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A stationary phase argument for the modal wave beam deviation in the time–space domain for anisotropic multilayered media

Catherine Potel *,[1], Philippe Gatignol, Jean-François de Belleval

Laboratoire Roberval, UMR UTC-CNRS, Université de Technologie de Compiègne, BP 20 529, 60205 Compiègne Cedex, France

Abstract

The aim of the paper is to describe the physical phenomenon of the excitation of modal waves, such as Lamb waves, in anisotropic multilayered media by a monochromatic incident beam and then by a time depending signal. A modal beam is generated in the structure and, due to the anisotropy of the media constituting the structure, is deviated with respect to the sagittal plane of the incident bounded beam. Using a stationary phase approach, it is possible to determine the deviation direction of the modal beam in the far field at a given frequency. This direction is normal to the modal curve, at the point corresponding to the main modal wave vector. Using Lagrange multipliers, it is possible to obtain the equation of an oblique plane in which the modal beam reradiates in the external fluid. As the modal waves are dispersive, the group velocity and the direction of propagation of the principal modal wave vary with the frequency. So, in the far field, for a time depending signal, the different monochromatic components of the main modal wave are found in different directions. In general, the main crest line of this modal wave packet is not a straight line.

Keywords: Anisotropic media; Bounded beam; Guided waves; Stationary phase

1. Introduction

For some configurations in the ultrasonic testing of anisotropic multilayered plane structures, modal waves [1] (which include guided waves such as Lamb waves, surface waves such as Rayleigh waves, or interface waves) can be locally excited in the structure by an emitter transducer immersed in an external fluid. Since the incident field is a bounded beam, a modal wave beam is generated in the structure. In fact, besides the main modal wave generated by the incident plane wave of the acoustic beam axis at a characteristic pair (angle θ of the acoustic beam axis, frequency), also are excited the neighbouring modal waves in the structure. Due to the anisotropy of the media constituting the structure, the most energetic part of the modal beam is deviated with respect to the sagittal plane of the incident bounded beam [2]. In the far field, this modal beam reradiates in the external fluid, along an oblique plane which contains the reflected direction of the acoustic beam axis (see Fig. 1).

2. Modal curves and dispersion

The structures considered here are very general and are made up of one or several plane anisotropic layers, the practical application being to composite media. In such structures, modal waves, which propagate with a phase velocity V_{ph} in the direction of the layers plane, can be brought to the fore: they can be analogous to Lamb waves when the structure is finite in depth, or Rayleigh-type when the structure is a semi-infinite periodic medium. In the last case, several Rayleigh wave families do exist and are dispersive, like Lamb waves; these waves have been called "*multilayered Rayleigh waves*" [3].

When the anisotropic multilayered structure is in contact with vacuum, the writing of the boundary conditions leads to a dispersion relation for modal waves:

$$\omega = F(k_x, k_y), \tag{1}$$

* Corresponding author. Fax: +33-34423-5287.
E-mail addresses: catherine.potel@utc.fr, catherine.potel@univ-lemans.fr (C. Potel).
[1] Present address: Laboratoire d'Acoustique de l'Université du Maine, UMR CNRS 6613, Avenue Olivier Messiaen, 72 085 Le Mans Cedex 9, France.

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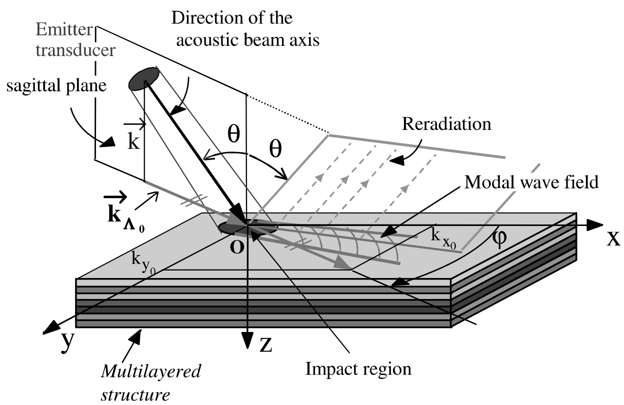

Fig. 1. Geometry of the problem.

where ω is the angular frequency of the waves, k_x and k_y are the components of the current wave vector on the x- and y-axis of the interface plane $z = 0$ (see Fig. 1). For a given propagation direction (angle φ), V_{ph} can be plotted as a multi-valued function of the angular frequency ω, which gives dispersion curves for the corresponding modal waves (Lamb or Rayleigh-type). For a given frequency ω and a given direction φ, there may be a number of possible values for V_{ph}, corresponding to different families of modal waves. When φ varies with ω constant, plotting V_{ph} leads to a number of modal curves $\mathscr{F}$, given implicitly by Eq. (1) in the (k_x, k_y) plane. It is more usual [4,5] to draw the so-called "slowness curves", by plotting $\|\vec{k}_A/\omega\|$ as a function of φ, where $\vec{k}_A$ is the modal wave vector, with the components k_x and k_y. As an example, Figs. 2 and 3 show, respectively, the slowness curves for Lamb modes in a structure made up of two layers of carbon/epoxy (with fibers orthogonal from one layer to the other) and the slowness curves for "*multilayered Rayleigh modes*" in an infinite periodically multilayered medium (with fibers of each layer 45° rotated with respect to the previous one). The characteristics of the media can be found in [3]. Contrary to the case of classical slowness surfaces for plane waves in infinite homogeneous media, these modal slowness curves depend on the frequency.

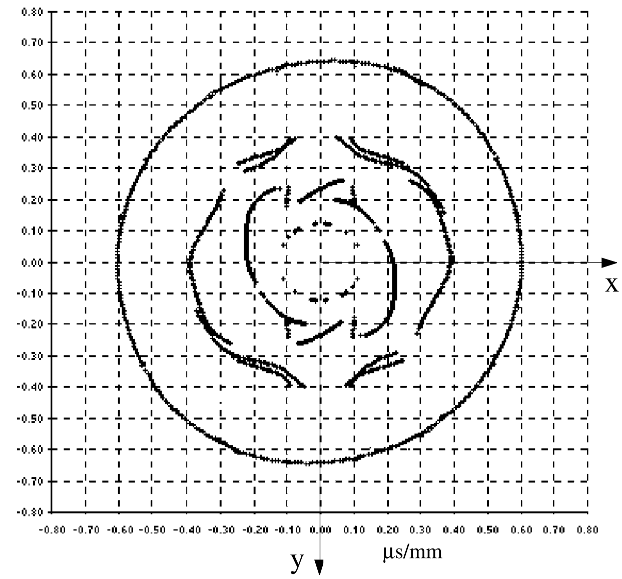

Fig. 3. Slowness curves for "*multilayered Rayleigh modes*", 0°/45°/90°/135° carbon/epoxy structure, $f = 2.5$ MHz.

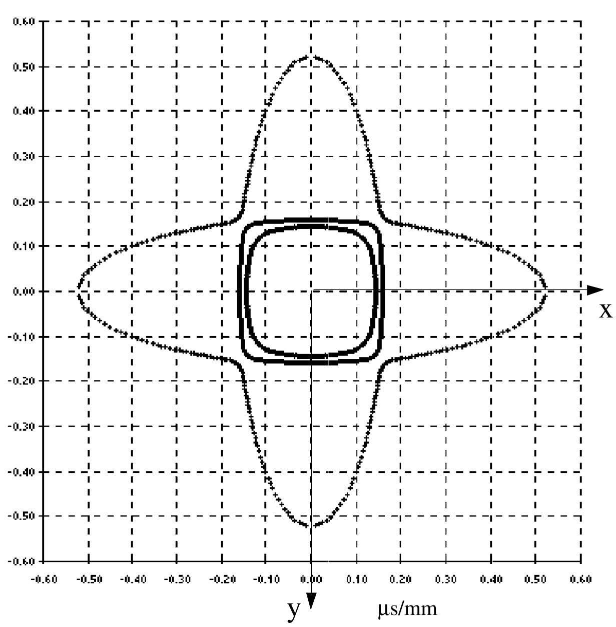

Fig. 2. Slowness curves for Lamb modes for two layers of a 0°/90° carbon/epoxy structure, $fH = 1$ MHz mm.

3. Beam effect

Generally speaking, the field generated by an emitter transducer can be obtained by a decomposition in plane waves which involves a double spatial Fourier transform [6]. For example, using a coordinate system attached to the insonified interface, the acoustical pressure can be written such that

$$\hat{P}_{\mathrm{inc}}(x, y, z, t; \omega) = \int\int \hat{A}^{\mathrm{E}}(k_x, k_y, a; \omega)\, \mathrm{e}^{\mathrm{i}(k_x x + k_y y + k_z z - \omega t)}\, \mathrm{d}k_x\, \mathrm{d}k_y, \qquad (2)$$

where a denotes the characteristic length of the impact region of the beam in the vicinity of the interface and k_z is given by the dispersion relation in the fluid:

$$c_0\sqrt{k_x^2 + k_y^2 + k_z^2} = \omega, \qquad (3)$$

where c_0 denotes the velocity of sound. $\hat{A}^{\mathrm{E}}$ is an amplitude function which depends on the particular boundary condition on the emitter. It presents a strongly marked maximum for a pair (k_{x_0}, k_{y_0}) which corresponds to the acoustic beam axis. Then, the reflected acoustic field in the fluid may be expressed in the following form:

$$\hat{P}_{\mathrm{ref}}(x,y,z,t;\omega)=\int\int \hat{A}^{\mathrm{E}}(k_x,k_y,a;\omega)\mathscr{R}(k_x,k_y,h,\tilde{\rho};\omega)\times \mathrm{e}^{\mathrm{i}(k_xx+k_yy-k_zz-\omega t)}\,\mathrm{d}k_x\,\mathrm{d}k_y, \quad (4)$$

where h denotes the characteristic thickness of the structure and the adimensional parameter $\tilde{\rho}$ is representative of the ratio of the density of the fluid to that of the layers. The reflection coefficient $\mathscr{R}(k_x,k_y,h,\tilde{\rho};\omega)$ of each plane wave depends on the geometrical and physical properties of the multilayered structure (and of the external fluid). Notably, the singularities of the reflection coefficient $\mathscr{R}(k_x,k_y,h,\tilde{\rho};\omega)$ are close to the solutions of the modal equation (1) in the complex domain $\mathbb{C}^2$ in (k_x,k_y). Thus, if the wave vector components (k_{x_0},k_{y_0}) of the acoustic beam axis belong to the modal curve at the given frequency ω, i.e. for an adequate pair of values (angle θ of the acoustic beam axis, frequency), the significant domain of integration for the double integral (4) contains a part of this modal curve, along which the reflection coefficient has quasi singularities. From a physical point of view, these quasi singularities correspond to a quasi resonance phenomenon, in the sense that the corresponding modal waves are generated in the structure by the incident bounded beam. The modal waves corresponding to the values (k_{x_0},k_{y_0}) are not the only waves which are excited, but also are excited all the modal waves in the neighbourhood of this point on the modal curve. In other words, all the modal waves in the domain of integration participate to this quasi resonant phenomenon. As a result, a bounded modal wave beam is generated in the structure (see Fig. 1). In the integral (4), besides the contribution which gives classically the specular acoustic beam in the direction symmetric to the acoustic beam axis, there is, in the present case, a second part which is significantly important along the modal curve. For the acoustic pressure of the external fluid, this part can be theoretically expressed in terms of a single integral along the modal curve $\mathscr{F}$ [2]:

$$\hat{P}_{\Lambda}(x,y,z,t;\omega)=\int_{\mathscr{F}}\hat{A}_{\Lambda}(k_x,a,h;\omega)\,\mathrm{e}^{\mathrm{i}(k_xx+k_yy-k_zz-\omega t)}\,\mathrm{d}k_x. \quad (5)$$

In the vicinity of the modal curve $\mathscr{F}$, the reduction of the twofold integral (4) in the single integral (5) may be justified under the assumption that $\tilde{\rho}\ll 1$. In this case, the phase term of the reflected field,

$$\Phi=k_xx+k_yy-k_zz-\omega t, \quad (6)$$

can be considered as a constant across a small strip region on both sides of the modal curve $\mathscr{F}$, whereas the phase of the reflection coefficient undergoes an abrupt change. In (5), ω is fixed, k_y is a function of k_x via the modal dispersion relation (1) and k_z depends on k_x and k_y through the fluid dispersion equation (3). However, in this integral, the main contribution remains that of the central point (k_{x_0},k_{y_0}) of the incident beam spectrum, which belongs, as a hypothesis, to the modal curve $\mathscr{F}$.

4. Angular deviation of the modal beam

Let us suppose now that the phase term (6) of the reflected field varies rapidly, compared to the amplitude function $\hat{A}_{\Lambda}(k_x,a,h;\omega)$, when $\vec{k}_{\Lambda}$ follows the slowness curve $\mathscr{F}$. This is the case under a far field hypothesis with respect to the impact region of the incident beam. An argument of stationary phase may then be introduced which permits the angular deviation of the modal beam to be predicted. This deviation occurs in the group direction, normal to the modal curve, at the point corresponding to the main modal wave vector $\vec{k}_{\Lambda_0}$ of the acoustic beam axis, associated to the main contribution of the beam spectrum in the integral (5) (see Fig. 4). Here, the angular dispersion due to the anisotropy plays a fundamental role: the modal curve is not circular and, as a consequence, the group direction is no more in the sagittal plane [2].

5. Analysis of the reradiation by using Lagrange multipliers

Let $M(x,y,z)$ be given in the far field. The main plane wave at this point (at a given time t) is given by the stationary phase method, using Eq. (6):

$$\mathrm{d}\Phi=0\Longleftrightarrow x\,\mathrm{d}k_x+y\,\mathrm{d}k_y-z\,\mathrm{d}k_z=0. \quad (7)$$

However, $\mathrm{d}k_x$, $\mathrm{d}k_y$, $\mathrm{d}k_z$ are not independent in Eq. (7). Two cases are now to be considered.

Let us first examine the case of the specular reflected beam, except modal excitation.

Besides Eq. (7), there is one constraint given by the dispersion equation in the fluid (for a given frequency):

$$\omega=\Omega(k_x,k_y,k_z)=c_0\|\vec{k}\|=\text{constant}. \quad (8)$$

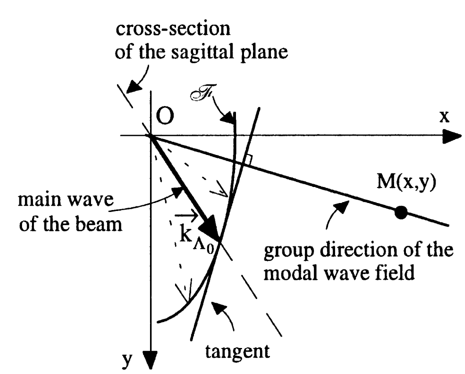

Fig. 4. Obtention of the main group direction of the modal wave field, for a particular branch of the modal curve.

Using Lagrange multiplier theory, the main local solution is among the solutions of the following stationary condition:

$$\mathrm{d}(\Phi - \tau_r \Omega) = 0 \Longleftrightarrow \left(x - \tau_r \frac{\partial \Omega}{\partial k_x}\right)\mathrm{d}k_x + \left(y - \tau_r \frac{\partial \Omega}{\partial k_y}\right)\mathrm{d}k_y - \left(z + \tau_r \frac{\partial \Omega}{\partial k_z}\right)\mathrm{d}k_z = 0, \tag{9}$$

where $\mathrm{d}k_x$, $\mathrm{d}k_y$, $\mathrm{d}k_z$ are now independent variations and τ_r is a Lagrange multiplier. Conversely, given k_x, k_y (and k_z through Eq. (8)), the set of points $M(x, y, z)$ where the wave vector (k_x, k_y, k_z) is locally dominant is a straight line (a ray) given by

$$x = \tau_r \frac{\partial \Omega}{\partial k_x}; \quad y = \tau_r \frac{\partial \Omega}{\partial k_y}; \quad z = -\tau_r \frac{\partial \Omega}{\partial k_z}. \tag{10}$$

Noticing that $(\partial\Omega/\partial k_x, \partial\Omega/\partial k_y, -\partial\Omega/\partial k_z)$ are the components of the (reflected) group velocity $\vec{V}_{\mathrm{spec}}$ in the fluid, it appears that the Lagrange multiplier τ_r may be interpreted as a time delay along the ray from the origin (the impact region in the far field hypothesis) to the observation point M. As the fluid is isotropic, Eq. (10) reduces to the classical ray equation

$$\overrightarrow{\mathrm{OM}} = \tau_r \vec{V}_{\mathrm{spec}} = \tau_r c_0 \frac{\vec{k}^{\mathrm{R}}}{k}, \tag{11}$$

where $\vec{k}^{\mathrm{R}}$ is the wave vector of the reflected acoustic beam axis.

Let us now examine the case of the reradiation of the excited modal wave beam.

In this case, the integration point (k_x, k_y) belongs to the modal curve $\mathscr{F}$ (see Fig. 4) where the reflection coefficient $\mathscr{R}$ undergoes an abrupt phase variation. As the current wave vector verifies now the two relations (1) and (8), two multipliers are now needed. Then the stationary principle takes the form

$$\mathrm{d}(\Phi - \tau_r \Omega - \tau_m F) = 0 \Longleftrightarrow \left(x - \tau_r \frac{\partial \Omega}{\partial k_x} - \tau_m \frac{\partial F}{\partial k_x}\right)\mathrm{d}k_x + \left(y - \tau_r \frac{\partial \Omega}{\partial k_y} - \tau_m \frac{\partial F}{\partial k_y}\right)\mathrm{d}k_y - \left(z + \tau_r \frac{\partial \Omega}{\partial k_z}\right)\mathrm{d}k_z = 0. \tag{12}$$

The set of points $M(x, y, z)$ of the physical space where a given wave vector is locally dominant is found in the plane defined parametrically by the equations

$$x = \tau_r \frac{\partial \Omega}{\partial k_x} + \tau_m \frac{\partial F}{\partial k_x}; \quad y = \tau_r \frac{\partial \Omega}{\partial k_y} + \tau_m \frac{\partial F}{\partial k_y}; \quad z = -\tau_r \frac{\partial \Omega}{\partial k_z}. \tag{13}$$

These equations may be written in the condensed form

$$\overrightarrow{\mathrm{OM}} = \tau_r \vec{V}_{\mathrm{spec}} + \tau_m \vec{V}_{g_{\mathrm{mod}}}, \tag{14}$$

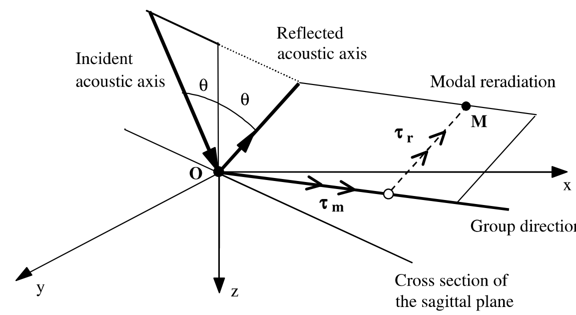

Fig. 5. Oblique plane in which the modal beam reradiates in the external fluid.

where $\vec{V}_{g_{\mathrm{mod}}}$ is the modal group velocity. Eq. (14) shows clearly that the multipliers τ_m and τ_r are time delays along the modal direction and the reradiated ray, respectively (see Fig. 5). As a conclusion, it can be seen in this case that all the nonspecular effects observed by Neubauer [7] and modelized by several authors, among them those of [8,9], are to be searched for in this oblique plane and not in the sagittal plane. In this context, a bidimensional modelization of the acoustic beam becomes insufficient.

6. Time depending signal

For a time depending signal, the group velocity and the propagation direction of the main modal wave vary with the frequency. Let ω_1 and ω_2 be two different frequential components of the signal. For the same direction of all the main modal wave vectors $\vec{k}_{\Lambda_0}$, these two monochromatic components will propagate following two different group directions δ_1 and δ_2 with two different group velocities $\vec{V}_{g_{\mathrm{mod}_1}}$ and $\vec{V}_{g_{\mathrm{mod}_2}}$ (see Fig. 6). In the far field, the monochromatic components of the signal are

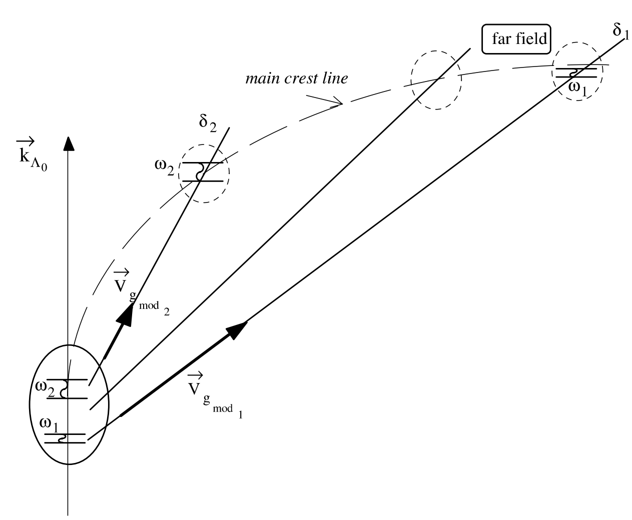

Fig. 6. Propagation of a time depending signal.

separated. Thus, at a given time, all the waves constituting the modal wave packet will be found on a main crest line which is not, in general, a straight line.

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