General Acoustics

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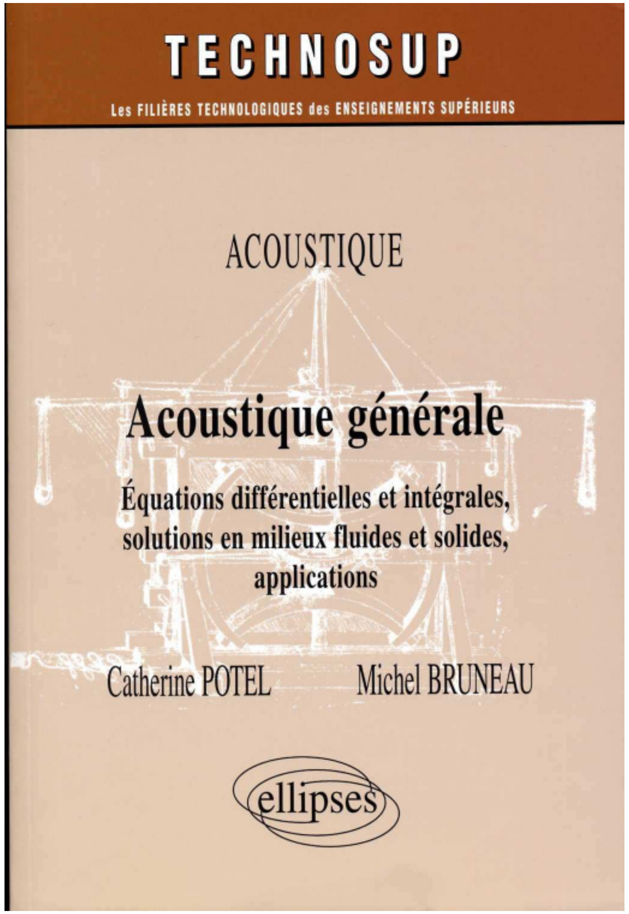
⚠ *These slides may sometimes appear incoherent when they are not associated to oral comments.*

Slides based upon

C. POTEL, M. BRUNEAU, *Acoustique Générale - équations différentielles et intégrales, solutions en milieux fluide et solide, applications*, Ed. Ellipse collection Technosup, 352 pages, ISBN 2-7298-2805-2, 2006



TECHNOSUP

Les FILIÈRES TECHNOLOGIQUES des ENSEIGNEMENTS SUPÉRIEURS

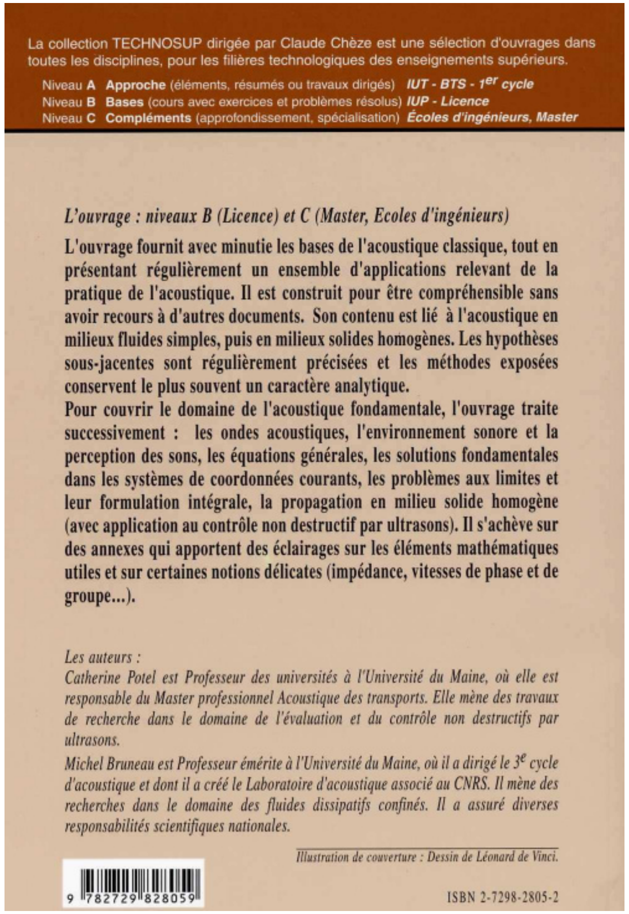
ACOUSTIQUE

Acoustique générale

Équations différentielles et intégrales, solutions en milieux fluides et solides, applications

Catherine POTEL   Michel BRUNEAU

ellipses



La collection TECHNOSUP dirigée par Claude Chèze est une sélection d'ouvrages dans toutes les disciplines, pour les filières technologiques des enseignements supérieurs.

Niveau **A** **Approche** (éléments, résumés ou travaux dirigés) ***IUT - BTS - 1er cycle***
Niveau **B** **Bases** (cours avec exercices et problèmes résolus) ***IUP - Licence***
Niveau **C** **Compléments** (approfondissement, spécialisation) ***Écoles d'ingénieurs, Master***

L'ouvrage : niveaux B (Licence) et C (Master, Ecoles d'ingénieurs)

L'ouvrage fournit avec minutie les bases de l'acoustique classique, tout en présentant régulièrement un ensemble d'applications relevant de la pratique de l'acoustique. Il est construit pour être compréhensible sans avoir recours à d'autres documents. Son contenu est lié à l'acoustique en milieux fluides simples, puis en milieux solides homogènes. Les hypothèses sous-jacentes sont régulièrement précisées et les méthodes exposées conservent le plus souvent un caractère analytique.

Pour couvrir le domaine de l'acoustique fondamentale, l'ouvrage traite successivement : les ondes acoustiques, l'environnement sonore et la perception des sons, les équations générales, les solutions fondamentales dans les systèmes de coordonnées courants, les problèmes aux limites et leur formulation intégrale, la propagation en milieu solide homogène (avec application au contrôle non destructif par ultrasons). Il s'achève sur des annexes qui apportent des éclairages sur les éléments mathématiques utiles et sur certaines notions délicates (impédance, vitesses de phase et de groupe...).

Les auteurs :

Catherine Potel est Professeur des universités à l'Université du Maine, où elle est responsable du Master professionnel Acoustique des transports. Elle mène des travaux de recherche dans le domaine de l'évaluation et du contrôle non destructifs par ultrasons.

Michel Bruneau est Professeur émérite à l'Université du Maine, où il a dirigé le 3^{e} cycle d'acoustique et dont il a créé le Laboratoire d'acoustique associé au CNRS. Il mène des recherches dans le domaine des fluides dissipatifs confinés. Il a assuré diverses responsabilités scientifiques nationales.

Illustration de couverture : Dessin de Léonard de Vinci.

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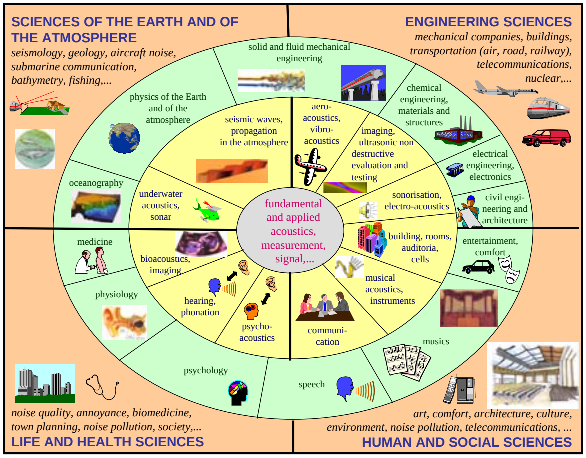
Université du Maine

Chapter 1

Acoustics and its applications: generalities

The world of acoustics



Synoptic view of acoustic skills: the four fields of activity, the engineering domains, the specialised aeras
(from outside to center). *Adapted from R.B. Lindsay, J. Acoust. Soc. Am.,* ***36****, 1964, by Michel Bruneau, Pr., and Catherine Potel, Pr., French Acoustical Society*

Music at prehistory

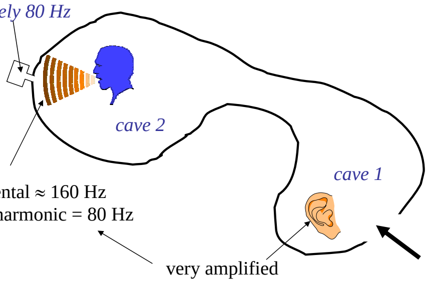
Among many bones and food reliefs of the men of Paleolithic, one finds some drilled phalanges of reindeer. They make whistles emitting clear and powerful sounds. These phalanges always carry the print of animal bites, men contenting themselves to arrange and make regular these holes. Oldest phalanges are approximately 100.000 years old. The smaller the phalanges are, the higher the produced sounds were.

Whilstling phalanges of reindeer

http://perso.wanadoo.fr/palladia/prehistoire/mediatheque.htm

Caves

cavity+tubular hole ≡ Helmholtz's resonator tuned to approximatively 80 Hz



fundamental ≈ 160 Hz
1st sub-harmonic = 80 Hz

very amplified

If the speaker is close to the resonator, the voice is enhanced at the tuning frequency of the resonator (modification of the radiation impedance of the speaker)

Pythagorean school

All the universe is music

Pythagore de Samos (-570?/-500?, Greek astronomer, philosophical, musicologist)

➡ Study of acoustics ≡ Study of musical acoustics

- Relation between the length of a vibrating cord and the pitch of the emitted sound
- “Mathematical” construction of the musical scale.

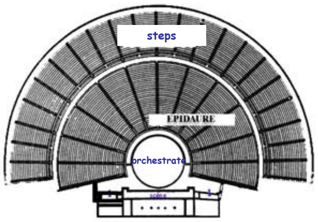
Acoustics of the open-air theaters

Arenas of Nîmes

Vitruve

Vitruve Marcus Pollio, (-88? to -26?, Roman architect and military engineer)

Orchestrates: reflective stone slab which remains open and plays the part of reflectors of sounds. These reflections and those of the back wall being added to the direct sound reinforce the transmitted energy and consequently improve the speech intelligibility.



Epidaure theater (Grèce)

One millenium and a half later (1/4)

Galileo Galilei
Italian, (1565-1642)

Marin Mersenne
French, (1588-1648)

Acoustics, which is considered under many aspects as a branch of mechanics, is henceforth detached from the musical art to become a ***real science of the sound phenomenon***.

Sound radiation of a small clock enclosed in a vessel where Boyle makes a partial vacuum

→ shows the ***need for the presence of air*** for the production and the transmission of noise.

Experiment of
Robert Boyle (1627-1691)

One millenium and a half later (2/4)

Pierre Gassendi
French, (1592-1655)

The sound results from a ***current of atoms emitted by the sound object***; celerity and frequency of the sound being interpreted respectively like the speed of the atoms and their number emitted per unit of times…(cf. scientific context of the time)

Christiaan Huygens
Deutch, (1629-1695)

Global explanation of the sound and luminous phenomena; Huygens interprets both of them as being due to the propagation of longitudinal waves, associated to the vibrations of the molecules of elastic media in the case of the sound, and to the oscillating movements of *ether*, hypothetical substrate of the luminous phenomena, in the case of the light.

One millenium and a half later (3/4)

Isaac Newton
English, (1642-1727)

The mathematical theory of the sound propagation starts. Physics of continuous media or fields theory (of which the sound field) started to reach its final mathematical structure.

Since then, the theories, so complex are they, are considered (for the greatest part) as refinements of those which date from this period.

Leonhard Euler
Swiss, (1707-1783)
Jean Bernoulli's student

Joseph Louis Lagrange (Comte de-)
French, (1736-1813)

Jean le Rond d'Alembert
French, (1717-1783)

One millenium and a half later (4/4)

Hermann L. von Helmholtz
German, (1821-1894)

The analysis of the complex sounds was experimentally carried out by the German physiologist and physicist, Hermann von Helmholtz (1821-1894), by means of resonators which bear its name :

any musical sound with a given frequency, is associated to a timbre which results from the ***superposition of a series of harmonics to the fundamental sound***.

The mathematical analysis of these complex sounds is based on famous work of the French mathematician Joseph Fourier (1768-1830), which always makes authority.

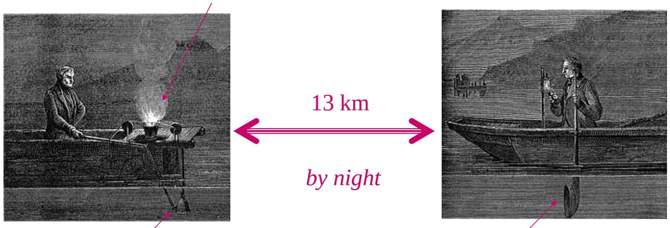
Jean-Baptiste Joseph Fourier
French, (1768-1830)

The speed of sound

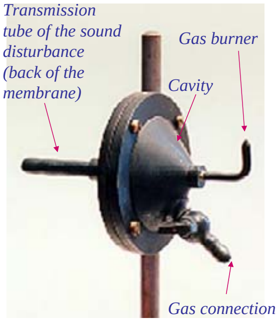
- Measurements made about 1860-1870 in tubes lengths going up to 4900 meters (sewers of Paris)

Victor Regnault
French, (1810-1878)

- Experiments made in 1826 on the Geneva Lake by the physicists Colladon and Sturm



Visualization of the vibrations of the acoustic wave (1/3)

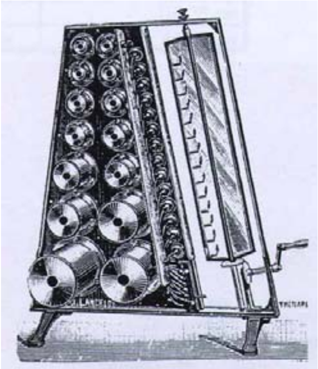


Any sound vibration, here transmitted by a tube to the back of the membrane (on the left on the figure) makes the membrane vibrating, which makes the pressure of gas contained in the cavity in front of the membrane varying. Consequently, the flow of gas at the exit of the nozzle (burner) is modulated at the frequency of the sound disturbance, which produces a height of flame which fluctuates at the same frequency. The variations of height of this flame allow, by an appropriated method, in particular by the use of a turning mirror, a visual appreciation of the nature of the analyzed sounds. The manometric cells are, with Helmholtz's resonators, one of the essential elements of *Koenig's apparatus for the analysis of the timbre of the musical sounds*.

Cell sensitive to the pressure variations (manometric cell)
from Karl Rudolf Koenig (1832-1901)
Specimen of the Montesquieu college, in Le Mans

http://www.inrp.fr/she/instruments/instr_aco_capsule_koenig.htm

Visualization of the vibrations of the acoustic wave (2/3)



Manometric analyzer
*from Karl Rudolf **Koenig** (1832-1901)*

The left part of the apparatus is made up of Helmholtz's resonators of increasing size top to downwards. Each one of them is connected to the back with a manometric cell which transmits the vibrations of the air in the resonators to a gas supplying the flames which are thus modulated. Using the device of the turning mirror, often used in the laboratories of physics at that time, the structure of the sound was visually analyzed. The rotation of the mirror with 4 faces (on the right) will cut out the movements of the flame and thus will slow down it: the vibration of the flame becomes visible for the eye.

http://www.inrp.fr/she/instruments/instr_aco_analyseur_koenig.

Visualization of the vibrations of the acoustic wave (3/3)

Rudolf Koenig was born in Koenigsberg in Eastern Prussia, but is established in Paris in 1851. Initially apprentice in the famous violin maker Jean Cambric Villaume (1798-1875), he left him in 1858 to found his own company and to manufacture acoustic apparatuses. One says of him: "*The name of Koenig is synonymous with the brilliant days of acoustics of the 19th century. Its instruments were among most beautiful, most effective and most precise of this time*".

http://misha1.u-strasbg.fr/AMUSS/OdS311.htm

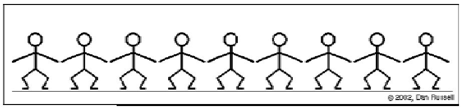
Lord Rayleigh

John William Strutt (Lord Rayleigh)
English, (1842-1919)

At the dawn of XXth century, the top of research in acoustics was marked by the masterly work of the English scientist John William Strutt, Lord Rayleigh (1842-1919), who, in particular, synthesized the knowledge obtained before in its treaty *A theory of sound*, whose first edition appeared in 1877 (T.I) and 1885 (T. II). The bases of acoustics then were posed.

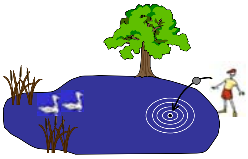
MECHANICAL WAVE (1/5)

- A mechanical wave is an oscillatory motion which is gradually transmitted in a material medium, by vicinity, like information, a change of position which one transmits to his neighbor.



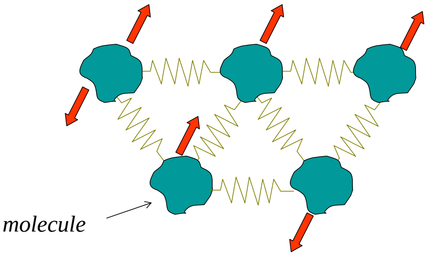
http://www.kettering.edu/~drussell

Animation courtesy of Dr. Dan Russell, Kettering University



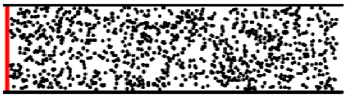
The water particle at the centre moves and transmits its motion to the others

MECHANICAL WAVE (2/5)



Schematic representation of matter made up of molecules (of given masses) with elastic interactions.

MECHANICAL WAVE: PRESSURE WAVE (3/5)

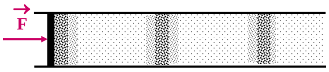


in a gas

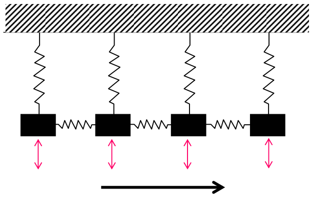
http://www.kettering.edu/~drussell

Animation courtesy of Dr. Dan Russell, Kettering University

in a spring

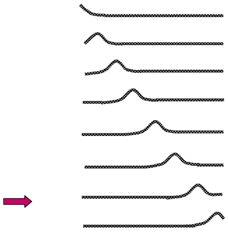


MECHANICAL WAVE: SHEAR WAVE (4/5)

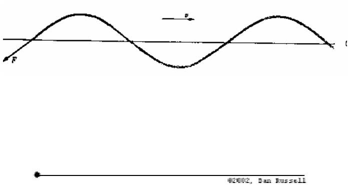


discrete system

Continuous system: propagation of a pulse along a spring. The sections of the spring move from top to bottom as the pulse moves frome left to right.



MECHANICAL WAVE: FLEXURE WAVE (5/5)



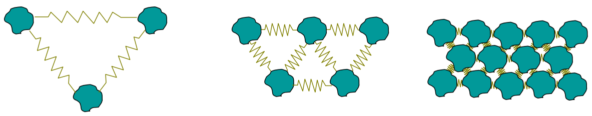
http://www.kettering.edu/~drussell

Animation courtesy of Dr. Dan Russell, Kettering University

flexure waves in a vibrating string

PROPAGATION CELERITY

GAS *LIQUID* *SOLID*



$V_{\text{air}} = 340$ m/s $V_{\text{water}} = 1500$ m/s $V_{\text{metal}} \cong 6000$ m/s

Schematic look of the three fundamental states of the matter, and order of magnitude of the propagation velocity of pressure waves for each one of them

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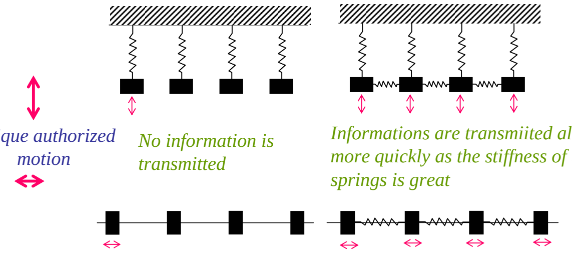
FROM DISCONTINUOUS MATTER...

SHEAR WAVE



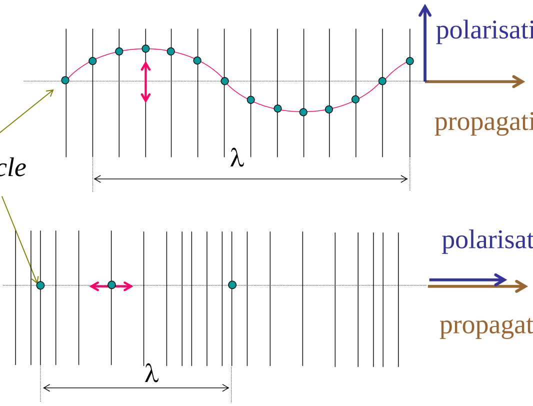
Unique authorized motion

No information is transmitted

Informations are transmiited all the more quickly as the stiffness of the springs is great

PRESSURE WAVE

... TO CONTINUOUS MATTER



polarisation

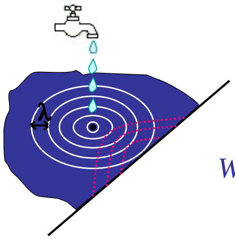
propagation

particle

polarisation

propagation

FREQUENCY



$$\lambda = \frac{V}{f}$$

m/s

m

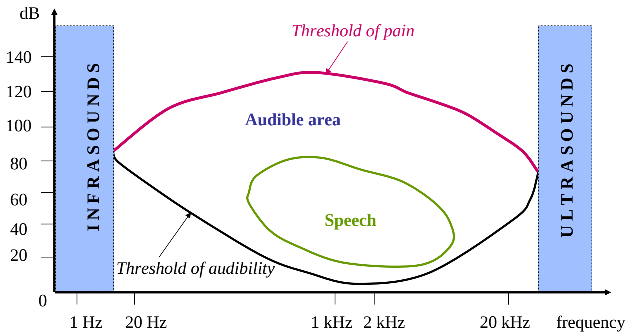
Hertz (Hz) ≡ 1/s

Wavelength

- The frequency of the wave is the speed with which the particle oscillates around its mean position.

f = numbers of to and fro motions of the particle / second

AUDIBLE AREA

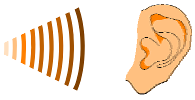


SONIC WAVE - ULTRASONIC WAVE

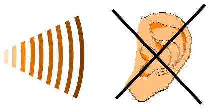
- In air:

20 Hz ≤ f ≤ 20 000 Hz

f > 20 000 Hz



Audible noise
Sonic wave

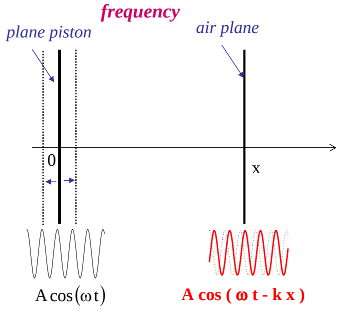


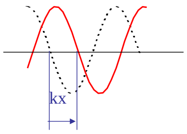
Inaudible noise (for human being)
Ultrasonic wave

CHARACTERISTICS OF A WAVE

$$u(x;t) = A\cos\left[\omega\left(t-\frac{x}{c}\right)\right] = A\cos\left[2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)\right] = A\cos(\omega t - k x)$$

amplitude ***angular frequency*** ***célérité*** ***wavelength*** ***wave number***



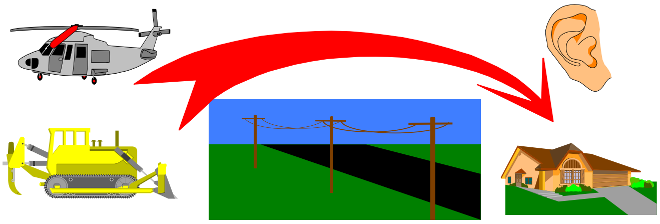


$$k = \omega/c = 2\pi/\lambda$$

$$\lambda = cT$$

ACOUSTIC TRANSMISSION PHENOMENON (1/2)

Example 1: noise path



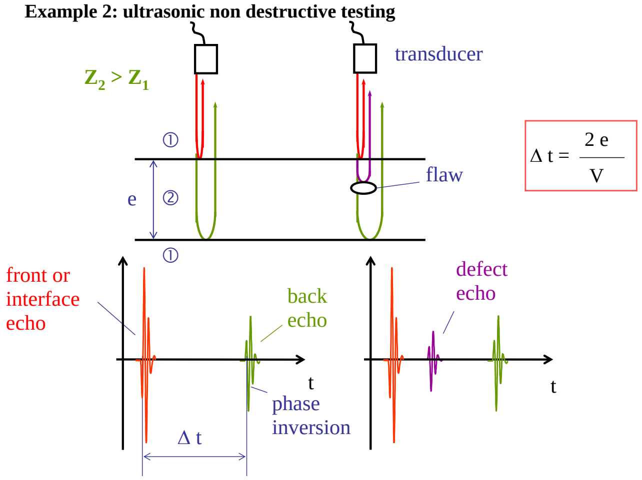
Emission
Sources of noise

Propagation
Atmosphere, ground, meteorology

Reception
Hearing

ACOUSTIC TRANSMISSION PHENOMENON (2/2)

Example 2: ultrasonic non destructive testing



$$\Delta t = \frac{2e}{V}$$

GENERATION OF A SONIC WAVE (1/4)

- Loudspeakers

broad band loudspeakder 215 RTF bicone (Supravox)

http://www.supravox.fr/haut-parleurs/intro_hp.html

tweeter

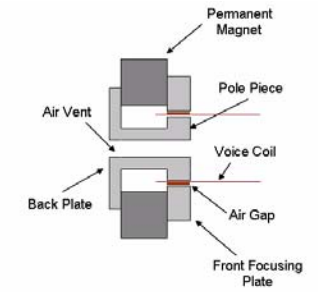
closed diaphram loudspeaker with acoustic horn

baffle

http://fr.audiofanzine.com/apprendre/glossaire/index,popup,,id_mot,49.html

GENERATION OF A SONIC WAVE (2/4)

- Electrodynamic loudspeaker



The Magnet Motor Drive System

Aim: establish a symmetrical magnetic field in which the voice coil will operate

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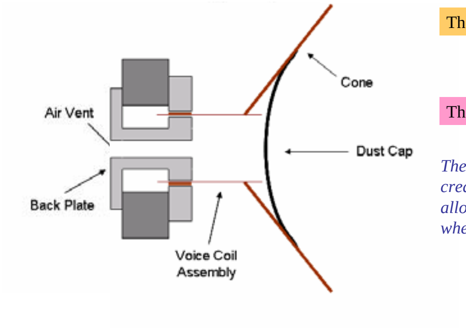
GENERATION OF A SONIC WAVE (3/4)

- Electrodynamic loudspeaker



The Magnet Motor Drive System

The Loudspeaker Cone System

The cone serves the purpose of creating a larger radiating area allowing more air to be moved when excited by the voice coil

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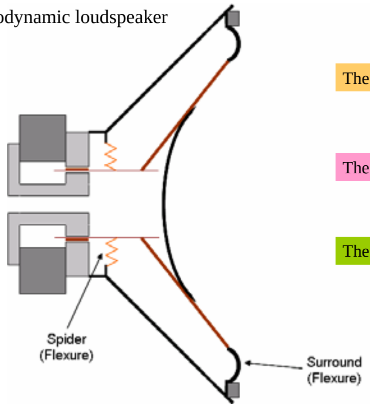
GENERATION OF A SONIC WAVE (4/4)

- Electrodynamic loudspeaker



The Magnet Motor Drive System

The Loudspeaker Cone System

The Loudspeaker Suspension

The combination of the two flexures allows the voice coil to maintain linear travel as the voice coil is energized and provide a restoring force for the voice coil system.

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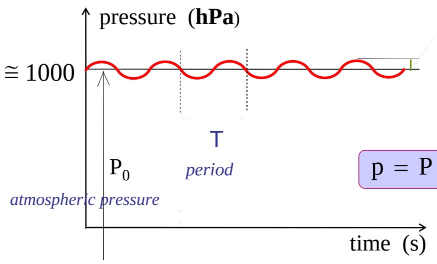
AUDIBLE AREA (1/4)

- Orders of magnitude (1/3)

✓**IF homogeneous** fluid, the characteristics of which do not depend on time



p < 2 hPa

$$P_0 \cong 1000 \quad \text{h Pa}$$

$$\Delta P_0 \cong 160 \quad \text{h Pa}$$

$$\boxed{p = P_{tot} - P_0 < 2 \quad \text{h Pa}}$$

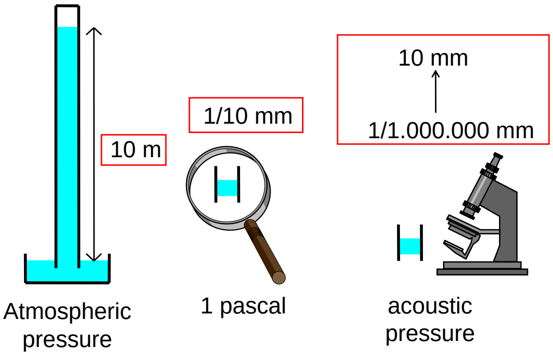
Static pressure = constant

✓**IF non homogeneous** fluid, the characteristics of which depend on time $P_0 \Rightarrow P_E(\vec{r};t)$

Driving

AUDIBLE AREA (2/4)

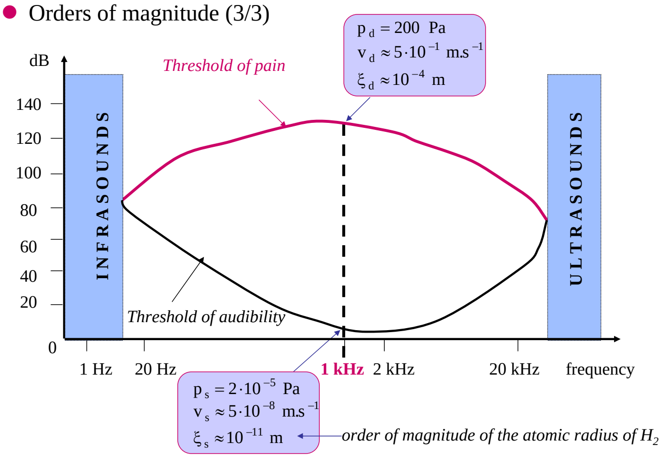
- Orders of magnitude (2/3)



2.10^{-5} Pa < p < 200 Pa → *very wide variation range*

AUDIBLE AREA (3/4)

- Orders of magnitude (3/3)



$p_d = 200$ Pa
$v_d \approx 5\cdot10^{-1}$ m.s^{-1}
$\xi_d \approx 10^{-4}$ m

$p_s = 2\cdot10^{-5}$ Pa
$v_s \approx 5\cdot10^{-8}$ m.s^{-1}
$\xi_s \approx 10^{-11}$ m

order of magnitude of the atomic radius of H_2

AUDIBLE AREA (4/4)

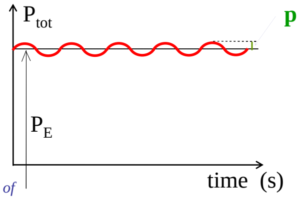
- Fundamental variables
  - ✓ acoustic pressure $p(\vec r;t)$
  - ✓ density $\rho(\vec r;t)$
  - ✓ particle velocity $\vec v(\vec r;t)$ → particle displacement $\vec\xi(\vec r;t)$
  - ✓ entropy $s(\vec r;t)$
  - ✓ temperature $\tau(\vec r;t)$

Variations around a reference state "E" :

$$p(\vec r;t) = P_{tot}(\vec r;t) - \boxed{P_E(\vec r;t)}$$
$$\rho(\vec r;t) = \rho_{tot}(\vec r;t) - \rho_E(\vec r;t)$$
$$\vec v(\vec r;t) = \vec v_{tot}(\vec r;t) - \vec v_E(\vec r;t)$$
$$s(\vec r;t) = S_{tot}(\vec r;t) - S_E(\vec r;t)$$
$$\tau(\vec r;t) = T_{tot}(\vec r;t) - T_E(\vec r;t)$$

$$\rightarrow \boxed{P_0 \quad \rho_0 \quad \vec v_0 \quad S_0 \quad T_0}$$

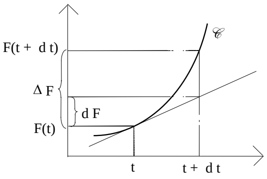
***homogeneous** fluid, the characteristics of which do not depend on time*



ELEMENTARY VARIATION - INSTANTANEOUS DIFFERENCE

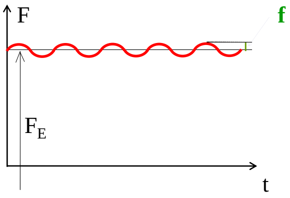
- Elementary variation

$$dF = \lim_{dt\to0}\left[F(t+dt) - F(t)\right]$$



- Instantaneous difference, at any given time, from a given origin F_E

$$f(t) = \int_{F_E}^{F} dF = F(t) - F_E(t)$$



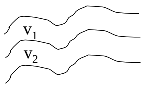
Application:

$$p(\vec r;t) = \int_{P_E}^{P_{tot}} dP = P_{tot}(\vec r;t) - P_E(\vec r;t)$$

DISSIPATIVE EFFECTS

- Viscosity

when two adjacent layers of fluid are animated with different speeds, the viscosity generates reaction forces between these two layers which tend to oppose the displacements and are responsible for the damping of the waves.

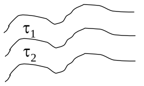


BUT
- ✓ v_2-v_1 very small
- ✓ very weak "viscous diffusion" velocity

→ very weak viscosity effects

- Thermal conduction

heat transfers between two fluid regions (with slightly different temperatures due to slightly different pressures) which tend to disrupt their motions (acoustical origin); this disruption is responsible for damping.



BUT
- ✓ τ_2-τ_1 very small
- ✓ very weak heat "propagation" velocity (diffusion)

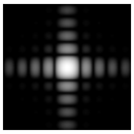
→ ***quasi adiabatic*** transformations

- Molecular relaxation

delay of return to equilibrium due to the fact that te effect of the input excitation is not instantaneous.

Some vocabulary

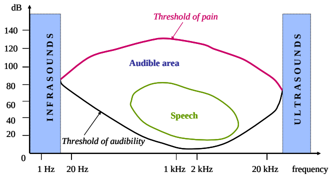
- **Absorption**
  Transformation of acoustical energy (ordered) into heat (disordered).
  Ordered: the motion can be random (forced by a source), but gets a certain kind of order which is recognized by the brain
  Disordered: entirely erractic motion, which is not a vibratory motion; macroscopic temperature and pressure notions are associated to this motion.
- **Dissipation** *(dissipatio(onis) : dissolution, annihilation, destruction)*
  Energy dissipation results from absorption.
  A medium can be called a dissipative or an absorbant medium (synonymous terms).
- **Attenuation**
  Decreasing of the amplitude of the considered phenomenon, due to one or several phenomena
  Examples: geometrical attenuation of a spherical wave (in one direction), attenuation due to dissipation, etc.
- **Diffusion** *(diffusio(onis): act of spreading)*
  Phenomenon of redistribution of energy in all directions (scattering)
- **Scattering** *(diffractum: pulled to pieces)*
  Behaviour of waves when they interact with an obstacle which is not entirely transparent to them



scattering by a square hole

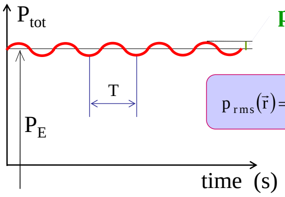
SOUND LEVELS (1/2)

- ✓ The sensitivity of human ear depends on the frequency
- ✓ The auditive sense is proportional to the ***logarithm of the acoustical intensity*** I of the wave



$\boxed{L = 10\log_{10}(I/I_s)}$ *with* $I_s = 10^{-12}$ W.m^{-2}

$I \propto p_{rms}^2 \rightarrow \boxed{L = 20\log_{10}(p_{rms}/p_s)}$ $\quad p_s = \sqrt{\rho_0 c_0 I_s} = \sqrt{400\cdot10^{-12}} = 2\cdot10^{-5}$ Pa



averages over time, on an acoustic period T :

$$\overline{p}(\vec r) = 0 \qquad \overline{\vec v}(\vec r) = \vec 0 \qquad \overline{\rho}(\vec r) = 0 \qquad \forall\ \vec r$$

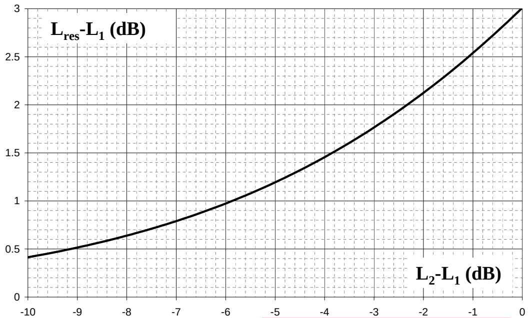
$$\boxed{p_{rms}(\vec r) = \sqrt{\overline{[p(\vec r;t)]^2}}} \quad \textit{with} \quad \overline{[p(\vec r;t)]^2} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}[p(\vec r;t)]^2\,dt$$

⚠ $$\overline{[p(\vec r;t)]^2} \neq \overline{[p(\vec r;t)]}^2$$

SOUND LEVELS (2/2)

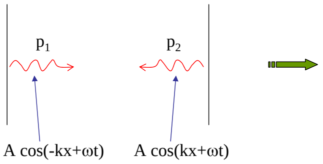
- sound level which results from two non-correlated sources

$$\boxed{L_{res} = 10\log_{10}\left(\frac{I_1+I_2}{I_s}\right) = 10\log_{10}\left(10^{L_1/10}+10^{L_2/10}\right)}$$



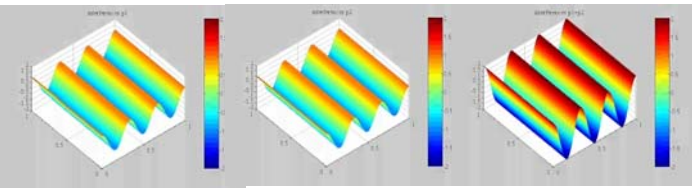
$L_{res} = \left(L_{res} - L_1\right) + L_1$ *with* $\boxed{L_{res} - L_1 = 10\log_{10}\left(1+10^{(L_2-L_1)/10}\right)}$

INTERFERENCE OF TWO WAVES

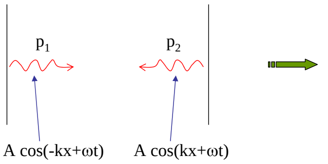


$$p_1 + p_2 = 2A\cos(kx)\cos(\omega t)$$

standing (stationnary) wave

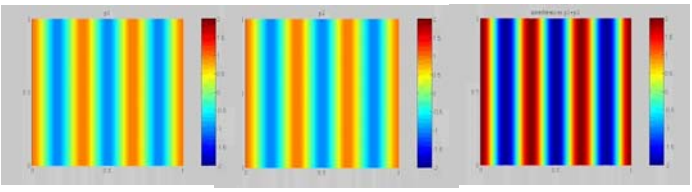


INTERFERENCE OF TWO WAVES



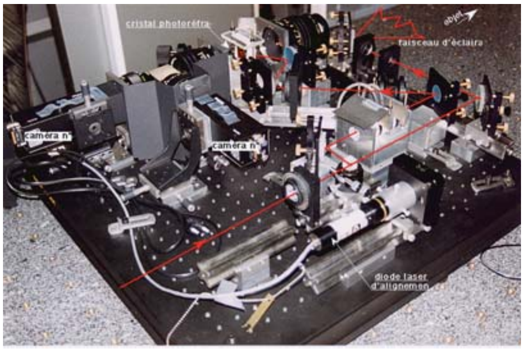
$$p_1 + p_2 = 2A\cos(kx)\cos(\omega t)$$

standing wave



GENERATION OF AN ULTRASONIC WAVE (1/2)

- Laser interferometry

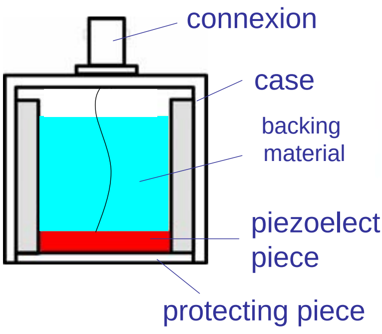


Interferometer which permits to measure the deformations of a piece between two laser exposures

Laboratoire Charles Fabry de l'Institut d'Optique, Orsay
http://www.iota.u-psud.fr/~roosen/capteur_interferometrique.html

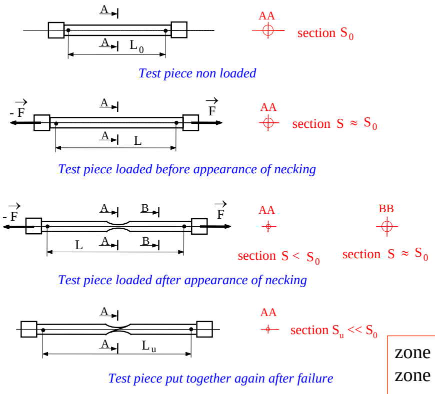
GENERATION OF AN ULTRASONIC WAVE (2/2)

- Ultrasonic transducers



Transforms an electrical signal into a mechanical vibration and conversely

Behavior law: example of traction test

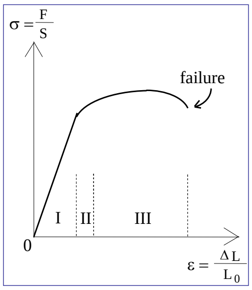


Test piece non loaded

Test piece loaded before appearance of necking

Test piece loaded after appearance of necking

Test piece put together again after failure



zone I	Linear elasticity
zone II	Non linear elasticity
zone III	Plasticity

Hooke's law: $\boxed{\mathbf{T = E\,S}}$

Relations in isotropic solids

	E, ν	E, μ	λ, μ	c_{11}, c_{12}
λ	$\frac{E\nu}{(1+\nu)(1-2\nu)}$	$\frac{\mu(E-2\mu)}{3\mu-E}$	λ	c_{12}
μ	$\frac{E}{2(1+\nu)}$	μ	μ	$\frac{c_{11}-c_{12}}{2}$
E	E	E	$\frac{\mu(3\lambda+2\mu)}{\lambda+\mu}$	$c_{11}-2\frac{c_{12}^2}{c_{11}+c_{12}}$
B	$\frac{E}{3(1-2\nu)}$	$\frac{\mu E}{3(3\mu-E)}$	$\lambda+\frac{2}{3}\mu$	$\frac{c_{11}+2c_{12}}{3}$
ν	ν	$\frac{E-2\mu}{2\mu}$	$\frac{\lambda}{2(\lambda+\mu)}$	$\frac{c_{12}}{c_{11}+c_{12}}$

c_{11}, c_{12} : Rigidity constants (Pa)

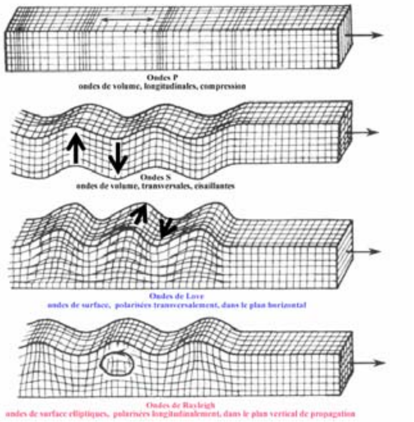
E : Young modulus (Pa)

ν : Poisson ratio (no unit)

B : Voluminal elasticity modulus (Pa/m^2)

λ, μ : Lamé coefficients (Pa)

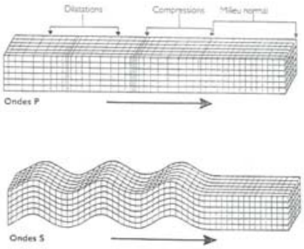
SEISMIC WAVES (1/2)



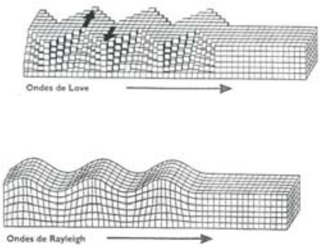
http://www.ens-lyon.fr/Planet-Terre/Infosciences/Geodynamique/Structure-interne/Sismologie/pendulum.html

SEISMIC WAVES (2/2)

- Bulk waves



- Surface waves

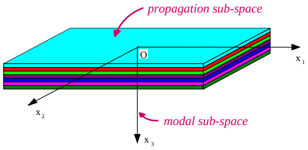


http://eost.u-strasbg.fr/pedago/fiche1/ondes_sismiques.fr.html

Modal waves (1/4)

Mechanical waves → local waves

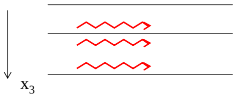
Mechanical waves → modal waves



- guided waves
- surface waves
- interface waves

acoustical energy:

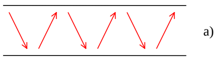
- propagates along the layers
- bounded in x_3



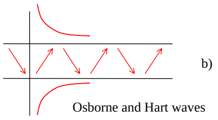
Modal waves (2/4)

guided waves

Vacuum/ rigid wall / reactive impedance

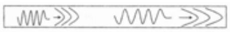


Vacuum/ rigid wall / reactive impedance

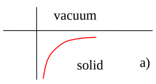


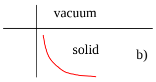
Osborne and Hart waves

Lamb wave

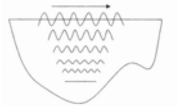


surface waves

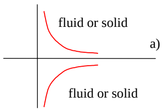


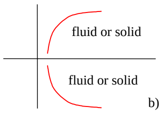


- Rayleigh wave a)
- anti-modal wave b)

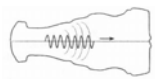


interface waves

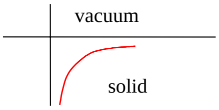


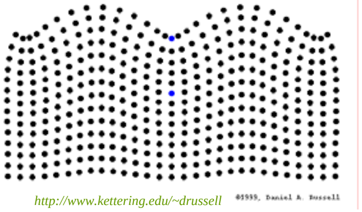


- Scholte wave, Stoneley wave, Rayleigh-Cezawa wave, etc... a)
- anti-modal wave b)



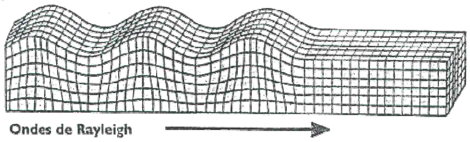
Modal waves (3/4): Rayleigh wave



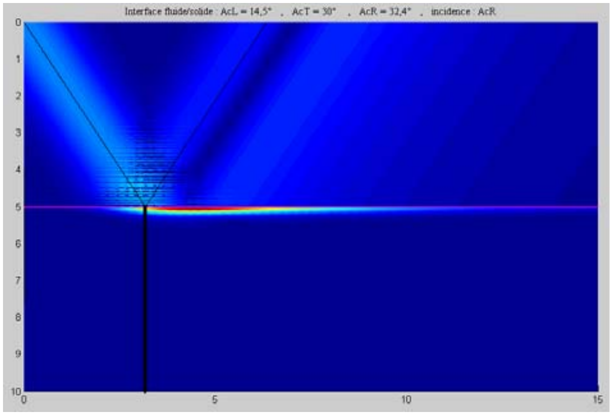


http://www.kettering.edu/~drussell

Animation courtesy of Dr. Dan Russell, Kettering University



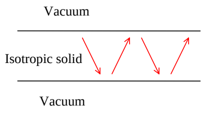
$k_0a = 60$, $k_La = 15$, $k_Ta=30$, $\theta = \theta_{Rayleigh}$



$\rho_0/\rho_1 = 0{,}1$

Programs realised by Ph. Gatignol, Pr., Université de Technologie de Compiègne

Displacements of Lamb waves (1/2)



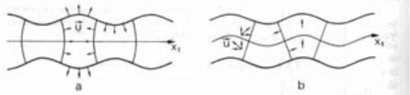
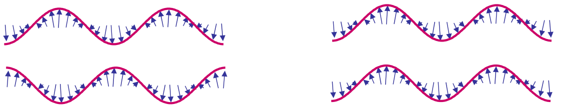


FIG. 5. 36. *Onde de Lamb. a) symétrique : de part et d'autre du plan médian, les composantes longitudinales sont égales et les composantes transversales sont opposées. b) antisymétrique : de part et d'autre du plan médian, les composantes transversales sont égales et les composantes longitudinales sont opposées.*



pressure waves (symmetric mode)

flexure waves (antisymmetric mode)

vectorial particle displacement field at the surface of the plate;
its effect on the shape of the plate

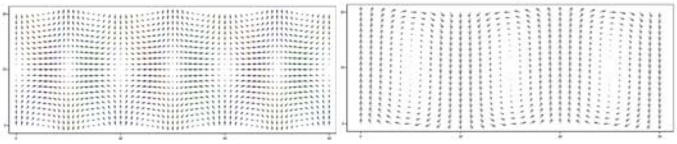
figures from:
D. Royer et E. Dieulesaint, "Ondes élastiques dans les solides", tome 1 : propagation libre et guidée, Masson, (1996)
J.L. Rose, "Ultrasonic waves in solid media",Cambridge Univ. Press, 1999

Displacements of Lamb waves (2/2)

pressure waves (symmetric mode)

flexure waves (antisymmetric mode)

vectorial particle displacement field at the surface of the plate;
its effect on the shape of the plate

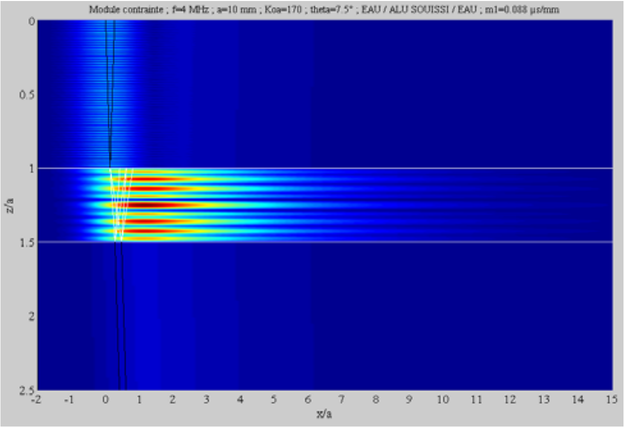


mode S_0

mode A_0

animations realised by Patrick Lanceleur, Université de Technologie de Compiègne
http://www.utc.fr/~lanceleu/links_CT04.html

Eau / Aluminium / Eau; ka=170 ; H=5 mm; before the 1st critical angle



Lamb mode