MODAL WAVE DECOHERENCE WITHIN A FLUID LAYER WITH A ROUGH SURFACE

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Modal wave decoherence within a fluid layer with a rough surface

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Abstract
The aim of the presentation is to study analytically the propagation of modal waves (like Lamb waves) in a rough fluid plate, in order to interpret the decay of Lamb modes which are observed experimentally and theoretically in solid plates with a rough interface. The analytical method employed here uses modal wave expansion, expressing locally the roughness as an operator acting on the acoustic pressure. The expansion coefficient corresponding to a given modal wave can be expressed as the sum of the direct field when interfaces are rigid plane (without roughness) and of a diffused field created by the mode coupling due to the roughness.

Introduction

Rough surfaces are of industrial interest, in the aim of improving the wetting of the glue in bounded structures for example. Guided waves like Lamb waves in solid structures are very useful to control such structures and may give some information on the quality of the wetting[1-3].

Former theoretical and experimental studies have permitted to bring to the fore a decay of a Lamb mode, with a probable energy transfer between modes.

In order to better understand these energy transfer phenomena, the present work aims to study in detail the case of the propagation of modal waves which propagate in a fluid layer with a rough surface, by expressing the field as a modal expansion. The coefficients of the expansion are function of the projection of the incident field on the eigenfunctions and of the energy transfer between modes (mode coupling) due to the roughness.

Former studies

Theoretical model in solid media

A 3D model has been developed for an anisotropic plate in vacuum (with a randomly rough surface on one side, the other side being considered as the reference side, see Fig. 1), characterized by its thickness d, its density ρ and its (6x6) elastic constant matrix $(c_{\alpha\beta})$, in order to study the propagation of Lamb waves[4].
A perturbation method permits to express the dispersion equation of the rough plate as a sum of the dispersion equation of the plate with roughless surfaces and of a perturbation :

$$F(k_1,\omega) \approx F_0(k_1,\omega) + \delta F(k_1,\omega) = 0 \quad , \qquad (1)$$

where the complex number k_1 is the projection of the wave number vector on the x_1-axis. For a given angular frequency ω, the solution of Eq. (1) is of the form

$$k_1 = k_0 + \delta k_1 \, , \qquad (2)$$

where k_0 is the (real) solution of the dispersion equation $F_0(k_1,\omega) = 0$ corresponding to the dispersion equation for Lamb modes in a plate with plane surfaces, and δk_1 is a small complex perturbation due to the roughness. The real and imaginary parts of the wavenumber k_1 are related respectively to the shift frequency and to the attenuation of the wave. Two mechanisms contribute to the decay of a Lamb mode: ***its decay into bulk elastic waves and its decay into other Lamb modes, with an energy transfer between modes***. This attenuation of the studied Lamb mode can be interpreted as a ***phenomenon of decoherence*** of the longitudinal and transversal waves, which cannot interfere constructively at each interface of the plate, in order to be recombined and to give Lamb waves.

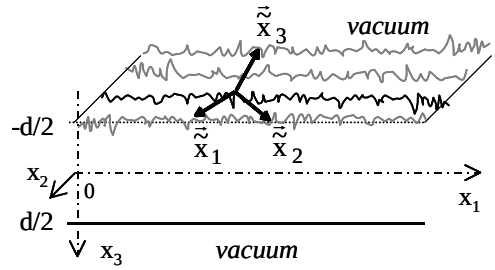

Figure 1: Geometry of the problem for an anisotropic plate.

Comparison with experimental results

Experimental[5] and numerical studies have been done on a rough shot blasted glass plate. They are in very good agreement for some modes, but are less good for other modes : the influence of the ***spatial wavelengths***, through the Power Density Spectrum of the rough profile which makes several spatial wavelengths appear, has to be taken into account.

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Finite element method simulation in a fluid plate

In the general case, when the wavelength of the Lamb wave is close to the grating spacing, reflected waves are observed and a phonon relation is written between the incident signal, the converted mode and the phonon related to the grating, like in the elastic case[6]. In order to visualize the phenomenon of decoherence, a simplified problem was studied using Finite Element Code, that of a "fluid" rough plate, with a periodically roughness (see Fig. 2).

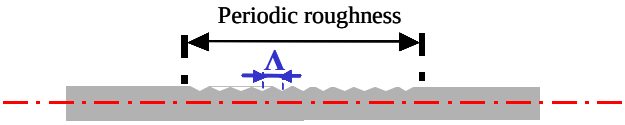

Figure 2: Fluid plate with a periodic roughness, of spatial period Λ.

Analytical model in a fluid plate, using modal wave decomposition

In order to better understand the transfer energy phenomena between modes, an analytical model is currently developed for the acoustic pressure in a fluid plate. The fluid plate is characterized by its thickness d, its density ρ_0, and the speed of sound c_0. The boundary surfaces $x_3 = d$ and $x_3 = 0$ are respectively a plane rigid wall, and a rough rigid wall with a weak variation $h(x_1,0)$, which can be locally expressed as a time operator $\hat{b}(x_1,x_3;t)$ acting on the acoustic pressure which depends on the spatial coordinates.

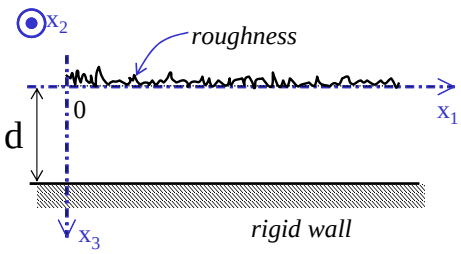

Figure 3: Fluid plate between a rigid wall and a rough boundary surface

The problem

The fundamental equations of the problem are written as

$$\begin{cases} \bullet \left(\Delta - \frac{1}{c_0^2}\partial_{tt}^2\right)\hat{p}(x_1,x_3;t) = -\hat{f}(x_1,x_3;t), & (3\text{-a}) \\ \qquad 0 \le x_3 \le d, x_1 \ge 0, & \\ \bullet \left(\partial_n + \hat{b}\right)\hat{p}(x_1,x_3;t) = \hat{u}(x_1,x_3;t), & (3\text{-b}) \\ \qquad x_3 = 0 \text{ and } x_3 = d, x_1 \ge 0, & \\ \bullet \text{Sommerfeld radiation condition when } x_1 \to \infty, & (3\text{-c}) \end{cases}$$

where ∂_{tt}^2 and ∂_n stand respectively for $\partial^2/\partial t^2$ and $\partial/\partial n$, $\vec{n}$ being the normal to the boundaries, $\hat{p}(\vec{r};t)$ is the complex pressure at a point $\vec{r}$ for a given time t, $\hat{f}$ and $\hat{u}$ are respectively bulk and surface sources factors, $\hat{b}(x_1,x_3;t)$ is an operator upon the time which modelises the effect of the roughness ($\hat{b} = 0$ for the plane rigid wall set at $x_3 = d$ in Fig. 3).

The 1D transverse eigenvalue problem, associated to the problem (3), for a Neuman condition at the boundaries, is expressed as

$$\begin{cases} \left(\partial_{x_3x_3}^2 + k_m^2\right)\psi_m(x_3) = 0, \; 0 \le x_3 \le d, & (3.\text{a}) \\ \partial_{x_3}\psi_m(x_3) = 0, \; x_3 = 0 \text{ and } x_3 = d, & (3.\text{b}) \end{cases}$$

where the eigenvalues k_m and the eigenfunctions (normalised orthogonal) ψ_m are given respectively by

$$k_m = m\pi/d, \; m \in \mathbb{N} \qquad (4\text{-a})$$

and $$\psi_m(x_3) = \sqrt{(2-\delta_{m0})/d}\cos(k_m x_3). \qquad (4\text{-b})$$

The pressure field $\hat{p}(x_1,x_3;t)$ can thus be written approximately as an expansion on these eigenmodes

$$\hat{p}(x_1,x_3;t) = \sum_m \hat{a}_m(x_1;t)\psi_m(x_3), \qquad (5)$$

where the coefficients $\hat{a}_m$ are given by

$$\hat{a}_m(x_1;t) = \int_0^d \hat{p}(x_1,x_3;t)\psi_m(x_3)dx_3, \qquad (6)$$

denoted $\langle m|\hat{p}(x_1,x_3;t)\rangle$ below.

Using the Green theorem (one dimension), Eq. (3-a) can be written as

$$\boxed{\begin{aligned} &\left(\frac{1}{c_0^2}\partial_{tt}^2 - \partial_{x_1x_1}^2 + \chi_m^2\right)\hat{a}_m(x_1;t) = \hat{\sigma}_m(x_1;t) \\ &\quad - \sum_{\mu\neq m}\left[\gamma_{\mu m}(x_1,d;t) + \gamma_{\mu m}(x_1,0;t)\right]\hat{a}_\mu(x_1;t), \end{aligned}} \qquad (7)$$

where $\hat{\sigma}_m(x_1;t)$ represents the energy transfer between the sources and the eigenmode m,

$$\hat{\sigma}_m(x_1;t) = \langle m|\hat{f}(x_1,x_3;t)\rangle + \hat{u}(x_1,d;t)\psi_m(d) + \hat{u}(x_1,0;t)\psi_m(0), \qquad (8\text{-a})$$

$\gamma_{\mu m}$ the coupling operator (acting on the time)

$$\gamma_{\mu m}(x_1,x_3;t) = \left(\hat{b}(x_1,x_3;t)\psi_\mu(x_3)\right)\psi_m(x_3), \qquad (8\text{-b})$$

χ_m depending on the wave number k_m and on the perturbation induced by the roughness :

$$\chi_m^2 = k_m^2 + \gamma_{mm}(x_1, d; t) + \gamma_{mm}(x_1, 0; t) \ . \qquad (9)$$

Note that when the wall set at $x_3 = d$ is rigid (without roughness), $\gamma_{\mu m}(x_1, d; t) = 0$.

The solution for a monochromatic source

A single monochromatic source (angular frequency ω) is set at $x_1 = 0$. Its strength is given by :

$$\hat{f}(x_1, x_3; t) = \delta(x_1)\hat{Q}(x_3)e^{i\omega t} \ , \qquad (10\text{-a})$$

thus $\hat{u}(x_1, x_3; t) = 0$, $x_3 = 0$ and $x_3 = d$. (10-b)

The operator $\hat{b}(x_1, x_3; t)$ becomes then a function denoted $\hat{B}_\omega(x_1, x_3)$.

Substituting Eqs. (10) into Eq. (7) leads to

$$\boxed{\left(\partial^2_{x_1 x_1} + k^2_{x_{1m}}\right)\hat{A}_m(x_1) = -\delta(x_1)\hat{S}_m + \hat{C}_m(x_1)} \ ,(11)$$

where $\hat{a}_m(x_1; t) = \hat{A}_m(x_1)e^{i\omega t}$, (12-a)

$$k^2_{x_{1m}} = k_0^2 - \chi_m^2 \ , \qquad (12\text{-b})$$

$$\hat{S}_m = \langle m | \hat{Q}(x_3) \rangle \ , \qquad (12\text{-c})$$

$$\hat{C}_m(x_1) = \sum_{\mu \neq m} \hat{A}_\mu(x_1)\hat{\Gamma}_{\mu m}(x_1) \ , \qquad (12\text{-d})$$

$$\hat{\Gamma}_{\mu m}(x_1) = \left(\hat{B}_\omega(x_1, d)\psi_\mu(d)\right)\psi_m(d) + \left(\hat{B}_\omega(x_1, 0)\psi_\mu(0)\right)\psi_m(0). \qquad (12\text{-e})$$

The term $\hat{C}_m(x_1)$ represents the mode coupling due to the roughness.

Using the 1D-Green function ($x_1 > 0$)

$$G(x_1, x'_0) = \frac{e^{-ik_{x_{1m}}|x_1 - x'_0|}}{2ik_{x_{1m}}} \ , \qquad (13)$$

the solution of Eq. (11), subject to the asymptotic condition (3-c), can be expressed as[7] :

$$\boxed{\hat{A}_m(x_1) = -\hat{S}_m \frac{e^{-ik_{x_{1m}}x_1}}{2ik_{x_{1m}}} + \int_0^\infty G(x_1, x'_0)\hat{C}_m(x'_0)dx'_0 \ ,} \qquad (14)$$

where the first term represents the direct field when interfaces are rigid planes (without roughness) and the second term the coupling due to the roughness.

Substituting Eq. (13) into Eq. (14) leads finally to

$$\boxed{\begin{aligned}\hat{A}_m(x_1) = &-\hat{S}_m \frac{e^{-ik_{x_{1m}}x_1}}{2ik_{x_{1m}}} \\ &+ \frac{e^{-ik_{x_{1m}}x_1}}{2ik_{x_{1m}}} \sum_{\mu \neq m} \int_0^{x_1} e^{ik_{x_{1m}}x'_0} \hat{A}_\mu(x'_0)\hat{\Gamma}_{\mu m}(x'_0)dx'_0 \\ &+ \frac{e^{ik_{x_{1m}}x_1}}{2ik_{x_{1m}}} \sum_{\mu \neq m} \int_{x_1}^\infty e^{-ik_{x_{1m}}x'_0} \hat{A}_\mu(x'_0)\hat{\Gamma}_{\mu m}(x'_0)dx'_0 \ .\end{aligned}} \qquad (15)$$

The case of a periodically rough profile : the regularly distributed "small cavities"

As a first approach, the rough profile is assumed to be periodically distributed cavities at the interface $x_3 = 0$ as shown in Fig. 4.

In each cavity (volume V_0), the pressure variation $\hat{p}_c$ is assumed to be uniform and created by the displacement ξ_c of the surface S_0 between the cavity and the fluid plate (Fig. 5). The pressure $\hat{p}_c$ in the small cavity can then be written as a function of the ratio S_0/V_0 and of the displacement ξ_c of the section S_0 :

$$\hat{p}_c = -\rho_0 c_0^2 \frac{S_0}{V_0}\xi_c \ . \qquad (15)$$

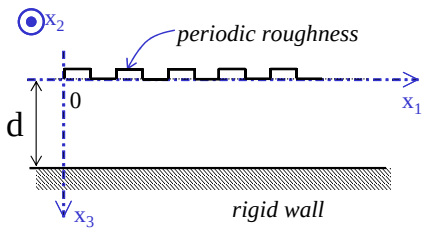

Figure 4: Fluid plate between a rigid wall and a periodically rough interface.

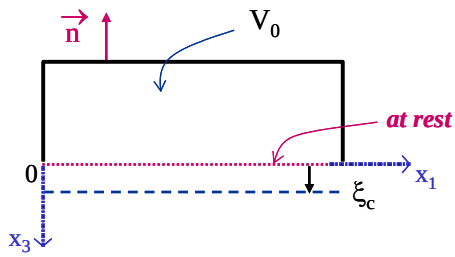

Figure 5: Geometry of a small cavity

Invoking Euler equation, this last expression (15) leads to the expression of the complex operator $\hat{b}(x_1, x_3; t)$ as following :

$$\boxed{\hat{b}(x_1, x_3; t) = \frac{1}{c_0^2}\frac{V_0}{S_0}\partial^2_{tt}} \ . \qquad (16)$$

When the sources are monochromatic (angular frequency ω), the operator $\hat{b}(x_1, x_3; t)$ becomes a function $\hat{B}_\omega(x_1, x_3)$:

$$\hat{B}_\omega(x_1, x_3) = k_0^2 \frac{V_0}{S_0} \quad \text{with} \quad k_0 = \omega / c_0 \quad . \qquad (17)$$

The ratio V_0/S_0 being the equivalent height $h(x_1, x_3)$ of the cavity, function $\hat{B}_\omega(x_1, x_3)$ can be expressed as

$$\hat{B}_\omega(x_1, x_3) = -k_0^2 h(x_1, x_3) \ , \qquad (18)$$

representing the reaction of any point of the surface $x_3 = 0$, and makes appear the Power Density Spectrum of the profile.

The particular case of a two modes coupling

When a monochromatic source generates only the mode $m = 0$ in the fluid plate, the function $\hat{Q}(x_3)$ (see Eq. (10-a)) being not orthogonal to $\psi_{m=0}(x_3)$, the only mode generated by the mode coupling due to the roughness being assumed to be the mode $m = 1$ (i.e. $\hat{S}_1 = 0$), therefore the coefficients $\hat{A}_0(x_1)$ and $\hat{A}_1(x_1)$ are solutions of the coupled integral equations

$$\begin{aligned}\hat{A}_0(x_1) = & -\hat{S}_0 \frac{e^{-ik_{x_{10}} x_1}}{2ik_{x_{10}}} \\ & + \frac{e^{-ik_{x_{10}} x_1}}{2ik_{x_{10}}} \int_0^{x_1} e^{ik_{x_{10}} x'_0} \hat{A}_1(x'_0) \hat{\Gamma}_{10}(x'_0) dx'_0 \\ & + \frac{e^{ik_{x_{10}} x_1}}{2ik_{x_{10}}} \int_{x_1}^{\infty} e^{-ik_{x_{10}} x'_0} \hat{A}_1(x'_0) \hat{\Gamma}_{10}(x'_0) dx'_0 \ , \end{aligned} \qquad (19)$$

and

$$\begin{aligned}\hat{A}_1(x_1) = & \frac{e^{-ik_{x_{11}} x_1}}{2ik_{x_{11}}} \int_0^{x_1} e^{ik_{x_{11}} x'_0} \hat{A}_0(x'_0) \hat{\Gamma}_{01}(x'_0) dx'_0 \\ & + \frac{e^{ik_{x_{11}} x_1}}{2ik_{x_{11}}} \int_{x_1}^{\infty} e^{-ik_{x_{11}} x'_0} \hat{A}_0(x'_0) \hat{\Gamma}_{01}(x'_0) dx'_0 \ . \end{aligned} \qquad (20)$$

Substituting the expression (20) of $\hat{A}_1(x'_0)$ into Eq. (19) leads to an integro-differential equation of $\hat{A}_0(x_1)$. The zero-order $\hat{A}_0^{(0)}(x_1)$ of $\hat{A}_0(x_1)$ is given by

$$\hat{A}_0^{(0)}(x_1) = -\hat{S}_0 \frac{e^{-ik_{x_{10}} x_1}}{2ik_{x_{10}}} \ , \qquad (21)$$

and the first order $\hat{A}_0^{(1)}(x_1)$ of $\hat{A}_0(x_1)$ is given by substituting $\hat{A}_0^{(0)}(x_1)$ for $\hat{A}_0(x_1)$ in the integro-differential equation of $\hat{A}_0(x_1)$.

Conclusion

The analytical model developed here has to be further investigated in particular to draw the profile of the amplitude of each mode along the interface, in order to give an interpretation of the phonon relationship, and then to compare with the results obtained using a Finite Element Method. From now on, this work permits to prove that, when a modal wave is generated in a fluid plate with a rough interface, the energy transfer between this mode and the other possible modes in the plate can be quantified, which leads to a decoherence of the initial mode. The adaptation of this kind of problem to more sophisticated investigation, especially when dealing with Lamb waves propagating into solid plates, with multimodal coupling or with random boundaries has also to be addressed.

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