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Numerical and experimental deviation of monochromatic Lamb wave beam for anisotropic multilayered media

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NUMERICAL AND EXPERIMENTAL DEVIATION OF MONOCHROMATIC LAMB WAVE BEAM FOR ANISOTROPIC MULTILAYERED MEDIA

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Abstract. The interaction of an incident monochromatic bounded beam with an anisotropic multilayered plate immersed in an external fluid has been modelised, using the decomposition method into monochromatic plane waves and the transfer matrix method, in a 3D configuration. This model permits the pressure of the reflected and/or transmitted field in the fluid to be simulated, notably in a plane parallel to the plate. The subject of this paper is the study of the deviation of a monochromatic Lamb wave beam due to the anisotropy of the structure. Comparisons between some theoritical, numerical and experimental results are given.

INTRODUCTION

For some ultrasonic testing configurations, a monochromatic Lamb wave beam can be locally excited in a structure by an emitter transducer immersed in a fluid : since the incident field is a bounded beam, a Lamb wave beam is generated in the structure. Due to the anisotropy of the media constituting the structure, the most energetic part of the Lamb wave beam is deviated with respect to the sagittal plane of the incident bounded beam, in a direction normal to the Lamb slowness curve. This phenomenon is illustrated both numerically and experimentally in the case of a single layer of unidirectional carbon/epoxy plate, the fibers of which being not necessarily parallel to the sagittal plane. According to the chosen Lamb mode, the deviation of the Lamb wave beam can occur in a direction very close to that of the fibers.

The deviation of this ultrasonic Lamb wave beam is predicted in this paper using two different methods. A theoritical one involves the slowness curves for Lamb modes. Another one is a numerical method which uses a decomposition of the ultrasonic beam in

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monochromatic plane waves. Then, some comparisons between numerical and experimental results are given.

GEOMETRY

We consider here a plate, made up of one or several anisotropic layers immersed in a fluid medium (Fig. 1). An ultrasonic monochromatic beam is generated in the fluid by an emitter. The sagittal plane is defined by the acoustic axis of the transducer and the normal to the plate. θ is the angle between these two directions. The position of the sagittal plane with respect to x-axis linked to the plate is defined by the azimuthal angle φ.

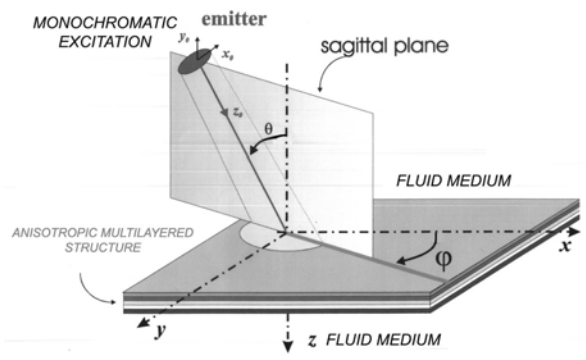


FIGURE 1. Geometry of the problem

A model has been developped which permits an ultrasonic reflection/transmission through the plate to be simulated, in order to obtain the field of pressure. Among various existing methods [1-4], the chosen method uses an ultrasonic beam decomposition method into monochromatic plane waves and a transfer matrix method, in order to obtain the reflected and transmitted coefficients for each monochromatic plane wave.

Before focusing on the propagation of Lamb waves, let us describe the general features of our model.

DESCRIPTION OF THE MODEL

The acoustic field is assumed to be known in a reference plane. In our case, we assume known particule displacement vector in a plane near the front face of the transducer emitter.

Generally speaking, the field generated by an emitter transducer can be obtained by a decomposition into plane waves which involves a double spatial Fourier transform [1] :

$$\vec{u}(x_0, y_0, z_0 = 0) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \vec{U}(k_x, k_y; \omega) \cdot e^{-j(k_x x_0 + k_y y_0 - \omega t)} dk_x dk_y \,, \qquad (1)$$

where k_x and k_y are the projections of the wave number on the x_0 and y_0 axis and ω is the frequency. Looking for the field of displacement, in a plane parallel to the reference one, needs to take into account the propagation along z_0 axis via the exponentional factor $e^{-j \cdot k_z z_0}$ [5,6] :

$$\vec{u}(x_0, y_0, z_0) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \vec{U}(k_x, k_y; \omega) \cdot e^{-jk_z z_0} \cdot e^{-j(k_x x_0 + k_y y_0 - \omega t)} dk_x dk_y \,. \quad (2)$$

The interaction of each monochromatic plane wave with the structure is taken into account using the transfer matrix method, which permits the reflected and transmitted coefficients, $R(k_x, k_y; \omega)$ and $T(k_x, k_y; \omega)$, to be calculated [7]. Then the reflected and transmitted fields are respectively given by Eq. (3) and Eq. (4) :

$$\vec{u}_{ref}(x_1, y_1, z_1 = 0) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \vec{U}_M(k_x, k_y, z_0 = 0; \omega) \cdot R(k_x, k_y; \omega) \cdot e^{-j\varphi_R} \cdot e^{-j(k_x x_1 + k_y y_1 - \omega t)} dk_x dk_y \,, \quad (3)$$

$$\vec{u}_{tra}(x_2, y_2, z_2 = 0) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \vec{U}(k_x, k_y, z_0 = 0; \omega) \cdot T(k_x, k_y; \omega) \cdot e^{-j\varphi_T} \cdot e^{-j(k_x x_2 + k_y y_2 - \omega t)} dk_x dk_y \,. \quad (4)$$

where $(x_1, y_1, z_1 = 0)$ is a plane symetric to the reference plane with respect to the normal of the plate and $(x_2, y_2, z_2 = 0)$ is a plane parallel to the reference plane. The phase change due to the propagation between emitter and receiver planes are respectively φ_R and φ_T for the reflected and transmitted cases. $\vec{U}_M$ corresponds to the field mirror to the incident one. All the Fourier transforms are numerically calculated by a Fast Fourier Transform algorithm.

As an example, Fig. 2 presents the reflected field in water for a 0.59 mm thick uniaxial carbon – epoxy plate. All computations have been done using the elastic constants given in Table 1. The configuration is chosen such that a Lamb mode S_0 is excited in the plate, with $\theta = 9.8°$ and the frequency $f = 1.35\,\text{MHz}$. The specular and non – specular parts can be observed on Fig. 2.

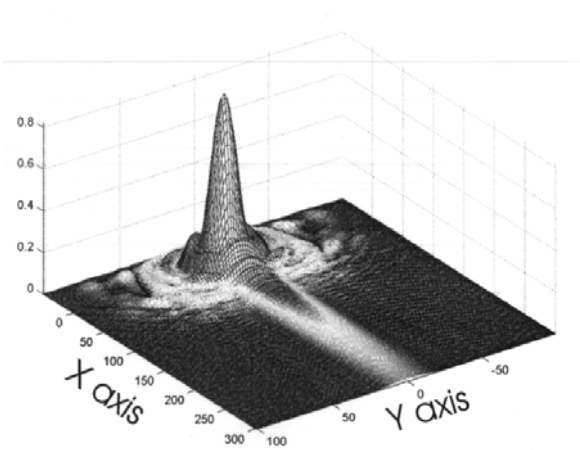


FIGURE 2 : Reflected field of pressure (linear scale). Uniaxial carbon – epoxy plate, azimuthal angle φ=0°, thickness = 0.59 mm, frequency=1.35 MHz, θ=9.8°

TABLE 1. Material characteristics of the carbon – epoxy medium. Elastic constants (GPa) such that 6th-order axis is parallel to the x-axis [8].

C_{11}	C_{12}	C_{22}	C_{23}	C_{44}	C_{55}	ρ
126	6.7	13.7	7.1	3.3	5.8	1580 kg/m^3

It should be noted that, in this case, the fibers of the plate are parallel to the x-axis, i.e. the direction of the fibers is contained in the sagittal plane. No deviation of the Lamb beam is

observed. As it will be seen subsequently, when the fiber direction is not contained in the sagittal plane, due to the bounded nature of the beam and to the anisotropy of the structure, there is a deviation of this Lamb beam.

The aim of the following section is to predict the dircetion of deviation (group direction) and the Lamb beam, using theoritical and numerical methods. Then, some experimental results are given.

PREDICTION OF THE DEVIATION DIRECTION OF THE LAMB BEAM

Dispersion curves for Lamb modes

When the anisotropic multilayered structure is in contact with vacuum, the writing of the boundary conditions leads to a dispersion relation for Lamb waves :

$$\omega = F(k_x, k_y)\,. \tag{5}$$

The dispersion curves can be drawn in the plane (ω, V_{Ph}) as well as in the plane (ω, θ) when V_{Ph} is the phase velocity of the Lamb waves and θ is a fictitious incident angle, defined with respect to a reference medium [9]. If this medium is water, V_{Ph} and θ are related by the following equation :

$$V_{Ph} = \frac{V_{water}}{\sin(\theta)}\,. \tag{6}$$

An example of these curves for an unidirectional carbon – epoxy plate is given on Fig. 3. For this kind of representation, the position of the sagittal plane is fixed, i.e. the azimuthal angle φ is constant, and the dispersion curves are function of the frequency ω.

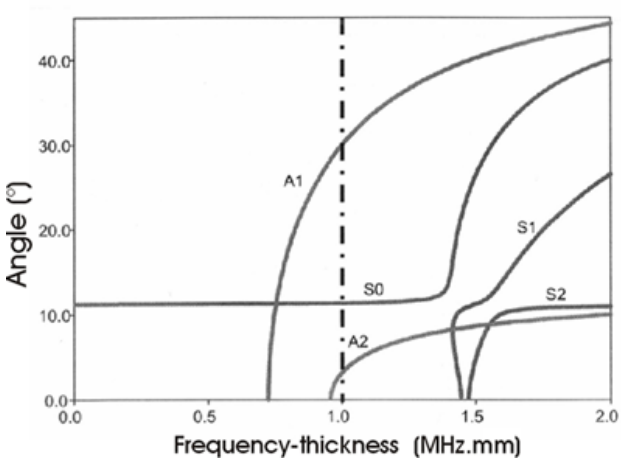


FIGURE 3. Dispersion curves for Lamb modes for an uniaxial carbon epoxy plate, φ = 30° calculatus done using Disperse software [10].

For a given frequency ω, and a given direction φ, there may be a number of possible values for V_{Ph}, corresponding to different families of Lamb waves. When φ varies, plotting V_{Ph} leads to a number of Lamb curves, given implicitly by Eq. (5) in the (k_x,k_y) plane. k_x and k_y are the projection of the wave number of the incident wave on the x and y-axis linked to the plate. It is more usual to draw the so-called « slowness curves », here in the (x, y) plane,

by plotting $\left\|\vec{k}_\Lambda / \omega\right\|$ as a function of φ, where $\vec{k}_\Lambda$ is the Lamb wave vector, with the components k_x and k_y. As an example, Fig. 4 presents the slowness curves for Lamb modes in a single carbon – epoxy layer. Contrary to the case of classical slowness surfaces for plane waves in infinite homogeneous media, these slowness curves depend on the frequency [9,11].

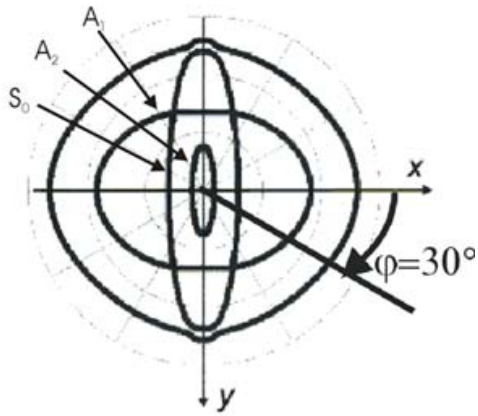


FIGURE 4. Slowness curves for Lamb modes. Uniaxial carbon epoxy plate. Frequency - thickness = 1 MHz.mm.

These two representations of dispersion curves are complementary. As an example, at 1 MHz.mm and φ=30°, the three modes A_2, S_0 and A_1 which are presented in Fig. 3 are found again in Fig. 4. If we want to generate one of these modes, due to the anisotropy of the structure, the Lamb beam is going to be deviated following a group direction.

Group direction

The most energetic part of the incident beam is found along the acoustic axis. $\vec{k}_\Lambda$ is the projection of the main wave number of this acoustic axis on the plane plate. It can be shown [11,12], that due to the bounded nature of the incident beam, the Lamb beam generated in the structure is centered on a direction given by the normal to the slowness curve at the point corresponding to the acoustic axis. Due to the anisotropy, slowness curves are not circular, so the normal does not coincide with the direction φ of the sagittal plane.

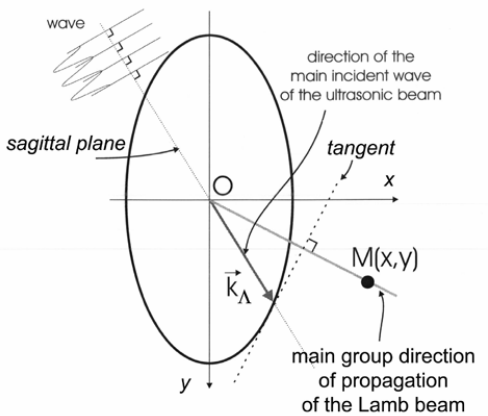
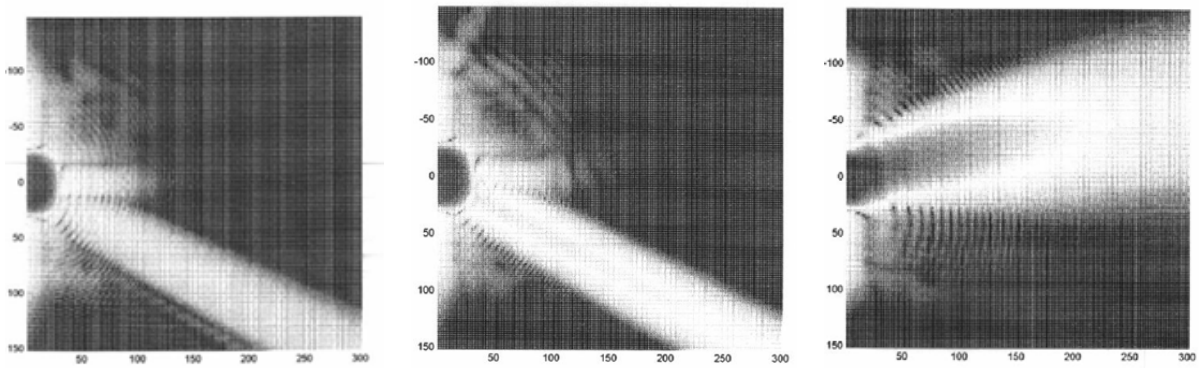


FIGURE 5. Obtention of the main group direction of the Lamb wave field

It is also possible to determine the group direction using numerical results from the above - described model of an ultrasonic beam interaction with a plate. Let us seeking for the

maximum amplitude of the reflected field of pressure. Considering only the non - specular part of the reflected field, we are able to determine the deviation angle of the Lamb beam.



a) Mode A_2 b) Mode S_0 c) Mode A_1

FIGURE 6. Reflected field of pressure (dB). Uniaxial carbon - epoxy plate, azimuthal angle $\varphi = 30°$, frequency - thickness =1 MHz.mm, excitation of mode A_2 a), mode S_0 b) and mode A_1 c).

As an example, Figs. 6 present the reflected fields of pressure for an uniaxial carbon – epoxy plate when the fiber direction is not contained in the sagittal plane (φ=30°). The excited Lamb mode is not the same for Figs 6-a, b and c. Firstly, it appears clearly that the deviation direction of the Lamb mode depends on the excited mode itself. Indeed, these three excited modes are those described before in Figs. 3 and 4. From Fig. 4, it can be observed that modes A_2 and S_0 have very similar slowness curves shapes, whereas that of mode A_1 is totally different. As a result, the direction of propagation of Lamb mode A_1 is different from that of modes A_2 and S_0 which roughly follow the fiber direction. The angles of deviation for each mode are given in Table 2, using the normal to the slowness curve (theoretical method) and the search for the maximum amplitude of the reflected field of pressure (numerical method). It can be seen that both methods are in excellent agreement.

TABLE 2. Lamb mode deviation obtained by both theoritical and numerical methods. Uniaxial carbon – epoxy plate, azimuthal angle $\varphi = 30°$, frequency - thickness =1 MHz.mm.

Lamb mode	A_2	S_0	A_1
Normal to slowness curve for Lamb	28.0°	28.0°	-12.0°
Maximum of magnitude of pressure	27.9°	28.3°	-10.6°

As a last observation, Lamb mode magnitude depends on the excited mode. That is due to a coupling fluid-structure effect which is connected to the mode attenuation value [13].

As conclusion, each excited Lamb mode has his own direction of propagation, and his own coupling behavior.

COMPARISON BETWEEN EXPERIMENTAL AND NUMERICAL RESULTS

Experiments have been made on a 0.59 mm thick uniaxial carbon – epoxy plate in order to validate our developments.

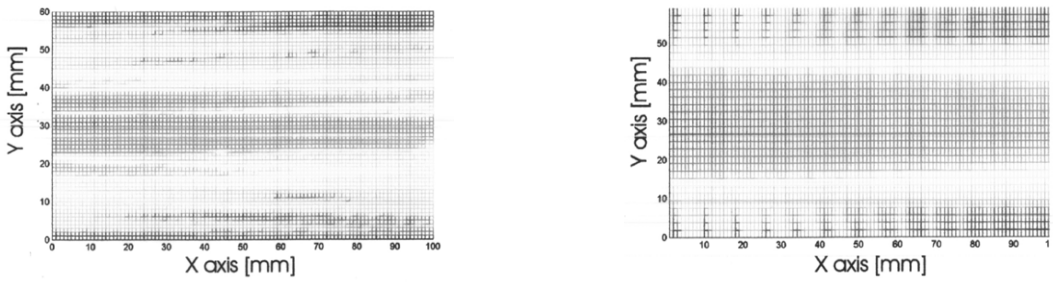
Experimental setup

The transducer emitter used to generate the ultrasonic field is a Panametrics Ref. V314, with nominal frequency 1 MHz and diameter 3/4 inc (19.05 mm). The receiver is a needle hydrophon, PVDF technology, diameter 1 mm from Precision Acoustics Ltd. The hydrophon has an integrated preamplifier with a sensitivity equal to 413 mV/MPa at 3 MHz. Then, the electrical output signal is amplified by a Matec TB-100 electronic card. Duration of excitation has been adjusted in order to obtain a ten cycles excitation.

The distance between the transducer emitter front face and the surface of the plate is equal to 90 mm. The hydrophon is positioned as closed as possible from the surface of the plate (roughly 1 mm). It is supported by an electronically controlled arm. Displacements accuracy is +/- 0.1 mm along both x and y axis.

In plane fiber configuration

Fig. 7 presents experimental and numerical results when the fiber direction is contained in the sagittal plane (in-plane fiber configuration), i.e. φ=0°. The excited mode is the mode S_0. Globally, a good agreement between experimental and numerical shapes of reflected pressure can be observed. The deviation angle of the Lamb beam is equal to zero in both cases : the Lamb beam propagation direction is parallel to the fiber direction and is contained in the sagittal plane.

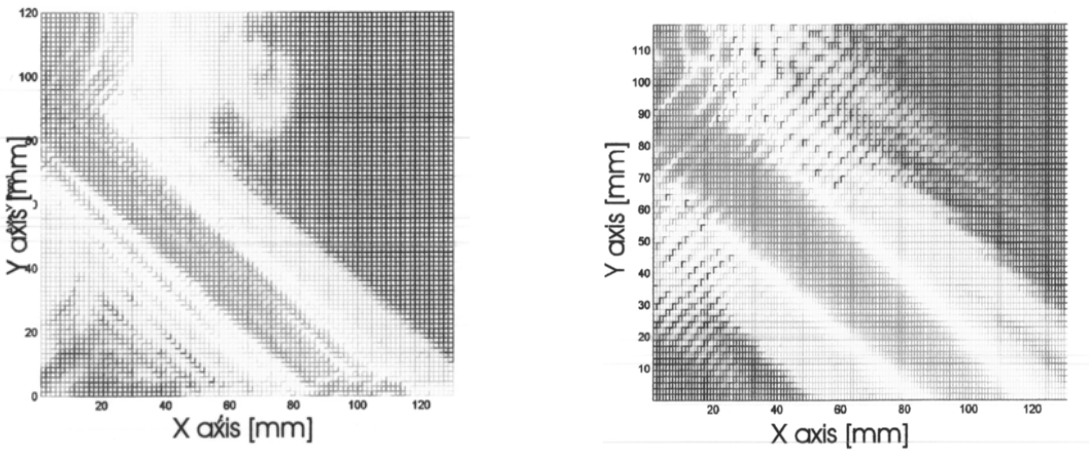


a) b)

FIGURE 7. Reflected field of pressure (dB). Experimental result a) and numerical result b). Mode S_0 excitation with an azimutal angle φ = 0°, frequency=1.35 MHz, incident angle=9.8°, plate thickness = 0.59 mm.

Out plane fiber configuration

Fig. 8 presents experimental and numerical results when the fiber direction is not contained in the sagittal plane (out-plane fiber configuration). Here φ=45°. The excited mode is still mode S_0. The fact that the direction of the ultrasonic reflected Lamb beam is not contained in the sagittal plane is highlighted in Fig 8. The deviation angle obtained by seeking for the maximum amplitude of the numerical and experimental reflected fields, are respectively 42.2° and 45.1°. Moreover pressure magnitudes are roughly similar. Lamb beam propagation direction is quite close to the fiber direction.



a) b)

FIGURE 8. Reflected field of pressure (dB). Experimental result a) and numerical result b). Mode S_0 excitation with an azimutal angle φ = 45°, frequency=1.20 MHz, incident angle=13.6°, plate thickness = 0.59 mm.

CONCLUSION

A 3D numerical model using plane wave decomposition has been developped. This model takes into account ultrasonic bounded beam and the excitation is monochromatic. The structure modelled can be an anisotropic multilayered medium.

Due to the anisotropy of the media constituting the structure, it has been shown that the Lamb wave beam is deviated with respect to the sagittal plane of the incident bounded beam, in a direction normal to the Lamb slowness curve. This physical phenomenon has been illustrated both numerically and experimentally in the case of a single layer of uniaxial carbon – epoxy plate, the fibers of which being not necessarily parallel to the sagittal plane. A very good coincidence between numerical and experimental results has been found for different configurations.

It should be noted that this numerical study needs imperatively a 3D model, in order to highlight phenomena which occur out of the sagittal plane. Though the work presented here uses monochromatic beams, the transient excitation case has also been developped [14].

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