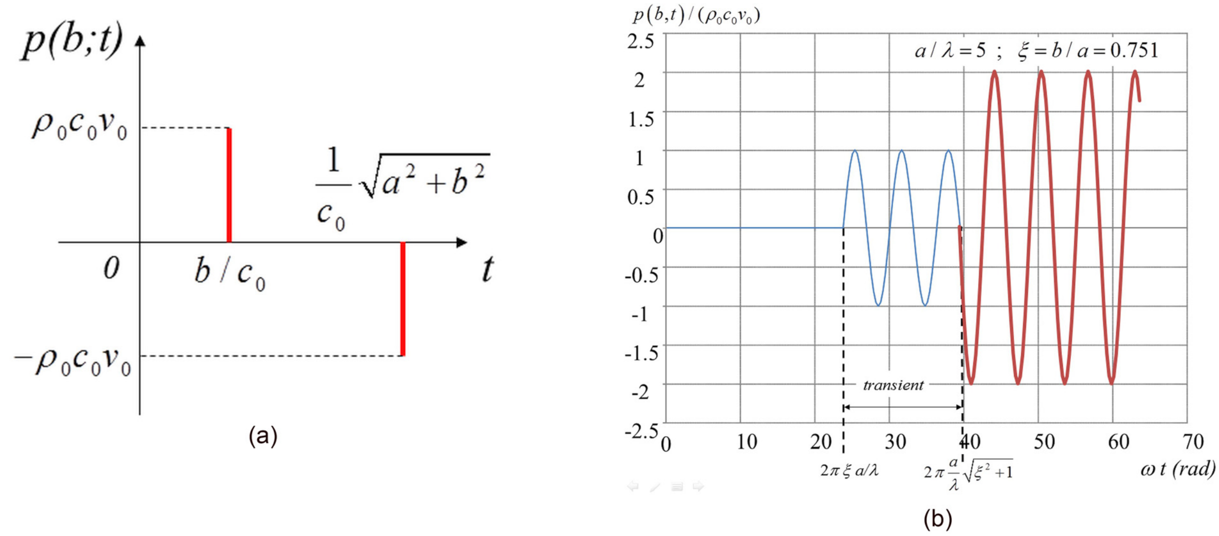

FIG. 7. (Color online) The pressure field radiated by the piston (a) for an impulse velocity at any receiving point on the axis of a circular piston and (b) for a sinusoidal velocity starting at $t=0$ (null before $t=0$) at a given location at the receiving point. Extracted from Refs. 10–12.

In the second exercise, we consider the case of a 2-D plane emitter for which the normal vibration speed is constant with the amplitude V_0 on the segment $-a \le x \le a$ and zero outside. The acoustic potential is sought in the form of the spatial Fourier integral

$$\hat{\Phi}(x,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{A}_0(k_x)\exp\left[-\mathrm{i}\left(k_x x + \hat{k}_z z\right)\right] \mathrm{d}k_x \quad (z \ge 0), \tag{26}$$

where $\hat{k}_z = \sqrt{k^2 - k_x^2}$ with the sign conditions (25), z is the coordinate normal to the plane of the baffle, k_x is the in-plane component of the wave number $\vec{k}$, $\hat{\Phi}(x,z)$ is the complex amplitude of the velocity potential $\hat{\varphi}(x,z,t) = \hat{\Phi}(x,z)\exp(\mathrm{i}\,\omega t)$. The amplitude $\hat{A}_0(k_x)$ is deduced from the data on the transmitter, namely,

$$\hat{A}_0(k_x) = 2\mathrm{i}V_0 \frac{\sin(k_x a)}{k_x \hat{k}_z}. \tag{27}$$

This representation is well-suited for asymptotic studies in the far field (Fraunhofer approximation) and high frequency near field (Fresnel approximation). Two dimensionless parameters are involved: the products ka and kr, where r denotes the distance from the observation point to the center of the emitter.

For the far-field approximation, we assume $ka = O(1)$ and $kr \gg 1$. Only the propagation exponential factor of the plane waves is concerned by this hypothesis. The application of the stationary phase method to the Fourier integral (see Ref. 24) leads to a description of the field as that of a beam of rays coming from the center of the emitter with an angular distribution of the amplitude or directivity diagram, whose form depends on the (finite) value of ka.

The Fresnel approximation is based on the asymptotic assumptions $r/a = O(1)$ and $ka \gg 1$. The plane wave propagation exponential factor and also the amplitude factor (27) are then concerned by the approximation. The Fourier integral, extended to the complex values of the variable k_x, is then evaluated by the saddle point method, which is the complex plane generalization of the stationary phase method. The acoustic field, then, appears as the sum of the geometric field generated by the rays perpendicular to the vibrating zone and the field diffracted by the edges of this zone, namely, two rays received at the observation point (Fig. 8). The interaction between these two or three rays generates the expected interference patterns of the Fresnel zone.

IV. CONCLUSION

In a first step, we hinted at a possible avenue for both raising students interest when practicing on academic exercises and properly training them at the beginning of their studies in acoustics. In a second step, we presented brief glimpses of the exercises, among many others,[10–12] to help students deepen the subjects by treating the topics as having several ways to be addressed: in Secs. III A–III D, four acoustical topics and the ways in which they were used to

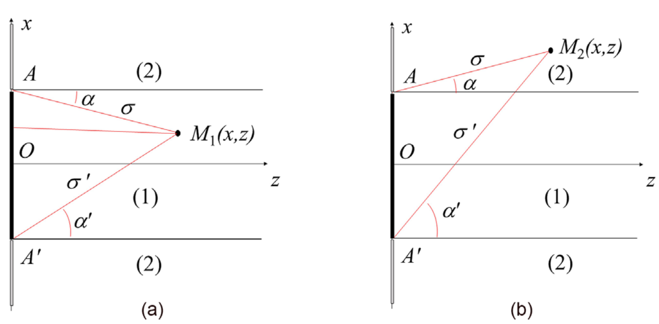

FIG. 8. (Color online) The rays for the pressure field under the Fresnel approximation. The (a) receiving point inside the domain labelled (1) $(-a \le x \le a)$ and (b) receiving point inside the domain labelled (2) are shown.

model them have been outlined. Overall, the general purpose of the paper was to examine the possible ways to attract the attention of the students and deepen their knowledge through exercises by providing some examples of the key techniques to meet the expectations of the students and the instructors.

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