

Behavior of Lamb waves and Multilayered Rayleigh Waves in an Anisotropic Periodically Multilayered Medium: Application to the Long-wave Length Domain

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Summary

The aim of this paper is to study the behavior of leaky Lamb waves and what we termed *multilayered Rayleigh waves* in anisotropic periodically multilayered media and particularly in the long wave length domain. The multilayered medium is first studied in its whole complexity and dispersion curves for *multilayered Rayleigh modes* are drawn for Carbon/Epoxy composites. The displacement vector of such a wave is studied in detail, enabling the examination of the change of the nature of a mode which is the result of a Floquet wave becoming propagative again. In the long wavelength domain, the *multilayered Lamb modes* are identical for all the multilayered media which have the same layers in a superlayer, irrespective of the stacking order of the layers. However, the stacking order of the layers in a period of the multilayered medium has an importance in the validity domain of homogenisation for *multilayered Rayleigh mode*. When the frequency tends towards zero, the velocity of the *multilayered Rayleigh mode* tends towards the velocity of the Rayleigh wave that is propagating in the corresponding homogenised medium. The validity domain of homogenisation therefore depends on the nature of the studied waves.

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Introduction

Guided waves in plates and surface waves have become a subject of intensive study, especially Lamb waves and Rayleigh waves. Although a complete review of the extensive literature on this subject cannot be undertaken here, it is worth mentioning the main contributions. Lamb waves are free waves in an isotropic plate within a vacuum, and Rayleigh waves are surface waves propagating at the interface separating the vacuum from an infinite isotropic medium (Dieulesaint *et al.*, 1974; Auld, 1973). In both cases, the registering of boundary conditions leads to the cancellation of a determinant which yields to the symmetric and anti-symmetric Lamb modes or to the Rayleigh modes. When the vacuum is replaced by a fluid, the Lamb waves become leaky Lamb waves and the Rayleigh wave becomes a leaky Rayleigh wave, due to the fact that their energy leaks into the fluid. As discussed at length by Schoch (Schoch, 1952) and Fiorito *et al.* (Fiorito *et al.*, 1979), it is assumed that the Lamb wave vector equals the wave vector projection of the longitudinal wave from the fluid onto the surface of the plate. The presence of Lamb waves in a plate is indicated by the sharp minima in the magnitude and rapid reversals in the phase of the reflection coefficient in the fluid (Chimenti *et al.*, 1985). Based on Cremer's "coincidence" hypothesis, this approximation is valid provided that the ratio of acoustic impedances of the fluid and plate is small (Nayfeh *et al.*, 1988; Merkulov, 1988; Chimenti and Rokhlin, 1990). When this ratio is close to the value one, Plona *et al.* (Plona *et al.*, 1975) have shown that the inertial effects of the fluid are significant and cannot be neglected. Let us call k_1 the projection of the wave number vector of the waves on the surface of the plate. In the complex plane of k_1 , the phase reversal clearly indicates the passage

beneath a Lamb wave pole or between a pole-zero pair. As far as the leaky Rayleigh wave is concerned, its behavior is related to the pole of the reflection coefficient. The wave numbers corresponding to the pole and to the zero of the reflection coefficient are conjugated complexes (Izbicki *et al.*, 1987). The modulus of the reflection coefficient is therefore equal to one, whereas its phase varies from π to $-\pi$. The Rayleigh mode can also be studied by a resonant formalism, using the Poynting vector (Duclos *et al.*, 1994). There exists a correspondence between Lamb modes and Rayleigh modes. Indeed, dispersion curves for Lamb modes can be drawn as a function of the product fd : the frequency of the incident wave multiplied by the thickness of the plate (de Billy *et al.*, 1984; Claeys *et al.*, 1981). Irrespective of whether the plate is an isotropic or less symmetry crystal system medium (Li *et al.*, 1990), when the thickness of the plate tends towards the infinity, the velocity of the waves which compose the two first symmetric and anti-symmetric Lamb modes tend toward the Rayleigh wave velocity. Moreover, when there is a propagation along a symmetry axis, Li *et al.* (Li *et al.*, 1990) have found a Rayleigh wave velocity equivalent to that obtained by Dieulesaint and Royer (Royer *et al.*, 1984). When the medium consists of a composite material instead of a homogeneous material, the important works of Chimenti and Nayfeh have led to a better understanding of the ultrasonic leaky Lamb wave propagation in fiber-reinforced unidirectional and further non-unidirectional composites (Nayfeh *et al.*, 1988; Chimenti *et al.*, 1985; Chimenti *et al.*, 1989; Chimenti and Nayfeh, 1990). Chimenti and Rokhlin (Chimenti and Rokhlin, 1990a; Chimenti and Rokhlin, 1990b; Chimenti and Rokhlin, 1991) have particularly studied the influence of the fluid/solid density ratio when it approaches or exceeds unity: instead of approaching the Rayleigh wave speed asymptotically, the two curves for the fundamental anti-symmetric A_0 and symmetric S_0 modes join and cross the Rayleigh wave speed at finite fd when the ratio is ap-

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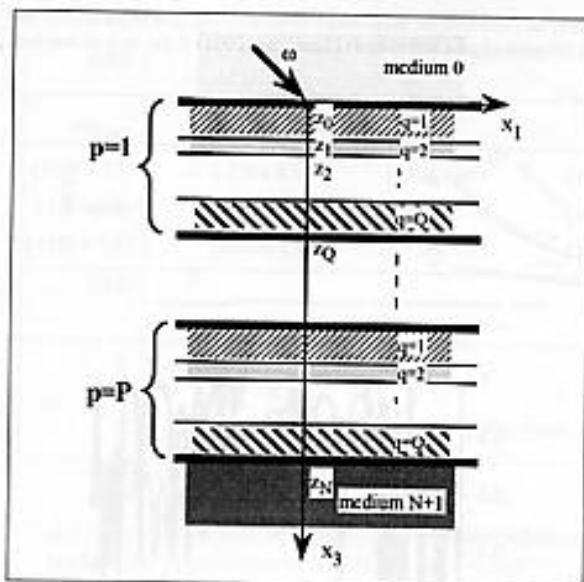


Figure 1. Periodically multilayered medium.

proximately 1. When the ratio increases, the curve is more distorted or disappears. The influence of liquid layers on Lamb waves has also been studied by Wu et al. (Wu et al., 1992). This is by no means an exhaustive list of the work carried out in this field.

The aim of this paper is to study the behavior of leaky Lamb waves and what we termed in a previous paper *multilayered Rayleigh waves* in anisotropic periodically multilayered media (see (Potel and de Belleval, 1995) and references contained therein) and particularly in the long wave length domain. As many works have dealt with the ultrasonic propagation in an homogenised multilayered medium (Dayal et al., 1989; Datta et al., 1988), the aim of the paper is not to consider the multilayered medium as an homogeneous medium; it is to study the multilayered medium in its whole complexity and to compare the results with those obtained when the medium is homogenised. Anisotropic periodically multilayered media consist of the reproduction of P anisotropic multilayered medium cells, termed "superlayer". These media have been extensively studied in previous works (see (Potel et al., 1993b; Potel et al., 1993a; Potel, 1994; Potel et al., 1994; Potel et al., 1995) and references contained therein). By registering the boundary conditions at the interface separating two successive layers, a transfer matrix is obtained. The solutions which permit us to go from one interface to another one by means of a diagonal matrix are the Floquet waves. These waves are linear combinations of the classical plane wave propagating in each layer of the multilayered medium. The long-wavelength approximation allows equivalent elastic constants for a multilayered medium to be obtained. In 1984, Helbig (Helbig, 1984) used this approximation and presented dispersive curves and slowness surfaces for transversely isotropic media. During the same year Shoenberg (Shoenberg, 1984), who studied alternating fluid/solid layers, specified that the homogenised medium

which models the behavior of the periodically multilayered medium in the long-wavelength domain must have the same slowness surfaces as those of the multilayered medium. In 1995, the present authors (Potel et al., 1994; Potel et al., 1995) have defined a Floquet polarisation vector at different interfaces. This vector tends towards a limit which is the polarisation of the classical plane wave in the homogenised medium.

The first aim of this paper is to advance the physical interpreting of the *multilayered Rayleigh waves*. After a background study of the ultrasonic propagation in multilayered media, and on *multilayered Rayleigh waves*, the dispersion curves of *multilayered Rayleigh modes* will be drawn in part 1. These curves allow the change in nature of *multilayered Rayleigh modes* to be studied. The reconstruction of the displacement vector of *multilayered Rayleigh waves* at any depth, measured from the first interface, permits a physical interpretation to be accomplished. The study of the behavior of these waves and of *multilayered Lamb waves* in the long wave length domain is then carried out in part 2. This is the second aim of the paper: the dispersion curves of *multilayered Lamb modes* are drawn for multilayered media that are first studied in their whole complexity and, secondly, homogenised. A comparison with the behavior of the *multilayered Rayleigh waves* is then performed.

1. Multilayered rayleigh modes

1.1. Summary

1.1.1. Anisotropic periodically multilayered media

Geometry of the medium

A periodically multilayered medium is the reproduction of P "superlayers", each one made by the stacking of Q distinct anisotropic media (see Figure 1). Each layer can have any thickness. A "superlayer" is, therefore, the period of the medium. Media 0 and $N + 1$ above and below the periodically multilayered medium are semi-infinite. The study of the acoustic propagation of the waves which are generated by an oblique incident wave propagating in the media 0 and contained in the plane $x_1 x_3$, defined in Figure 1, has been carried out in previous works (Potel et al., 1993b; Potel et al., 1993a; Potel, 1994; Potel et al., 1994; Potel et al., 1995); we will therefore commence with a quick summary of these main results.

Floquet analysis

By writing the boundary conditions at each interface separating two successive layers, the transfer matrix $[\Phi]$ of one superlayer can be found. This matrix is defined in (Potel et al., 1993b) and (Potel et al., 1993a). It allows the displacement amplitudes of the plane waves in the first layer of a superlayer to be expressed as a function of those in the first layer of the next superlayer. The transfer matrix of the whole periodically multilayered medium is $[\Phi]^P$, where P is the number of superlayers. Due to the fact that, locally, the acoustical state is characterised by six quantities, this matrix

Table I. Elastic constants in GPa for a Carbon/Epoxy medium from (Hosten et al., 1993) if the sixth-order symmetry A_6 axis is parallel to the x_3 -axis and for homogenised $0^\circ/90^\circ$ and $0^\circ/45^\circ/90^\circ/135^\circ$ media from (Hosten et al., 1993) and (Lhermitte, 1991) if the axis of stacked layers is parallel to the x_3 -axis.

	C_{11}	C_{12}	C_{13}	C_{33}	C_{44}	C_{66}
constants A	$13.7 + 0.13j$	$7.1 + 0.04j$	$6.7 + 0.04j$	$126 + 0.73j$	$5.8 + 0.1j$	$3.3 + 0.05j$
$0/90$	$69.8 + 0.43j$	$6.7 + 0.04j$	$6.9 + 0.04j$	$13.7 + 0.13j$	$4.2 + 0.06j$	$5.8 + 0.1j$
$0/45/90/135$	$57.0 + 0.38j$	$19.6 + 0.09j$	$6.9 + 0.04j$	$13.7 + 0.13j$	$4.2 + 0.06j$	$18.7 + 0.15j$

is of the 6th order, and the waves which correspond to the eigen vectors of this matrix are the Floquet waves. If the general solution is decomposed on the Floquet wave basis, the transfer matrix becomes a diagonal matrix. These six Floquet waves are the propagation modes of an infinite periodically multilayered medium. They are linear combinations of the classical waves propagating in each layer of the multilayered medium. The linear combination differs simply according to the layer (Potel et al., 1994; Potel et al., 1995).

One of the reasons for using the Floquet analysis is that the displacement and stress vectors are expressed in the Floquet wave basis. Subsequently, Floquet waves propagating in a multilayered medium will be used as classical waves propagating in a homogeneous medium, which is physically more transparent. The major difference results from the dispersive character of the Floquet waves.

Examples of anisotropic periodically multilayered media

Although the media constituting the layers in a "superlayer" can be distinct in the most general case, we will subsequently illustrate this work by the study of composite media made up of stacked identical transversely isotropic layers of Carbon/Epoxy. When each layer is at 90° to the previous one, the composite is called a $0^\circ/90^\circ$ medium. If the 6th-order symmetry axis of the first layer is parallel to the x_1 -axis and contained in the incident plane, (hence at 0° to this axis), the composite is called a $\{0^\circ/90^\circ\}_{0^\circ}$ medium. If this symmetry axis is, for instance, at 30° to the x_1 -axis, the composite is thus called a $\{0^\circ/90^\circ\}_{30^\circ}$ medium. One "superlayer" thus consists of two layers and is 0.23 mm thick. When each layer is at 45° to the previous one, the composite is called a $0^\circ/45^\circ/90^\circ/135^\circ$ medium. One "superlayer" therefore consists of four layers and is 0.47 mm thick. In the same manner, if the 6th-order symmetry axis of the first layer is, for instance, at 45° to the x_1 -axis and contained in the incident plane, the composite is called a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{45^\circ}$ medium. The elastic constants subsequently used were determined by Castaings (Hosten et al., 1993; Castaings, 1993) (see constants A in Table I); these constants are complex, which amounts to saying that the medium is a lossy one. A non lossy medium will thus be obtained by equalling the imaginary part of the elastic constants to zero. The volumetric mass of each layer is equal to 1577 kg/m^3 and the thickness is equal to 0.117 mm.

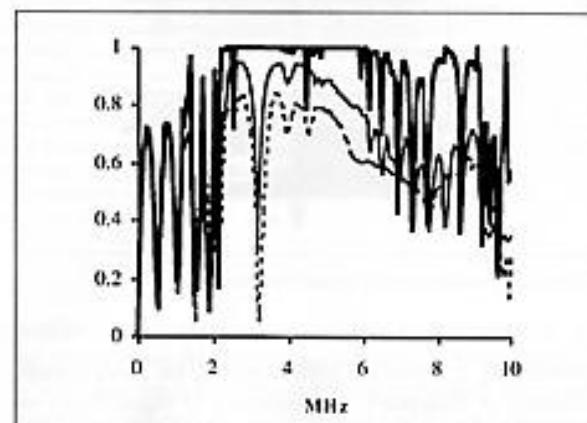


Figure 2. Modulus of the reflection coefficient for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium made of 6 superlayers at $\theta = 28.2^\circ$. Figure extracted from (Potel and de Belleval, 1995). Thick line: non lossy model, thin line: lossy model, dashed line: experiment.

1.1.2. Multilayered Rayleigh waves

The results presented in this part of the article were carried out in (Potel and de Belleval, 1995).

Experimental trough of a reflection coefficient

From experiments on a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium consisting of 6 superlayers submerged in water, it was shown that the presence of a trough of the experimental reflection coefficient results from the propagation of what we called a *multilayered Rayleigh wave*. Indeed, this trough was not found on the modelled coefficient when the medium is considered as non lossy (i.e. the elastic constants are real), whereas it is found on the modelled coefficient when the medium is considered as lossy (see Figure 2 extracted from (Potel and de Belleval, 1995)). Moreover, there is a phase variation from $+\pi$ to $-\pi$. By analogy with the behavior of the Rayleigh wave in isotropic media, we have studied the properties of this wave.

Properties of the multilayered Rayleigh wave

1. The propagation of a *multilayered Rayleigh wave* is characterised by the cancellation of a (3×3) determinant. This determinant corresponds to the boundary conditions for a *vacuum/infinite anisotropic periodically multilayered medium* structure. Indeed, equating to zero the normal

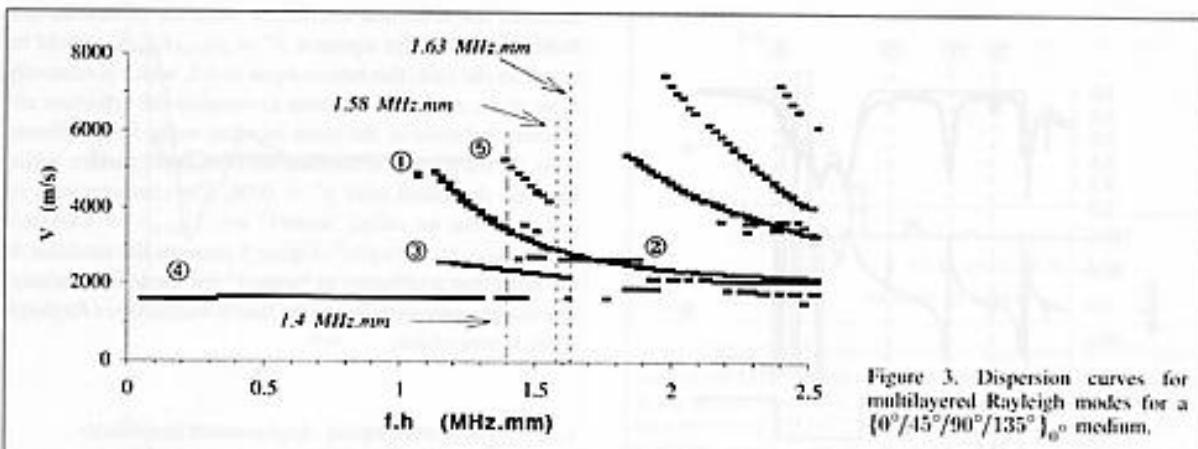


Figure 3. Dispersion curves for multilayered Rayleigh modes for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium.

- and tangential stresses, in order to have non zero solutions, leads to the cancellation of a (3×3) determinant.
2. A multilayered Rayleigh wave is a linear combination of three inhomogeneous Floquet waves.
 3. As the Floquet waves are dispersive, the multilayered Rayleigh wave is also dispersive. The propagation of this wave is therefore linked to both an angle and a frequency, which is not the case for the Rayleigh wave.
 4. A critical attenuation, where the depth of the trough of the reflection coefficient is maximum, exists. For this attenuation, the reflection coefficient is equal to zero.
 5. When the medium is thick enough, it behaves as if it were infinite. This is why the modulus of the reflection coefficient is equal to one when the medium is non lossy. When the medium is thin, the beginning of a multilayered Lamb mode can be seen on the reflection coefficient, even if the medium is considered as non lossy. We will return to this topic later on in the article, in section 1.3.

1.2. Obtaining of dispersion multilayered Rayleigh modes curves

1.2.1. Conditions

We have just seen that the propagation of a multilayered Rayleigh wave is characterised by the cancellation of a (3×3) determinant that corresponds to the boundary conditions for a vacuum/infinite anisotropic periodically multilayered medium structure. This determinant depends on k_1 , the projection on the x_1 -axis of the wave number vector. k_1 is real. However, it is more convenient to represent this determinant as a function of a fictitious angle of incidence θ , which can be defined in relation to a medium of reference. If this medium is water, k_1 is given by:

$$k_1 = \frac{\omega}{V_{\text{water}} \sin \theta} \quad (1)$$

with $V_{\text{water}} = 1480 \text{ m/s}$.

The value of k_1 for which the (3×3) determinant is equal to zero corresponds to the propagation of a multilayered Rayleigh wave. The graphical representation of the determinant for a non lossy medium, as a function of k_1 or

of θ is thus equal to zero at $\theta = \theta_R$ and $f = f_R$ where θ_R is the multilayered Rayleigh angle and f_R is the multilayered Rayleigh frequency. Numerically, when $\theta \approx \theta_R$ and $f \approx f_R$, the determinant is close to zero. Due to the fact that a multilayered Rayleigh wave is the linear combination of three inhomogeneous Floquet waves, dispersion curves for the multilayered Rayleigh modes can therefore be drawn by searching both in angle and in frequency for the cancellation point of the (3×3) determinant, but when all the Floquet waves are inhomogeneous in the medium. If the medium of reference is water, the "multilayered Rayleigh wave" speed is given by:

$$V_R = \frac{V_{\text{water}}}{\sin \theta_R} \quad (2)$$

1.2.2. Angular and frequential scanning

As it will be detailed in section 2.2.1, for Lamb modes, it is necessary to search for the cancellation point of the determinant by using two different scanning methods:

- for a fixed incident angle, as a function of the frequency: this is the frequential scanning;
- for a fixed frequency, as a function of the incident angle: this is the angular scanning.

1.2.3. Example of dispersion curves for $0^\circ/45^\circ/90^\circ/135^\circ$ media

The velocity of the multilayered Rayleigh wave defined by equation (2) is drawn as a function of the product $f \cdot h$, where f is the natural frequency and h the thickness of a superlayer. Figure 3 presents the multilayered Rayleigh modes for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium. Notice that for $f \cdot h = 1.52 \text{ MHz} \cdot \text{mm}$, the multilayered Rayleigh mode found experimentally at $\theta = 28.2^\circ$ in Figure 2 can be found again on the curve for the corresponding wave speed $V_R = 3132 \text{ m/s}$.

1.2.4. Influence of the fluid/solid density ratio

The cancellation of the determinant, when all the Floquet waves are inhomogeneous, is associated to either a trough of

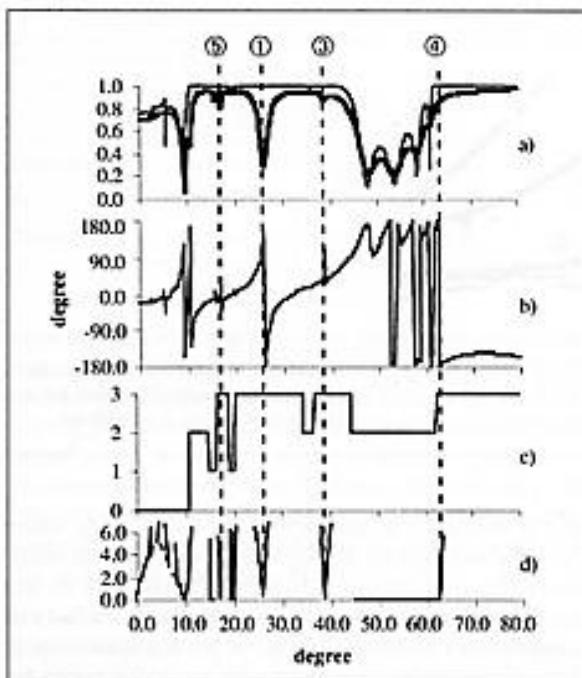


Figure 4. $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium consisting of 6 periods; $f h = 1.40 \text{ MHz mm}$; modulus a) and phase b) of the reflection coefficient in water. Thin line: non lossy medium, thick line: lossy medium. Number of the Floquet waves which are inhomogeneous in the non lossy medium c) and zoom of the modulus of the (3×3) determinant d) corresponding to the boundary conditions for a vacuum/infinite non lossy multilayered medium structure.

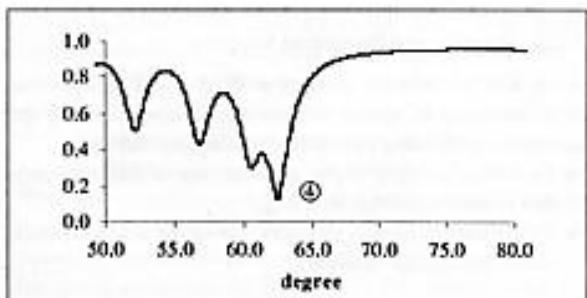


Figure 5. Modulus of the reflection coefficient in "water1" for a lossy $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium consisting of 6 periods; $f h = 1.40 \text{ MHz mm}$.

the modulus of the reflection coefficient in the fluid when the medium is lossy, or to a modulus equal to one and a rapid reversal of the phase when the medium is not lossy (see section 1.1.2.). This can be verified for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium for instance at $f = 3 \text{ MHz}$, i.e. $f h = 1.40 \text{ MHz mm}$ and $P = 6$. It can be seen in Figure 3 that 4 modes exist. Indeed, a trough of the modulus of the coefficient for the lossy medium corresponds to the first three modes. However, although the phase goes from π to $-\pi$ for the fourth mode, there is no trough (see Figure 4). We have seen in the introduction that the ratio of the solid density to the fluid density

modifies the reflection coefficient behavior (Chimenti and Rokhlin, 1990). The equation $\rho^* = \rho_{\text{fluid}}/\rho_{\text{solid}}$ should be noted. In our case, this ratio is equal to 0.6, which is relatively close to the value one. In order to compare the reflection coefficient behavior at the same incident angle for a different ratio, the wave speed in the fluid has been kept constant, while the ratio decreased until $\rho^* = 0.06$. The characteristics of this fluid that we called "water1" are: $V_{\text{water1}} = 1480 \text{ m/s}$ and $\rho_{\text{water1}} = 100 \text{ kg/m}^3$. Figure 5 presents the modulus of the reflection coefficient in "water1" for the lossy medium. The trough corresponding to the fourth *multilayered Rayleigh mode* is now visible.

1.3. Physical interpreting: displacement amplitudes

1.3.1. Lack of radiation

The *multilayered Rayleigh mode*, observed in Figure 2, does not appear when the medium is considered as a non lossy medium. The vanishing of this mode can be interpreted as a lack of radiation in the medium opposed to the insonification (Benelmostafa et al., 1990; Benelmostafa, 1990). As this mode appears for an incident angle greater than the critical angles in each layer of the medium, the side opposed to the insonification of the plate has very small displacement amplitudes and so has a very weak radiation. The radiation is all the more weak due to the fact that the thickness is sizeable.

Generally speaking, the displacement and stresses vector in the layer q of a "superlayer" p of the wave can be expressed as a function of these in the previous layer, and therefore as a function of these in the first layer (Potel et al., 1993b; Potel et al., 1993a). The amplitudes of the displacements of the six plane waves propagating or decreasing in the first layer are given by the registering of the boundary conditions. Gradually, it is possible to reconstruct the displacement vector at any depth measured out from the first interface. In order to verify that, at the angle and natural frequency for which the *multilayered Rayleigh mode* appears, the amplitudes of the displacement vector are very weak at the last interface, Figure 6 presents the components of the amplitude of the displacement vector of the wave decreasing in the non lossy $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium submerged in water, as a function of the depth. When the medium consists of 6 periods (see Figure 6a), which is the case for our experiments, the displacement amplitudes are very weak at the last interface. When the medium is three times thinner (see Figure 6b), the displacement amplitudes do not remain negligible. Due to the fact that the thickness of the medium is not sizeable, this mode is a *multilayered Lamb mode*. Above a certain thickness, the medium responds as if it were infinite, hence the *multilayered Lamb mode* becomes a *multilayered Rayleigh mode*.

Although all the Floquet waves are inhomogeneous in the periodically multilayered medium, some classical plane waves are homogeneous (or propogative) in some layers. Indeed, it has been shown (Potel et al., 1995) that as the Floquet waves are linear combinations of the classical plane waves in each layer, there is a reconstruction of the plane

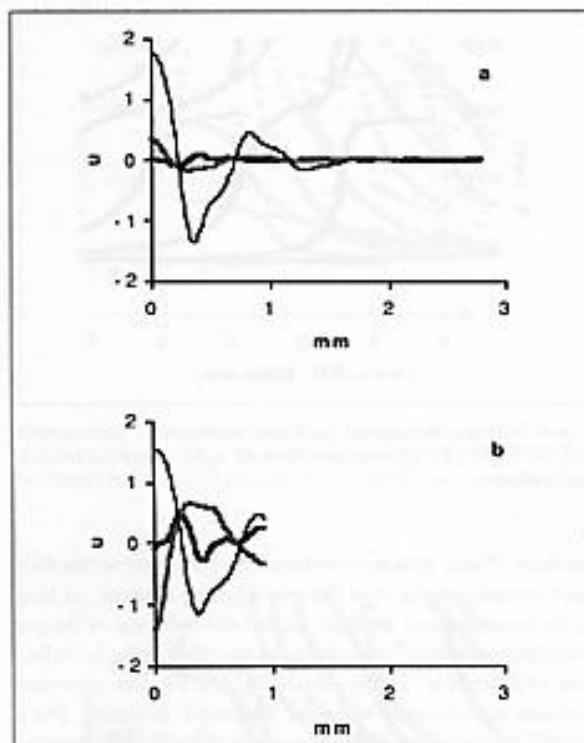


Figure 6. Components of the amplitude of the displacement vector of the wave decreasing in a non lossy $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium submerged in water, made of 6 periods a) and of 2 periods b); $f_h = 4.52 \text{ MHz/mm}$, $\theta = 28.2^\circ$. Thick line: u_1 , grey line: u_2 , thin line: u_3 .

waves of which some are inhomogeneous in one layer, to give Floquet waves which are all propagative. Conversely, there is a reconstruction of the plane waves of which some are propagative in one layer, to give Floquet waves which are all inhomogeneous. Suppose that the classical plane wave (η) in the layer q of a superlayer p is propagative. Its displacement vector can be written as:

$$\begin{aligned} {}^{(n)}u^{p,q} = & {}^{(n)}a^{p,q} {}^{(n)}p^q e^{-i\omega^{(n)}m_3^q(x_3-h_q)} \\ & \times e^{-i\omega^{(n)}m_1x_1-t}, \end{aligned} \quad (3)$$

with:

${}^{(n)}m^q$: the slowness vector of the wave (η) in the layer q

${}^{(n)}p^q$: the polarisation vector of the wave (η) in the layer q

${}^{(n)}a^{p,q}$: the complex amplitude of the particle displacement corresponding to the wave (η) in the layer $n = (p-1)Q + q$.

${}^{(n)}u^{p,q}$: the displacement vector of the wave (η) in the layer n

h_q : the thickness of the layer q .

As the wave (η) is propagative, the projection ${}^{(n)}m_3^q$ of the slowness vector on the x_3 -axis is real. The modulus of the amplitude of each of the three components of the displacement vector is therefore constant in the layer q of the "superlayer" p . However, as the *multilayered Rayleigh wave*

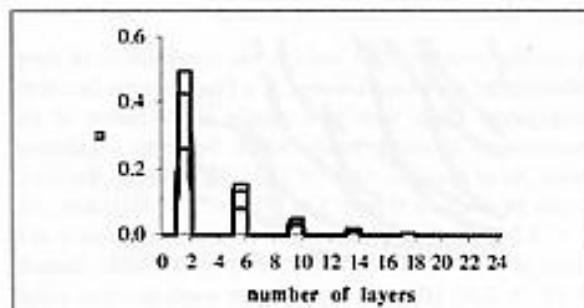


Figure 7. Modulus of components of the amplitudes of the displacement vector of the propagative wave such as ${}^{(n)}m_3^q = 0.23 \mu\text{s/mm}$ in the second layer of a "superlayer", as a function of the depth for a non lossy $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium; $P = 6$, $f_h = 4.52 \text{ MHz/mm}$, $\theta = 28.2^\circ$. Thick line: u_1 , grey line: u_2 , thin line: u_3 .

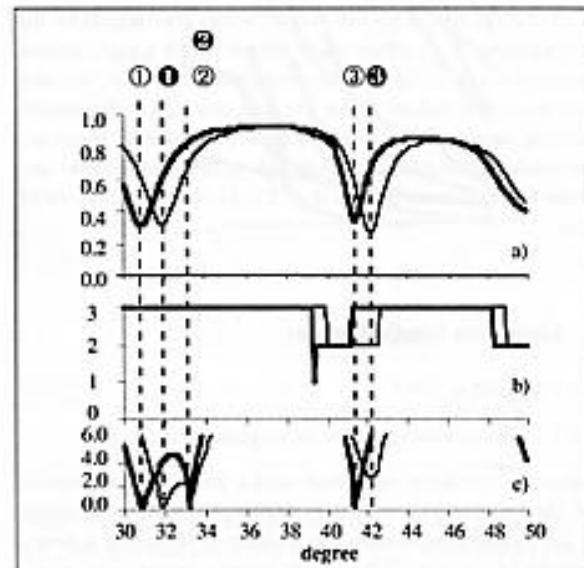


Figure 8. Modulus of the reflection coefficient in water for a lossy semi-infinite $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium a), number of the Floquet waves which are inhomogeneous in the non lossy medium b), zoom of the modulus of the (3×3) determinant corresponding to the boundary conditions for a *vacuum/non lossy infinite multilayered medium* structure c). Thick line: $f_h = 1.58 \text{ MHz/mm}$, thin line: $f_h = 1.63 \text{ MHz/mm}$.

globally decreases as a function of the depth, the amplitude of the wave (η) will also decrease in the layer q of the next period $p+1$ and so forth. As an example, Figure 7 displays the modulus of the amplitudes of the three components of the displacement vector of a propagative wave in the second layer of a "superlayer", i.e. the layer of which the 6th order symmetry axis is at 45° to the x_1 -axis. The propagative wave chosen is characterised by ${}^{(n)}m_3^q = 0.23 \mu\text{s/mm}$.

1.3.2. Change of the nature of a mode

A *multilayered Rayleigh wave* is the combination of three inhomogeneous Floquet waves. If a Floquet wave becomes propagative again, there is a change in the nature of the *multilayered Rayleigh mode*, which becomes a radiating mode. As an example, for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium, it can be seen in Figure 3 at $fH = 1.63 \text{ MHz mm}$, i.e. $f = 3.5 \text{ MHz}$, the interruption of the mode whereas it still exists at $fH = 1.58 \text{ MHz mm}$, i.e. $f = 3.4 \text{ MHz}$. Indeed, at $fH = 1.63 \text{ MHz mm}$ one Floquet wave becomes propagative again, as can be seen in Figure 8b. The curves of Figure 8 are drawn as a function of the incident angle corresponding to the wave speed of the surface wave (see equation (1) in section 1.2.1.). Though the (3×3) determinant corresponding to the boundary conditions for a *vacuum/infinite $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium structure* does not cancel out (see Figure 8c), there is a trough of the modulus of the reflection coefficient for a lossy medium at $fH = 1.63 \text{ MHz mm}$ and $\theta = 42.2^\circ$, as can be seen in Figure 8a. Moreover, there is a rapid phase reversal at this angle which corresponds to $V = 2203.3 \text{ m/s}$. This can be physically interpreted by the propagation of a surface wave which is not a *multilayered Rayleigh wave*. If the multilayered medium is finite, this surface wave will radiate in the medium opposed to the insoufication because one Floquet wave is propagative. However, the mode ② does not correspond to any trough or to any phase reversal, even though the (3×3) determinant is about zero.

2. Long wavelength domain

2.1. Summary

2.1.1. Characteristics of the homogenised medium

Generally speaking, stratified media are dispersive media, i.e. the velocities of the waves which propagate or decrease in the medium depend on the frequency. However, a development in the long-wavelength domain permits the stratified medium to be homogenised. Indeed, when the wavelength of the waves increases to a much greater thickness than that of one "superlayer", the less important the disruptions due to the layers become. Elastic constants of the homogenised medium can then be deduced from a development in the long wavelength domain. Such a development has been done by Lhermitte (Lhermitte, 1991) for $0^\circ/90^\circ$ and $0^\circ/45^\circ/90^\circ/135^\circ$ media. The equivalent constants are given in Table I. The homogenised $0^\circ/90^\circ$ medium has the properties of tetragonal crystal system media, whereas the homogenised $0^\circ/45^\circ/90^\circ/135^\circ$ medium has the properties of hexagonal crystal system media. The later is therefore transversally isotropic.

2.1.2. Slowness surfaces

As equivalent constants can be calculated, it is possible to draw slowness surfaces in the same manner as homogeneous

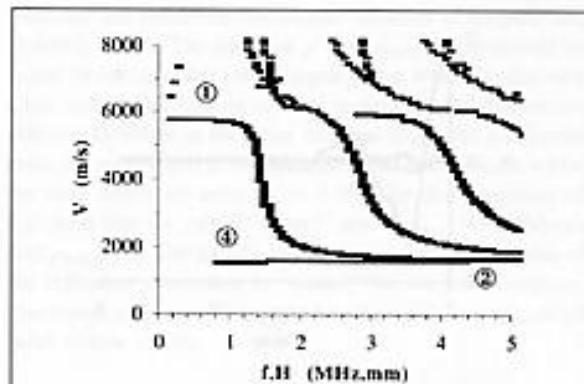


Figure 9. Dispersion curves for Lamb modes for a homogenised $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium; here, H is the whole thickness of the medium.

medium. These slowness surfaces are the same as the Floquet slowness surfaces of the multilayered medium, as long as the homogenised medium models the behavior of the periodically multilayered medium, as specified in the introduction (Schoenberg, 1984). Gradually, the Floquet slowness surfaces are distorted when the frequency increases. For a $0^\circ/90^\circ$ medium, the homogenisation is valid until approximately $fH = 0.65 \text{ MHz mm}$, whereas it is valid until approximately $fH = 0.52 \text{ MHz mm}$ for a $0^\circ/45^\circ/90^\circ/135^\circ$ medium (Potel et al., 1995).

2.1.3. Polarisation vector

The polarisation vector of a classical plane wave propagating in a homogeneous medium remains constant in a plane perpendicular to the direction of propagation vector of the wave. This is not the case for the Floquet polarisation vector defined in (Potel et al., 1995), because the Floquet waves are not plane waves: the Floquet polarisation vector varies according to its position in the multilayered medium. However, in the long wavelength domain, the Floquet polarisation vector tends towards a limit which is the polarisation of the classical plane wave in the homogenised medium.

2.2. Multilayered Lamb modes

2.2.1. Principle

As specified in the introduction, the presence of Lamb waves in a plate is indicated by the sharp minima in the magnitude and rapid reversals in the phase of the reflection coefficient in the fluid (Chimenti et al., 1985). This approximation is valid so long as the ratio of acoustic impedances of the fluid and plate is small (Nayfeh et al., 1988; Merkulov, 1988; Chimenti and Rokhlin, 1990). The reflection coefficient will thus be calculated as previously stated in section 1.2.4, in the fluid called "water". As an example, let us take the dispersion curves of Lamb modes for an homogenised $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium (see Figure 9). The curves are now plotted as a function of the product fH , where H

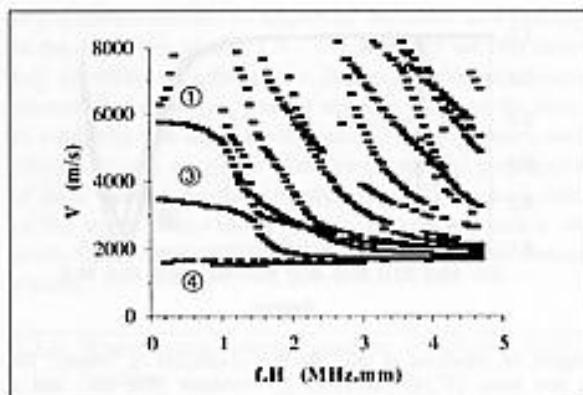


Figure 10. Dispersion curves for Lamb modes for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium or a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{45^\circ}$ medium consisting of 2 periods; the two curves are exactly the same.

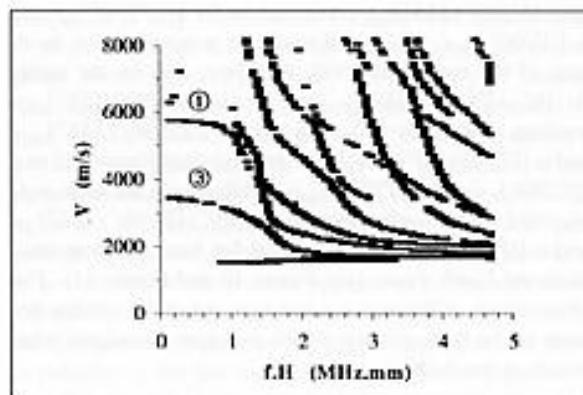


Figure 12. Dispersion curves for Lamb modes for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium consisting of 2 periods.

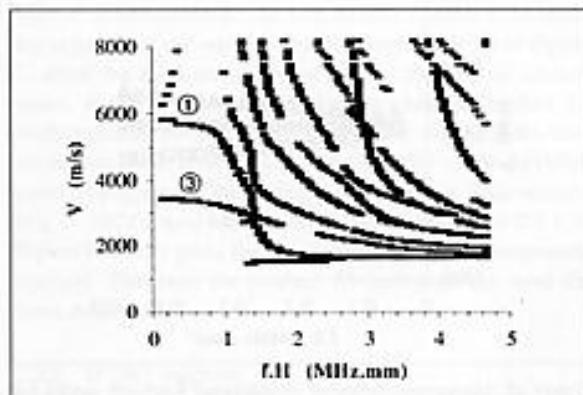


Figure 11. Dispersion curves for Lamb modes for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{90^\circ}$ medium or a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{135^\circ}$ medium consisting of 2 periods; the two curves are exactly the same.

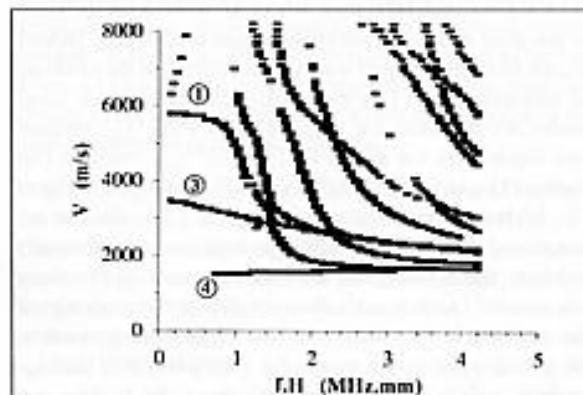


Figure 13. Dispersion curves for Lamb modes for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{90^\circ}$ medium consisting of 2 periods.

is the total thickness of the plate. It can be immediately noted that the horizontal mode (2) corresponds to a Rayleigh mode ($\theta_R = 65.8^\circ$, $V_R = 1622.6$ m/s). Moreover, the dispersion curves for *multilayered Lamb modes* have to be drawn by two different scannings: a frequential scanning and an angular scanning (see section 1.2.2). Indeed, a mode parallel to the frequencies-axis will be detected only by angular scanning, whereas a mode parallel to the velocities-axis will be detected only by frequential scanning (Benelmostafa *et al.*, 1990; Benelmostafa, 1990).

2.2.2. Stacking order of the layers

The stacking order of the layers in a "superlayer" has no importance in the long-wavelength domain for the calculation of the elastic constants of the homogenised multilayered medium (Lhermitte, 1991). Indeed, this calculation can be viewed as an average. For example, a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium, a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{45^\circ}$ medium, a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{90^\circ}$ medium and a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{135^\circ}$ medium will have the

same homogenised elastic constants. In the same way, a $\{0^\circ/90^\circ\}_{0^\circ}$ medium and a $\{0^\circ/90^\circ\}_{90^\circ}$ medium will share the same homogenised elastic constants.

In any frequency range

Moreover, if each layer of a "superlayer" is a monoclinic crystal system media with a second-order axis perpendicular to the interfaces (or with a mirror plane parallel to these), the Floquet waves propagate in the same manner in two directions symmetrical with respect to a perpendicular, which is at a right angle to the interfaces. This can be stated in a different manner, by stating that the third components of the Floquet slowness vectors are opposed or conjugated to each other (Potel *et al.*, 1993b). A $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$, a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{45^\circ}$, a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{90^\circ}$, and a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{135^\circ}$ medium have thus the same Floquet waves. However, when the medium is submerged in a fluid, it has been observed that the modulus of the reflection coefficients are exactly the same for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ and for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{45^\circ}$ medium. The same phenomenon also occurs for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{90^\circ}$ and a

$\{0^\circ/45^\circ/90^\circ/135^\circ\}_{135^\circ}$ medium, or for a $\{0^\circ/90^\circ\}_{0^\circ}$ and a $\{0^\circ/90^\circ\}_{90^\circ}$. The position of the minima of the modulus of the coefficients will, therefore, also be the same. A $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ and a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{45^\circ}$ medium (see Figure 10), such as a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{90^\circ}$ and a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{135^\circ}$ medium (see Figure 11) or a $\{0^\circ/90^\circ\}_{0^\circ}$ and a $\{0^\circ/90^\circ\}_{90^\circ}$ will thus have the same *multilayered Lamb waves*. However, a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ and a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{90^\circ}$ will not have the same *multilayered Lamb waves* (see Figure 10 and Figure 11). The observations of this part are true even out of the validity domain of the homogenisation. We will now investigate what occurs in this domain.

In low frequency range

As far as *multilayered Lamb modes* are concerned, it can be seen in Figure 10, Figure 11, and Figure 12 that the stacking order of the layers in a superlayer has no importance in the validity domain of the homogenisation (for $fH < 1 \text{ MHz mm}$ i.e. for $fh < 0.5 \text{ MHz mm}$, where H is the total thickness of the plate and h the thickness of one superlayer). Indeed, except when the Floquet waves which compose the mode are all inhomogeneous (see mode ④), the *multilayered Lamb modes* are the same for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium (see Figure 10), for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{90^\circ}$ medium (see Figure 11) and for a $0^\circ/90^\circ/45^\circ/135^\circ$ medium (see Figure 12). Moreover, we have seen in section 2.1.1, that the homogenised $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium is transversally isotropic, that is to say that any rotation around its symmetry axis parallel to the x_3 -axis does not change the properties of the medium. In the validity domain of the homogenisation, the *multilayered Lamb modes* for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium which is rotated by 30° about the x_3 -axis, i.e. for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{30^\circ}$ medium (see Figure 13), are thus the same as those for a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ or a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{90^\circ}$ medium. However, the comparison of all these dispersion curves for *multilayered Lamb modes* with those of Figure 9 for the homogenised medium shows that the mode ③ is only found in the multilayered case. This amounts to saying that according to the type of waves and to the required precision, the ultrasonic propagation can be studied for a homogenised medium. However, if the studied case needs more precise results, the multilayered medium must be studied in its whole complexity.

To summarise, it can be said that, as far as *multilayered Lamb waves* are concerned, in the low frequency range, the stacking order of the layers in a superlayer has no importance and that all the multilayered media which have the same layers in a superlayer have the same behavior. Moreover, when the homogenised medium is transversally isotropic, any rotation around the axis of stacked layers of the multilayered medium does not change its *multilayered Lamb modes*. However, some modes are not found for the corresponding homogenised medium.

2.2.3. Modes at the validity limit of homogenisation

At the validity limit of homogenisation, the more the incident angle increases, which is equivalent to saying the

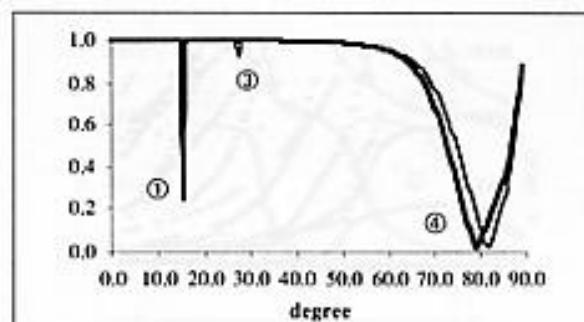


Figure 14. Modulus of the reflection coefficient in "water1" for a non lossy $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{0^\circ}$ medium (thin line) and a $\{0^\circ/45^\circ/90^\circ/135^\circ\}_{90^\circ}$ medium (thick line) consisting of 2 periods at $fh = 0.373 \text{ MHz mm}$ i.e. $fH = 0.747 \text{ MHz mm}$.

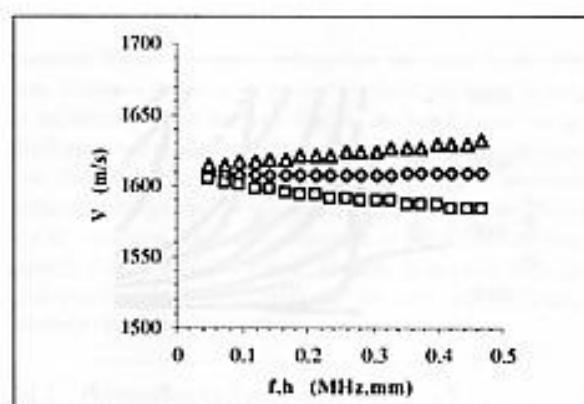


Figure 15. Dispersion curves of multilayered Rayleigh modes for $0^\circ/45^\circ/90^\circ/135^\circ$ media rotated by 0° (Δ), 30° (\circ) or 45° (\square) around the x_3 -axis.

more the velocity decreases, the less the *multilayered Lamb modes* superimpose. Indeed, the Floquet slowness surfaces at $fH = 0.747 \text{ MHz mm}$, i.e. $fh = 0.373 \text{ MHz mm}$ do not superimpose for great incident angles. The homogenisation is therefore only valid for weak angles. Indeed, a wave generated by an oblique incident wave covers a greater distance than a wave generated by a normal incident wave. If the velocity of the Floquet wave decreases when the incident angle increases, the wavelength also decreases. In this case, the more the incident angle increases, the more the validity domain of homogenisation shrinks. This is why a shift of modes ③ and ④ can be observed in Figure 14 for an incident angles of approximately 28° and 75° respectively, whereas there is none for the mode ①.

2.3. Multilayered Rayleigh modes

We have just seen that as far as *multilayered Lamb modes* are concerned, the stacking order of the layers in a superlayer has no importance in the long-wavelength domain. The aim of this section is to illustrate that this is not the case for *multilayered Rayleigh modes*. Indeed, a *multilayered Rayleigh wave*

is a surface wave and its amplitude decreases as a function of the depth (see section 1.3.). The nature of the first layers has, therefore, an influence on the propagation of a *multilayered Rayleigh wave* and the stacking order of the layers in a superlayer is therefore important. On the contrary, *multilayered Lamb waves* are plate waves and the propagation of these waves is the same irrespective of the stacking order of the layers. The validity domain of homogenisation depends on the nature of the studied waves. In fact, this domain shrinks.

2.3.1. Transversally isotropic medium: $0^\circ/45^\circ/90^\circ/135^\circ$ medium

For a transversally isotropic medium, any rotation around its symmetry axis does not change its properties. Figure 15 presents the dispersion curves of *multilayered Rayleigh modes* for $0^\circ/45^\circ/90^\circ/135^\circ$ media, rotated by 0° , 30° , or 45° around the x_3 -axis, in the long wavelength domain ($fh < 0.5 \text{ MHz mm}$). The aim of this section is to study the behavior of the *multilayered Rayleigh mode* ① of Figure 3, when the medium is rotated around the axis of stacked layers. It can be seen immediately that when the product fh tends towards zero, the velocities of the three modes tend toward the same limit, which is the velocity of the Rayleigh wave propagating in the homogenised medium. This velocity ($V_R = 1622.6 \text{ m/s}$) has already been found in section 2.2.1, in Figure 9, which gives the Lamb modes of the homogenised medium. The more the product fh increases, the more the three modes divide.

2.3.2. $0^\circ/90^\circ$ medium

We have seen in section 2.2.2, that the propagation of *multilayered Lamb modes* is the same in a $\{0^\circ/90^\circ\}_{90^\circ}$ medium and $\{0^\circ/90^\circ\}_{90^\circ}$ medium. However, the *multilayered Rayleigh modes* are not the same, even in the validity domain of homogenisation. As previously stated, when the product fh tends towards zero, the velocities of the two modes tend toward a limit which is the velocity of the Rayleigh wave propagating in the homogenised medium.

3. Conclusion

From observing an experimental trough of the reflection coefficient for a composite plate, a wave that was termed *multilayered Rayleigh wave* was defined, by analogy with the Rayleigh wave which propagates in isotropic media. This *multilayered Rayleigh wave* is the combination of three inhomogeneous Floquet waves and results from the cancellation of a (3×3) determinant. The scanning, both in angle and in frequency, of the cancellation of this determinant allows dispersion curves for the *multilayered Rayleigh modes* to be drawn. When the composite plate is loaded by a fluid, the propagation of a *multilayered Rayleigh wave* is also characterised by a trough of the modulus of the reflection coefficient and a rapid reversal of its phase. For some *multilayered Rayleigh modes*, the trough of the modulus of the reflection

coefficient is only visible when the fluid/solid density ratio is small. Examples are given for Carbon/Epoxy composite plates consisting of $0^\circ/45^\circ/90^\circ/135^\circ$ and $0^\circ/90^\circ$ media. The calculation of the displacement vector of the *multilayered Rayleigh wave*, as a function of the depth, allows a physical interpretation to be given: the amplitudes of the three components of the displacement vector are very weak at the last interface of the stratified medium when it is thick enough. It therefore behaves as if it were infinite and there is no radiation in the medium opposed to the insonification. On the contrary, when the medium is thin, the amplitudes of the displacement vector are no longer negligible and there is radiation in the last medium. There is the propagation of a *multilayered Lamb mode* which becomes a *multilayered Rayleigh mode* when the medium is sizeable. Although a *multilayered Rayleigh wave* is the combination of three inhomogeneous Floquet waves, some classical plane waves can be propagative in some layers of a "superlayer". Globally, as the *multilayered Rayleigh wave* decreases as a function of the depth, the amplitude of a propagative classical plane wave is constant in one layer, but also decreases in the next period. When a Floquet wave becomes propagative again, there is a change in the nature of the mode, which thus radiates in the medium opposed to the insonification.

In the long wavelength domain, the multilayered medium can be homogenised, and the stacking order of the layers in a "superlayer" is no longer important for the *multilayered Lamb modes*. These modes are identical for all the multilayered media which have the same layers in a superlayer, irrespective of the stacking order of the layers. When the homogenised medium is transversally isotropic, any rotation around the axis of stacked layers of the multilayered medium does not change its *multilayered Lamb modes*. However, when the Floquet waves which compose the *multilayered Lamb mode* are all inhomogeneous, the latter are not identical in each multilayered medium. Moreover, although most of the *multilayered Lamb modes* are identical in the validity domain of the homogenisation when the multilayered medium is studied in its whole complexity, some modes are not found for the corresponding homogenised medium. On the contrary, as far as *multilayered Rayleigh modes* are concerned, the stacking order of the layers in a superlayer is of importance, even in the validity domain of homogenisation. Indeed, the amplitude of a *multilayered Rayleigh wave* decreases as a function of the depth. When the frequency tends towards zero, the velocity of the modes tend toward a limit, which is the velocity of the Rayleigh wave propagating in the homogenised medium. The more the frequency increases, the more the modes divide.

To conclude, it can be said that the validity domain of homogenisation depends on the nature of the studied waves. This domain is reduced for *multilayered Rayleigh waves*. Moreover, according to the type of waves and to the required precision, the ultrasonic propagation can be studied for a homogenised medium. However, if the studied case needs more precise results, the multilayered medium must be studied in its whole complexity.

References

- Auld, B. A. (1973). *Acoustic fields and waves in solids*. Wiley, New York.
- Benelmostafa, Y. (1990). *Etude de la propagation des ondes ultrasonores dans un milieu multicouche. Application à l'évaluation non destructive de collage*. PhD thesis, Univ. Techn. Compiegne.
- Benelmostafa, Y., de Belleval, J. F., Mercier, N., and Molinero, I. (1990). Modélisation numérique de la propagation des ultrasons dans un milieu multicouche. Application aux collages. *J. Phys.* **51**, 1990. First French Conf. on Acoust., Suppl. C2.
- Castaings, M. (1993). *Propagation ultrasonore dans les milieux stratifiés plans constitués de matériaux absorbants et orthotropes*. PhD thesis, Univ. Bordeaux I.
- Chimenti, D. E. and Nayfeh, A. H. (1985). Leaky lamb waves in fibrous composite laminates. *J. Acoust. Soc. Am.* **58**, 12, 4531–4538.
- Chimenti, D. E. and Nayfeh, A. H. (1989). Ultrasonic leaky waves in a solid plate separating a fluid and vacuum. *J. Acoust. Soc. Am.* **85**, 2, 555–560.
- Chimenti, D. E. and Nayfeh, A. H. (1990). Ultrasonic reflection and guided waves in fluid-coupled composite laminates. *J. Nondestruct. Eval.* **9**, 2/3, 51–69.
- Chimenti, D. E. and Rokhlin, S. I. (1990a). Reflection coefficient of a fluid-coupled elastic layer. Thompson, D. O. and Chimenti, D. E., editors, *Review of Progress in Quantitative NDE*, volume 9, Plenum Press, New-York.
- Chimenti, D. E. and Rokhlin, S. I. (1990b). Relationship between leaky lamb modes and reflection coefficient zeroes for a fluid-coupled elastic layer. *J. Acoust. Soc. Am.* **88**, 3, 1603–1611.
- Chimenti, D. E. and Rokhlin, S. I. (1991). Influence of fluid loading on reflection coefficient zeroes and leaky lamb modes for a composite plate. Thompson, D. O. and Chimenti, D. E., editors, *Review of Progress in Quantitative NDE*, volume 10A, Plenum Press, New-York, 209–216.
- Claeys, J. M., Leroy, O. J., Ngoc, T. D. K., and Mayer, W. G. (1981). Reducibility of plane wave reflectivity from a solid plate in liquid to a liquid-solid interface. *Acoustics Letters* **5**, 3, 48–54.
- Datta, S. K., Shah, A. H., and Bratton, R. L. (1988). Wave propagation in laminated composite plates. *J. Acoust. Soc. Am.* **83**, 6, 2020–2026.
- Dayal, V. and Kinra, V. K. (1989). Leaky lamb waves in an anisotropic plate. I: An exact solution and experiments. *J. Acoust. Soc. Am.* **85**, 6, 2268–2276.
- de Billy, M. and Quentin, G. (1984). Experimental investigation of reflection coefficients for lossy liquid-solid-liquid systems. *Ultrasonics* **22**(5), 249–252.
- Dieuleveult, E. and Royer, D. (1974). *Ondes élastiques dans les solides*. Masson, Paris.
- Duelos, J., Izquierdo, J. L., Lenoir, O., and Conoir, J. M. (1994). Resonant formalism for the liquid-solid interface rayleigh mode. *Acta Acustica* **2**, 375–378.
- Fiorito, R., Madigosky, W., and Überall, H. (1979). Resonance theory of acoustic waves interacting with an elastic plate. *J. Acoust. Soc. Am.* **66**, 1857–1866.
- Helbig, K. (1984). Anisotropy and dispersion in periodically layered media. *Geophysics* **49**, 4, 364–373.
- Hosten, B. and Castaings, M. (1993). Transfer matrix of multilayered absorbing and anisotropic media. Measurements and simulations of ultrasonic wave propagation through composite materials. *J. Acoust. Soc. Am.* **94**, 3, 1488–1495.
- Izquierdo, J. L., Maze, G., and Riposte, J. (1987). Diffusion acoustique par un plan. Onde de Rayleigh. Gespa, N., editor, *La diffusion acoustique par des cibles élastiques de forme géométrique simple. Théories et expériences*. Cedocar, Paris, chap. 2.
- Lhermitte, T. (1991). *Anisotropie des propriétés élastiques des composites carbone-époxy – étude de la propagation, de la dispersion et de la radiodiffusion ultrasonores*. PhD thesis, Univ. Pierre et Marie Curie, Paris.
- Li, Y. and Thompson, R. B. (1990). Influence of anisotropy on the dispersion characteristics of guided ultrasonic plate modes. *J. Acoust. Soc. Am.* **87**, 5, 1911–1931.
- Merkulov, L. G. (1964). Damping of normal modes in a plate immersed in a liquid. *Sov. Phys. Acoust.* **10**, 2, 169–173.
- Nayfeh, A. H. and Chimenti, D. E. (1988). Propagation of guided waves in fluid-coupled plates of fiber-reinforced composite. *J. Acoust. Soc. Am.* **83**, 5, 1736–1743.
- Piona, T. J., Baharvash, M., and Mayers, W. G. (1975). Rayleigh and lamb waves at liquid solid boundaries. *Ultrasonics* **13**, 4, 171–174.
- Potel, C. (1994). *Propagation des ultrasons dans les milieux multicouches anisotropes – modélisation et expérimentations*. PhD thesis, Univ. Techn. Compiegne.
- Potel, C. and de Belleval, J. F. (1993a). Acoustic propagation in anisotropic periodically multilayered media. A method to solve numerical instabilities. *J. Appl. Phys.* **74**, 4, 2208–2215.
- Potel, C. and de Belleval, J. F. (1993b). Propagation in an anisotropic periodically multilayered medium. *J. Acoust. Soc. Am.* **93**, 5, 2669–2677.
- Potel, C. and de Belleval, J. F. (1994). Interprétation physique des ondes de floquet se propageant dans un milieu multicouche périodique anisotrope. *J. Phys.* **IV**, 685–688. Third French Conf. on Acoust., vol. 4, C5.
- Potel, C., de Belleval, J. F., and Gargouri, Y. (1995). Floquet waves and classical plane waves in an anisotropic periodically multilayered medium: application to the validity domain of homogenization. *J. Acoust. Soc. Am.* **97**, 5, 2815–2825.
- Potel, C. and de Belleval, J. F. (1995). Surface waves in an anisotropic periodically multilayered medium: influence of the absorption. *J. Appl. Phys.* **77**, 2, 6152–6161.
- Royer, D. and Dieuleveult, E. (1984). Rayleigh wave velocity and displacement in orthorhombic, tetragonal, hexagonal, and cubic crystals. *J. Acoust. Soc. Am.* **76**, 5, 1438–1444.
- Schoch, A. (1952). Der Schalldurchgang durch Platten (Sound transmission in plates). *Acustica* **2**, 1, 1–17.
- Schoenberg, M. (1984). Wave propagation in alternating solid and fluid layers. *Wave Motion* **6**, 302–320.
- Wu, J. and Zhu, Z. (1992). The propagation of lamb waves in a plate bordered with layers of liquid. *J. Acoust. Soc. Am.* **91**, 2, 861–867.