C. Potel, M. Bruneau, "Acoustique générale (...)", Ed. Ellipse, collection Technosup, ISBN 2-7298-2805-2, 2006

Université du Maine

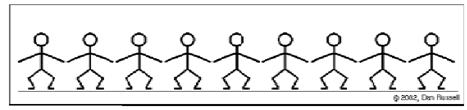
Chapter 8

ELASTIC WAVES IN ISOTROPIC SOLIDS

APPLICATION TO ULTRASONIC NON DESTRUCTIVE TESTING

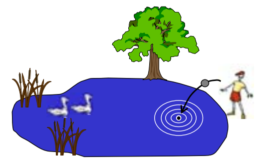
Mechanical wave (1/5)

- A mechanical wave is an oscillatory motion which is gradually transmitted in a material medium, by vicinity, like information, a change of position which one transmits to his neighbor.



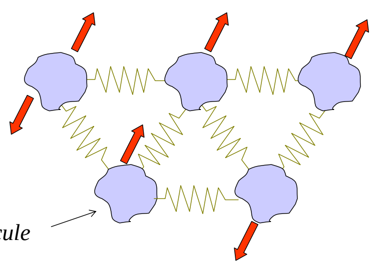
http://www.kettering.edu/~drussell

Animation courtesy of Dr. Dan Russell, Kettering University



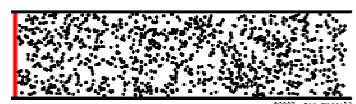
The water particle at the centre moves and transmits its motion to the others

Mechanical wave (2/5)



Schematic representation of matter made up of molecules (of given masses) with elastic interactions.

Mechanical wave : pressure wave (3/5)



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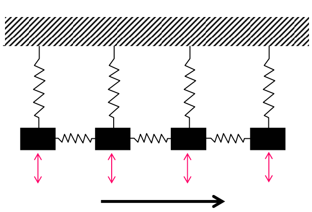
Animation courtesy of Dr. Dan Russell, Kettering University

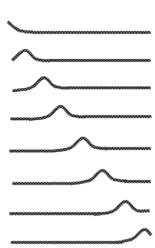
in a gas

in a spring

Mechanical wave: shear wave (4/5)

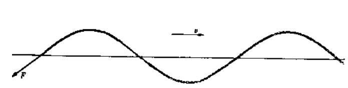


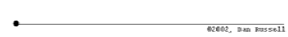
discrete system



Continuous system: propagation of a pulse along a spring. The sections of the spring move from top to bottom as the pulse moves frome left to right.

Mechanical wave: flexure wave (5/5)





http://www.kettering.edu/~drussell

Animation courtesy of Dr. Dan Russell, Kettering University

flexure waves in a vibrating string

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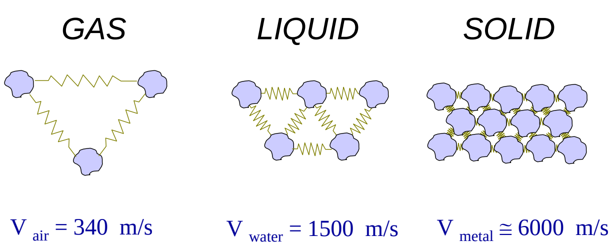
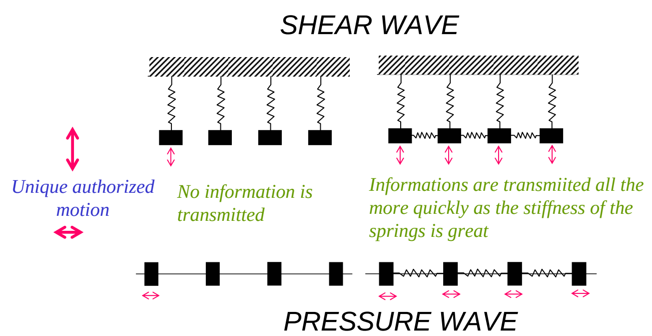
Propagation velocity

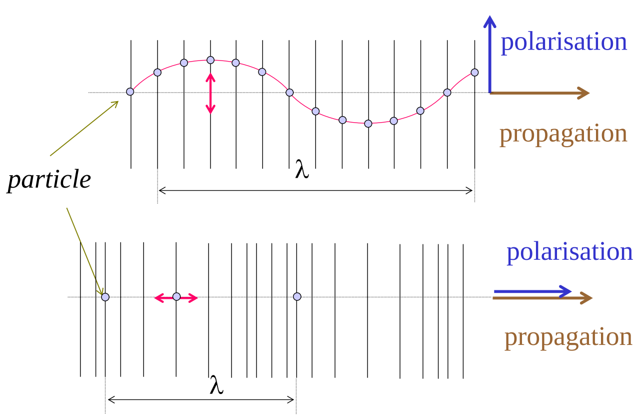


Schematic look of the three fundamental states of the matter, and order of magnitude of the propagation velocity of pressure waves for each one of them

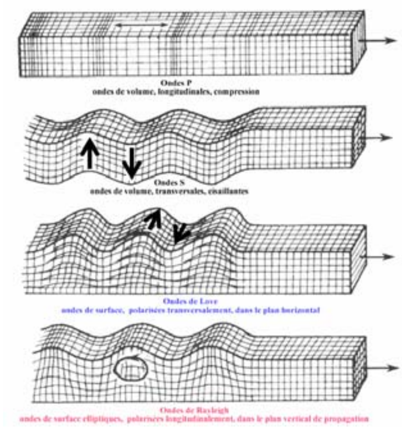
From discontinuous matter...



... to continuous matter

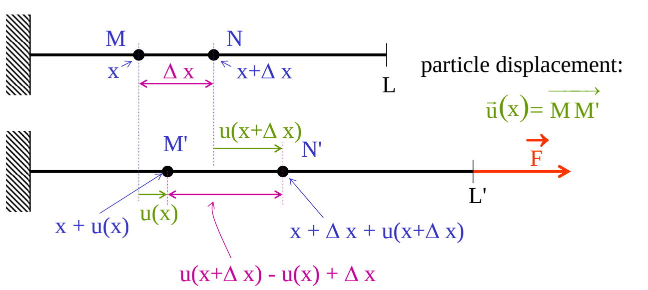


Different types of waves



http://www.ens-lyon.fr/Planet-Terre/Infosciences/Geodynamique/Structure-interne/Sismologie/pendulum.html

Elongation of an extensible line



relative variation length of the little piece MN:

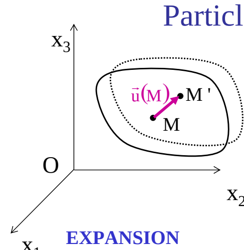
$$\frac{[u(x+\Delta x)-u(x)+\Delta x]-\Delta x}{\Delta x}=\frac{\Delta u}{\Delta x}$$

$\Delta u = u(x+\Delta x) - u(x)$

$\Delta u = 0$ simple translation

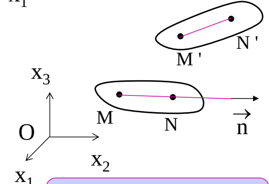
$\Delta u \neq 0$ deformation

Particle displacement vector



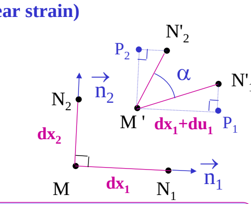
$$\vec{u}(M)=\overrightarrow{MM'}=\begin{vmatrix} u_1(x_1,x_2,x_3) \\ u_2(x_1,x_2,x_3) \\ u_3(x_1,x_2,x_3) \end{vmatrix}_{\mathscr{B}}$$

EXPANSION



$$S(M,\vec{n}) = \lim_{N\to M} \frac{M'N' - MN}{MN}$$

SHEAR (shear strain)



$$\gamma(M,\vec{n}_1,\vec{n}_2) \approx \lim_{\substack{dx_1\to 0\\ dx_2\to 0}} \left(\frac{\pi}{2}-\alpha\right) \approx \frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}$$

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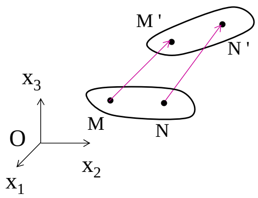
Displacement



$$\vec{u}(M)=\overrightarrow{MM'}=\begin{vmatrix} u_1(x_1,x_2,x_3)\\ u_2(x_1,x_2,x_3)\\ u_3(x_1,x_2,x_3)\end{vmatrix}_{\mathcal{B}}$$

$$\vec{u}(N)=\overrightarrow{NN'}=\begin{vmatrix} u_1+du_1\\ u_2+du_2\\ u_3+du_3\end{vmatrix}_{\mathcal{B}}$$

$$du_1=\frac{\partial u_1}{\partial x_1}dx_1+\frac{\partial u_1}{\partial x_2}dx_2+\frac{\partial u_1}{\partial x_3}dx_3=\left(\overrightarrow{\text{grad}}\,u_1\right)\cdot d\overrightarrow{OM}$$

$$du_2=\frac{\partial u_2}{\partial x_1}dx_1+\frac{\partial u_2}{\partial x_2}dx_2+\frac{\partial u_2}{\partial x_3}dx_3=\left(\overrightarrow{\text{grad}}\,u_2\right)\cdot d\overrightarrow{OM}$$

$$du_3=\frac{\partial u_3}{\partial x_1}dx_1+\frac{\partial u_3}{\partial x_2}dx_2+\frac{\partial u_3}{\partial x_3}dx_3=\left(\overrightarrow{\text{grad}}\,u_3\right)\cdot d\overrightarrow{OM}$$

$$d\vec{u}=\left(\overline{\overline{\text{grad}\,\vec{u}}}\right)\cdot d\overrightarrow{OM}$$

$$\Rightarrow\quad \vec{u}(N)=\vec{u}(M)+\underbrace{\left(\overline{\overline{\text{grad}\,\vec{u}}}\right)\cdot d\overrightarrow{OM}}_{d\vec{u}}$$

Strain tensor $\overline{\overline{S}}$

$$d\vec{u}=\frac{\partial\vec{u}}{\partial x_1}dx_1+\frac{\partial\vec{u}}{\partial x_2}dx_2+\frac{\partial\vec{u}}{\partial x_3}dx_3=\left(\overline{\overline{\text{grad}\,\vec{u}}}\right)\cdot d\vec{x}\qquad\text{with}\qquad \overline{\overline{\text{grad}\,\vec{u}}}=\overline{\overline{S}}+\overline{\overline{\Omega}}$$

$$\overline{\overline{S}}=\begin{bmatrix}\frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right)\\ \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}\right)\\ \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}\right) & \frac{\partial u_3}{\partial x_3}\end{bmatrix}$$

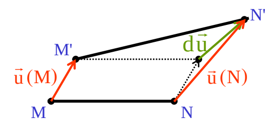
symmetric

$$\overline{\overline{\Omega}}=\begin{bmatrix}0 & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}-\frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)\\ -\frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}-\frac{\partial u_2}{\partial x_1}\right) & 0 & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3}-\frac{\partial u_3}{\partial x_2}\right)\\ -\frac{1}{2}\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right) & -\frac{1}{2}\left(\frac{\partial u_2}{\partial x_3}-\frac{\partial u_3}{\partial x_2}\right) & 0\end{bmatrix}$$

antisymmetric

$$\vec{u}(N)=\vec{u}(M)+d\vec{u}\quad\Longleftrightarrow\quad \vec{u}(N)=\vec{u}(M)+\overline{\overline{\Omega}}\cdot d\overrightarrow{OM}+\overline{\overline{S}}\cdot d\overrightarrow{OM}$$

$$u_i(x_j+dx_j)=u_i(x_j)+S_{ij}\,dx_j+\Omega_{ij}\,dx_j$$

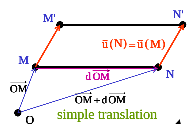


$$S_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$

Interpretation (1/3)

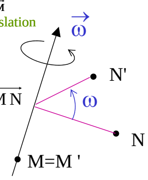
$$\vec{u}(N)=\vec{u}(M)+\overline{\overline{\Omega}}\cdot d\overrightarrow{OM}+\overline{\overline{S}}\cdot d\overrightarrow{OM}$$

- If $\overline{\overline{\Omega}}=\overline{\overline{0}}$ and $\overline{\overline{S}}=\overline{\overline{0}}$ then $\vec{u}(N)=\vec{u}(M)$



- If $\vec{u}(M)=\vec{0}$ and $\overline{\overline{S}}=\overline{\overline{0}}$ then $\vec{u}(N)=\overline{\overline{\Omega}}\cdot d\overrightarrow{OM}$

$$\vec{u}(N)=\begin{vmatrix}du_1\\ du_2\\ du_3\end{vmatrix}_{\mathcal{B}}=\begin{bmatrix}0 & -\omega_3 & \omega_2\\ \omega_3 & 0 & -\omega_1\\ -\omega_2 & \omega_1 & 0\end{bmatrix}_{\mathcal{B}}\begin{vmatrix}dx_1\\ dx_2\\ dx_3\end{vmatrix}=\begin{vmatrix}\omega_1\\ \omega_2\\ \omega_3\end{vmatrix}_{\mathcal{B}}\wedge\begin{vmatrix}dx_1\\ dx_2\\ dx_3\end{vmatrix}_{\mathcal{B}}=\vec{\omega}\wedge\overrightarrow{MN}$$



$\Rightarrow$ $\vec{u}(M)+\overline{\overline{\Omega}}\cdot d\overrightarrow{OM}$: displacement of the solid (mechanical meaning) **simple rotation**

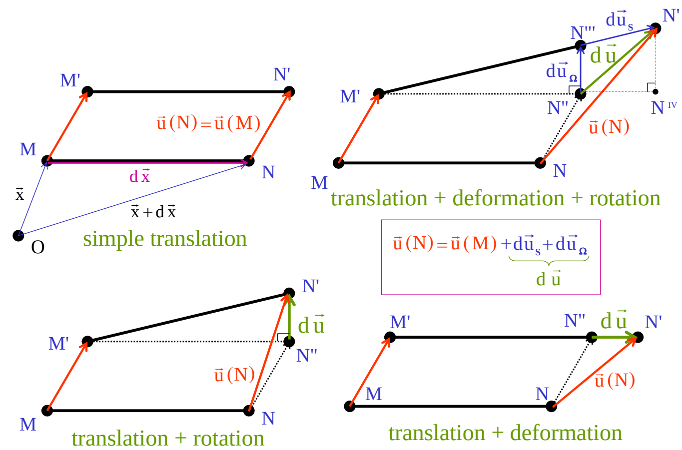
- If $\vec{u}(M)=\vec{0}$ and $\overline{\overline{\Omega}}=\overline{\overline{0}}$ then $\vec{u}(N)=\overline{\overline{S}}\cdot d\overrightarrow{OM}$ **deformation**

$$\vec{u}(N)=\vec{u}(M)+\overline{\overline{\Omega}}\cdot d\overrightarrow{OM}+\overline{\overline{S}}\cdot d\overrightarrow{OM}$$

translation *rotation* *pure deformation*

mechanics

Interpretation (2/3): local displacement of two points

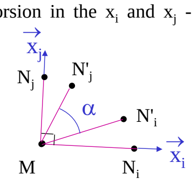


Interpretation (3/3)

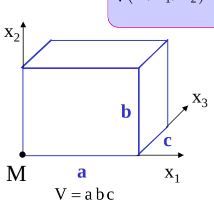
$$\overline{\overline{S}}=\begin{bmatrix}\frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right)\\ \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}\right)\\ \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}\right) & \frac{\partial u_3}{\partial x_3}\end{bmatrix}$$

S_{ii} : deformation in the x_i -direction

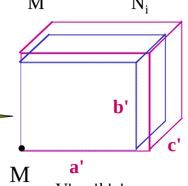
S_{ij} : demi distorsion in the x_i and x_j -directions



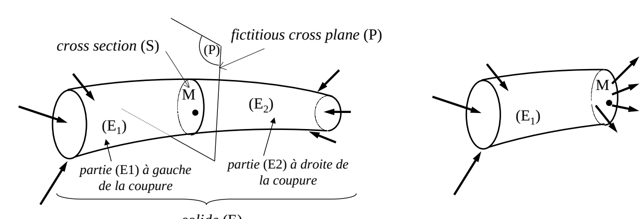
$$\gamma(M,\vec{n}_1,\vec{n}_2)=\lim_{\substack{dx_1\to 0\\ dx_2\to 0}}\left(\frac{\pi}{2}-\alpha\right)=\frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}$$

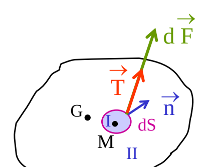


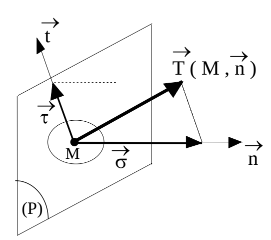
deformation assumed without shearing



$$V'\approx V\left(1+\partial u_1/\partial x_1+\partial u_2/\partial x_2+\partial u_3/\partial x_3\right)\quad\Rightarrow\quad (V'-V)/V\approx\text{div}\,\vec{u}=\text{trace}\,\overline{\overline{S}}$$







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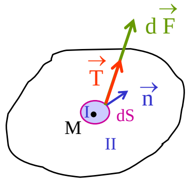
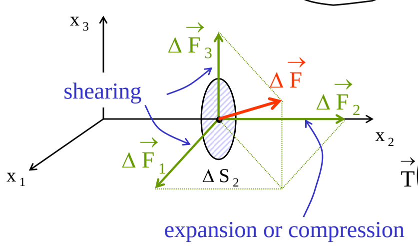
Stress tensor $\bar{\bar{T}}$ (1/2)

$$\vec{T}(M,\vec{n}) = \lim_{dS\to 0} \frac{d\vec{F}}{dS} = \bar{\bar{T}}.\vec{n}$$

$$\mathscr{T}_i = T_{ik}\, n_k$$



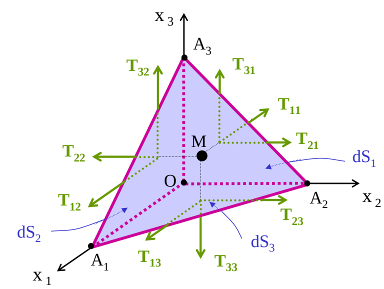
$$\bar{\bar{T}} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}$$

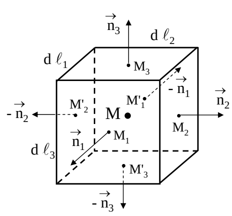


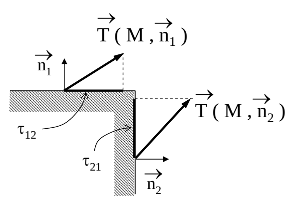
$$T_{ik} = \lim_{\Delta s_k\to 0} \frac{\Delta F_i}{\Delta S_k} = T_{ki}$$

$$\vec{T}(M,\vec{e}_{x_2}) = \bar{\bar{T}}\cdot\vec{e}_{x_2} = \mathscr{B}\left|\begin{matrix} T_{12} \\ T_{22} \\ T_{32} \end{matrix}\right.$$

Stress tensor $\bar{\bar{T}}$ (2/2)

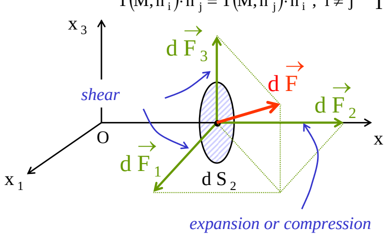


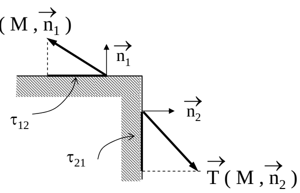




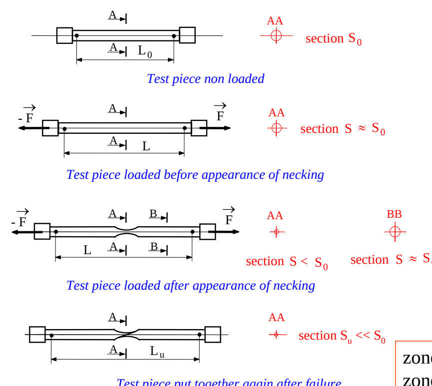
$\tau_{12} = \tau_{21}$

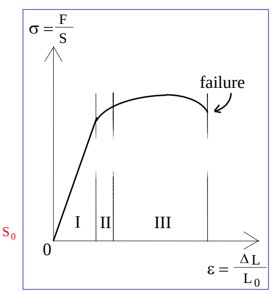
$$\vec{T}(M,\vec{n}_i)\cdot\vec{n}_j = \vec{T}(M,\vec{n}_j)\cdot\vec{n}_i\ ,\ i\neq j$$





Traction test





zone I	Linear elasticity
zone II	Non linear elasticity
zone III	Plasticity

Hooke's law for isotropic media

- Normal stresses:

$$T_{ii} = \lambda\,(S_{11} + S_{22} + S_{33}) + 2\,\mu\,S_{ii} \qquad \textit{i given}$$

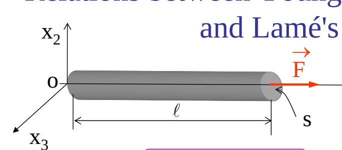
$$= \lambda\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\right) + 2\mu\frac{\partial u_i}{\partial x_i}$$

with $S_{11} + S_{22} + S_{33} = \mathrm{div}\,\vec{u} = \frac{\Delta V}{V}$ $\qquad \lambda, \mu$: Lamé's coefficients

- Tangential stresses:

$$T_{ij} = 2\,\mu\,S_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad i \neq j$$

Relations between Young's modulus, Poisson's ratio and Lamé's coefficients



$\Delta\ell/\ell = F/(ES)$ ⟺ $S_{11} = \frac{1}{E}T_{11}$

$\Delta d/d = -\nu\,\Delta\ell/\ell = -\nu\,F/(ES)$ → $S_{22} = S_{33} = -\frac{\nu}{E}T_{11}$

with $T_{22} = 0$ et $T_{33} = 0$

- Hooke's law for normal stresses

$T_{22} = \lambda(S_{11} + S_{22} + S_{33}) + 2\mu S_{22}$ ⟹ $T_{22} = \lambda\left(\frac{1}{E} - \frac{2\nu}{E}\right)T_{11} - \frac{2\mu\nu}{E}T_{11}$

$$\nu = \frac{\lambda}{2(\lambda+\mu)}$$

$T_{11} = \lambda(S_{11} + S_{22} + S_{33}) + 2\mu S_{11}$ ⟹ $T_{11} = \lambda\left(\frac{1}{E} - \frac{2\nu}{E}\right)T_{11} + \frac{2\mu}{E}T_{11}$

$$E = \lambda(1-2\nu) + 2\mu$$

Relations in isotropic solids

	E,ν	E,μ	λ,μ	c_{11},c_{12}
λ	$\frac{E\nu}{(1+\nu)(1-2\nu)}$	$\frac{\mu(E-2\mu)}{3\mu-E}$	λ	c_{12}
μ	$\frac{E}{2(1+\nu)}$	μ	μ	$\frac{c_{11}-c_{12}}{2}$
E	E	E	$\frac{\mu(3\lambda+2\mu)}{\lambda+\mu}$	$c_{11}-2\frac{c_{12}^2}{c_{11}+c_{12}}$
B	$\frac{E}{3(1-2\nu)}$	$\frac{\mu E}{3(3\mu-E)}$	$\lambda+\frac{2}{3}\mu$	$\frac{c_{11}+2c_{12}}{3}$
ν	ν	$\frac{E-2\mu}{2\mu}$	$\frac{\lambda}{2(\lambda+\mu)}$	$\frac{c_{12}}{c_{11}+c_{12}}$

c_{11} , c_{12} : Rigidity constants (Pa)

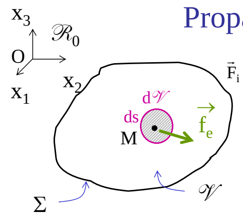
E : Young modulus (Pa)

ν : Poisson ratio (no unit)

B : Voluminal elasticity modulus (Pa/m²)

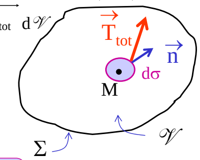
λ , μ : Lamé coefficients (Pa)

Propagation equation (1/2)



$$\vec F_i = \oint\!\!\oint_\Sigma \bar{\bar T}_{tot}.\vec n\, d\sigma = \iiint_{\mathscr V} \overrightarrow{\mathrm{div}\,\bar{\bar T}_{tot}}\; d\mathscr V$$

$$\vec T_{tot}(M,\vec n) = \bar{\bar T}_{tot}.\vec n$$



$$\vec F_e = \iiint_{\mathscr V} \vec f_e\, d\mathscr V$$

with $\vec f_e = \vec f_{e_0} + \delta \vec f_e$

- Dynamic resultant $\quad \vec d(\mathscr V/\mathscr R_0) = \iiint_{\mathscr V} \rho \frac{\partial^2 \vec u}{\partial t^2}\, d\mathscr V$
- Resultant of external forces $\quad \vec{\mathfrak R}(ext \to \mathscr V) = \iiint_{\mathscr V} \left(\vec f_e + \overrightarrow{\mathrm{div}\,\bar{\bar T}}\right) d\mathscr V$

with
$$\overrightarrow{\mathrm{div}\,\bar{\bar T}_{tot}} = \begin{vmatrix} \mathrm{div}\left({}^t\bar{\bar T}_{tot}.\vec e_{x_1}\right) \\ \mathrm{div}\left({}^t\bar{\bar T}_{tot}.\vec e_{x_2}\right) \\ \mathrm{div}\left({}^t\bar{\bar T}_{tot}.\vec e_{x_3}\right) \end{vmatrix}_{\mathscr B_0} = \begin{vmatrix} \frac{\partial T_{11}}{\partial x_1} + \frac{\partial T_{12}}{\partial x_2} + \frac{\partial T_{13}}{\partial x_3} \\ \frac{\partial T_{21}}{\partial x_1} + \frac{\partial T_{22}}{\partial x_2} + \frac{\partial T_{23}}{\partial x_3} \\ \frac{\partial T_{31}}{\partial x_1} + \frac{\partial T_{32}}{\partial x_2} + \frac{\partial T_{33}}{\partial x_3} \end{vmatrix}_{\mathscr B_0} = \begin{vmatrix} \sum_{j=1}^{3} \frac{\partial T_{1j}}{\partial x_j} \\ \sum_{j=1}^{3} \frac{\partial T_{2j}}{\partial x_j} \\ \sum_{j=1}^{3} \frac{\partial T_{3j}}{\partial x_j} \end{vmatrix}_{\mathscr B_0} = \sum_{i=1}^{3}\left(\sum_{j=1}^{3} \frac{\partial T_{ij}}{\partial x_j}\right) \vec e_{x_i}$$

Propagation equation (2/2)

- Fundamental Relation of Dynamics for resultants: $\vec{\mathfrak R}(ext \to \mathscr V) = \vec d(\mathscr V/\mathscr R_0),\ \forall\ \mathscr V$

$$\iiint_{\mathscr V} \left(\vec f_e + \overrightarrow{\mathrm{div}\,\bar{\bar T}_{tot}}\right) d\mathscr V = \iiint_{\mathscr V} \rho \frac{\partial^2 \vec u}{\partial t^2}\, d\mathscr V,\ \underline{\underline{\forall\ \mathscr V}} \Rightarrow \boxed{\vec f_e + \overrightarrow{\mathrm{div}\,\bar{\bar T}_{tot}} = \rho \frac{\partial^2 \vec u}{\partial t^2}}$$

with $\vec f_e = \vec f_{e_0} + \delta \vec f_e$

- At equilibrium $\quad \vec f_{e_0} + \overrightarrow{\mathrm{div}\,\bar{\bar T}_0} = \vec 0 \qquad$ change of variable: $\bar{\bar T} = \bar{\bar T}_{tot} - \bar{\bar T}_0$

stress variation about equilibrium position

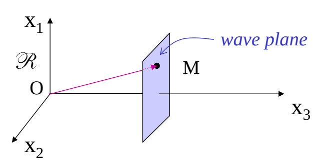
- Substituting in propagation equation $\quad \vec f_{e_0} + \delta \vec f_e + \overrightarrow{\mathrm{div}\left(\bar{\bar T} + \bar{\bar T}_0\right)} = \rho \frac{\partial^2 \vec u}{\partial t^2}$ ($\vec f_{e_0} + \overrightarrow{\mathrm{div}\,\bar{\bar T}_0} = \vec 0$)

$\Rightarrow \boxed{\delta \vec f_e + \overrightarrow{\mathrm{div}\,\bar{\bar T}} = \rho \frac{\partial^2 \vec u}{\partial t^2}}$ *with sources*

and $\boxed{\overrightarrow{\mathrm{div}\,\bar{\bar T}} = \rho \frac{\partial^2 \vec u}{\partial t^2}}$ *ouside sources*

using $\overrightarrow{\mathrm{div}\,\bar{\bar T}} = \sum_{i=1}^{3}\left(\sum_{j=1}^{3} \frac{\partial T_{ij}}{\partial x_j}\right) \vec e_{x_i} \Rightarrow \boxed{\rho \frac{\partial^2 u_i}{\partial t^2} = \sum_{j=1}^{3} \frac{\partial T_{ij}}{\partial x_j},\quad i = 1,2,3}$

Isotropic solid (1/5): decoupling of the propagation equation



$$\vec u(\vec x;t) = \vec u(x_3;t) \Rightarrow \begin{cases} \partial u_i/\partial x_1 = 0 \\ \partial u_i/\partial x_2 = 0 \end{cases}$$

- Hooke Law

$$T_{ii} = \lambda\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\right) + 2\mu \frac{\partial u_i}{\partial x_i} \qquad \begin{cases} T_{11} = \lambda\, \partial u_3/\partial x_3 \\ T_{22} = \lambda\, \partial u_3/\partial x_3 \\ T_{33} = (\lambda + 2\mu)\, \partial u_3/\partial x_3 \end{cases} \quad \textit{normal stresses}$$

$$T_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad \begin{cases} T_{12} = 0 \\ T_{13} = \mu\, \partial u_1/\partial x_3 \\ T_{23} = \mu\, \partial u_2/\partial x_3 \end{cases} \quad \textit{tangential stresses}$$

Isotropic solid (2/5): decoupling of the propagation equation

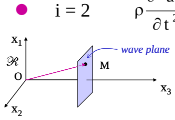
- $i = 3 \quad \rho \frac{\partial^2 u_3}{\partial t^2} = \sum_{j=1}^{3} \frac{\partial T_{3j}}{\partial x_j} \Rightarrow \frac{\partial^2 u_3}{\partial t^2} = \left(\frac{\lambda + 2\mu}{\rho}\right) \frac{\partial^2 u_3}{\partial x_3^2}$ *homogeneous to* L^2T^{-2}

let $\boxed{V_L = \sqrt{\frac{\lambda + 2\mu}{\rho}}} \Rightarrow \boxed{\frac{\partial^2 u_3}{\partial x_3^2} - \frac{1}{V_L^2} \frac{\partial^2 u_3}{\partial t^2} = 0}$ *pressure waves (longitudinal)*

- $i = 1 \quad \rho \frac{\partial^2 u_1}{\partial t^2} = \sum_{j=1}^{3} \frac{\partial T_{1j}}{\partial x_j} \Rightarrow \frac{\partial^2 u_1}{\partial t^2} = \frac{\mu}{\rho} \frac{\partial^2 u_1}{\partial x_3^2}$ *homogeneous to* L^2T^{-2}

let $\boxed{V_T = \sqrt{\frac{\mu}{\rho}}} \Rightarrow \boxed{\frac{\partial^2 u_1}{\partial x_3^2} - \frac{1}{V_T^2} \frac{\partial^2 u_1}{\partial t^2} = 0}$ *shear waves (transversal)*

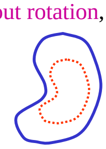
- $i = 2 \quad \rho \frac{\partial^2 u_2}{\partial t^2} = \sum_{j=1}^{3} \frac{\partial T_{2j}}{\partial x_j} \Rightarrow \frac{\partial^2 u_2}{\partial t^2} = \frac{\mu}{\rho} \frac{\partial^2 u_2}{\partial x_3^2}$



$\Rightarrow \boxed{\frac{\partial^2 u_2}{\partial x_3^2} - \frac{1}{V_T^2} \frac{\partial^2 u_2}{\partial t^2} = 0}$ *shear waves (transversal)*

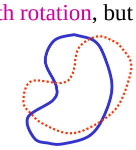
Isotropic solid (3/5): deformations

Without rotation, but with a variation of volume $\boxed{\overrightarrow{\mathrm{rot}}\ \vec u_L = \vec 0}$



scalar potential ψ : $\vec u_L = \overrightarrow{\mathrm{grad}}\ \psi$

With rotation, but without variation of volume $\boxed{\mathrm{div}\,\vec u_T = \vec 0}$



vectorial potential $\vec\chi$: $\vec u_T = \overrightarrow{\mathrm{rot}}\ \vec\chi$

$\Rightarrow$ particle displacement : $\boxed{\vec u = \overrightarrow{\mathrm{grad}}\ \psi + \overrightarrow{\mathrm{rot}}\ \vec\chi = \vec\nabla \psi + \vec\nabla \wedge \vec\chi}$

C. Potel, M. Bruneau, "Acoustique générale (...)", Ed. Ellipse, collection Technosup, ISBN 2-7298-2805-2, 2006

Isotropic solid (4/5): any waves

- **Propagation equation:** $\rho \dfrac{\partial^2 \vec{u}}{\partial t^2} = (\lambda+\mu)\overrightarrow{\text{grad}}(\text{div}\,\vec{u}) + \mu \Delta \vec{u}$

  with $\vec{u} = \overrightarrow{\text{grad}}\,\Psi + \overrightarrow{\text{rot}}\,\vec{\chi} = \vec{u}_L + \vec{u}_T$

- **Decoupling of the propagation equation:**

  pressure wave propagating with the velocity $V_L = \sqrt{\dfrac{\lambda+2\mu}{\rho}} = \sqrt{\dfrac{c_{11}}{\rho}}$

  $$\frac{\partial^2 \vec{u}_L}{\partial t^2} - V_L^2 \Delta \vec{u}_L = \vec{0}$$

  shear wave propagating with the velocity $V_T = \sqrt{\dfrac{\mu}{\rho}} = \sqrt{\dfrac{c_{11}-c_{12}}{2\rho}}$

  $$\frac{\partial^2 \vec{u}_T}{\partial t^2} - V_T^2 \Delta \vec{u}_T = \vec{0}$$

- **Case of plane waves,** in connection with a propagation direction $\vec{n}$

  pressure wave = longitudinal wave
  shear wave = transversale wave

Isotropic solid (5/5): monochromatic waves

- Wave equations $\Delta\psi - \dfrac{1}{V_L^2}\dfrac{\partial^2\psi}{\partial t^2} = 0$ and $\Delta\vec{\chi} - \dfrac{1}{V_T^2}\dfrac{\partial^2\vec{\chi}}{\partial t^2} = \vec{0}$

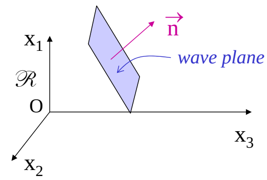
- $\hat{\psi}(\vec{x};t) = \hat{\Psi}(\vec{x})e^{+i\omega t}$ and $\hat{\vec{\chi}}(\vec{x};t) = \hat{\vec{X}}(\vec{x})e^{+i\omega t}$

  → $\Delta\hat{\psi} - \dfrac{1}{V_L^2}\dfrac{\partial^2\hat{\psi}}{\partial t^2} = 0$ and $\Delta\hat{\vec{\chi}} - \dfrac{1}{V_T^2}\dfrac{\partial^2\hat{\vec{\chi}}}{\partial t^2} = \vec{0}$

- Helmholtz equations $\Delta\hat{\Psi} + k_L^2\hat{\Psi} = 0$ and $\Delta\hat{\vec{X}} + k_T^2\hat{\vec{X}} = \vec{0}$

  with $k_L = \omega/V_L$ and $k_T = \omega/V_T$

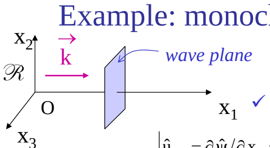
- Monochromatic plane waves



$\vec{k}_L = k_L\,\vec{n}$ and $\vec{k}_T = k_T\,\vec{n}$

$$\hat{\psi}(\vec{x}) = \hat{A}_L e^{-i(\vec{k}_L\cdot\vec{x}-\omega t)}$$
$$\hat{\vec{\chi}}(\vec{x}) = \hat{\vec{A}}_T e^{-i(\vec{k}_T\cdot\vec{x}-\omega t)}$$

Example: monochromatic plane waves (1/2)



- Longitudinal waves

✓ $\hat{\vec{u}}_L(x_1;t) = \overrightarrow{\text{grad}}\,\hat{\psi}(x_1;t)$ *with* $\hat{\psi}(x_1;t) = \hat{A}_L e^{-i(k_L x_1 - \omega t)}$

→ $\hat{\vec{u}}_L(x_1;t) = \left|\begin{array}{l} \hat{u}_{L_1} = \partial\hat{\psi}/\partial x_1 = -i k_L \hat{A}_L e^{-i(k_L x_1-\omega t)} \\ \hat{u}_{L_2} = \partial\hat{\psi}/\partial x_2 = 0 \\ \hat{u}_{L_3} = \partial\hat{\psi}/\partial x_3 = 0 \end{array}\right._{\mathscr{B}}$

→ $\vec{u}_L(x_1;t) = \mathscr{R}e\left[\hat{\vec{u}}_L(x_1;t)\right] = k_L\left|\hat{A}_L\right|\sin(\omega t - k_L x_1 + \alpha_L)\vec{e}_{x_1}$ *with* $\hat{A}_L = \left|\hat{A}_L\right| e^{i\alpha_L}$

✓ $\hat{\vec{T}}_L = \hat{\bar{\bar{T}}}\cdot\vec{e}_{x_1} = \hat{T}_{11}\vec{e}_{x_1} + \hat{T}_{21}\vec{e}_{x_2} + \hat{T}_{31}\vec{e}_{x_3}$

with $\hat{T}_{11} = (\lambda+2\mu)\partial\hat{u}_{L_1}/\partial x_1 = -(\lambda+2\mu)k_L^2\hat{A}_L e^{-i(k_L x_1-\omega t)}$

$\hat{T}_{21} = \mu\partial\hat{u}_{L_2}/\partial x_1 = 0$ *and* $\hat{T}_{31} = \mu\partial\hat{u}_{L_3}/\partial x_1 = 0$

→ $\hat{\vec{T}}_L = \hat{T}_{11}\vec{e}_{x_1} = -(\lambda+2\mu)k_L^2\hat{A}_L e^{-i(k_L x_1-\omega t)}\vec{e}_{x_1}$

→ $\vec{T}_L = \mathscr{R}e\left[\hat{\vec{T}}_L\right] = -(\lambda+2\mu)k_L^2\left|\hat{A}_L\right|\cos(\omega t - k_L x_1 + \alpha_L)\vec{e}_{x_1}$ // $\vec{n} = \vec{e}_{x_1}$

→ **compression / expansion motion**

Example: monochromatic plane waves (2/2)

- Shear waves

✓ $\hat{\vec{u}}_T(x_1;t) = \overrightarrow{\text{rot}}\,\hat{\vec{\chi}}(x_1;t)$ *with* $\hat{\vec{\chi}}(x_1;t) = \hat{\vec{A}}_T e^{-i(k_T x_1-\omega t)}$

$$\hat{\vec{u}}_T(x_1;t) = \left|\begin{array}{l}\partial/\partial x_1 \\ \partial/\partial x_2 \\ \partial/\partial x_3\end{array}\right._{\mathscr{B}} \wedge \left|\begin{array}{l}\hat{\chi}_1 \\ \hat{\chi}_2 \\ \hat{\chi}_3\end{array}\right._{\mathscr{B}} = \left|\begin{array}{l}\partial\hat{\chi}_3/\partial x_2 - \partial\hat{\chi}_2/\partial x_3 \\ \partial\hat{\chi}_1/\partial x_3 - \partial\hat{\chi}_3/\partial x_1 \\ \partial\hat{\chi}_2/\partial x_1 - \partial\hat{\chi}_1/\partial x_2\end{array}\right._{\mathscr{B}} = \left|\begin{array}{c}0 \\ -\partial\hat{\chi}_3/\partial x_1 \\ \partial\hat{\chi}_2/\partial x_1\end{array}\right._{\mathscr{B}} = \left|\begin{array}{l}\hat{u}_{T_1} = 0 \\ \hat{u}_{T_2} = i k_T \hat{A}_{T_3} e^{-i(k_T x_1-\omega t)} \\ \hat{u}_{T_3} = -i k_T \hat{A}_{T_2} e^{-i(k_T x_1-\omega t)}\end{array}\right._{\mathscr{B}}$$

$$\vec{u}_T(x_1;t) = \mathscr{R}e\left[\hat{\vec{u}}_T(x_1;t)\right] = -k_T\left|\hat{A}_{T_3}\right|\sin(\omega t - k_T x_1 + \alpha_{T_3})\vec{e}_{x_2} + k_T\left|\hat{A}_{T_2}\right|\sin(\omega t - k_T x_1 + \alpha_{T_2})\vec{e}_{x_3}$$

with $\hat{A}_{T_2} = \left|\hat{A}_{T_2}\right|e^{i\alpha_{T_2}}$, $\hat{A}_{T_3} = \left|\hat{A}_{T_3}\right|e^{i\alpha_{T_3}}$

✓ $\hat{\vec{T}}_T = \hat{\bar{\bar{T}}}\cdot\vec{e}_{x_1} = \hat{T}_{11}\vec{e}_{x_1} + \hat{T}_{21}\vec{e}_{x_2} + \hat{T}_{31}\vec{e}_{x_3}$

with $\hat{T}_{11} = (\lambda+2\mu)\partial\hat{u}_{T_1}/\partial x_1 = 0$; $\hat{T}_{21} = \mu\partial\hat{u}_{T_2}/\partial x_1 = \mu k_T^2\hat{A}_{T_3}e^{-i(k_T x_1-\omega t)}$

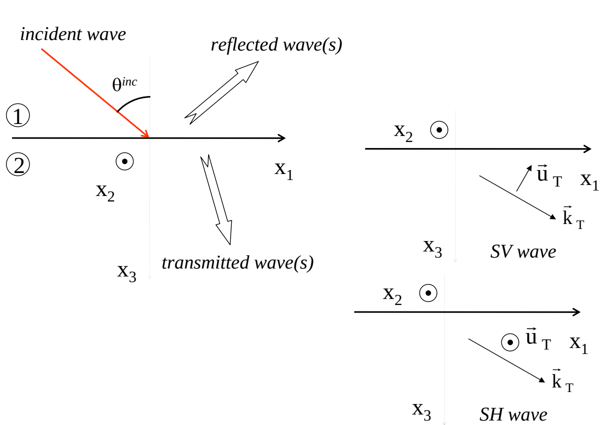
and $\hat{T}_{31} = \mu\partial\hat{u}_{T_3}/\partial x_1 = -\mu k_T^2\hat{A}_{T_2}e^{-i(k_T x_1-\omega t)}$

→ $\hat{\vec{T}}_T = \hat{T}_{21}\vec{e}_{x_2} + \hat{T}_{31}\vec{e}_{x_3} = \mu k_T^2\hat{A}_{T_3}e^{-i(k_T x_1-\omega t)}\vec{e}_{x_2} - \mu k_T^2\hat{A}_{T_2}e^{-i(k_T x_1-\omega t)}\vec{e}_{x_3}$

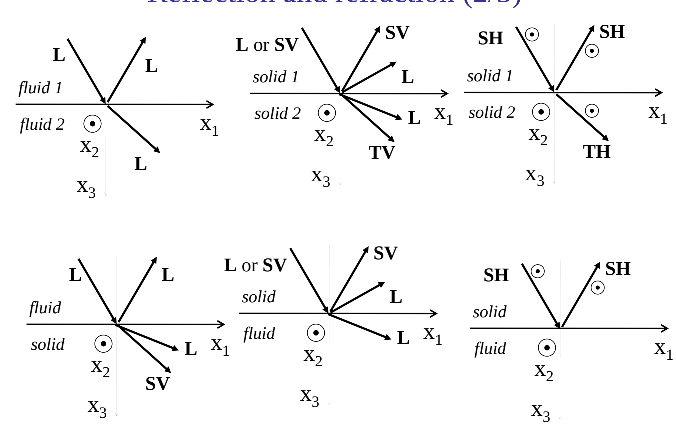
$$\vec{T}_T = \mathscr{R}e\left[\hat{\vec{T}}_T\right] = \mu k_T^2\left|\hat{A}_{T_3}\right|\cos(\omega t - k_L x_1 + \alpha_{T_3})\vec{e}_{x_2} - \mu k_T^2\left|\hat{A}_{T_2}\right|\cos(\omega t - k_L x_1 + \alpha_{T_2})\vec{e}_{x_3}$$

// $(O\,x_2\,x_3)$ ⊥ $\vec{n} = \vec{e}_{x_1}$ → **shear motion**

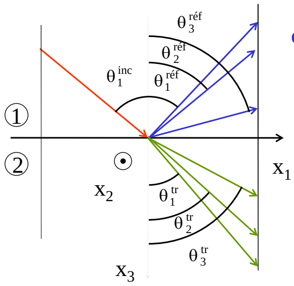
Reflection and refraction (1/3)



Reflection and refraction (2/3)



Reflection and refraction (3/3)



continuity of displacements at $x_3 = 0$:

$$u_i^{inc} + \sum_{réf} u_i^{réf} = \sum_{tr} u_i^{tr}$$

continuity of stress at $x_3 = 0$:

$$T_{i3}^{inc} + \sum_{réf} T_{i3}^{réf} = \sum_{tr} T_{i3}^{tr}$$

stresses acting on a surface element at the interface

$$\vec{T} = \overline{\overline{T}} \cdot \vec{e}_{x_3} = \begin{vmatrix} T_{13} = T_5 \\ T_{23} = T_4 \\ T_{33} = T_3 \end{vmatrix}_{\mathscr{B}}$$

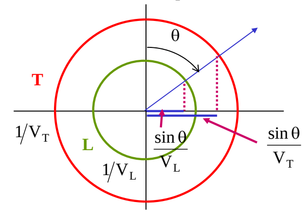
Visualisation of Snell-Descartes' law: slowness surface

Location of the ends of the slowness vector $\vec{m}$, drawn from a fixed point O, when the propagation $\vec{n}$ varies.

Slowness vector : $\vec{m} = \dfrac{\vec{n}}{V}$

Isotropic medium: 2 velocities for a given propagation direction
same velocities in any direction

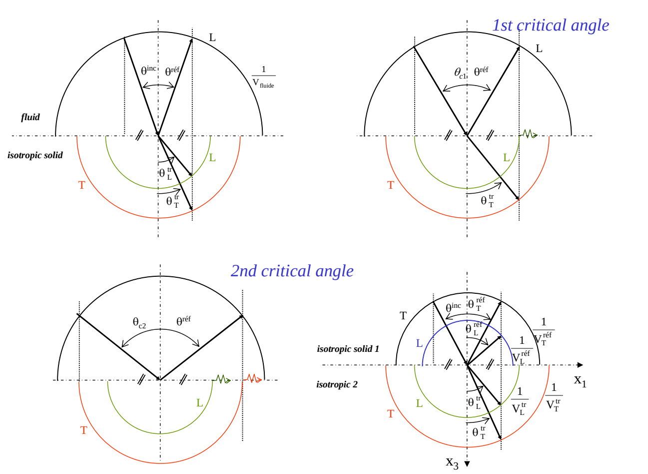
slowness surfaces = spheres



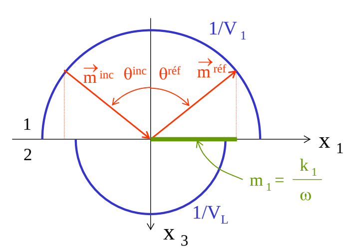
Snell-Descartes' law:

$$\frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2}$$

Isotropic media: critical angles -evanescent waves



Isotropic medium: evanescent waves



slowness vector: $\vec{m} = \dfrac{\vec{k}}{\omega}$

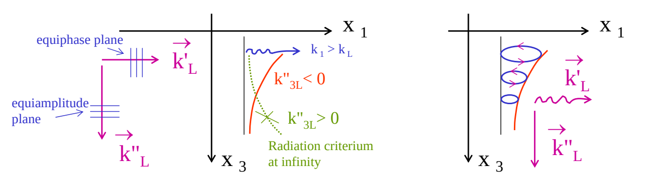
$$m_1 = \frac{k_1}{\omega} > \frac{1}{V_L} = \frac{k_L}{\omega} \Rightarrow k_1 > k_L$$

Dispersion relation:

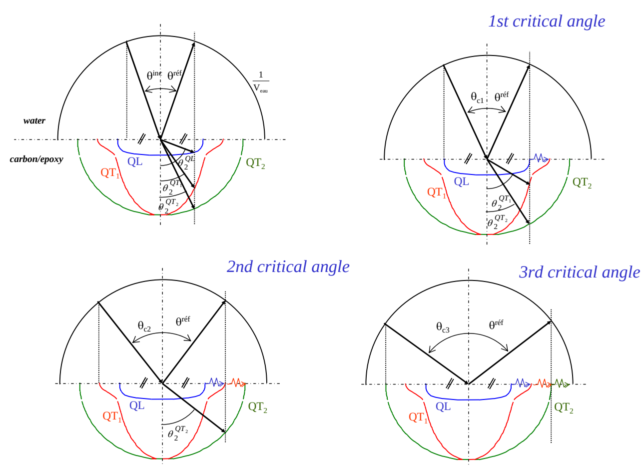
$$k_1^2 + k_{3L}^2 = k_L^2 \Rightarrow k_{3L}^2 = k_L^2 - k_1^2 < 0$$

$k_{3L} = i\,k''_{3L}$ $\qquad k''_{3L} = \pm\sqrt{k_1^2 - k_L^2}$

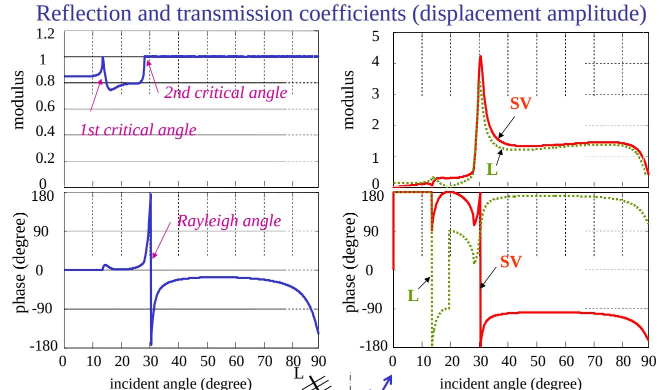
$$\hat{\psi} = \hat{A}_L\, e^{k''_{3L} x_3}\, e^{-i(k_1 x - \omega t)}$$



Anisotropic media: critical angles - evanescent waves



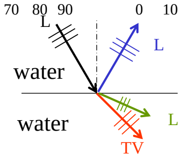
Reflection and transmission coefficients (displacement amplitude)



Water

ρ = 1000 kg/m³

V_L = 1480 m/s



Aluminium

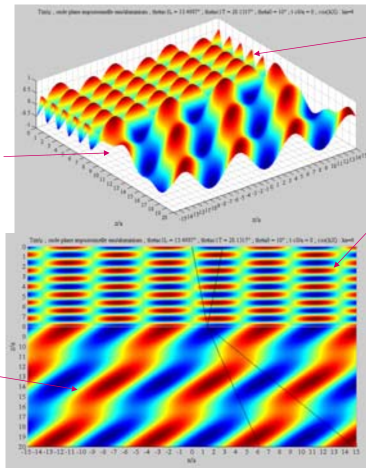
ρ = 2786 kg/m³

V_L = 6650 m/s

V_T = 3447 m/s

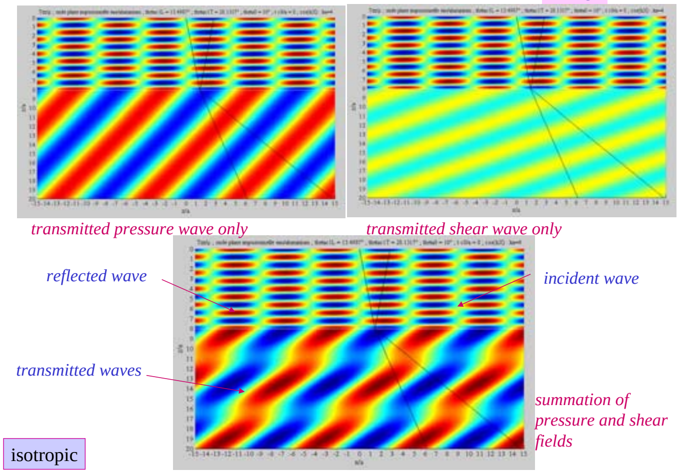
C. Potel, M. Bruneau, "Acoustique générale (...)", Ed. Ellipse, collection Technosup, ISBN 2-7298-2805-2, 2006

Example (T_{zz}/μ): f(X) = cos(kX), ka = 4, $\theta<\theta_{cL}$



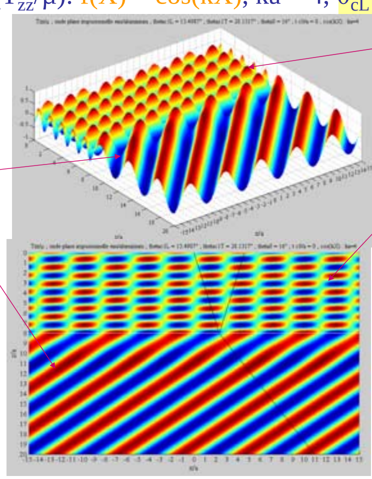
isotropic

Example (T_{zz}/μ): f(X) = cos(kX), ka = 4, $\theta<\theta_{cL}$



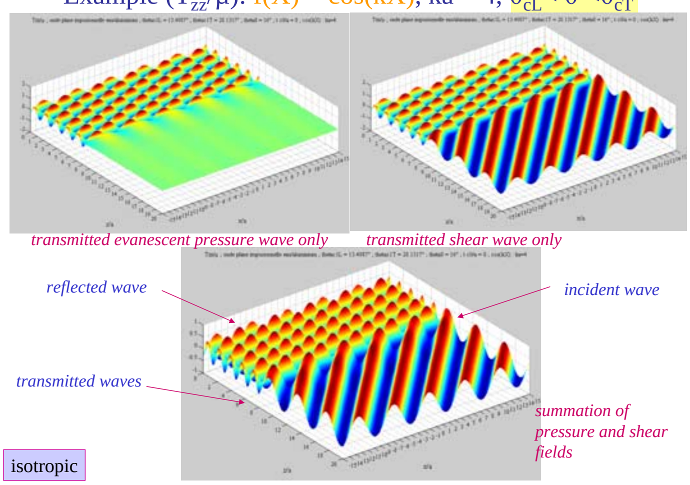
isotropic

Example (T_{zz}/μ): f(X) = cos(kX), ka = 4, $\theta_{cL}< \theta <\theta_{cT}$



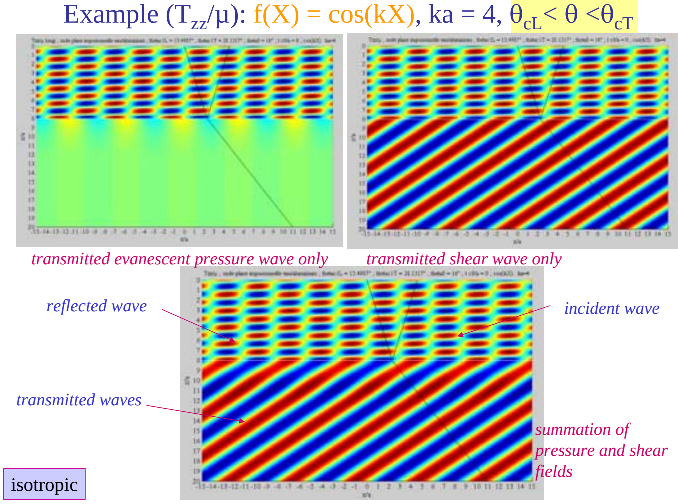
isotropic

Example (T_{zz}/μ): f(X) = cos(kX), ka = 4, $\theta_{cL}< \theta <\theta_{cT}$



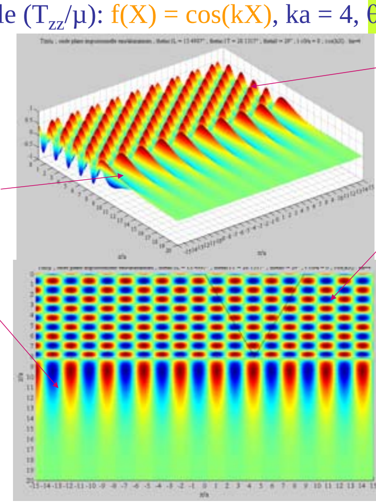
isotropic

Example (T_{zz}/μ): f(X) = cos(kX), ka = 4, $\theta_{cL}< \theta <\theta_{cT}$



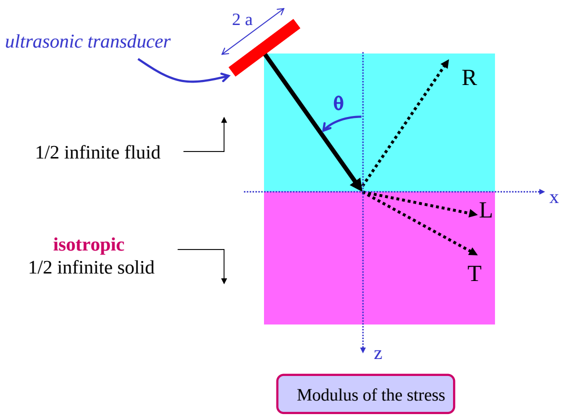
isotropic

Example (T_{zz}/μ): f(X) = cos(kX), ka = 4, $\theta >\theta_{cT}$



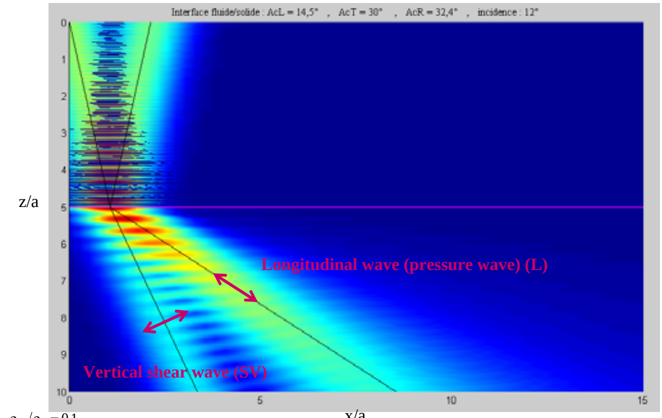
isotropic

Gaussian incident beam onto an interface fluid/solid



Programs realised by Ph. Gatignol, Pr., Université de Technologie de Compiègne

Before the 1st critical angle

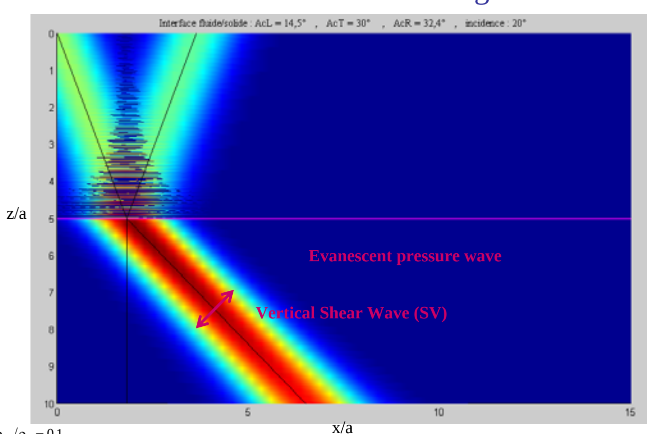


$\rho_0/\rho_1 = 0{,}1$

$k_0a = 60$, $k_La = 15$, $k_Ta=30$, $\theta < \theta_{c1}$

Programs realised by Ph. Gatignol, Pr., Université de Technologie de Compiègne

Between the two critical angles

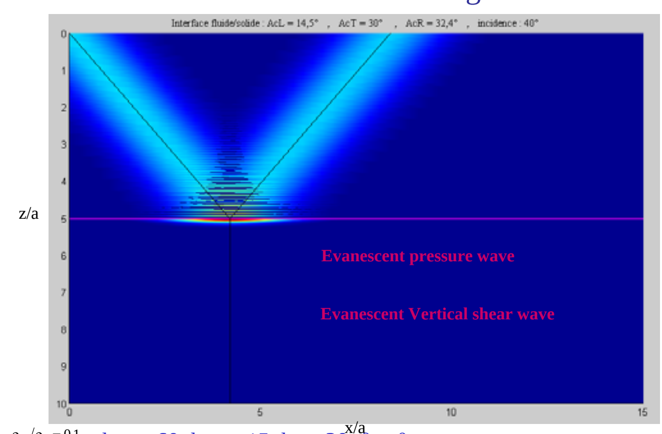


$\rho_0/\rho_1 = 0{,}1$

$k_0a = 60$, $k_La = 15$, $k_Ta=30$, $\theta_{c1} < \theta < \theta_{c2}$

Programs realised by Ph. Gatignol, Pr., Université de Technologie de Compiègne

After the two critical angles



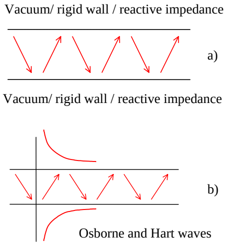
$\rho_0/\rho_1 = 0{,}1$

$k_0a = 60$, $k_La = 15$, $k_Ta=30$, $\theta > \theta_{c2}$

Programs realised by Ph. Gatignol, Pr., Université de Technologie de Compiègne

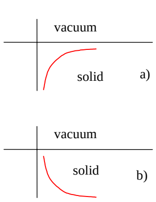
Modal waves (1/3)

guided waves

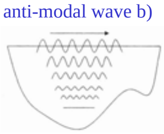


Lamb wave

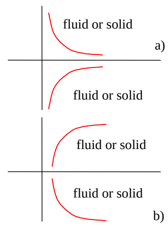
surface waves



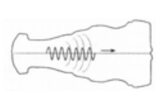
- Rayleigh wave a)
- anti-modal wave b)



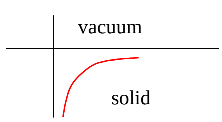
interface waves

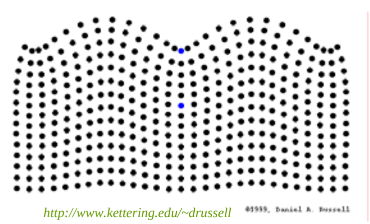


- Scholte wave, Stoneley wave, Rayleigh-Cezawa wave, etc... a)
- anti-modal wave b)



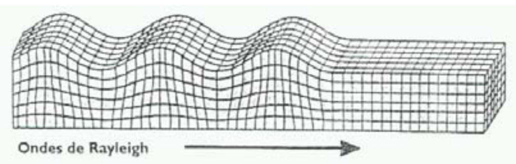
Modal waves (2/3): Rayleigh wave





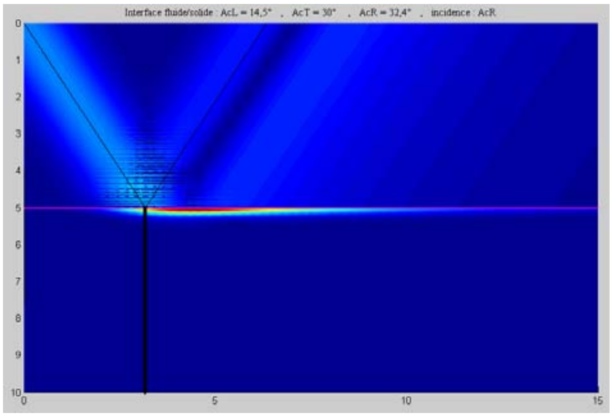
http://www.kettering.edu/~drussell

Animation courtesy of Dr. Dan Russell, Kettering University



C. Potel, M. Bruneau, "Acoustique générale (...)", Ed. Ellipse, collection Technosup, ISBN 2-7298-2805-2, 2006

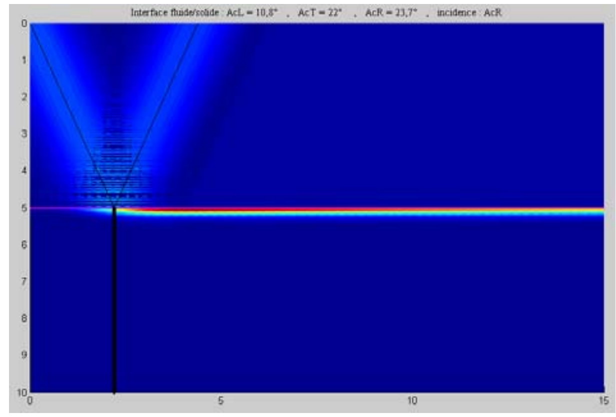
$k_0a = 60$, $k_La = 15$, $k_Ta=30$, $\theta = \theta_{Rayleigh}$



$\rho_0/\rho_1 = 0{,}1$

Programs realised by Ph. Gatignol, Pr., Université de Technologie de Compiègne

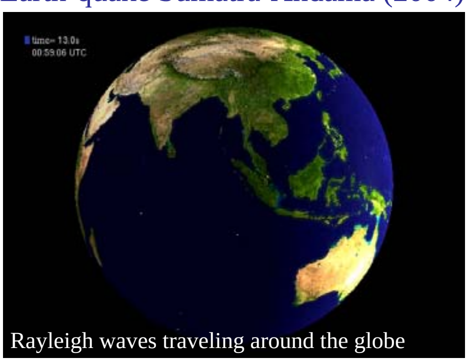
$k_0a = 80$, $k_La = 15$, $k_Ta=30$, $\theta = \theta_{Rayleigh}$



$\rho_0/\rho_1 = 0{,}02$

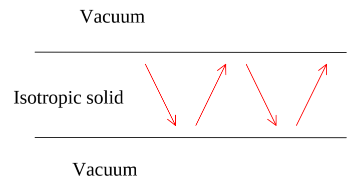
Programs realised by Ph. Gatignol, Pr., Université de Technologie de Compiègne

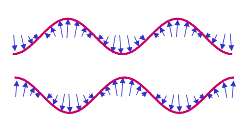
Earth-quake Sumatra-Andama (2004)



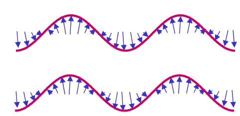
***Courtesy** http://www.sciencemag.org/cgi/content/full/308/5725/1133/DC1*
Charles J. Ammon, Chen Ji, Hong-Kie Thio, David Robinson, Sidao Ni, Vala Hjorleifsdottir, Hiroo Kanamori, Thorne Lay, Shamita Das, Don Helmberger, Gene Ichinose, Jascha Polet, David Wald ; The animation was made with the help of Santiago Lombeyda at the Center for Advanced Computing Research, Caltech

Modal waves (3/3): Lamb waves





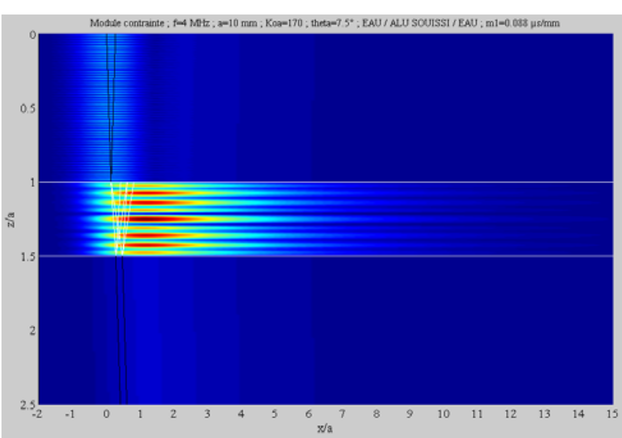
pressure waves (symmetric mode)



flexure waves (antisymmetric mode)

vectorial particle displacement field at the surface of the plate; its effect on the shape of the plate

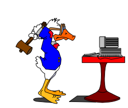
Water / Aluminium / Water ; ka=170 ; H=5 mm ; before the 1st critical angle

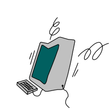


Lamb mode

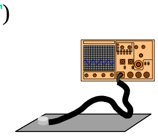
Testing of material

- **Aim:** Detect **defects** in a piece
- **2 types:**
  - **Destructive** Testing
    ⇨ non-reusable piece



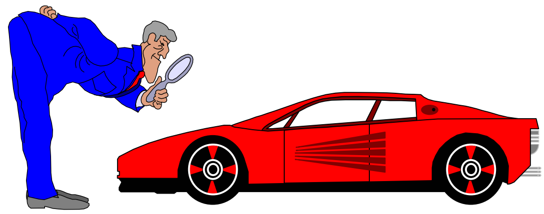


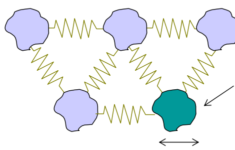
  - **Non Destructive Testing** (**NDT**)
    ⇨ reusable piece



- **Different NDT:**
  - Eddy currents
  - X-Rays, γ-Rays
  - **Ultrasounds**
    - not very expensive
    - access to one side of the piece
    - detection in defects located **deeply** in the piece

Non Destructive Testing (NDT) by ultrasounds





The more the defect is small, the more the frequency has to be high

NDT and NDE

NON DESTRUCTIVE TESTING

Presence or not of **defects**

Understanding of propagation phenomena

Determination of elastic (or viscoelastic) **properties**

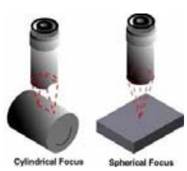
NON DESTRUCTIVE EVALUATION

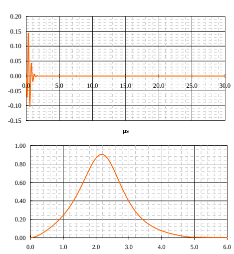
Transducer types

- **Transducers**

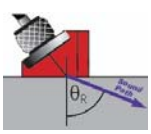
http://www.ndt-ed.org

- **Immersion transducters**

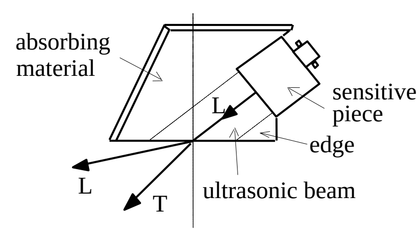




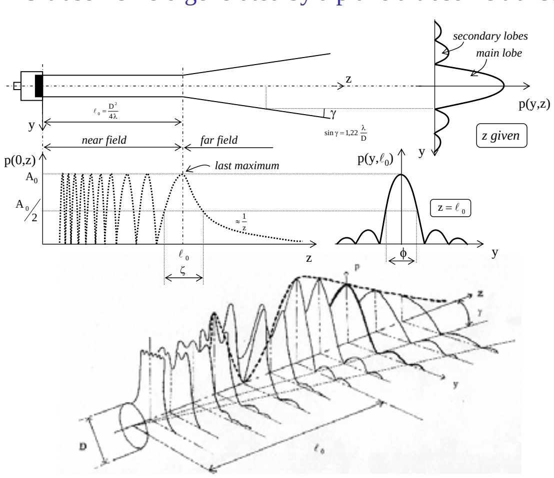
- **Angle beam transducers**



Angle beam transducer

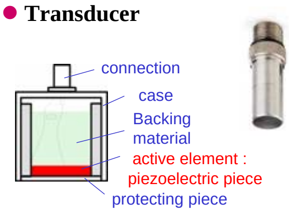


Ultrasonic field generated by a plane ultrasonic transducer



NDT by ultrasounds: principle

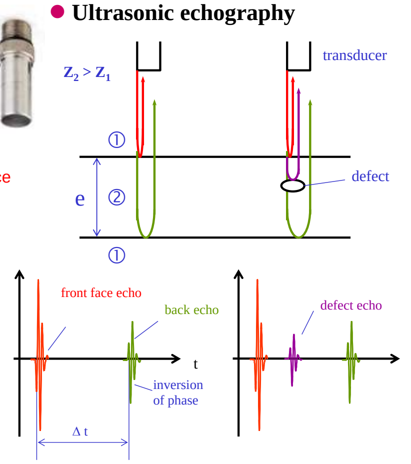
- **Transducer**



Transforms an electrical signal into a mechanical vibration and conversely

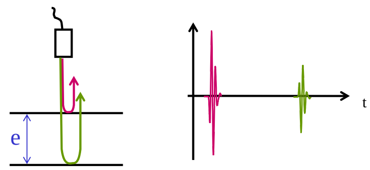
$$\Delta t = \frac{2\,e}{V}$$

- **Ultrasonic echography**

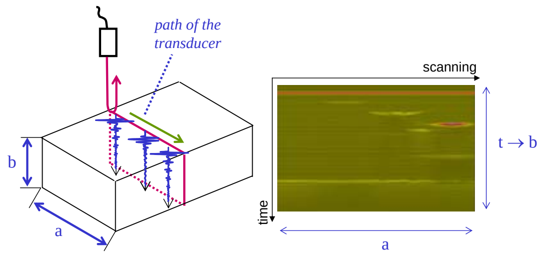


A-scan; B-scan

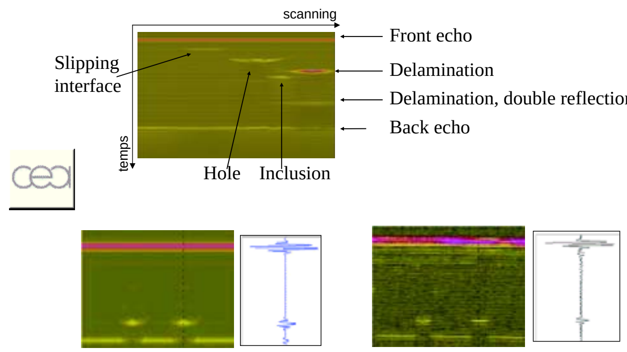
- A-Scan



- B-Scan: corresponds to a **cut of the material**



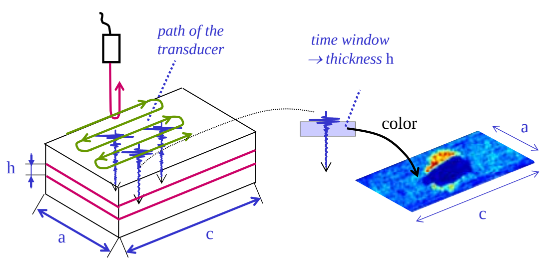
Identification of defects



Comparison modeling / experiment (CIVA from CEA)

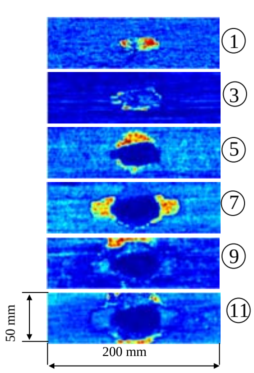
C-Scan

- C-scan: corresponds to a **representation of a material slice**

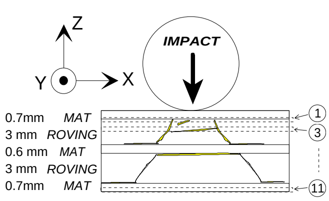


C-scan on a coin of currency

C-scan on an impacted beam

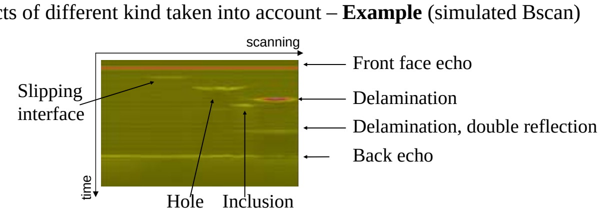


Impact on a pultruded beam

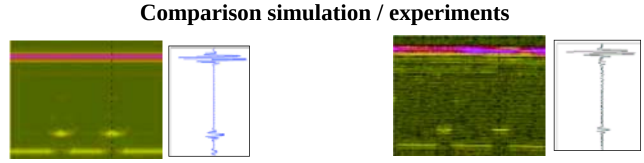


Hybrid model

Defects of different kind taken into account – **Example** (simulated Bscan)



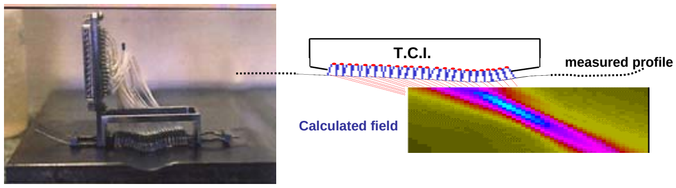
Comparison simulation / experiments



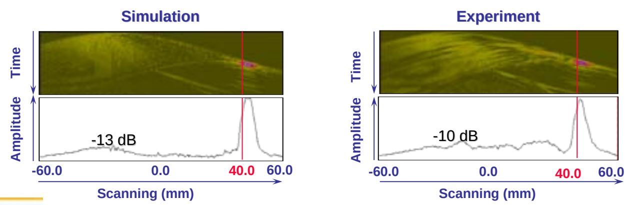
CEA courtesy.

DETECS / Service simulation et systèmes pour la Surveillance et le Contrôle

B-scan with a multi-element transducer



Contact flexible multi-element transducer (T.C.I., CEA)

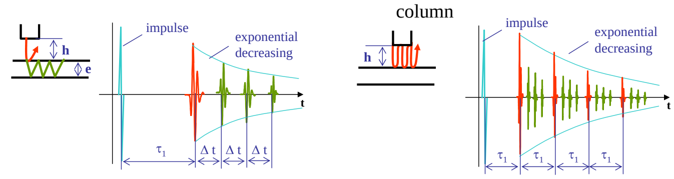


CEA courtesy.

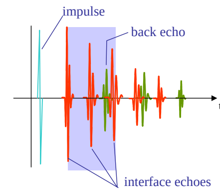
DETECS / Service simulation et systèmes pour la Surveillance et le Contrôle

Precautions of adjustment (1/2)

- Multiples reflections in a plate
- Multiples reflections in the water column



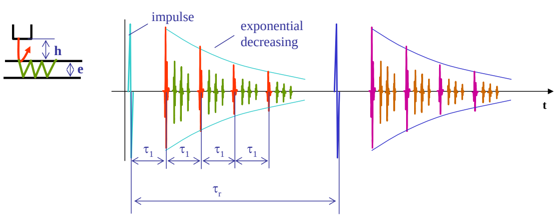
- Conquences of a bad adjustment of the water column



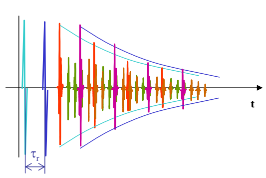
Insertion of a back echo between the interface echos. This back echo could be mixed up with a defect echo.

Precautions of adjustment (2/2)

- Adjustement of the pulse repetition frequency

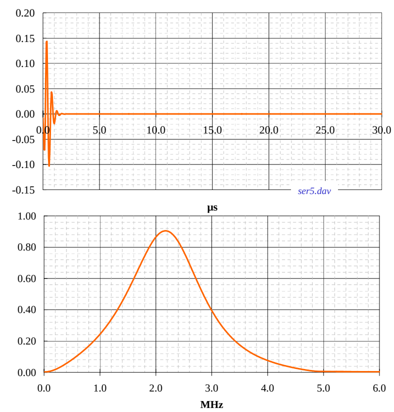


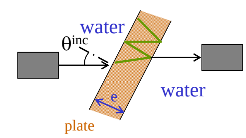
- Consequences of a bad adjustment of the pulse repetition frequency

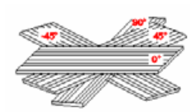


Insertion of interface or back echoes of the 2nd repetition between interface echoes of the 1st recurrence, which could be mixed up with a defect echo.

Example of input signal

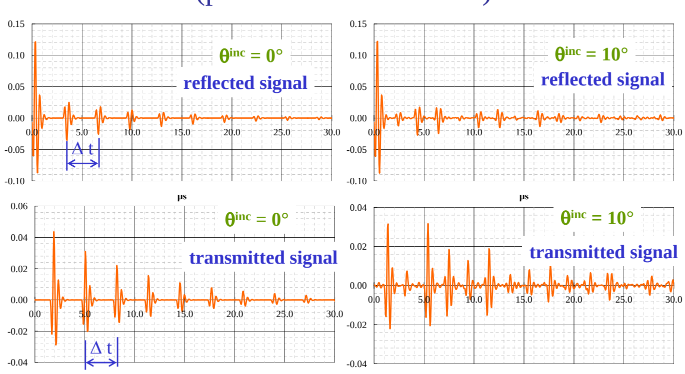






0°/45°/90°/135°
carbon-epoxy plate

Aluminium plate immersed in water (plane wave simulation)

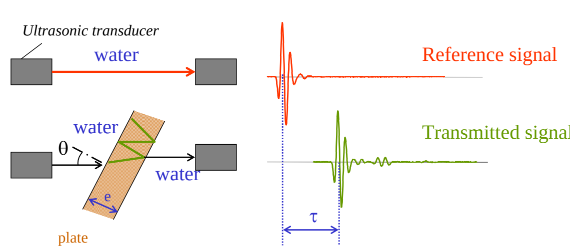


$$V_L = \frac{2\,e}{\Delta t}$$

e = 10 mm ; λ_L = 2.8 mm ; λ_T = 1.4 mm
V_L = 6340.4 m/s
V_T = 3138.9 m/s

1st critical angle = 13.5°
2nd critical angle = 28°

Measures of velocities (1/2)

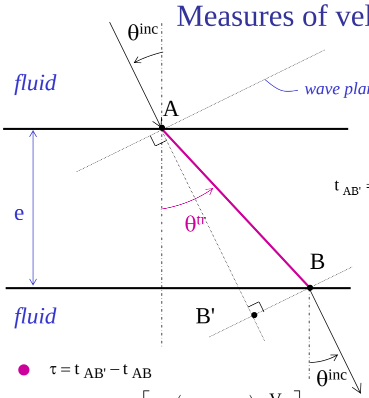


Propagation velocity of a wave in the medium, for a given direction

$$V = \frac{V_{eau}}{\sqrt{1+\frac{\tau V_{eau}}{e}\left(\frac{\tau V_{eau}}{e}-2\cos\theta\right)}}$$

Real elastic constants

Measures of velocities (2/2)



- Path in the plate

$$t_{AB} = \frac{AB}{V} = \frac{e}{V\cos\theta^{tr}}$$

- Path in the fluid

$$t_{AB'} = \frac{AB'}{V_0} = \frac{AB\cos\left(\theta^{tr}-\theta^{inc}\right)}{V_0} = \frac{e\cos\left(\theta^{tr}-\theta^{inc}\right)}{V_0\cos\theta^{inc}}$$

with $n = \frac{\sin\theta^{inc}}{\sin\theta^{tr}} = \frac{V_0}{V} \Rightarrow V = \frac{V_0}{n}$

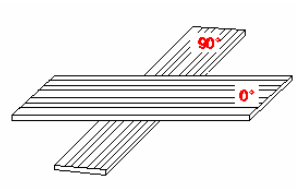
and $\sin\theta^{tr} = \frac{1}{n}\sin\theta^{inc}$, $\cos\theta^{tr} = \frac{1}{n}\sqrt{n^2-\sin^2\theta^{inc}}$

- $\tau = t_{AB'} - t_{AB}$

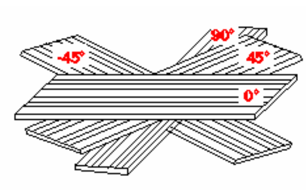
$$= \frac{e}{V\cos\theta^{tr}}\left[\cos\left(\theta^{tr}-\theta^{inc}\right) - \frac{V_0}{V}\right] = \frac{e}{V\cos\theta^{tr}}\left[\cos\theta^{tr}\cos\theta^{inc} + \sin\theta^{tr}\sin\theta^{inc} - n\right]$$

$$\Rightarrow \left(\frac{\tau V_0}{e} - \cos\theta^{inc}\right)^2 = n^2 - \sin^2\theta^{inc} \Rightarrow V = \frac{V_0}{n} = V_0\Big/\sqrt{1+\frac{\tau V_0}{e}\left(\frac{\tau V_0}{e} - 2\cos\theta^{inc}\right)}$$

Composite materials (1/2): example of carbon/epoxy composites

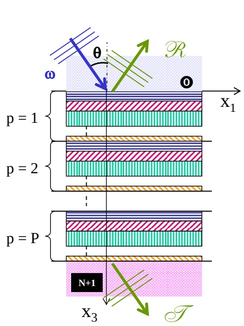


0°/90°

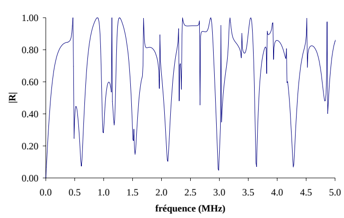


0°/45°/90°/135°

Composite materials (2/2): example of carbon/epoxy composites

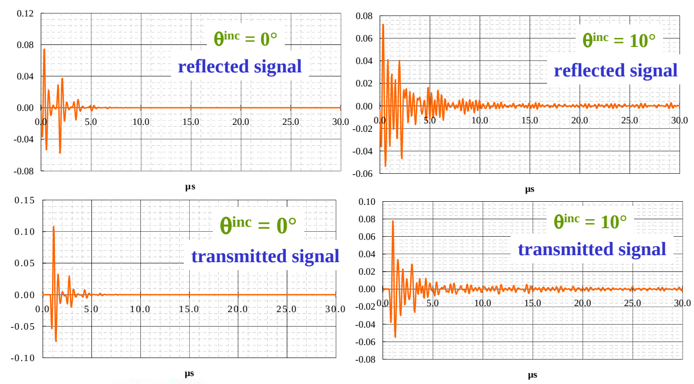


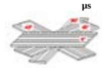
- Reflection coefficient(s) ℛ
- Transmission coefficient(s) 𝒯
- characteristics of all the waves



0°/45°/90°/135°
P=5 ; θ = 10°

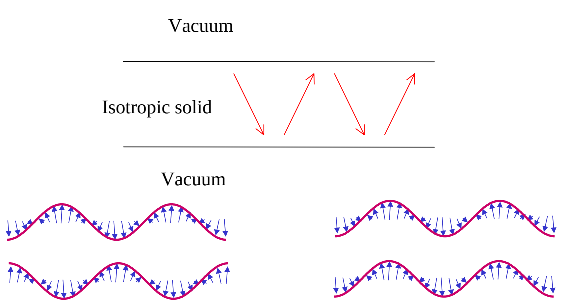
Composite plate immersed in water (Plane wave simulation)





0°/45°/90°/135° ; 5 periods (20 layers) ; carbon/epoxy plate

Lamb waves

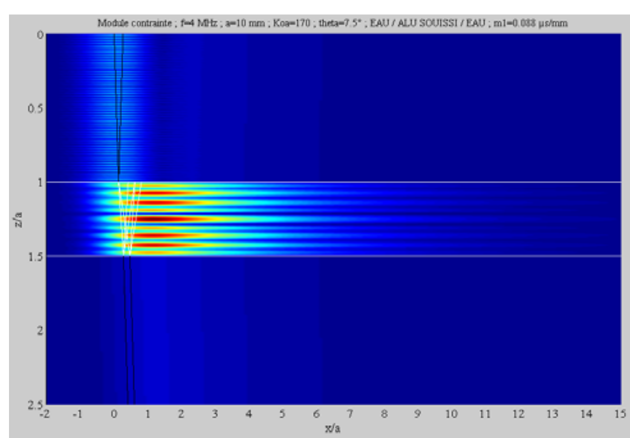


pressure waves (symmetric mode)

flexure waves (antisymmetric mode)

vectorial particle displacement field at the surface of the plate; its effect on the shape of the plate

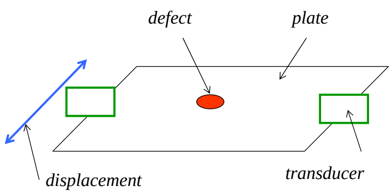
Water / Aluminium / Water ; ka=170 ; H=5 mm ; before the 1st critical angle



Lamb mode

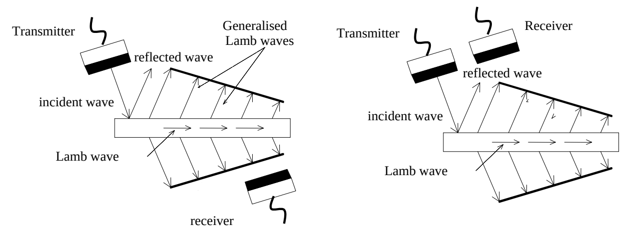
C. Potel, M. Bruneau, "Acoustique générale (...)", Ed. Ellipse, collection Technosup, ISBN 2-7298-2805-2, 2006

Lamb waves: interest in NDT

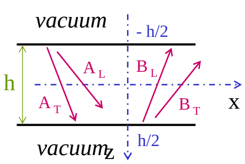


defect ⇒ perturbation of the wave propagation

Cartographies using Lamb waves



Lamb waves in isotropic media



Boundary conditions:

$$\begin{cases} T_{xz}\left(-\frac{h}{2}\right)=0 \\ T_{zz}\left(-\frac{h}{2}\right)=0 \end{cases} \qquad \begin{cases} T_{xz}\left(\frac{h}{2}\right)=0 \\ T_{zz}\left(\frac{h}{2}\right)=0 \end{cases}$$

homogeneous 4th order system

$$\begin{cases} (\ \ )A_L+(\ \ )A_T+(\ \ )B_L+(\ \ )B_T=0 \\ (\ \ )A_L+(\ \ )A_T+(\ \ )B_L+(\ \ )B_T=0 \\ (\ \ )A_L+(\ \ )A_T+(\ \ )B_L+(\ \ )B_T=0 \\ (\ \ )A_L+(\ \ )A_T+(\ \ )B_L+(\ \ )B_T=0 \end{cases}$$

⇒ determinant $(4\times 4)=0$

⇒

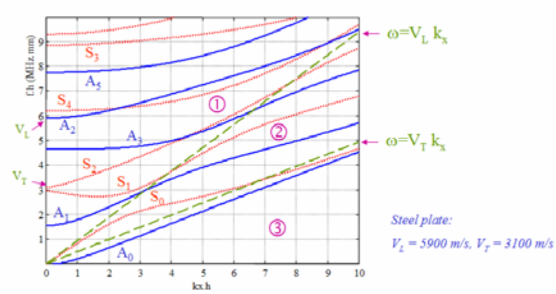
$$\left[4k_x^2 k_{z_L} k_{z_T}\cos\left(k_{z_L}\frac{h}{2}\right)\sin\left(k_{z_T}\frac{h}{2}\right)+\left(2k_x^2-k_T^2\right)^2\cos\left(k_{z_T}\frac{h}{2}\right)\sin\left(k_{z_L}\frac{h}{2}\right)\right]\bullet$$

antisymmetric modes

$$\left[4k_x^2 k_{z_L} k_{z_T}\cos\left(k_{z_T}\frac{h}{2}\right)\sin\left(k_{z_L}\frac{h}{2}\right)+\left(2k_x^2-k_T^2\right)^2\cos\left(k_{z_L}\frac{h}{2}\right)\sin\left(k_{z_T}\frac{h}{2}\right)\right]=0$$

symmetric modes

Dispersion curves for Lamb waves plane ($k_x.h$, f.h) or ($k_x.h/(2\pi)$, $\omega h/(2\pi)$)

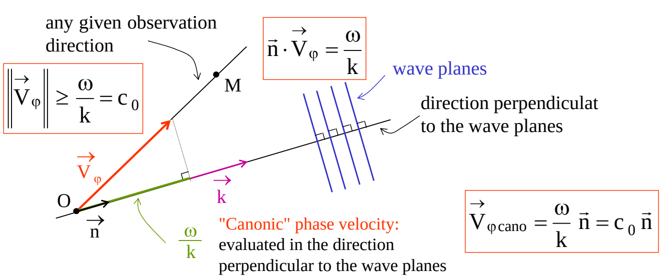


① k_{zL} and k_{zT} real *propagative OL and OT*

② k_{zL} pure imaginary and k_{zT} real *evanescent OL and propagative OT*

③ k_{zL} and k_{zT} pure imaginaries *evanescentes OL and OT*

Phase velocity

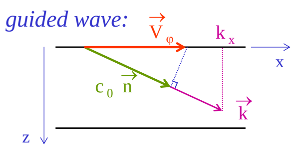


$$\left\|\vec{V}_\varphi\right\| \geq \frac{\omega}{k}=c_0$$

$$\vec{n}\cdot\vec{V}_\varphi=\frac{\omega}{k}$$

$$\vec{V}_{\varphi\,cano}=\frac{\omega}{k}\vec{n}=c_0\,\vec{n}$$

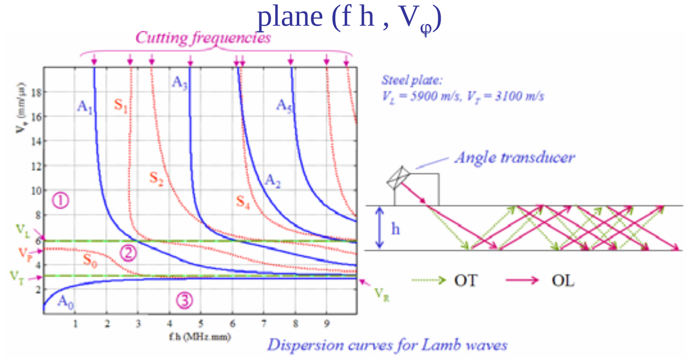
guided wave:



Phase velocity evaluated parallely to the guide

$$V_\varphi=\frac{\omega}{k_x}$$

Dispersion curves for Lamb waves plane (f h , V_φ)



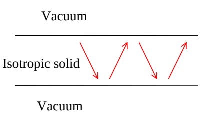
Dispersion curves for Lamb waves

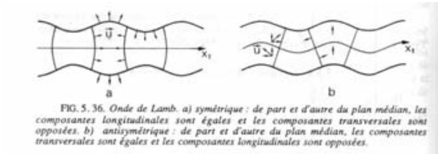
① k_{zL} and k_{zT} real *propagative OL and OT*

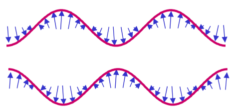
② k_{zL} pure imaginary and k_{zT} real *evanescent OL and propagative OT*

③ k_{zL} and k_{zT} pure imaginaries *evanescentes OL and OT*

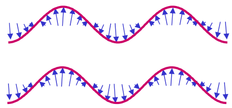
Displacements of Lamb waves (1/2)







pressure waves (symmetric mode)

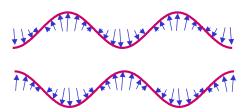


flexure waves (antisymmetric mode)

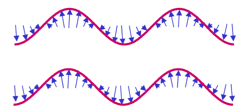
vectorial particle displacement field at the surface of the plate ; its effect on the shape of the plate

figures from:
D. Royer et E. Dieulesaint, "Ondes élastiques dans les solides", tome 1 : propagation libre et guidée, Masson, (1996)
J.L. Rose, "Ultrasonic waves in solid media",Cambridge Univ. Press, 1999

Displacements of Lamb waves (2/2)

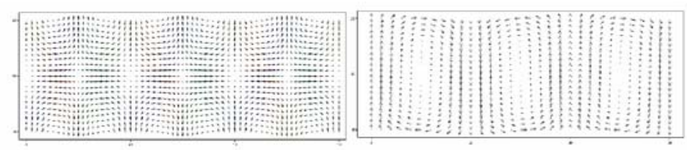


pressure waves (symmetric mode)



flexure waves (antisymmetric mode)

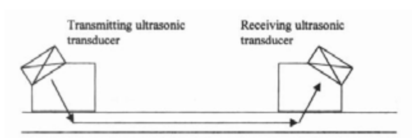
vectorial particle displacement field at the surface of the plate ; its effect on the shape of the plate

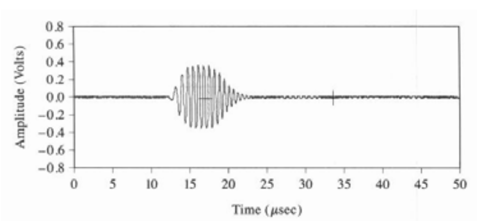


mode S_0 *mode A_0*

animations realised by Patrick Lanceleur, Université de Technologie de Compiègne
http://www.utc.fr/~lanceleu/links_CT04.html

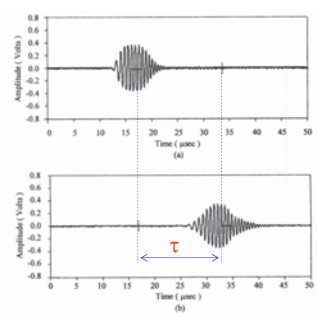
Experimental set-up

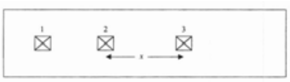




figures from: J.L. Rose, "Ultrasonic waves in solid media",Cambridge Univ. Press, 1999

Measure of the group velocity





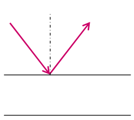
Aluminium plate

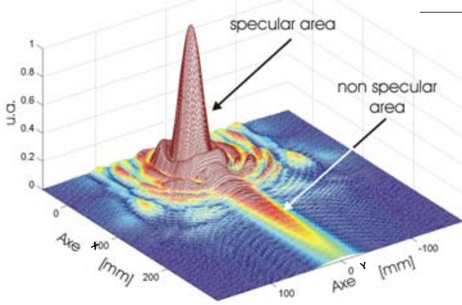
Mode S_0, fh=1.434 MHz.mm

x = 76.2 mm
τ = 15.875 µs ==> V_g = 4800 m/s

figures from: J.L. Rose, "Ultrasonic waves in solid media",Cambridge Univ. Press, 1999

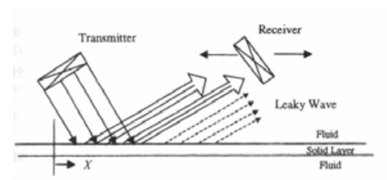
Reflected field

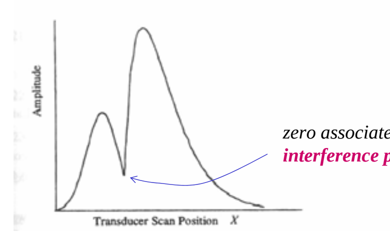




unidirectional carbon/epoxy plate
θ = 9.8°, f = 1.35 MHz ; e = 0.59 mm

Non specular reflection

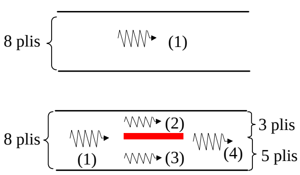




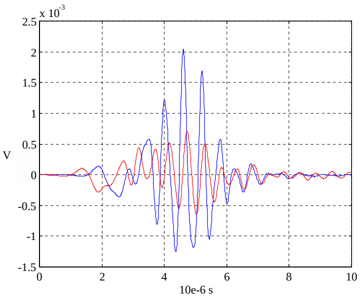
zero associated to an ***interference phenomenon***

figures from: J.L. Rose, "Ultrasonic waves in solid media",Cambridge Univ. Press, 1999

Detection of a defect using Lamb waves (1/6)

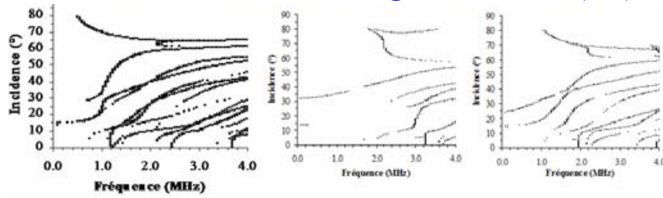


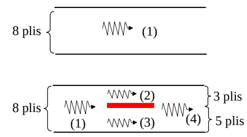
plaques en carbone/époxyde : défaut entre le 3ème et le 4ème pli



{0°/45°/90°/135°}$_{0s}$ en carbone/époxyde, comportant 8 plis en symétrie miroir. Incidence de 10,3°, fréquence de 2 MHz.
bleu : plaque saine
rouge : plaque avec défaut

Detection of a defect using Lamb waves (2/6)





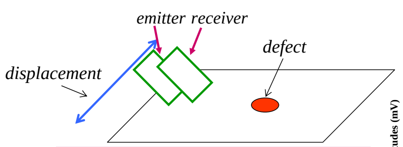
Defect between 3rd and 4th layer

Conversion of mode (1) into (2) and (3) then (4)

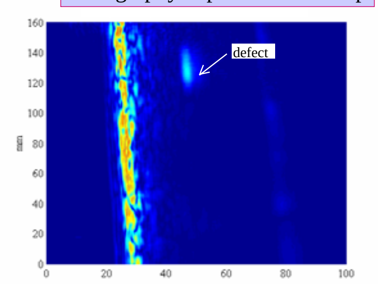
If mode (2) or mode (3) near of mode (1) then mode (4) ≈ mode (1) ➡ non detected defect

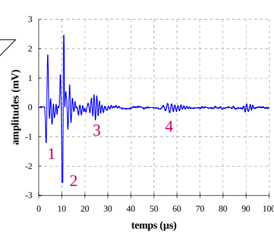
If mode (2) or mode (3) differents from mode (1) then mode (4) ≠ mode (1) ➡ detected defect

Detection of a defect using Lamb waves (3/6): cartography



Echography experimental set-up

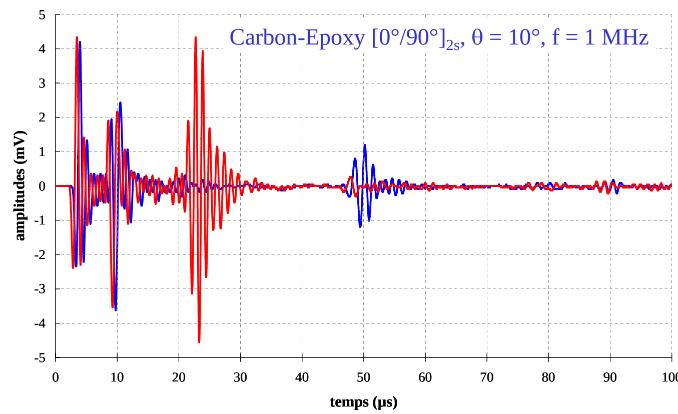




1 : surface echo
2 : radiation of Lamb wave
3 : defect echo
4 : edge plate echo

0°/90° mirror - 8 layers
f=1 MHz, θ=10°

Detection of a defect using Lamb waves (4/6)



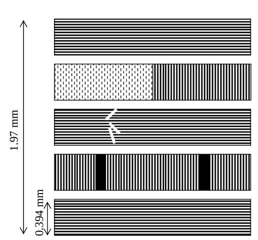
blue: plate without defect   red: plate without defect

Detection of defect (5/6): cartography using Lamb waves

5 layers 0°/90° mirror
SCS-6
matrix Ti - 6 Al - 4 V

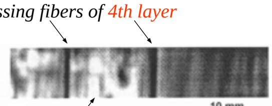
T. Kundu et al., Ultrasonics, 1996 and 1997

L-Scan



layer 1 : 0°
no defect
layer 2 : 90°
debond
layer 3 : 0°
broken fibers
layer 4 : 90°
missing fibers
layer 1 : 0°
no defect

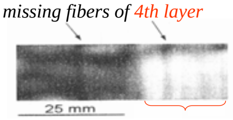
missing fibers of 4th layer



broken fibers of 3rd layer

θ = 20° ; f = 5.05 MHz

missing fibers of 4th layer

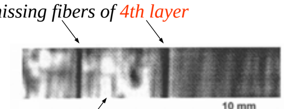


debond in 2nd layer

θ = 21° ; f = 5.15 MHz

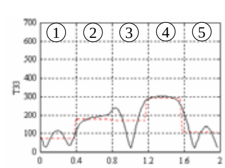
Detection of defect (6/6): cartography using Lamb waves

missing fibers of 4th layer

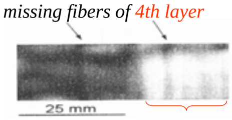


broken fibers of 3rd layer

θ = 20° ; f = 5.05 MHz

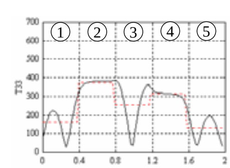


missing fibers of 4th layer



debond in 2nd layer

θ = 21° ; f = 5.15 MHz



repartition of normal stress as a function of thickness

Slides based upon

C. POTEL, M. BRUNEAU, *Acoustique Générale - équations différentielles et intégrales, solutions en milieux fluide et solide, applications*, Ed. Ellipse collection Technosup, 352 pages, ISBN 2-7298-2805-2, 2006