Why is Old Workers' Labor Market more Volatile? Unemployment Fluctuations over the Life-Cycle

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Abstract

Using CPS data, we document volatilities of fluctuations in worker flows and real hourly wage across age groups. We find that old worker's job flows are characterized by a higher responsiveness to business cycles than their younger counterparts. In contrast, their wage cyclicality is lower. We then show that the Mortensen & Pissarides (1994) model extended to introduce life-cycle features is well suited to explain these facts: with a shorter horizon on the labor market, old workers' outside options endogenously fall, which leads their wages to be less sensitive to the business cycle, whereas their job findings and separations are more responsive to the business cycle.

JEL Classification : E32, J11, J23

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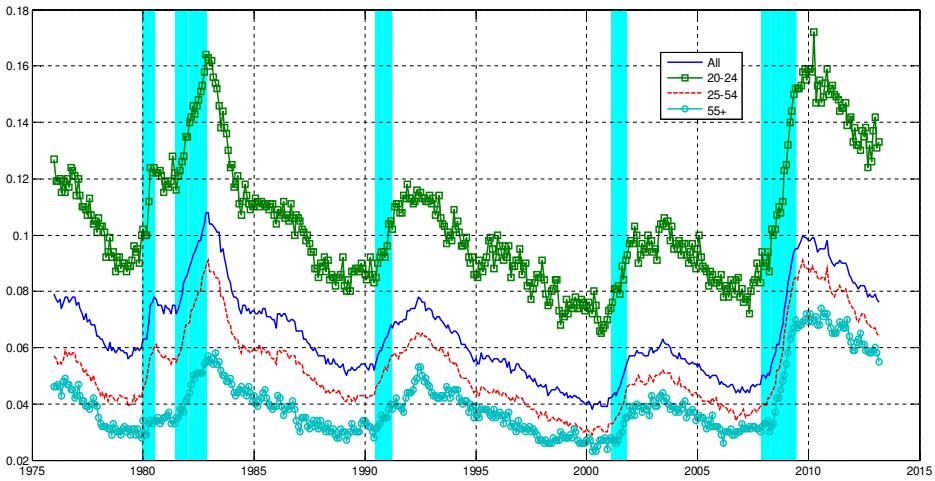
1 Introduction

The last recession has dramatically deteriorated labor market conditions. Figure 1 displays monthly unemployment rate across age groups since 1976. The unemployment ramp up in the aftermath of the financial crisis shows the depth of the so-called Great Recession. Figure 1 shows substantial differences across age groups.

Figure 1: Unemployment rate by age. BLS monthly data. 1976 January - 2013 March



It is often stressed that young workers were hit harder by the economic downturn, and more generally by the business cycle (Elsby et al. (2010)). This view seems to be reinforced by the last recession: their unemployment rate has dramatically risen by 8 percentage points, from 9% at the end of 2007 to 17% in April 2010. On the other hand, prime-age workers' unemployment rate has risen by only 5.5 percentage points, from 3.5% in 2007 to 9% in 2009, whereas the unemployment rate of workers aged 55+ displays an increase of 4 percentage points (from 3% to 7% in the same period). However, focusing on one recession or on changes in absolute value is not enough to establish stylized facts. In this paper, we adopt an agnostic approach and let the data speak. We propose a systematic and quantitative measure of the relative volatility of unemployment rates across age groups based on unconditional relative standard deviations calculated over a large sample. Moreover, as unemployment rate delivers a partial view of labor market adjustments, we document the respective roles of job separation and job finding rates in shaping the change in unemployment across age groups using monthly CPS data.

Beyond this empirical contribution, we investigate whether the Mortensen & Pissarides (1994) model extended to introduce life-cycle feature is well suited to explain flow and stock labor market volatility across age groups. Are age-specific horizons on the labor market enough to explain the relative volatility across age groups? We aim to answer this question by filling the gap

between two strands of literature: the one aiming at matching *levels* of employment rates across age groups without considering business cycle features (Ljungqvist & Sargent (2008), Cheron et al. (2013), Menzio et al. (2012), Gervais et al. (2012)) and the one looking at *fluctuations* in unemployment rates, without considering age heterogeneity (Shimer (2005) and Fujita & Ramey (2012) among others).

In this paper, we first document the relative volatility of transition and unemployment rates across ages using 3 age-groups (16-24, 25-54 and 55-61) in the US labor market. Following Shimer (2012)'s methodology, we measure workers' flows between employment and unemployment using CPS monthly male data between January 1976 and March 2013. Historical averages suggest that older workers are characterized by slower employment exit and longer unemployment spell: job finding and job separation rates fall with age. These results are consistent with the age-decreasing transitions found in the male population in Choi et al. (2013), Menzio et al. (2012), Gervais et al. (2012), Elsby et al. (2010) and Elsby et al. (2011). Beyond these long-run features, we focus on fluctuations in worker flows across age groups. Surprisingly enough, the volatility of transition rates display an age-increasing pattern: older workers appear to be more responsive to the business cycle than their younger counterparts. This feature appears robust to detrending methods, gender, skill groups. The data shows that the higher relative sensitivity of older workers to the business cycle is more striking for the job separation rate than for the job finding rate. We also investigate the business cycle volatility of real hourly wage, using monthly CPS data, and then show that the life-cycle pattern of wage volatility is actually age-decreasing, which mirrors the larger labor market adjustments found for old worker.

Our empirical investigation extends the existing literature along 3 dimensions: firstly, previous papers (Gomme et al. (2005), Jaimovich & Siu (2009)) have focused only on age-specific cyclicality of worked hours and employment using annual data. We extend their work by looking at infra-annual data and documenting labor flows. Secondly, Elsby et al. (2010) and Elsby et al. (2011) study the age pattern of worker flows but they did not investigate business cycle features. Finally, we also characterize wage cyclicality across age groups at business cycle frequencies, using infra-annual data. Jaimovich & Siu (2009) use annual data only. In our view, since the length of recession is usually less than a year (with the exceptions of the 1981 and 2008 episodes), it is paramount to document business cycle features using infra-annual data.

We then propose a Mortensen & Pissarides (1994) (hereafter MP) model including aggregate uncertainty and extended to introduce life-cycle feature to account for the life cycle pattern of the cyclicality of labor market stock and flows workers. All workers are the same except for their work-life expectancy. We then argue that the short work-life expectancy plays a key role in accounting for the high responsiveness of old workers' job flows to the business cycle. The basic intuition is the following: a short horizon prior to retirement age creates no incentive to search, the outside option is then reduced to zero for older workers, leading wages to be more

rigid at the end of the life cycle. Intuitively, older workers act as static agents with no future whereas younger workers behave like infinitively-lived agents. Our theoretical results point out that Shimer (2005)'s view on the MP model[1] is consistent with prime-age workers' labor market while aging endogenously introduces more real wage inertia by disconnecting the bargained wage from labor market tightness. This allows us to match what we observe for old workers, without specific assumptions or calibrations as in Hagendorn & Manovskii (2008) or in Hall & Milgrom (2008).

We first provide an analytical characterization of the age specific impact of the business cycle on labor market outcomes in the model. After calibrating the key parameters of the model to match age-specific levels of job finding and separation rates, we show that the model matches the age-increasing volatility in transition rates, especially between prime-age and old workers. The distance to retirement plays a key role in accounting for old workers' high responsiveness to the business cycle. Let us acknowledge that considering this canonical model leaves no chance to differentiate prime-age workers and young workers, as they share a similar long horizon, ie. the horizon gap is negligible with respect to the duration of their job. We then can only address the relative volatility of older workers with respect to prime-age workers. We choose to follow this route as a first attempt to analyze the fluctuations of the labor market flows into the canonical search and matching model without introducing any other source of heterogeneity across age groups. To some extent, this approach extends to a life cycle environment the analysis proposed Shimer (2005) for the aggregate unemployment volatility.

The paper is organized as follows. Section 2 documents workers' transitions rates by age groups and age patterns in their responsiveness to business cycles on US data. We also investigate wage fluctuations by age. In Section 3, we present the search and matching model with life-cycle features. We provide a steady state analysis (section 4) and an analytical characterization of the age-specific impact of the business cycle on labor market outcomes (Section 5). Section 6 brings the model to the data, after calibrating the key parameters of the model.

2 Measuring fluctuations in unemployment, workers' transition rates and hourly wages by age in US data

We aim at analyzing the cyclical behavior of workers' transition between employment and unemployment by age. We compute the age profile of transitions from employment to unemployment

[1]Shimer (2005) underlines that, following an expansion, the increase in outside opportunities puts an upward pressure on wages (wage reservation effect), which reduces firms' incentives to post new vacancies: the MP model generates a large adjustment in prices and small changes in quantities on the labor market, which is not consistent with the aggregate data. Shimer (2005)'s criticism deals with the consistency of the MP model with aggregate data, under the restriction of a representative infinitively-lived agent.

(job separation) and transitions from unemployment to employment (job finding). [2] The working life cycle before retirement is divided into 3 age groups: 16-24, 25-54 and 55-61. We chose the same age groups as in Elsby et al. (2010), except that we consider only individuals prior to retirement. Since we do not consider retirement choices in the model, we discard individuals aged 62 and more. Finally, we restrict our attention to men, since female transitions are also linked to fertility and child rearing, that are not modeled in the paper. Appendix A.3 shows that the main business cycle facts remain relevant when we include women in the sample.

2.1 Measuring fluctuations in unemployment and workers' transition rates by age in US data

Using monthly CPS data, between January 1976 and March 2013, we follow all the steps described in Shimer (2012). We compute sample-weighted gross flows between labor market states and seasonally adjust the time series using the same ratio-to-moving average technique as in Shimer (2012). We then correct for time aggregation in order to take into account transitions occurring within the month. We get time series for the instantaneous transition rates for each age group and consider quarterly averages to smooth out the noise. We then get quarterly data on workers' instantaneous transition rates (JSR_t and JFR_t) and the corresponding unemployment conditional steady state $\left(u_t = \frac{JSR_t}{JSR_t+JFR_t}\right)$. The time series are displayed in figures 2 and 3.

The mean of the time series is reported in Table 1. We report in tiny characters the mean relative to the mean of prime age workers: for instance, as the average separation rate for young workers is 4.9% per month versus 1.7% for prime-age workers, young workers are characterized by a separation rate that is 2.82 times higher than the one prevailing for prime age workers.

Like Elsby et al. (2010), we find large differences in levels of separation rates across age groups. Young workers who face high unemployment rates are characterized by high rates of entry into unemployment. The average job tenure when young amounts to 21 months ($\frac{1}{1-e^{-0.049}}$) versus 7.6 years when old (91.4 months, $\frac{1}{1-e^{-0.011}}$). Differences in job finding rates are less striking but significant: the length of unemployment spell is 2.6 months ($\frac{1}{1-e^{-0.49}}$) for young workers versus 3.6 months ($\frac{1}{1-e^{-0.33}}$) for old workers. The levels of inflow and outflow rates of unemployment fall with age. As stressed by Elsby et al. (2011) on UK data, with faster exit from employment and shorter unemployment spell, youth face a more fluid labor market than their older counterparts. This faster exit from employment and unemployment is missed when one simply looks at unemployment rates across age groups. In addition, the differences in job finding rates would actually predict an age increasing profile for unemployment. As a result, Table 1 suggests that

[2] We deal with inactivity in Appendix A.2. When we consider the participation margin, flows between employment and unemployment account for a large share of unemployment fluctuations, for all age groups. In addition, even when we account for flows to and from inactivity, the age profile of mean and volatility of labor market transition rates are qualitatively consistent with the results below.

Figure 2: Job Separation Rate by age group, JSR, CPS quarterly averages of monthly data, Men, 1976 Q1 - 2013Q2. Authors' calculations. Recession in shaded area.

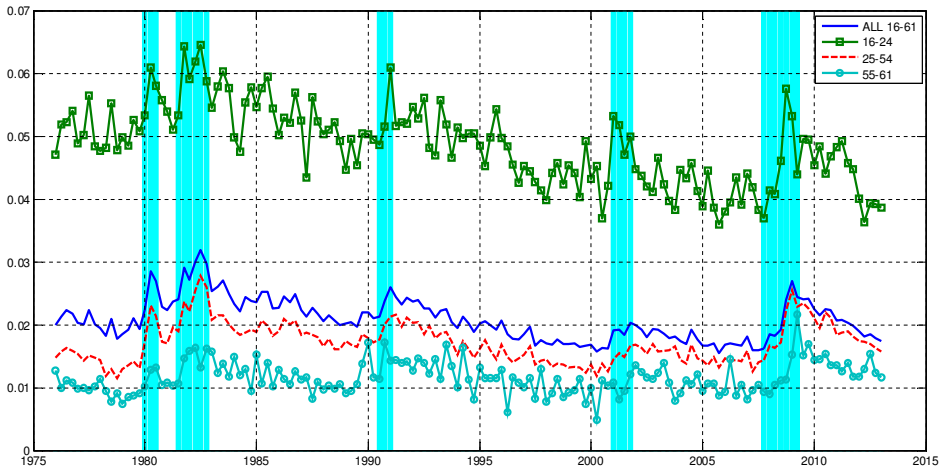
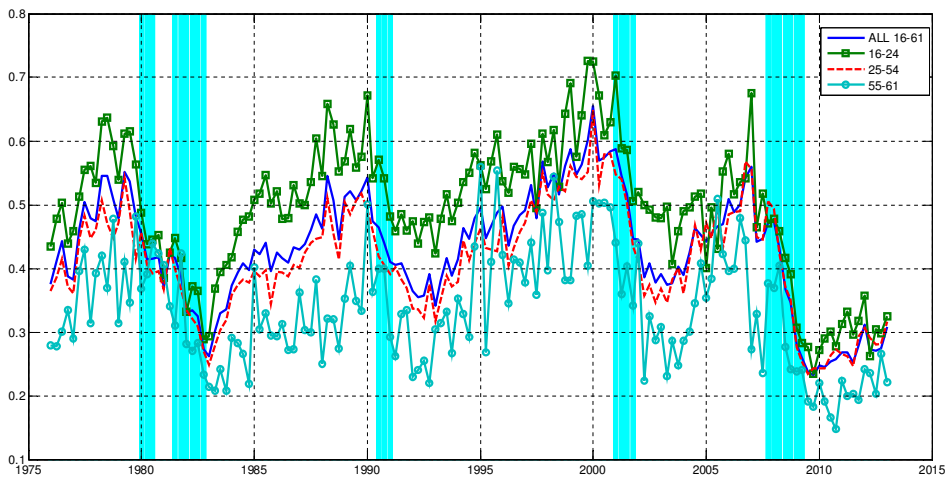


Figure 3: Job Finding Rate by age group, JFR, CPS quarterly averages of monthly data, Men, 1976 Q1 - 2013Q2 Sept. Authors' calculations. Recession in shaded area.



the high level of unemployment rate for the youth is actually driven by their high level of exit from employment. This is consistent with Elsby et al. (2011) and Gervais et al. (2012).

Estimated means are consistent with the decreasing transitions with age found in the male population in Choi et al. (2013), Menzio et al. (2012) and Gervais et al. (2012). The level of our transition rates is larger in our data than in their calculations because we discard labor market transition that are considered in the previous papers (namely inactivity for Choi et al. (2013) and job-to-job transitions for Menzio et al. (2012)). In addition, Menzio et al. (2012) restrict their sample to individuals with a high school degree. [3] Elsby et al. (2010) also compute unemployment outflows and inflows by age, using employment and unemployment stocks in which separations are captured by the flows of workers who report having been unemployed for less than one month (as in Shimer (2012)). We check in Appendix A.4 that our business facts are relevant on Elsby et al. (2010)'s time series.

Table 1: Mean. CPS quarterly averages of monthly data, Men. 1976Q1 - 2013Q1. Authors' calculations.

	All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR	0.021	0.049	0.017	0.011
		2.82	1	0.68 [a]
JFR	0.43	0.49	0.41	0.33
		1.20	1	0.81
u	0.049	0.094	0.0427	0.037
		2.19	1	0.86

[a] Old workers' average JSR is 68% of prime-age workers'.

The cyclical behavior transition rates are characterized in recession by a rise in separations (figure 2) and a significant drop in the job finding rate (figure 3). It is striking that, in recession, the fall in the outflow rate seems of the same magnitude across age groups. For instance, in the 2001 recession, the job finding rate fell by 0.2 for all age groups (from 0.7 to 0.5 for young workers, from 0.6 to 0.4 for prime age and from 0.5 to nearly 0.3 for old workers). However, the magnitude of this fall is very different across age groups when scaled it by the average job finding rate of each age group. Since old (young) workers are characterized by lower (higher) levels of job finding rates, the fall in unemployment outflows in the 2001 recession is large (small) for this age group. Data show that the higher relative sensitivity of the older workers to the business cycle is even more striking for the job separation rate than for the job finding rate. This high (low) responsiveness of old (young) workers, relative to their younger (older) counterparts, is the

[3] The unemployment steady state is consistent with the BLS unemployment rate across age groups (0.11076 for 20-24, 0.051714 for 25-54,0.041289 for 55+ and 0.064696 for 16+. Source: BLS monthly SA data, 1976 Jan-2013 March, Men). In appendix A.8, we also check that steady state unemployment rate by age is consistent with BLS time series

key business cycle fact that we want to investigate.

In order to propose a more comprehensive measure, we then identify the cyclical component of each time series using the Hodrick Prescott filter, with smoothing parameter 10^5 on monthly logged data. Business cycle facts on the de-trended data are reported in Table 2. We find that the volatility of transitions rates increases with age. The increase in volatility in the job finding rate is small from young to prime - aged individuals (the volatility increases from 0.16 to 0.17) than from prime - aged to old individuals (the volatility goes up from 0.17 to 0.22). Old workers' transition rates are highly responsive to the business cycle, much more than young and prime-aged individuals'. This is the key business cycle feature that we want to understand using our theoretical model.

Table 2: Standard deviation. CPS quarterly averages of monthly data, Men, 1976Q1 - 2013Q1, HP filter with smoothing parameter 10^5. Authors' calculations.

	All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR	0.11	0.089	0.14	0.20
		0.62	1	1.40
JFR	0.16	0.16	0.17	0.22
		0.94	1	1.32
u	0.24	0.188	0.274	0.326
		0.69	1	1.19

[a] Old workers' JSR standard deviation is 1.4 times larger than prime-age workers'.

We show in appendix A that this salient feature is robust when we consider an alternative smoothing parameter for the HP filter, inactivity, Elsby et al. (2010) data or transition for men and women. In appendix A.5, we compute rolling six-year standard deviation of de-trended data and show the evolution of changes in relative volatilities. In Appendix A.6, in the spirit of Jaimovich & Siu (2009), we also evaluate the impact of changing age decomposition on our aggregate results. The high sensitivity of old workers' transition rates remains a robust feature. Finally, we check that our age effect is not a composition due, e.g. to a higher proportion of low-skill individuals in the population of older workers. We show in appendix A.7 that the level and the volatilities have the same age profiles within each sub-group, "High school degree and less" and "High school degree and more". Thus our age effect is not a skill effect.

2.2 Real hourly wage

Using monthly CPS data, between January 1995 and March 2013, we document the business cycle response of male real hourly wage across age groups.[4] Hourly wage w is usual weekly earnings

[4]Due to the CPS Re-design in 1994, we only consider data from 1995 onwards. Jaimovich et al. (2013) use the annual March Supplement to look at wage cyclicality. Monthly CPS provide information on earnings from

divided by usual weekly hours. We then deal with outliers, deflate by inflation and technology growth, seasonally adjust, and then take quarterly averages (see Appendix B for further details). Table 3 reports the descriptive statistics for male real hourly wage. The levels are age-increasing, which is consistent with the view that experience makes workers more productive. Interestingly, wage cyclical volatility actually falls with age.

Table 3: CPS quarterly averages of monthly data, male, 1995Q1-2013Q1. Authors' calculations.

	All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
Mean(w)	16.37	10.53	17.69	19.42
		0.6	1	1.1
Std(w)	0.017	0.026	0.019	0.018
		1.37	1	0.93

The empirical evidence seems to point at different patterns of cyclicality across age groups : old worker's job flows are characterized by larger volatility than their younger counterparts and lower wage responsiveness to the business cycle. Our findings on the volatilities per age of the hourly wage are consistent with those on worker flows and unemployment stock. Indeed, in a market with more rigid prices, the large part of adjustments fall on quantities.

3 A life-cycle matching model with aggregate uncertainty

In this section, we present an extension of the Mortensen & Pissarides (1994) model that introduce life-cycle features and aggregate shocks.[5] Beyond the life-cycle pattern of job flows and stocks, our objective is to evaluate the impact of age heterogeneity on business cycle elasticities. Thus, in our economy à la MP, there is stochastic aging, as in Castañeda et al. (2003), Ljungqvist & Sargent (2008) and Hairault et al. (2010). Unlike Cheron et al. (2013), we have aggregate shocks, as in e.g. Fujita & Ramey (2012).[6] In addition, unlike Cheron et al. (2013), we have age-directed search as in Menzio et al. (2012). This implies that there is no externality coming form heterogenous workers in the matching function. The decentralized allocation is then

only one-fourth of the CPS total sample of approximately 60,000 households. We check that our levels of wage are consistent with Heathcote et al. (2010) and BLS weekly earnings by age and that the HP filtered volatility is consistent with Jaimovich et al. (2013)'s.

[5]Since Mortensen & Pissarides (1994) or Mortensen (1994), there have been a lot of papers that provide empirical evaluation of the MP model with respect to its ability to explain labor market fluctuations. This model has been also tested in the context of Dynamic Stochastic General Equilibrium models: see Merz (1994), Langot (1995) or Andolfatto (1996) with exogenous separations, Denhaan et al. (2000) with endogenous separations. Pissarides (2009)'s paper have revived this literature.

[6]Job-to-job transitions (as in Fujita & Ramey (2012)) and savings (as in Lise (2013)) are discarded. These extensions are left for future research.

efficient if the Hosios condition is satisfied.[7] Moreover, this simplifying assumption allows us to solve the model more easily, in the spirit of the solution provided by Menzio & Shi (2010).

Adopting this theoretical setting means that we do not intend to provide a comprehensive explanation of the cyclical properties of the labor market flows and stock over the life cycle: we leave aside the younth labor market to focus on the relative volatility of older workers with respect of prime-age workers. This choice is consistent with our use of the canonical search and matching model and our focus on the horizon effect alone. We believe that it is worth evaluating such a model first before considering further extensions. Moreover, this will allow us to provide analytical results based on conditional steady state analysis along the lines of Shimer (2005) or Nagypal & Mortensen (2007).

3.1 Demographic setting and aggregate shock

The Life Cycle. Unlike the large literature following MP, we consider a life-cycle setting characterized by an age at which workers exit the labor market, interpreted as the exogenous retirement age. There are 2 age groups i: prime age and old workers, respectively A and O. All prime-age workers A enter the labor market as unemployed workers. We assume stochastic aging. The probability of remaining a prime age worker in the next period is (π_A). Conversely, the probability of becoming old is $1-\pi_A$. We divide the periods as old workers into 7 years, one for each year: $O = \{O_i\}_{i=1}^{7}$, with a probability of remaining in age class O_i in the next period is π_{O_i}. With probability $1-\pi_{O7}$, old workers reach the retirement age T: in order to have a constant population size, we assume that this mass of exiting workers is replaced by an equal mass of prime-age workers.

More formally, we assume for simplicity that the matrix Π governing the age markov-process is

$$\Pi = \begin{bmatrix} \pi_A & 1-\pi_A & 0 & \cdots & 0 \\ 0 & \pi_{O_1} & 1-\pi_{O_1} & \cdots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 1-\pi_{O_7} & 0 & 0 & \cdots & \pi_{O_7} \end{bmatrix}$$

With $\pi_i \neq \pi_j$, for $i, j = 1, ..., 8$ and $i \neq j$, the size of age groups are not equal. The size of each group is deduced from Π_∞, the matrix of the unconditional probabilities, given that the total size of the population is normalized to unity. The population of each group can be divided into two types of agent: the unemployed u_i and the employed n_i, such that $m_i = u_i + n_i$, with $1 = \sum_i m_i$. We thereby discard the participation margin. In our view, this is not a very restrictive assumption

[7] Future research will be devoted to reducing the gap between the observed and predicted fluctuations by our first rank allocation. Moreover, given that the undirected search equilibrium is inefficient, the additional trade externality gives some foundations for age-specific labor market policies (See Cheron et al. (2011) and Cheron et al. (2013)).

because we introduce an age-specific search effort which can converge towards zero at the end of the working life, before the retirement age. These old unemployed workers with a zero-search can thus be considered as non-participant. Their number is endogenously determined at the equilibrium.

Shocks. A worker-firm match can produce an output level of $z\epsilon$ during month t, where z and ϵ are respectively the aggregate and the match-specific productivity factor. The aggregate productivity component follows the exogenous process:

$$\log(z') = \rho \log(z) + \nu' \tag{1}$$

where ν' is an i.i.d. normal disturbance with mean zero and standard deviation σ_ν.

Firms are small and each has one job. For a common aggregate component of the productivity z, the destruction flows derive from idiosyncratic productivity shocks that hit jobs at random. At the end of each month t, a new productivity level for month $t+1$ is drawn with probability $\lambda_i \leq 1$ in the distribution $G(\epsilon)$, with $\epsilon \in [0, 1]$. The higher λ_i, the lower the persistence of the current productivity draw. The probability to draw a new match-specific productivity depends on workers' age. This assumption allows to account for the heterogeneous persistence across age groups: when persistence is high, it is more difficult to improve match-specific productivity.

Once a shock occurs, the firm either keeps on producing or destroys the job. Each week, employed workers are faced with layoffs when their job becomes unprofitable. The firms decide to close down any job whose productivity is below a productivity threshold (the reservation productivity) denoted $R_i(z)$.

Finally, unlike MP, new jobs are not opened at the highest productivity. Their productivity level is also drawn in the distribution $G(\epsilon)$.[8] The age-$i-1$ workers who become age-i workers (with probability $1 - \pi_{i-1}$), whereas they have been contacted at the age $i-1$, will be hired if and only if their productivity is larger than the threshold $R_i(z)$, ie. the reservation productivity of an age-i worker, because the productivity value is revealed after the firm has met the worker.

Matching with directed search. We consider an economy where labor market frictions imply that there is a costly delay in the process of filling vacancies. We assume that age is perfectly observed and that a worker who applies to a job not matching its age-characteristic, will have a nil production, and thus a nil surplus. Firms choose how many and what type of vacancies to

[8] This assumption is made in order to make the model more tractable. Indeed, with this productivity draw whether the worker is in the firm or in the pool of job seekers, the comparison of future opportunities reduces to comparing the efficiency of unemployment search $\gamma e_i p(\theta_i)$ to labor hoarding $(1 - s_e)\lambda_i$ (see Section 4.2.). If we had adopted MP's assumption, this would have given a comparative advantage to job seekers relative to workers within the firms. The quantitative results would not have been greatly affected by the switch to MP's assumption.

open, where the type of a vacancy is simply defined by worker's age. Since search is directed, the probability for a worker to meet a firm depends on his age. This assumption clearly makes solving the model much simpler, without altering the quantitative business cycle implications.

Since firms can *ex-ante* age-direct their search, there is one matching function by age. Let $v_i(z)$ be the number of vacancies, $u_i(z)$ the number of unemployed workers, and $e_i(z)$ the endogenous search effort for a worker of age i. The matching function gives the number of contact, $M(v_i(z), e_i(z)u_i(z))$, where M is increasing and concave in both its arguments, and with constant returns-to-scale. From the perspective of a firm, the contact probability is $q(\theta_i(z)) \equiv \frac{M(v_i(z),e_i(z)u_i(z))}{v_i(z)} = M(1, \theta_i^{-1}(z))$ with $\theta_i(z) = \frac{v_i(z)}{e_i(z)u_i(z)}$ the corresponding labor market tightness. The probability for unemployed workers of age i of being employed is then defined by $e_i(z)p(\theta_i(z))[1 - G(R_i(z))]$ with $p(\theta_i(z)) \equiv \frac{M(v_i(z),e_i(z)u_i(z))}{e_i(z)u_i(z)} = M(\theta_i(z), 1)$ the contact probability of the effective unemployed worker. Note that the hiring process is then age-differentiated via an age-specific search intensity ($v_i(z)$), an age-specific reservation productivity ($R_i(z)$) and an age-specific search effort ($e_i(z)$).

3.2 Firms' and workers' intertemporal values

Firms' problem. Any firm is free to open a job vacancy directed to an age-specific labor market and engage in hiring. c denotes the flow cost of hiring a worker and $\beta \in [0, 1]$ the discount factor. Let $V_i(z)$ be the expected value of a vacant position in age-i labor market, given the aggregate state of the economy z at time t, and $J_i(z, \epsilon)$ the value of a job filled by a worker of age i with productivity ϵ in aggregate state z. The firm's search value is given by:

$$V_i(z) = -c + q(\theta_i(z))\beta E_z \left[\pi_i \int_0^1 J_i(z', x)dG(x) + (1 - \pi_i)V_i(z')\right] + (1 - q(\theta_i(z)))\beta E_z V_i(z')$$

where the operator E_z denote the expectation with respect to the aggregate productivity component z. Given that search is directed, if the worker ages between the meeting and the production processes (with a probability $1 - \pi_i$), the job is not filled. We will assume hereafter the standard free-entry condition, ie. $V_i(z) = 0$, $\forall i, z$, which leads to:

$$\frac{c}{q(\theta_i(z))} = \beta \pi_i E_z \int_0^1 J_i(z', x)dG(x)$$

Vacancies are determined according to the expected value of a contact with an age-i unemployed worker, which depends on the uncertainty in the hiring process arising from the two components of productivity z and ϵ.

Given a state vector (z, ϵ) and for a bargained wage $w_i(z, \epsilon)$, the expected value $J_i(z, \epsilon)$ of a filled

job by a worker of age i, $\forall i \in [A, ..., O_6]$, is defined by:

$$J_i(z,\epsilon) \quad = \quad \max \left\{ \begin{array}{l} z\epsilon - w_i(z,\epsilon) \\ +\beta\pi_i(1-s_e)\left(\begin{array}{c} \lambda_i E_z \int_0^1 J_i(z',x)dG(x) \\ +(1-\lambda_i)E_z J_i(z',\epsilon) \end{array} \right) \\ +\beta(1-\pi_i)(1-s_e)\left(\begin{array}{c} \lambda_{i+1} E_z \int_0^1 J_{i+1}(z',x)dG(x) \\ +(1-\lambda_{i+1})E_z J_{i+1}(z',\epsilon) \end{array} \right) \end{array} ;0 \right\}$$

Notice that, for $i = O_7$, aging implies retirement. The value function becomes:

$$J_{O_7}(z,\epsilon) \quad = \quad \max \left\{ \begin{array}{l} z\epsilon - w_{O_7}(z,\epsilon) \\ +\beta\pi_{O_7}(1-s_e)\left(\lambda_{O_7} E_z \int_0^1 J_{O_7}(z',x)dG(x) + (1-\lambda_{O_7})E_z J_{O_7}(z',\epsilon)\right) \end{array} ;0 \right\}$$

The short horizon reduces the value of a filled job, for a given wage.

Workers' problem. Values of employed (on a match of productivity ϵ) and unemployed workers of any age $i \neq O_7$, are respectively given by:

$$\mathcal{W}_i(z,\epsilon) \quad = \quad \max \left\{ \begin{array}{l} w_i(z,\epsilon) \\ +\beta\pi_i \left[\begin{array}{l} (1-s_e)\left(\begin{array}{c} \lambda_i E_z \int_0^1 \mathcal{W}_i(z',x)dG(x) \\ +(1-\lambda_i)E_z \mathcal{W}_i(z',\epsilon) \end{array} \right) \\ +s_e E_z \mathcal{U}_i(z') \end{array} \right] \\ +\beta(1-\pi_i) \left[\begin{array}{l} (1-s_e)\left(\begin{array}{c} \lambda_{i+1} E_z \int_0^1 \mathcal{W}_{i+1}(z',x)dG(x) \\ +(1-\lambda_{i+1})E_z \mathcal{W}_{i+1}(z',\epsilon) \end{array} \right) \\ +s_e E_z \mathcal{U}_{i+1}(z') \end{array} \right] \end{array} ;\mathcal{U}_i(z) \right\}$$

$$\mathcal{U}_i(z) \quad = \quad \max_{e_i(z)} \left\{ \begin{array}{l} b - \phi(e_i(z)) \\ +\beta\pi_i \left(\begin{array}{c} e_i(z)p(\theta_i(z))E_z \int_0^1 \mathcal{W}_i(z',x)dG(x) \\ +(1-e_i(z)p(\theta_i(z)))E_z \mathcal{U}_i(z') \end{array} \right) \\ +\beta(1-\pi_i) \left(\begin{array}{c} e_{i+1}(z)p(\theta_{i+1}(z))E_z \int_0^1 \mathcal{W}_{i+1}(z',x)dG(x) \\ +(1-e_{i+1}(z)p(\theta_{i+1}(z)))E_z \mathcal{U}_{i+1}(z') \end{array} \right) \end{array} \right\}$$

with $b \geq 0$ denoting the instantaneous opportunity cost of employment and $\phi(.)$ the convex function capturing the disutility of search effort e_i. For $i = O_7$, these values are simply given by

$$\mathcal{W}_{O_7}(z,\epsilon) \quad = \quad \max \left\{ \begin{array}{l} w_{O_7}(z,\epsilon) \\ +\beta\pi_{O_7} \left[\begin{array}{l} (1-s_e)\left(\begin{array}{c} \lambda_{O_7} E_z \int_0^1 \mathcal{W}_{O_7}(z',x)dG(x) \\ +(1-\lambda_{O_7})E_z \mathcal{W}_{O_7}(z',\epsilon) \end{array} \right) \\ +s_e E_z \mathcal{U}_{O_7}(z') \end{array} \right] \end{array} ;\mathcal{U}_{O_7}(z) \right\}$$

$$\mathcal{U}_{O_7}(z) \quad = \quad \max_{e_{O_7}(z)} \left\{ \begin{array}{l} b - \phi(e_{O_7}(z)) \\ +\beta\pi_{O_7} \left(\begin{array}{c} e_{O_7}(z)p(\theta_{O_7}(z))E_z \int_0^1 \mathcal{W}_{O_7}(z',x)dG(x) \\ +(1-e_{O_7}(z)p(\theta_{O_7}(z)))E_z \mathcal{U}_{O_7}(z') \end{array} \right) \end{array} \right\}$$

The optimal search effort decision of the worker then satisfies the following condition:

$$\phi'(e_i(z)) = \beta\pi_i p(\theta_i(z))E_z\left[\int_0^1 \mathcal{W}_i(z',x)dG(x) - \mathcal{U}_i(z')\right]$$

The marginal cost of search effort at age i is equalized to its expected marginal return.

3.3 Job surplus, Nash sharing rule and reservation productivity

The surplus $S_i(z,\epsilon)$ generated by a job of productivity $z\epsilon$ is the sum of the worker's and the firm's surplus: $S_i(z,\epsilon) \equiv \mathcal{W}_i(z,\epsilon) - \mathcal{U}_i(z) + J_i(z,\epsilon)$, given that $V_i(z) = 0$ at the equilibrium. Thus, using the definitions of $J_i(z,\epsilon)$, $\mathcal{W}_i(z,\epsilon)$, and $\mathcal{U}_i(z)$, the surplus is given by:

$$S_i(z,\epsilon) \quad = \quad \max\left\{\begin{array}{l} z\epsilon - b + \phi(e_i(z)) \\ +\beta\pi_i(1-s_e)\left(\begin{array}{l}\left[\lambda_i - \frac{\gamma e_i(z)p(\theta_i(z))}{1-s_e}\right]E_z\int_0^1 S_i(z',x)dG(x) \\ +(1-\lambda_i)E_zS_i(z',\epsilon)\end{array}\right) \\ +\beta(1-\pi_i)(1-s_e)\left(\begin{array}{l}\left[\lambda_{i+1} - \frac{\gamma e_{i+1}(z)p(\theta_{i+1}(z))}{1-s_e}\right]E_z\int_0^1 S_{i+1}(z',x)dG(x) \\ +(1-\lambda_{i+1})E_zS_{i+1}(z',\epsilon)\end{array}\right) \end{array}\right. ;0\Bigg\}$$

The reservation productivity $R_i(z)$ can then be defined by the condition $S_i(z,R_i(z)) = 0$. As in MP, a crucial implication of this rule is that the job destruction is mutually optimal, for the firm and the worker. $S_i(z,R_i(z)) = 0$ indeed entails $J_i(z,R_i(z)) = 0$ and $\mathcal{W}_i(z,R_i(z)) = \mathcal{U}_i(z)$. Note that the lower bound of any integral over $S_i(z,\epsilon)$ is actually the reservation productivity, as no productivity levels below the reservation productivity yield a positive job surplus. Given $S_i(z,\epsilon)$, the Nash bargaining leads to

$$\mathcal{W}_i(z,\epsilon) - \mathcal{U}_i(z) = \gamma S_i(z,\epsilon) \quad \text{and} \quad J_i(z,\epsilon) = (1-\gamma)S_i(z,\epsilon)$$

Using this sharing and the definitions of the value functions, the wage rule is

$$w_i(z,\epsilon) = \gamma\left(z\epsilon + ce_i(z)\theta_i(z) + \frac{1-\pi_i}{\pi_{i+1}}ce_{i+1}(z)\theta_{i+1}(z)\right) + (1-\gamma)\left(b - \phi(e_i(z))\right) \tag{2}$$

Because workers age, the returns on search activity is an average between age i and age $i+1$.

3.4 Equilibrium

Definition 1. *The labor market equilibrium with directed search in a finite-horizon environment is defined by the search efforts of vacant jobs and unemployed workers, respectively $\theta_i(z)$ and*

$e_i(z)$, and the separation rule (the reservation productivity), $R_i(z)$:

$$\frac{c}{q(\theta_i(z))} = (1-\gamma)\beta\pi_i E_z \overline{S}_i(z') \tag{3}$$

$$\phi'(e_i(z)) = \gamma p(\theta_i(z))\beta\pi_i E_z \overline{S}_i(z') \tag{4}$$

$$\begin{aligned} zR_i(z) = {} & b - \phi(e_i(z)) \\ & -\beta\pi_i(1-s_e)\left(\begin{array}{l} \left[\lambda_i - \frac{\gamma e_i(z)p(\theta_i(z))}{1-s_e}\right] E_z\overline{S}_i(z') \\ +(1-\lambda_i)E_z S_i(z', R_i(z)) \end{array}\right) \\ & -\beta(1-\pi_i)(1-s_e)\left(\begin{array}{l} \left[\lambda_{i+1} - \frac{\gamma e_{i+1}(z)p(\theta_{i+1}(z))}{1-s_e}\right] E_z\overline{S}_{i+1}(z') \\ +(1-\lambda_{i+1})E_z S_{i+1}(z', R_i(z)) \end{array}\right) \end{aligned} \tag{5}$$

given the average and individual surpluses :

$$\overline{S}_i(z') \equiv \int_{R_i(z')}^{1} S_i(z', x)dG(x) \tag{6}$$

$$S_i(z,\epsilon) = \max\left\{\begin{array}{l} z(\epsilon - R_i(z)) + \beta\pi_i(1-\lambda)(1-s_e)E_z[S_i(z',\epsilon) - S_i(z', R_i(z))] \\ +\beta(1-\pi_i)(1-\lambda)(1-s_e)E_z[S_{i+1}(z',\epsilon) - S_{i+1}(z', R_i(z))] \end{array} ; 0\right\} \tag{7}$$

The stock-flow dynamics on the labor market are given in Appendix C.1 by equations (25), (26), (27) and (28), whereas the dynamics of the aggregate shock is given by (1).

Proposition 1. *Directed search implies that the problem is block-recursive.*

Proof. As in Menzio & Shi (2010), if we find a fix point for $S_{O_7}(z,\epsilon)$ which is a function of choices at the age O_7 (the terminal age) only, we then obtain $\{S_{O_7}(z,\epsilon), \theta_{O_7}(z), R_{O_7}(z), e_{O_7}(z)\}$, $\forall z, \epsilon$, using the equations (3), (5) and (4). Given these solutions for the labor market of the age-O_7 workers, we can solve for the age-O_6 workers using the system given in definition 1 until age $i = A$. □

The algorithm for this first step is the same as the one described in Fujita & Ramey (2012) for the solution of the MP model in the infinite horizon case. It is extended to account for the endogenous search effort from unemployed workers. See Appendix C.5 for a description of the algorithm.

Proposition 2. *If the Hosios condition is satisfied, the equilibrium is efficient.*

Proof. By backward induction, it is trivial to show that, if $\gamma = \eta$, then the labor market allocation for older workers is efficient. Given this result, the same conclusion applies on prime-age workers' labor market. □

In this paper, we assume that this restriction is satisfied. This simplifying assumption, in the spirit of Menzio & Shi (2010), is also used in Menzio et al. (2012). It then allows us to measure

the gap between optimal fluctuations and the observed business cycle. Cheron et al. (2011) discuss the impact of a non-directed search equilibrium per age. They show that non-directed search generates additional trade externality giving some foundations to age-specific labor market policies: because older workers create a negative externality in the hiring process, it could be optimal to retain them inside the firms or to subsidy their non-participation.

4 Steady state analysis: Accounting for the age-decreasing levels of transition rates

In this section, we show that the model may replicate the age-decreasing levels of transition rates found on the data (Table 1) only under some conditions. We consider the steady state of our economy, and stress the key mechanisms that allow to match the pattern of labor reallocation of prime-age and old workers. This steady state analysis is a first step for the validation of our theory: the model is required to match the average transition rates measured over all the sample. This analysis will also be very helpful to simplify the exposition of economic mechanisms when we consider the stochastic economy (section 5).

The fall of transition rates with age in the data implies that, at the steady state, for age i, the model must generate an age-pattern of transition rates such that

$$JSR_i \approx s_e + (1 - s_e)\lambda_i G(R_i) > JSR_{i+1} \tag{8}$$

$$JFR_i \approx e_i p(\theta_i)[1 - G(R_i)] > JFR_{i+1} \tag{9}$$

If we only focus on endogenous variables[9], the job separation rate is only a function of the reservation productivity (R_i), whereas the job finding rate is a function of search efforts ($\{e_i; \theta_i\}$ and the reservation productivity. This observed pattern is puzzling since, in the model, the value of a match is determined by a single variable, its surplus. Using this single variable, the model must produce an equilibrium outcome in which it is optimal to have old workers kept inside the firm while, at the same time, firms are less willing to hire them.

4.1 Steady state equilibrium by age

Since Mortensen & Pissarides (1994)'s seminal paper, the information in the surplus (equation (7) in definition 1) can be decomposed into two parts: *(i)* the expected average value of the surplus gives the incentive to search ($\overline{S}_i$ in equations (3) and (4)), *(ii)* whereas its marginal value provides information on the separation decision (equation (5)). Given that $e_i(z)$ can be easily

[9]The job separation rate is also a function of the age-specific probability to draw an new match-specific productivity (λ_i).

deduced from $\theta_i(z)$ (equations (4) and (3)), we only focus on the labor market tightness and reservation productivity. Our analysis in this section relies heavily on Cheron et al. (2013).

At the conditional steady state, ie. a steady state indexed by a permanent level of z, we have, omitting z for the sake of simplifying the notations:

$$\frac{c}{q(\theta_i)} = (1-\gamma)\beta\pi_i\overline{S}_i \tag{10}$$

$$R_i = b + \Sigma_i - \Lambda_i - \Gamma_i(R_i) \tag{11}$$

where Σ_i, Λ_i and $\Gamma_i(R_i)$ denote respectively the value of search opportunities $\Sigma_i = \pi_i\gamma e_i p(\theta_i)\beta\overline{S}_i + (1-\pi_i)\gamma e_{i+1}p(\theta_{i+1})\beta\overline{S}_{i+1} - \phi(e_i)$, labor hoarding $\Lambda_i = \pi_i(1-s_e)\lambda_i\beta\overline{S}_i + (1-\pi_i)(1-s_e)\lambda_{i+1}\beta\overline{S}_{i+1}$, and the continuation value $\Gamma_i(R_i) = (1-s_e)(1-\lambda_i)(1-\pi_i)\beta S_{i+1}(R_i)$. Given that $\overline{S}_i$ is a function of R_i and Σ_i a function of θ_i, equation (10) represents the job creation curve (JC), whereas (11) is the job destruction curve (JD). At each age i, and thus for a given set of variable representing the equilibrium at age $i+1$, JC defines a negative relationship between θ_i and R_i: the firm's investments in search are large when a wide set of jobs is profitable (low value of R_i). JD defines a positive relationship between θ_i and R_i: the set of profitable jobs is reduced when the search process allows to be more selective (high value of θ_i).

We will show in the following sections that there exists an equilibrium such that reservation productivity falls with age (section 4.2) as well as labor market tightness, and hence search effort (section 4.3). Both elements play a role in generating an age-decreasing profile of transition rates.

4.2 The steady-state life-cycle pattern of reservation productivity

Equation (8) suggests that the age-decreasing job separation rate can be obtained as soon as the reservation productivity falls with age. The reservation productivity (equation (11)) is the sum of unemployed worker's current surplus (unemployment benefit and home production, (b) and the net value of new opportunities. The latter consists of the difference between the return on opportunities outside the firm (Σ_i) and within the firm at the current productivity level ($\Gamma_i(R_i)$) or after a change in productivity (Λ_i). The reservation productivity differs across age groups because workers differ with respect to their expected time on the labor market. For workers who are close to retirement, only current surplus matters. The reservation productivity converges to the unemployed worker's current surplus b as the worker ages. In contrast, prime-age workers have a long work-life expectancy on the labor market. We have basically two cases. In the first one, the value of the labor hoarding Λ_i is larger than the search returns Σ_i. In this case, $R_i < b$ and the reservation productivity can only be age-increasing, which is not in accordance with the data of the job separation rates. In the other case, we have $\Sigma_i > \Lambda_i$, leading to $R_i > b$: an age-decreasing pattern of the job separation is then possible.

Thus, reservation productivity for prime age workers is larger than that of their older counterparts ($R_i > R_{i+1}$) as long as the expected return on their search activity is larger than the expected gain from opportunities within the firm ($\Sigma_i > \Lambda_i + \Gamma_i(R_i)$). If we consider the marginal job, we have $\Gamma_i(R_i) \to 0$. Thus, a sufficient condition for $R_i > R_{i+1}$ is $\Sigma_i > \Lambda_i$ or $\gamma e_i p(\theta_i) > (1 - s_e)\lambda_i$, $\forall i$. This gap between $\gamma e_i p(\theta_i)$ and λ_i simply measures the efficiency of unemployment search relative to labor hoarding. This condition underlines that the value of search can be manipulated by agents through their choices on $\{e_i, \theta_i\}$ whereas the labor hoarding value is driven by an exogenous process λ_i: the younger they are, the larger the incentive to invest in the labor market because a longer horizon allows them to recoup more easily their search costs. This age-dynamic of reservation productivity is consistent with the evidence found in the US data reported in Table 1. [10]

4.3 Hirings and search efforts along the life-cycle

According to Equation (9), the decline in the reservation productivity tends to increase the job finding rate along the life cycle, since more jobs become acceptable. This is not consistent with the data (Table 1). For the job finding rate to decline with age, the model must predict a large decline in search efforts ($\{e_i; \theta_i\}$) and thus a large decline in the meeting rate (through the contact probability of the unemployed worker ($e_i p(\theta_i)$)) at the end of the working life. The fall in e_i and θ_i must outweight the decline in the reservation productivity. Recall that, for age group i, $\{e_i, \theta_i\}$ are positively linked to the expected surplus $\overline{S}_i$, which is given by equation (6). We determine below the age profile of the expected surplus from a match.

If the reservation productivity is age-decreasing, implying an age-decreasing separation rates as in the data, then we have $S_i(\epsilon) = \Omega_i(\epsilon - R_i)$ where Ω_i is a polynomial which accounts for actualization, leading to $\Omega_i > \Omega_{i+1}$. In this case, we have $\overline{S}_i = \int_{R_i}^1 S_i(x)dG(x) = \Omega_i \int_{R_i}^1 (x - R_i)dG(x) = \Omega_i \int_{R_i}^1 [1 - G(x)]dx$. Thus we have [11]:

$$\overline{S}_i - \overline{S}_{i+1} = (\Omega_i - \Omega_{i+1}) \int_{R_i}^1 [1 - G(x)]dx - \Omega_{i+1} \int_{R_{i+1}}^{R_i} [1 - G(x)]dx$$

where *(i)* the first term accounts for the "horizon effect", leading the expected surplus to be age-decreasing ($\overline{S}_i > \overline{S}_{i+1}$) because, for a given draw of match-specific productivity ϵ, a shorter horizon prior to retirement leads each match-specific surplus to decline with the worker's age ($S_i(\epsilon) > S_{i+1}(\epsilon) \Leftrightarrow \Omega_i - \Omega_{i+1} > 0$), and *(ii)* the second term captures a "selection effect" leading surplus to be age-increasing ($\overline{S}_i < \overline{S}_{i+1}$) because, when $R_i > R_{i+1}$, older workers are less selective

[10] In Cheron et al. (2013), all the other cases are analyzed: the age-increasing reservation productivity case, implying $\Sigma_i < \Lambda_i$ $\forall i$, and the U-shape pattern of the reservation productivity, implying $\Sigma_i > \Lambda_i$, for $i < \widehat{i}$ and $\Sigma_i < \Lambda_i$, for $i \geq \widehat{i}$. Given that the US data are not in accordance with these two last cases, we restrict our analysis to case where the steady state of the model match the long run value of JSR.

[11] see Appendix C.2

than younger workers when new opportunities are available for them. If the number of new jobs that become profitable after aging, relative to the number of jobs that are already profitable at younger ages, is small enough, the *expected* surplus can be age-decreasing. In this case, where the "horizon effect" dominates the "selection effect", the expected surplus declines with age, leading search efforts (e_i and θ_i) to be age-decreasing. [12]

5 The age-specific impact of the business cycle in the model

Does the restrictions allowing to match the steady state per age of the worker flows compatible with the stylized facts by which older worker flows' are more sensitive to the business cycle than their younger counterparts? Indeed, in the previous section, we have shown that the valuation of future options depends on the workers' age. This necessarily affects the business cycle elasticities which become age-dependent. Following Shimer (2005) or Nagypal & Mortensen (2007), we derive comparative statics results that describe how the equilibrium market tightness changes with aggregate productivity across steady states. For this type of analysis, it is sufficient to focus on conditional steady states (see Nagypal & Mortensen (2007)), ie. the equilibrium contingent to a permanent realization of the aggregate productivity z, and to approximate the impact of shock on z as a permanent change. In the same line, we present analytical results under the approximation that transitions between ages do not matter for the fluctuations within an age class: the high persistence in an age class leads to neglect the low probability of aging at business cycle frequencies[13].

5.1 Log-linearizing the model

For expositional simplicity, we also assume in what follows that the disutility of search effort is such that $\phi(e_i) = \frac{e_i^{1+\phi}}{1+\phi}$ and that the matching function is given by $M(v_i, e_i u_i) = v_i^{1-\eta}(e_i u_i)^{\eta}$, with $\phi > 0$ and $\eta \in (0; 1)$.

[12] This restriction seems to be realistic simply because new jobs becoming profitable with aging are actually in a *small* interval ($[R_i, R_{i+1}]$), whereas the other jobs are in a *large* interval (at least $[R_i(z), 1]$). Empirically, we need the selection effect to be small enough. In which case, the age-dynamics of the expected surplus is driven by the horizon effect. The model can then generate an age-decreasing JFR. Notice that beyond its impact on the expected surplus, the selection effect has also a direct impact on the JFR though $1 - G(R_i)$: after a meeting, it is more likely to be productive for old workers. Thus, at the end, we need a large enough horizon effect in order to dominate the other two channels (the direct channel through e_i and θ_i and the indirect effect through R_i) through which the selection effect generates an age-increasing profile for the JFR.

[13] This assumption is also consistent with the measures of the worker flows in the data where we consider that the transitions within an age class are not influenced by the fraction of workers who age in a period (the month). This approximation is acceptable if $\frac{1-\pi_i}{\pi_{i+1}} \to 0$. In the calibrated model, we have $\frac{1-\pi_i}{\pi_{i+1}} \in [7e-4; 2e-2]$ showing that this approximation is acceptable.

Volatilities can be inferred from a log-linear approximation of equations (8) and (9)

$$\widehat{JSR}_i = \frac{JSR_e}{JSR}\varepsilon_{G|R_i}\widehat{R}_i \tag{12}$$

$$\widehat{JFR}_i = \widehat{e}_i + (1-\eta)\widehat{\theta}_i - \frac{G(R_i)}{1-G(R_i)}\varepsilon_{G|R_i}\widehat{R}_i \tag{13}$$

where $\varepsilon_{G|R_i}$ denotes the elasticity of the function G with respect to R_i. The job separation rate is driven by counter-cyclical movements in $\widehat{R}_i$ while the counter-cyclical behavior of the reservation productivity reinforces the pro-cyclical behavior of the job finding rate. To determine the age profile of volatility in JSR and JFR, we will examine below the responsiveness of reservation productivity (section 5.2) and search efforts (section 5.3). Both variables, reservation productivity and labor market tightness, are the solution of a system of equations involving the free entry condition and zero surplus.

More specifically, in order to decompose the macroeconomic impact of the aggregate productivity shock, we consider the following system:[14]

$$(JC) \quad \begin{cases} \frac{c}{q(\theta_i(z))} = \beta\pi_i J_i(z) \\ J_i(z) = ze(R_i(z)) - w_i(z) + (1-\gamma)[(1-G(R_i(z)))\Lambda_i(z) + \Gamma_i(z)] \\ w_i(z) = \gamma ze(R_i(z)) + (1-\gamma)\left(b + \Sigma_i(z)\right)(1-G(R_i(z))) \end{cases}$$

$$(JD) \quad \begin{cases} zR_i(z) = w_i(z, R_i(z)) - (1-\gamma)[\Lambda_i(z) + \Gamma_i(z, R_i(z))] \\ w_i(z, R_i(z)) = \gamma z R_i(z) + (1-\gamma)\left(b + \Sigma_i(z)\right) \end{cases}$$

where $J_i(z) = \int_{R_i(z)}^1 J_i(z,x)dG(x)$, $w_i(z) = \int_{R_i(z)}^1 w_i(z,x)dG(x)$, $e(R_i(z)) = \int_{R_i(z)}^1 xdG(x)$ and $\Gamma_i(z) = \int_{R_i(z)}^1 \Gamma_i(z,x)dG(x)$. These variables denote respectively the expected value of a filled vacancy for the firm, the *ex-ante* average wage cost, the average productivity and the average of the continuation value.

The Log-linear approximation of the previous system leads to the following dynamic of the (JC) curve: [15]

$$(JC) \quad \begin{cases} \widehat{\theta}_i = \frac{1}{1-\eta}\widehat{J}_i \\ \widehat{J}_i = \frac{ze(R_i)}{ze(R_i(z))-w_i(z)}\widehat{z} - \frac{w_i}{ze(R_i(z))-w_i(z)}\widehat{w}_i - R_i G'(R_i)\frac{zR_i+(1-\gamma)\Lambda_i}{ze(R_i(z))-w_i(z)}\widehat{R}_i \\ \widehat{w}_i = \gamma\frac{ze(R_i)}{w_i}\widehat{z} + (1-\gamma)\frac{\Sigma_i(1-G(R_i))}{w_i}\widehat{\Sigma}_i - R_i G'(R_i)\frac{w(R_i)}{w_i}\widehat{R}_i \end{cases}$$

[14] Given that all the variables are evaluated conditionally on a particular level of z, we have:

$$\begin{aligned} \Sigma_i(z) &= -\phi(e_i(z)) + \beta\gamma\left[\pi_i e_i(z)p(\theta_i(z))\overline{S}_i(z) + (1-\pi_i)e_{i+1}(z)p(\theta_{i+1}(z))\overline{S}_{i+1}(z)\right] \\ \Lambda_i(z) &= \beta(1-s_e)\left[\pi_i\lambda_i\overline{S}_i(z) + (1-\pi_i)\lambda_{i+1}\overline{S}_{i+1}(z)\right] \\ \Gamma_i(z,\epsilon) &= \beta(1-s_e)(1-\pi_i)(1-\lambda_{i+1})S_{i+1}(z,\epsilon) \end{aligned}$$

[15] Using the approximation $\frac{1-\pi_i}{\pi_{i+1}} \to 0$, we obtain $\Lambda_i(z) = \beta(1-s_e)\pi_i\lambda_i\frac{J_i(z)}{1-\gamma}$. This leads to

$$J_i(z) = \frac{ze(R_i(z)) - w_i(z)}{1-\beta(1-s_e)\pi_i\lambda_i\frac{1-(1-\lambda_i)G(R_i(z))}{1-\lambda_i}} \quad \text{and} \quad \widehat{\Lambda}_i = \widehat{J}_i$$

Fluctuations in the search effort of the firm, $\widehat{\theta}_i$ are driven by the dynamics of the job value $\widehat{J}_i$. The wage equation then shows that changes in (JC) are dampened if the search value is highly pro-cyclical: high wages in boom reduce the increase in the firm value and thus moderates the incentive for firms to hire more workers. Another crucial point is the fact that, when we combine the $\widehat{J}_i$ and the $\widehat{w}_i$ equations, the term on $\widehat{R}_i$ disappears: indeed, the ex-ante productivity draw leads firms to compare expected productivity to expected labor costs, both take into account the dynamics of the selective process accounting by R_i. Hence, this model with endogenous destructions shares the same properties with respect to the JC curve dynamics as the simple model with exogenous destructions. This (JC) system then provides the intuition behind Shimer (2005)'s criticism of the DMP model: ex-ante wage variations dampen the firm surplus volatility, thereby reducing the incentive to invest in vacancy in economic booms. This leads the DMP model to predict small changes in labor market quantities.

The Log-linear approximation of the (JD) system are given by:

$$(JD) \qquad \left\{ \begin{array}{rcl} \widehat{R}_i & = & -\widehat{z} + \frac{w_i(R_i)}{b+\Sigma_i}\widehat{w}_i^r - \frac{(1-\gamma)\Lambda_i}{b+\Sigma_i}\widehat{\Lambda}_i - \frac{(1-\gamma)\Gamma_i(R_i)}{b+\Sigma_i}\widehat{\Gamma}_i^r \\ \widehat{w}_i^r & = & \gamma\frac{zR_i}{w_i(R_i)}(\widehat{z} + \widehat{R}_i) + (1-\gamma)\frac{\Sigma_i}{w_i(R_i)}\widehat{\Sigma}_i \end{array} \right.$$

The system shows that movements in (JD) curve are also dampened if the wage is highly pro-cyclical.

The size of the age-specific ex-ante wage response to the business cycle is then at the heart of the age-specific volatilities of worker flows. In the two wage equations, intertemporal behaviors, summarized by fluctuations in the search value $\widehat{\Sigma}_i$, magnify the wage sensitivity to the business cycle. The longer the worker's opportunity to find a job, the larger the ex-ante wage sensitivity to changes in the value of the labor market prospects. Thus, others things being equal, if, for older workers, Σ_i is small, the pro-cyclical behavior of ex-ante wage is dampened. Given that our steady state analysis leads to the restrictions $\Sigma_i > \Sigma_{i+1}$ (the horizon effect), the elasticity of ex-ante wages (the average wage for the (JC) curve, or the reservation wage for the (JD) curve) to the search option (Σ_i) declines with the worker's age. Hence, ex-ante wages become more rigid when workers age. Other thinks being equal, this generate more volatility in the quantities (worker inflows and outflows) in the labor market for older workers.

5.2 Job separation rate volatility

Given that the job separation rate is a monotonic function of the reservation productivity, the analysis of this latter variable is sufficient to characterize the age pattern of separation fluctuations. Combining the wage reservation equation and the reservation productivity of the (JD)-curve, using the definition of the surplus and its approximation[16], we have $\widehat{\Gamma}_i^r(z) \approx \widehat{R}_i$,

[16]See Appendix C.4.

leading to

$$\widehat{R}_i = -\frac{b+\Sigma_i}{b+\Sigma_i+\Gamma_i(R_i)}\widehat{z} + \frac{\Sigma_i}{b+\Sigma_i+\Gamma_i(R_i)}\widehat{\Sigma}_i - \frac{\Lambda_i}{b+\Sigma_i+\Gamma_i(R_i)}\widehat{\Lambda}_i$$

Given that the Log-linear approximations of the free entry condition and the FOC w.r.t e lead to $\widehat{\Sigma}_i \approx \frac{1+\phi}{\phi}\widehat{\theta}_i$ and $\widehat{\Lambda}_i \approx \eta\widehat{\theta}_i$ respectively, we deduce that $\widehat{\Sigma}_i > \widehat{\Lambda}_i$. Thus, assuming that Σ_i is age-decreasing (our assumption in order to match the observed first order moments characterizing the steady state), the variations in search values ($\widehat{\Sigma}_i$) dominate the volatility in labor hoarding values ($\widehat{\Lambda}_i$). The expression of $\widehat{R}_i$ then shows that fluctuations in inter-temporal values, dominated by changes in $\widehat{\Sigma}_i$, dampen fluctuations in $R_i(z)$. In recession, prime-age workers are highly sensitive to the decrease in labor market opportunities because they have a future. The change in their reservation wage dampens the impact of the business cycle. For old workers, we have $\Sigma_i \to 0$ and thus $\Lambda_i \to 0$ because $\Sigma_i > \Lambda_i$, leading R_i to be highly sensitive to current aggregate shocks. In other words, $\widehat{R}_i = -\widehat{z}$ for old workers: old workers' reservation productivity respond to the aggregate shock on a one-for-one basis. In contrast, for prime-age workers, the reservation productivity responsiveness is less than one, in absolute value.

5.3 Job finding rate volatility

Let us first analyze the volatility of labor market tightness that depends on the expected value of a filled vacancy.[17] By introducing the log-linear approximation of the ex-ante wage in the log-linear approximation of the expected firm surplus, we obtain:

$$\widehat{J}_i = \frac{ze(R_i)}{ze(R_i(z)) - (b+\Sigma_i)(1-G(R_i))}\widehat{z} - \frac{\Sigma_i(1-G(R_i))}{ze(R_i(z)) - (b+\Sigma_i)(1-G(R_i))}\widehat{\Sigma}_i \tag{14}$$

Recall that Σ_i is an increasing function of γ (a large bargaining power leads to a high search value). Using equation (14), we can recover all the cases discussed in the literature.

In Shimer (2005), without age-heterogeneity, equation (14) boils down to :

$$\widehat{J} = \frac{z}{z-(b+\Sigma(\gamma))}\widehat{z} - \frac{\Sigma(\gamma)}{z-(b+\Sigma(\gamma))}\widehat{\Sigma}(\gamma) \tag{15}$$

In Shimer (2005), b the value of leisure is restricted to measure the replacement rate in the United-States: thus, b is small and denoted by $b = b^-$. Concerning the bargaining power, Shimer (2005) assumes that the Hosios condition is satisfied and thus sets γ to be equal to η. For the calibration, Shimer (2005) uses an information on η and sets it at the upper end of the range of estimates reported in Petrongolo & Pissarides (2001). Given that $\eta = \gamma$, this

[17] Equation (13) suggests that in order to understand the volatility age profile of JFR, we need to characterize the elasticity of search efforts for firms and workers. Labor market tightness and search effort ($\{\theta_i, e_i\}$) depend on the expected value of a filled vacancy J_i. The Log-linear approximations of the free entry condition and the FOC w.r.t the search effort of the unemployed workers leads to $\widehat{e}_i = \frac{1}{\phi}\widehat{\theta}_i$. Thus, we focus in this section on θ_i. The volatility of e_i behaves in a similar way.

implies that the bargaining power is large, $\gamma = \gamma^+$, and thus the value of the search is also large ($\Sigma(\gamma^+) >> 0$). With Shimer (2005)'s calibration [18], the direct impact of the productivity shock is small ($\frac{z}{z-(b^- + \Sigma(\gamma^+))}$) because the direct impact of b is larger than the indirect impact of γ on this multiplier, whereas the impact of fluctuation in search opportunities are not negligible ($\Sigma(\gamma^+) \neq 0$). This calibration leads to minimize the predicted impact of an aggregate shock on hiring decision in the DMP model.

In Hagendorn & Manovskii (2008), in equation (15), a new calibration of this model is proposed. First, the value of leisure includes unemployment benefits as well as home production, leading to a high value for b, denoted b^+ and such that $b^+ > b^- + \Sigma(\gamma^+)$. Concerning the bargaining power, Hagendorn & Manovskii (2008) do not assume that the Hosios condition is satisfied and set a low value for γ such that $\gamma \approx 0 \Rightarrow \Sigma(\gamma^-) \to 0$. With this calibration, the impact of search opportunities is negligible ($\Sigma \approx 0$) and the direct impact of $\widehat{z}$ is maximized $\frac{z}{z-b^+}$. With this calibration strategy [19], Hagendorn & Manovskii (2008) show that the DMP model can generate the observed fluctuation of worker flows. Nevertheless, their solution is obtained in a economy where the unemployment rates are lower than their optimal counterparts, because $\gamma < \eta$, suggesting that something is missing in the model.

An alternative solution, suggested in Hall (2005), is provided in Hall & Milgrom (2008): this setup provides a game where the weight of the search value Σ in the wage equation can be arbitrary very small. [20].

In our case, prime-age workers look like Shimer (2005)'s workers:

$$\widehat{J}_A = \frac{ze(R_A(z))}{ze(R_A(z)) - (b + \Sigma_A)(1 - G(R_A(z)))}\widehat{z} - \frac{\Sigma_A(1 - G(R_A(z)))}{ze(R_A(z)) - (b + \Sigma_A)(1 - G(R_A(z)))}\widehat{\Sigma} \quad (16)$$

In order to reproduce the average volatility of workers flows, it is necessary to calibrate the value of home production at a larger value than the one proposed by Shimer (2005), which is consistent with Hall (2005). The important point is that, unlike Hagendorn & Manovskii (2008), we preserve the Hosios condition and we endogenously obtain varying changes in the search value in the wage dynamics through another channel than the one proposed by Hall & Milgrom (2008). We have for old workers:

$$\widehat{J}_O = \frac{ze(R_O(z))}{ze(R_O(z)) - b(1 - G(R_O(z)))}\widehat{z} \quad (17)$$

[18] The values in Shimer (2005) are $\gamma = 0.72$ and $b = 0.4$.

[19] The values in Hagendorn & Manovskii (2008) are $\gamma = 0.061$ and $b = 0.943$.

[20] The values in Hall & Milgrom (2008) are $\gamma = 0.5445$ and $b = 0.71$. Nevertheless, the solution of the wage equation depends on additional parameters. Indeed, in their "strategic bargaining", if for simplicity it is assumed that *(i)* the worker receives payoff b and the employer incurs no cost while bargaining continues and *(ii)* they renegotiate the division of the match product $z\epsilon$ whenever it changes, then the outcome of a symmetric ($\gamma = 0.5$) alternating-offers game is $w = b + 0.5(z\epsilon - b)$. This shows that it exists a calibration of the costs incurred during the bargaining process leading the wage to be independent of the search opportunity value Σ.

With aging, the returns on search is nil for older workers, leading $\Sigma_O \to 0$, and reducing the elasticity of wages w.r.t. $\widehat{\Sigma}_i$ when workers age. Then, with a calibration for the value of leisure such that $b > b^-$, the aging endogenously leads average flows to look like Hall & Milgrom (2008)'s workers (even if $\gamma = \eta > 0$). In contrast, prime-age workers have highly procyclical value for search opportunities ($\Sigma_A > 0$), which moderates the elasticity of the job value w.r.t. aggregate shocks in equation (16). This then explains the lower elasticity of search efforts to the business cycle for prime-age workers than for old workers.

5.4 Worker flow volatility: the equilibrium analysis

In the two previous sections, we have presented the results on the impact of the business cycle using partial equilibrium analysis in order to decompose in an intuitive way the economic forces at work in the model. In this section, using the same set of simplifying assumptions (*(i)* conditional steady states and *(ii)* negligible impact of age transitions on fluctuations within an age class, ie. $\frac{1-\pi_i}{\pi_{i+1}} \to 0$), we show that the results are robust to the equilibrium analysis. Indeed, we obtain:

$$\widehat{R}_i \approx -\frac{b + \Sigma_i \left(1 - \frac{1+\phi}{\phi}\frac{1}{\eta}\right)}{b + \Sigma_i \left(1 + \frac{1+\phi}{\phi}\frac{1}{\eta}\varepsilon_{I|R}\right) - \Lambda_i(1+\varepsilon_{I|R})}\widehat{z} \tag{18}$$

$$\widehat{\theta}_i \approx \frac{1}{\eta}\left[1 + \varepsilon_{I|R}\frac{b + \Sigma_i \left(1 - \frac{1+\phi}{\phi}\frac{1}{\eta}\right)}{b + \Sigma_i \left(1 + \frac{1+\phi}{\phi}\frac{1}{\eta}\varepsilon_{I|R}\right) - \Lambda_i(1+\varepsilon_{I|R})}\right]\widehat{z} \tag{19}$$

$$\widehat{e}_i \approx \frac{1}{\phi}\frac{1}{\eta}\left[1 + \varepsilon_{I|R}\frac{b + \Sigma_i \left(1 - \frac{1+\phi}{\phi}\frac{1}{\eta}\right)}{b + \Sigma_i \left(1 + \frac{1+\phi}{\phi}\frac{1}{\eta}\varepsilon_{I|R}\right) - \Lambda_i(1+\varepsilon_{I|R})}\right]\widehat{z} \tag{20}$$

where $\varepsilon_{I|R} = \left|\frac{I'R}{I}\right|$, with $I(R_i(z)) = \int_{R_i(z)}^{1}(1 - G(x))dx$.

Proposition 3. *When JFR_i and JSR_i are age-decreasing, older workers are more sensitive to the business cycle than prime-age workers.*

Proof. Using (18), (19) and (20), if we assume that the values of Σ_i and Λ_i are at their lowest level for older workers $\Sigma_O = \Lambda_O = 0$, and at their highest level for prime-age workers $\overline{\Sigma}_A = \frac{b}{\frac{1+\phi}{\phi}\frac{1}{\eta}-1}$, we deduce that

$$\begin{array}{lll} \widehat{R}_O \approx -\widehat{z} & & \widehat{R}_A \approx 0 \\ \widehat{\theta}_O \approx \frac{1}{\eta}\left[1+\varepsilon_{I|R}\right]\widehat{z} & & \widehat{\theta}_A \approx \frac{1}{\eta}\widehat{z} \\ \widehat{e}_O \approx \frac{1}{\phi}\frac{1}{\eta}\left[1+\varepsilon_{I|R}\right]\widehat{z} & & \widehat{e}_A \approx \frac{1}{\phi}\frac{1}{\eta}\widehat{z} \end{array}$$

therefore $|\widehat{R}_O| > |\widehat{R}_A|$, $|\widehat{\theta}_O| > |\widehat{\theta}_A|$ and $|\widehat{e}_O| > |\widehat{e}_A|$. □

Proposition 3 shows that, if the model can reproduce the shape of worker per age at the steady state, then this set of restrictions is sufficient to be in accordance with fluctuations of these data around this steady state.

6 Quantitative assessment

In this section, we bring the model to the data. In order to match the observed age-increasing pattern of real hourly wages, we introduce a deterministic human capital accumulation. In spite of a shorter horizon on the labor market, older workers' value can increase because of human capital accumulation. Accordingly, we assume that hiring costs, unemployment benefits and the monetary value of search disutility are indexed on this age-specific human capital: formally, we have $c_i = ch_i$, $b_i = bh_i$ and $\phi_i(e_i) = h_i\phi(e_i)$. Note that this assumption is very conservative, as it creates a force which dampens the horizon effect by giving more value to workers at the end of the life cycle.

Under the steady state restrictions, our goal is to explain the volatility by age, while matching the one observed at the aggregate level, which is computed on workers of all age groups. More specifically, in the simulations below, the calibration is based on macroeconomic targets that include all age groups in order to place the aggregate environment into a labor market that matches the values found in the literature. However, we will only present the model's prediction on prime-age and old workers that are the primary focus of the paper. Indeed, as the "horizon effect" is not discriminating to explain the differences between the young and the prime-age workers, we chose to discard from the specificities of the youth labor market (16-24 years old).

We gauge the model's ability to replicate the age volatility profile when we calibrate the model to fit historical averages. In doing so, we first uncover structural parameters that allow the model to match the first moments found in the data (section 6.1). Under this calibration, we assess the model's ability to generate second order moments consistent with the data. The challenge is quantitative: will the model's predicted volatilities be close to the ones observed in the data? Our result indeed show that the parameter restrictions imposed by the first order moments are sufficient to generate age-increasing elasticities (Section 6.2). Finally, in section 6.3, a sensitivity analysis is performed.

6.1 Benchmark calibration: Matching the age-increasing volatility in transition rates

The vector of the model's parameter is $\Phi = \{\Phi_1, \Phi_2\}$ with $dim(\Phi) = 40$. All the usual parameters provided in the literature are in

$$\Phi_1 = \{\beta, \{\pi_i\}_{i=Y}^{O_7}, s_{e,A}, s_{e,O}, c, \gamma, \eta, \rho, \sigma_\nu\} \quad dim(\Phi_1) = 17$$

For these parameters, we follow Fujita & Ramey (2012) and Shimer (2005). The parameters $\{\beta, \gamma = \eta, c, \rho, \sigma_\nu\}$ are calibrated as follows. They are calibrated to match a monthly discount factor consistent with an annual interest rate of 4%, whereas the elasticity parameter of the matching function η and the bargaining weight of workers γ are both set to 0.7. These two values are close to the values of η and γ (0.72) used in Shimer (2005). The parameters for the aggregate productivity process ρ and σ_ν are set to the values proposed by Shimer (2005). We set π_i such that an age class correspond to the age groups in the data: $i = A$ are the $25 - 54$ years old workers and $i = O_j$ for $j = 1, ..., 7$ are the $55, ..., 61$ years old workers. The calibration of the vacancy posting cost c uses the results of Barron et al. (1997) and Barron & Bishop (1985). These authors suggest that an amount to 17 percent of a 40-hour workweek (nine applicants for each vacancy filled, with two hours of work time required to process each application). For the exogenous job separation rate $s_{e,i}$, for $i = A, O$, we follow Fujita & Ramey (2012): at each age, the exogenous job separations represent 34 percent of total separations.

For the other parameters, we have:

$$\Phi_2 = \{H, \phi, \chi, \sigma_\epsilon, b, \{h_i, \lambda_i\}_{i=A}^{O_7}\} \quad dim(\Phi_2) = 23$$

where H and χ are the scale parameters, respectively, of the matching function $M(v_i, e_i u_i) = H v_i^{1-\eta}(e_i u_i)^\eta$, and search cost $\phi(e_i) = \chi \frac{e_i^{1+\phi}}{1+\phi}$. The parameters h_i account for the increase in labor efficiency when workers age. We need some restrictions in order to identify these parameters using our first order moments. We assume that:

- Older workers share the same level of human capital, leading to $\{h_i\}_{i=A}^{O_7} = \{h_A, h_O\}$
- Older workers share the same λ_i, leading to $\{\lambda_i\}_{i=A}^{O_7} = \{\lambda_A, \lambda_O\}$,

The calibration procedure finds parameter values Φ_2 that minimize the distance between theoretical and observed moments $\Psi^{theo}(\Phi_2) - \Psi$. The numerical solution for $\Psi^{theo}(\cdot)$ is provided by the algorithm described in Appendix C.5. The 9 free parameters are

$$\Phi_2 = \{H, \chi, \phi, h_A, h_O, b, \lambda_A, \lambda_O, \sigma_\epsilon\} \quad dim(\Phi_2) = 9$$

whereas the 9 first-order moments provided by the data are:

$$\Psi = \{\overline{JFR}, \overline{JSR}, \overline{w}, JFR_A, JFR_O, JSR_A, JSR_O, w_A, w_O\} \quad dim(\Phi_2) = 9$$

where $X_O = \frac{1}{7}\sum_{i=O_1}^{O_7} X_i$ for $i = O_1, ..., O_7$, $\forall X = JFR, JSR, w$.

These targeted moments are reported in Table 4.[21] Table 5 summarizes the calibration.

[21] In Table 4, aggregate targets include young workers' characteristics in order to place the model in a macroeconomic environment that is consistent with the one found in the literature. However, we report only values for prime-age and old workers. For young workers, w_Y is normalized to 1. $h_Y = 1$ and π_Y is based on age group 16-24 years old.

Table 4: First order moments: Ψ

JFR_A	JFR_O	JSR_A	JSR_O	
0.41	0.33	0.017	0.011	
$\overline{JFR}$	$\overline{JSR}$	$\overline{w}$	w_A	w_O
0.43	0.021	1.55	1.68	1.84

Overbar refers to aggregate average

Table 5: Benchmark calibration

External information Φ_1				
	β	$\eta = \gamma$	c	$s_{e,i}$
	$r = 4\%$	0.7	0.17	34% JSR
	ρ	σ_ν	π_A	$\{\pi_i\}_{i=Y}^{O_7}$
	0.9895	0.0034	25-54	55-56...60-61
Calibration Φ_2				
	H	b	σ_ϵ	χ	ϕ
HLS	0.115	0.92	0.085	0.125	0.7
FR	0.061	0.934	0.124	-	-
	λ_A	λ_O	h_A	h_O	
HLS	0.0475	0.052	1.7	1.8	
FR	0.085	-	-	-	

HLS = our weekly calibration

FR = Fujita and Ramey (2011)'s weekly calibration

Two important elements must be stressed. First, prime-age workers are less likely to benefit from changes in productivity during their years of work experience. If we reduce the interpretation of λ to a job-to-job mobility opportunity, our calibration seems to contradict the idea that job-to-job mobility is more important at the early stage of a career (e.g. see the data discussed by Menzio et al. (2012)). However, as the model has only this source of heterogeneity between jobs at each stage of the life-cycle, it may reflect different phenomena for each age: a less restrictive interpretation is needed. Thus, the phenomenon of mismatch can be more relevant in the prime-age labor market and gradually decrease with experience in the labor market. A smaller λ for prime-age workers can indicate an inability to move up within the firm and improve match-specific productivity on the job (mismatch looks like a permanent shock). The best option for these workers is then to look for another job, thereby reducing the distance between the required skill and the worker's skills. We deduce that our smaller value for λ_A reflects that mismatch is dominant at this stage of the life-cycle, whereas, for older workers, the higher value of λ may reflect their ability to adapt to new tasks in the firms. This interpretation of $\{\lambda_A, \lambda_O\}$ is consistent with Menzio et al. (2012)'s results on the estimations of the production parameters of a match. Indeed, they find that the quality of a match changes once every 8 and a half years.

The second comment deals with the value of b. Results reported in table 5 show that our calibrated value for b is lower than the one used by Fujita & Ramey (2012) in their calibration à

Table 6: Implied values of the outside option

$\sum_{i=A}^{O_7} m_i \frac{b_i-\phi_i(e_i^{ss})}{h_i}$	$\frac{b_A-\phi_A(e_A^{ss})}{h_A}$	$\sum_{i=O_1}^{O_7} m_i \frac{b_O-\phi_O(e_i^{ss})}{h_O}$				
0.807	0.805	0.82				
$\frac{b_O-\phi_O(e_{O_1}^{ss})}{h_O}$	$\frac{b_O-\phi_O(e_{O_2}^{ss})}{h_O}$	$\frac{b_O-\phi_O(e_{O_3}^{ss})}{h_O}$	$\frac{b_O-\phi_O(e_{O_4}^{ss})}{h_O}$	$\frac{b_O-\phi_O(e_{O_5}^{ss})}{h_O}$	$\frac{b_O-\phi_O(e_{O_6}^{ss})}{h_O}$	$\frac{b_O-\phi_O(e_{O_7}^{ss})}{h_O}$
0.8111	0.8112	0.8114	0.8122	0.8140	0.8228	0.8654

la Hagendorn & Manovskii (2008). Our value is also greater than the more realistic one proposed by Hall & Milgrom (2008), which is $b = 0.7$.[22] Nevertheless, in our model with endogenous search effort, the instantaneous value of leisure is actually $b_i - \phi_i(e_i(z))$, not b. The net value of home production is reported in table 6. Our calibration is closed to Hall & Milgrom (2008)'s estimated value for outside opportunities. If, on average, we are not in the extreme case proposed by Hagendorn & Manovskii (2008), it is not the case for particular workers. Hence, for older workers, $e_{O_7} \to 0$, the value of leisure endogenously converges to b, a value close to Hagendorn & Manovskii (2008)'s calibration. Even if $e_i(z)$ is an endogenous variable which adjusts along the business cycle, the calibration suggests that the model could generate more volatility in the labor market of older workers than for prime-age workers through the net value of outside opportunities.

6.2 Quantitative results

Table 7: Second order moments: data versus theory

		All: 25-61	Prime-age: 25-54	Old: 55-61
JSR:	US Data	0.15	0.14	0.20
	Model	0.16	0.15	0.18
JFR:	US Data	0.18	0.17	0.22
	Model	0.125	0.12	0.14
u:	US Data	0.28	0.27	0.32
	Model	0.21	0.20	0.24

US data: Standard deviation, CPS Monthly data, Men, 1976Q1-2013Q1

HP filter with smoothing parameter 10^5. Authors' calculations.

Table 7 reports the 2nd order moments predicted by the model and those computed using our data set. The main result is that the model predicts that the volatilities of the labor market are age-increasing (stock and flows), as in the data. This result shows that the parameter restrictions imposed by the first order moments are consistent with age-increasing elasticities. Thus, changes in search value dominate all the other components in the expected job surplus, including the exogenous age-increasing profile in human capital introduced to match life-cycle

[22] This value is also used by Menzio et al. (2012).

pattern of wages. This leads prime-age workers to smooth the business cycle because their outside options are also affected by aggregate shocks. In contrast, older workers, with a shorter horizon on the labor market, do not have any outside option and thus are very responsive to the business cycle. This feature makes their wage less responsive to the business cycles. It creates a kind of wage stickiness, in the spirit of Hall & Milgrom (2008), but without removing the Nash bargaining hypothesis, or in the line of Hagendorn & Manovskii (2008), but without imposing a quasi nil worker bargaining power. These two channels come from the same mechanism : the shorter horizon reduces old unemployed workers' search intensity, which makes the outside opportunities almost unconditional to the state of the economy.

Beyond the increase of volatility with worker age, the fit of the model is quite good. Indeed, for prime-age and older workers, the model is able to approximatively match the volatilities per age of job separations rates, but it slightly underestimates the volatilities of the job finding rates and unemployment. These results come from the endogenous increase in the outside options with worker's age: the horizon effect reduces the search value and thus leads older workers to mimic Hagendorn & Manovskii (2008)'s infinitely lived agents.

The simulations results are encouraging and support the view that adding a horizon effect in the DMP model is relevant to explain a large part of the differences between prime-age and older workers along the business cycle: the MP is able to explain why older workers are more sensitive to business cycle than their younger counterparts.

6.3 Sensitivity analysis

In the theoretical part of the paper, only horizon heterogeneity matters: we then discuss the case of a "pure" horizon effect. Nevertheless, in the quantitative analysis, the objective to match the average level of the data per age leads us to introduce two exogenous components in our model: firstly, an age-increasing human capital in order to match the age-increasing pattern of the labor efficiency (reflected in the hourly real wage) and secondly a age-specific change of the match-specific productivity. Hence, in a first experiment, we simulate the model under the assumption that workers' labor productivity are homogenous across ages: this implies $h_i = h$, $\forall i$. In a second experience, we simulate the model without any exogenous age heterogeneity ($h_i = h$, and $\lambda_i = \lambda$, $\forall i$): only the horizon effect induces a endogenous age heterogeneity.

Simulation results are reported in table 8. All simulations display the same age-increasing pattern of the volatilities on the labor market, when taking into account no other heterogeneity than the one related to age. This confirms the crucial role of the horizon effect in generating the higher cyclicality of older worker labor market.

Table 8: Second order moments: Sensitivity analysis

		All: 25-61	A: 25-54	O: 55-61
JSR:	Benchmark	0.16	0.15	0.18
	λ	0.135	0.15	0.181
	h & λ	0.145	0.14	0.17
JFR:	Benchmark	0.125	0.12	0.14
	λ	0.125	0.12	0.14
	h & λ	0.115	0.11	0.13
u:	Benchmark	0.21	0.20	0.24
	λ	0.205	0.20	0.23
	h & λ	0.20	0.196	0.22

λ: Model with homogenous $\lambda = \lambda_A$

h & λ: Model with homogenous h & λ: $h_i = 1$ and $\lambda = \lambda_A$

6.4 The implied wage dynamics

If the model can explain a large part of the volatilities of the quantities on the labor market (job flows and stocks), is it able to capture wage adjustments observed in the data? We first present the theoretical implications in terms of wage age-dynamics. We then compare the model's cyclical properties to the observed volatility of real hourly wage, computed using CPS data.

6.4.1 From theory to the data

In our MP-life-cycle model, there is heterogeneity within each age group because each job is characterized by a match-specific productivity. Thus, the model generates a wage distribution per age. If we want to compare the model predictions with the stylized facts, it is then necessary to compute an *Average wage* per age, which is:

$$\mathcal{W}_i(z) \quad = \quad \gamma z \mathcal{G}(R_i(z)) + (1-\gamma)\left(b + \Sigma_i(z)\right) \tag{21}$$

where $\mathcal{G}(R_i(z)) = \frac{1}{n_i(z)} \int_{R_i(z)}^{1} x dn_i(z,x)$ denotes the average productivity of age-i workers, whereas $dn_i(z,x)$ is the mass of age-i employees on a x-productivity job. This distribution is endogenous: it depends on the worker movements in the job distribution (entries, exits and productivity changes), and more importantly $dn_i(z,\epsilon)$ change with the business cycle. Given that the average wage depends on both individual wages and the wage distribution, one can have a rigid individual wage and a volatile average wage. The log-linear approximation of equation (21) leads to

$$\widehat{\mathcal{W}}_i \quad = \quad \gamma \frac{z\mathcal{G}(R_i)}{\mathcal{W}_i}\left(\widehat{z} + \widehat{\mathcal{G}}_i\right) + (1-\gamma)\frac{\Sigma_i}{\mathcal{W}_i}\widehat{\Sigma}_i \quad \text{with } \widehat{\mathcal{G}}_i = \Gamma_i^z \widehat{z} + \Gamma_i^r \widehat{R}_i + \Gamma_i^s \widehat{\Sigma}_i$$

where the term $\widehat{\mathcal{G}}_i$ gives the impact the changes in the job composition on wage fluctuations.[23] The parameters Γ_i^z, Γ_i^r and $\Gamma_i^s \equiv \gamma_i^s \Sigma_i$ are the elasticities of the average productivity with respect to z, R_i and Σ_i. [24] This approximation underlines the channels through which average productivity $\mathcal{G}_i$ depends on the business cycle. The signs of these elasticity Γ_i^x, for $x = z, r, s$, are ambiguous. Aggregate productivity (z) has a positive impact on aggregate employment, and thus lowers average productivity, but also raises employment at each level of the productivity distribution through its impact on search efforts ($e_i(z)$ and $\theta_i(z)$). In boom, the fall in R_i increases the set of job (the integral has a larger span) but lowers its quality (more jobs are concentrated at the bottom), Moreover, when R_i declines, employment increases, thus average productivity falls. Finally, a rise in the search value Σ_i reduces the incentive to post new vacancies. This has a negative impact on the two dimensions of employment: by reducing its aggregate level, this raises average productivity, whereas its negative effect on each point lowers average productivity. Finally, we deduce that the average wage dynamics is given by:

$$\widehat{\mathcal{W}}_i = \gamma \frac{z\mathcal{G}(R_i)}{\mathcal{W}_i} \left((1+\Gamma_i^z)\widehat{z} + \Gamma_i^r \widehat{R}_i + \Gamma_i^s \widehat{\Sigma}_i \right) + (1-\gamma) \frac{\Sigma_i}{\mathcal{W}_i} \widehat{\Sigma}_i$$

Thus, for older workers, ie. when $\Sigma_O \to 0$, the average wage can be proxied by

$$\widehat{\mathcal{W}}_O = \gamma \frac{z\mathcal{G}(R_i)}{\mathcal{W}_i} \left((1+\Gamma_O^z)\widehat{z} + \Gamma_O^r \widehat{R}_O \right)$$

This shows that this average wage can be highly pro-cyclical if $\Gamma_i^r > 0$ and large enough. Moreover, given that the volatility of the JSR is age-increasing, implying $\widehat{R}_i < \widehat{R}_{i+1}$, the impact of $\widehat{R}_O$ on $\widehat{\mathcal{W}}_O$ can be reinforced by the large volatility in the reservation productivity.

In a nutshell, in our model, individual wage differs from average wage, as the latter takes into account changes in employment composition. For old workers, the horizon effect generates wage rigidity in individual wages. However, the composition of old workers' employment also responds to the business cycle. If the average real wage in the data displays less volatile real wage fluctuations when workers age, we can deduce than the composition effect is dominated by the dynamics of individual contracts. An inspection of the data is then interesting.

6.4.2 Real hourly wage fluctuation

Table 9 allows to compare the model prediction with the data. Notice that the promising results with respect to the cyclicality of quantities are obtained with a model that predicts a volatility of the real wage of the same order of magnitude as the observed one, which cannot be the case with a model with rigid real wage. Moreover, as in the data, the theory predicts a slight decrease in wage volatility at the end of the life-cycle.

[23] See the appendix C.4 for the derivation of the formula for $\widehat{\mathcal{G}}_i$

[24] The notation $\Gamma_i^s \equiv \gamma_i^s \Sigma_i$ allows us to use the property that $\Gamma_O^s \to 0$ for older worker, simply because $\Sigma_O \to 0$, as previously, and γ_O^s is bounded.

Table 9: Second order moments: data versus theory

		All: 25-61	Prime-age: 25-54	Old: 55-61
w:	US Data	0.0188	0.019	0.018
	Model	0.01239	0.0124	0.0123

US data: Standard deviation, CPS Monthly data, Men, 1995Q1-2013Q1

HP filter with smoothing parameter 10^5. Authors' calculations.

In a nutshell, the empirical evidence seems to point that there exists differences in cyclicality between age groups: old worker's job flows are characterized by larger volatility than their younger counterparts and lower hourly wage responsiveness to the business cycle. Our findings on the volatilities per age of the hourly wage can be viewed as consistent with our finding on worker flows and unemployment stock. Indeed, in a market with more rigid prices, the large part of the adjustments is borne by quantities. Nevertheless, one cannot consider real wage rigidity as an acceptable assumption : In the data, there is a decline in the wage volatility with the worker age, but the lowest volatility is still larger than zero.

7 Conclusion

In this paper, we contribute to the existing literature along two dimensions. First, we document business cycle fluctuations in worker flows across age groups in the US. While previous papers focused on differences in *average* transition rates across age groups, we extend the current literature by looking at the age profile of *volatilities* in workers' transition rates for young, prime-age and old workers on CPS data. We find that old worker's are characterized by a higher responsiveness to business cycles than their younger counterparts. We perform several checks to ensure that this is a robust stylized fact. The analysis of the hourly wage dynamics, using monthly data, shows that the cyclicality is age-decreasing. Then, the data presents a consistent picture of labor market adjustments: the market with the most rigid price adjusts largely through quantities. This is the case for older workers.

Secondly, we propose a life-cycle Mortensen & Pissarides (1994) model with age-directed search. First, we show that old workers' short horizon can quantitatively explain the observed gap between their job finding and separations rates and the ones for the prime-age workers. Older workers's shorter horizon endogenously reduces their outside options, thereby leading their wages to be less sensitive to the business cycle. Thus, in a market where wage adjustments are small, quantities vary a lot: this is the case for older workers, whereas their younger counterparts behave like infinitively-lived agents. Older workers do not smooth the impact of aggregate shocks through an expected low return on search, because they have no future (a short horizon reduces to zero the option value of search). We point out that older workers look like Hagendorn &

Manovskii (2008)'s workers, but, contrary to the proposition of these authors, we show that it is not necessary to calibrate the value of home production at a larger value than the one proposed by Hagendorn & Manovskii (2008) in order to match volatilities in workers flows. The important point is that, unlike Hagendorn & Manovskii (2008), we preserve the Hosios condition and we endogenously obtain varying changes in the search value in the wage dynamics through another channel than the one proposed by Hall & Milgrom (2008). Indeed, with aging, the returns on search is nil for older workers, which reduces the elasticity of wages and raises the responsiveness of worker flows. We show that this mechanism allows us to explain why older workers are more sensitive to the business cycle than their younger counterparts.

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A Stylized facts on worker flows

A.1 HP-filtering with $\lambda = 1600$

Table 10 reports business cycle facts when the smoothing parameter is 1600 rather than 10^5. The age-increasing pattern in volatility is robust.

A.2 Taking into account inactivity

Using Shimer (2012)'s methodology for 3 employment states (Employment, Unemployment and Inactivity), on CPS data for Men, we get results reported in Tables 11 and 12. With 3 employment states, the steady state unemployment includes all transitions rates, including those involving inactivity. Tables 11 and 12 suggest that our business cycle facts across age group remain robust when separations and findings are purged from transition to and from inactivity.

Table 10: Standard deviation. CPS data, quarterly averages of monthly instantaneous transition rates,1976Q1-2013Q1, Men, HP filter with smoothing parameter 1600. Authors' calculations.

	All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR	0.083638	0.077361	0.10856	0.17571
		0.7126	1	1.6186
JFR	0.11848	0.1219	0.12439	0.18647
		0.98004	1	1.4991
u	0.16876	0.13661	0.19427	0.25264
		0.7032	1	1.3004

The level of exit from employment as well as the job finding rate fall with age with their volatility increases with age.

Table 11: Mean. Quarterly averages of monthly CPS data, 3 states (Employment, Unemployment, Inactivity), 1976Q1 - 2013Q1, Men. Authors' calculations.

	All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR	0.021973	0.0498	0.01774	0.012265
		2.8073	1	0.69139
JFR	0.38674	0.41812	0.38077	0.30457
		1.0981	1	0.79988
u	0.061169	0.12544	0.048787	0.044766
		2.5712	1	0.91759
EI	0.020969	0.060632	0.00908	0.017287
UI	0.24631	0.38836	0.17323	0.2194
IE	0.050005	0.1067	0.089367	0.039833
IU	0.045497	0.12421	0.097117	0.026462

When we decompose unemployment fluctuations using β computations as in Shimer (2012), based on hypothetical unemployment rates. We find that the transitions between unemployment and unemployment account for 76% of unemployment fluctuations between January 1976 and March 2013. [25]

[25] We compute counterfactual steady states predicted by time varying finding and separation rates, while other transition rates are set at their historical mean. We log and HP-filter the time series using a smoothing parameter of 10^5 and compute the variance decomposition of the cyclical component of steady state unemployment based on βs. We find that $\beta^{EU} + \beta^{UE} = 0.7575$

Table 12: Standard deviation. Quarterly averages of monthly CPS data, 3 states (Employment, Unemployment, Inactivity), 1976Q1 - 2013Q1, Men. Authors' calculations.

	All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR	0.06116	0.04786	0.081897	0.094083
		0.5844	1	1.1488
JFR	0.09291	0.092489	0.097474	0.11835
		0.94885	1	1.2142
u	0.12747	0.0925	0.15478	0.15735
		0.59764	1	1.0166
EI	0.020701	0.03227	0.033263	0.048082
UI	0.077768	0.051111	0.098526	0.13237
IE	0.031257	0.051667	0.047445	0.050046
IU	0.048784	0.042098	0.062416	0.10822

A.3 Employment and Unemployment for all workers

Using Shimer (2012)'s methodology for 2 employment states (Employment, Unemployment), on CPS data for Men and women, we get results reported in Tables 13 and 14. The main stylized facts remain relevant : the mean transition rates fall with age while their volatility increases with age.

Table 13: Mean. Quarterly averages of monthly CPS data, Employment and Unemployment, 1976Q1 - 2013Q1, Men and Women. Authors' calculations.

	All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR	0.018822	0.04208	0.015499	0.010614
		2.715	1	0.68479
JFR	0.42647	0.49789	0.40141	0.33929
		1.2404	1	0.84524
u	0.044507	0.080907	0.039397	0.03252
		2.0536	1	0.82546

A.4 Robustness check on Elsby et al. (2010)'s data

We use the data for Figure 8 of their paper. Notice that there are several differences with our computations. First, they include men and women. Their numbers shall then be compared with our results in Appendix A.3. Secondly, Elsby et al. (2010) use Shimer (2012)'s formula based on stocks of unemployed and employed workers (in which separations are proxied by short-term unemployment) rather than disaggregated data in our case. Their approach yields higher levels

Table 14: Standard deviation. Quarterly averages of monthly CPS data, Employment and Unemployment, 1976Q1 - 2013Q1, Men and Women, HP filter with smoothing parameter 10^5. Authors' calculations.

	All: 16-61	Young: 16-24	Prime-age: 25-54	Old: 55-61
JSR	0.048779	0.034236	0.065052	0.081366
		0.52629	1	1.2508
JFR	0.093705	0.086096	0.096386	0.11683
		0.89324	1	1.2121
u	0.12716	0.095361	0.14464	0.15333
		0.65931	1	1.0601

of transition rates. Finally, old workers are 55 years old and more while we restrict our sample to workers prior to retirement (55-61). We compute business cycle statistics on their database. Since the time series are quarterly, the smoothing parameter on the HP filter is 1600. The results are reported in tables 15 and 16. We find that the mean of transition rates fall with age while the standard deviation increases with age.

Table 15: Mean. Elsby et al. (2010) data, 1977Q2 - 2009Q4, Quarterly data, Men and Women.

	All: 16+	Young: 16-24	Prime-age: 25-54	Old: 55+
JSR	0.03527	0.10047	0.023898	0.015542
		4.2043	1	0.65034
JFR	0.54514	0.7111	0.46652	0.43876
		1.5243	1	0.94049

Table 16: Standard deviation. HP-filtered, smoothing parameter 1600, 1977Q2 - 2009Q4, Quarterly data, , Men and Women. Elsby et al. (2010) data.

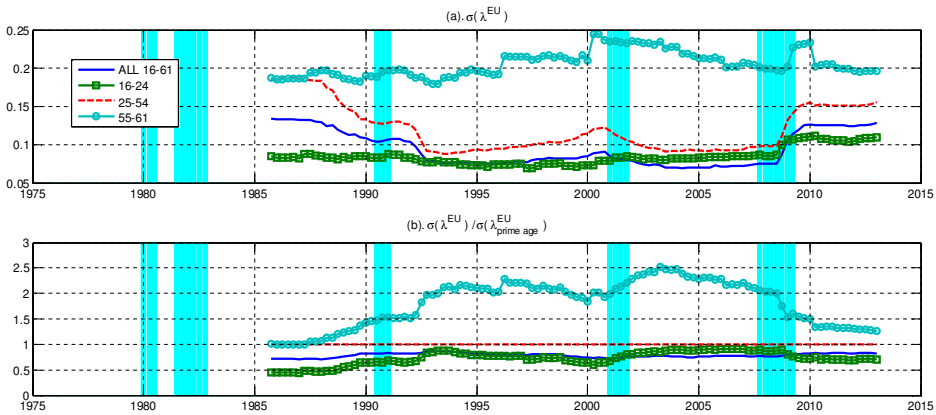
	All: 16+	Young: 16-24	Prime-age: 25-54	Old: 55+
JSR	0.044485	0.046366	0.060651	0.092973
		0.76447	1	1.5329
JFR	0.10627	0.095022	0.113	0.15079
		0.8409	1	1.3344
u	0.11521	0.086532	0.13605	0.14323
		0.63601	1	1.0528

A.5 Rolling standard deviation

In this appendix, we present rolling standard deviations computed over a 6-year window, using the same dataset as in the main text (CPS data, Men, 2 states (E, U), Log HP-filtered monthly

time series). Results are displayed in figures 4 and 5. The window is 6 years meaning that each point in the graph provides a measure of volatility over the 6 previous years.

Figure 4: Rolling standard deviation on separation rates by age group, 6 year window, Log HP-filtered Monthly CPS data, Men. Recession in shaded area.

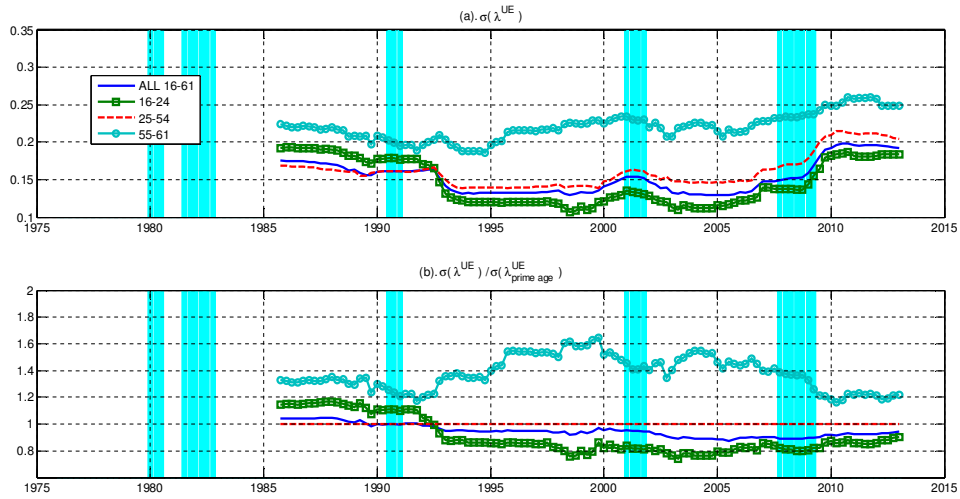


Panel (a) of each figure displays the volatility of each age group while panel (b) provides this measure relative to the volatility of prime-aged workers. Figures 4 and 5 confirm recessions are periods of sharp increases in volatility for all age groups. After the mid-80s, the responsiveness of old workers to business cycles is larger than the ones found for their younger counterparts. In the last recession, the increase in separation volatility is striking for old workers (Panel (a), figure 4).

A.6 Taking into account the change in age composition

In this section, we present a simple decomposition exercise to get a sense of how much of our results on aggregate data is attributable to the change in the age distribution of the workforce. In order to do so, as in Jaimovich & Siu (2009), we first assess the age structure of the workforce using OECD Labour Force Statistics annual database on unemployed and employed populations.

Figure 5: Rolling standard deviation on finding rates by age group, 6 year window, Log HP-filtered Monthly CPS data, Men. Recession in shaded area.



At any point in time t, the aggregate JSR_t is

$$\begin{aligned} JSR_t &= \frac{EU_{Yt} + EU_{At} + EU_{Ot}}{E_t} \\ JSR_t &= \frac{EU_{Yt}}{E_{Yt}}\frac{E_{Yt}}{E_t} + \frac{EU_{At}}{E_{At}}\frac{E_{At}}{E_t} + \frac{EU_{Ot}}{E_{Ot}}\frac{E_{Ot}}{E_t} \\ JSR_t &= JFR_{Yt}\frac{E_{Yt}}{E_t} + JSR_{At}\frac{E_{At}}{E_t} + JSR_{Ot}\frac{E_{Ot}}{E_t} \end{aligned} \tag{22}$$

with E the total employment stock ($E = E_Y + E_A + E_O$), and EU_i the number of transitions from employment to unemployment for workers of age i at time t. Equation (22) states that the aggregate JSR is a weighted average of JSR by age, the weights being given by each age group share in total employment. To isolate the effect due purely to the change in composition, we construct counterfactual JSR series that holds the age structure fixed at the values observed either at the beginning

$$\widetilde{JSR}_{1976,t} = JFR_{Yt}\frac{E_{Y,1976}}{E_{1976}} + JSR_{At}\frac{E_{A,1976}}{E_{1976}} + JSR_{Ot}\frac{E_{O,1976}}{E_{1976}}$$

or at the end of the sample.

$$\widetilde{JSR}_{2012,t} = JFR_{Yt}\frac{E_{Y,2012}}{E_{2012}} + JSR_{At}\frac{E_{A,2012}}{E_{2012}} + JSR_{Ot}\frac{E_{O,2012}}{E_{2012}}$$

Doing this for every month generates counterfactual time series $\widetilde{JSR}_{1976,t}$ and $\widetilde{JSR}_{2012,t}$. The

OCDE data are consistent with population aging: $\frac{E_{Y,1976}}{E_{1976}} = 22.27\%$, $\frac{E_{Y,2012}}{E_{2012}} = 13.65\%$ and $\frac{E_{O,1976}}{E_{1976}} = 8.3\%$, $\frac{E_{O,2012}}{E_{2012}} = 11\%$.

We compare the standard deviation of filtered counterfactual JSR with the earlier and later demographic structure. As expected, the counterfactual time series with the larger share of young people $\widetilde{JSR}_{1976}$ is characterized by a larger sample mean (13% higher) and a lower standard deviation after HP-filtering with smoothing parameter of 10^5 (10% lower).

A similar decomposition is performed for the JFR where the weights are given by each age group share in total unemployment. The OCDE data are consistent with population aging: $\frac{U_{Y,1976}}{U_{1976}} = 50\%$, $\frac{U_{Y,2012}}{U_{2012}} = 31.5\%$ and $\frac{U_{O,1976}}{U_{1976}} = 4.4\%$, $\frac{U_{O,2012}}{U_{2012}} = 8\%$. As expected from tables 1 and 2, the counterfactual time series with the larger share of young people $\widetilde{JFR}_{1976}$ is characterized by a larger sample mean (4% higher) and a lower standard deviation after HP-filtering with smoothing parameter of 10^5 (3% lower). The effects might not seem large. In our view, this is due to the age groups chosen in our analysis. The demographic changes have affected the group of prime age workers that also include workers in their early 50s.

A.7 Employment and Unemployment per skill

The data per age can mix an age effect and a skill effect. Indeed, older workers are in average less educated than younger workers. Thus, our age effect can be a skill effect. In order to deal with this identification problem, we propose to distinguish two skill groups (high school diploma and less, and high school degree or more).

Table 17: Mean. Monthly CPS data, Employment and Unemployment, 1976Q1 - 2013Q1, Men. Authors' calculations.

	High school and less			High school and more		
	All: 16+	16-54	Old: 55+	All: 16+	16-54	Old: 55+
JSR	0.034297	0.036999	0.016455	0.014686	0.015344	0.0085209
		1	0.44473		1	0.55534
JFR	0.43205	0.43819	0.35042	0.41524	0.42485	0.30959
		1	0.7997		1	0.72871
u	0.077717	0.082009	0.050573	0.03659	0.037343	0.029889
		1	0.61667		1	0.80039

After controlling for educational attainment, the level are age-decreasing and the volatilities are age-increasing. Thus, our stylized facts account for a phenomena liked to worker age, and is not the result of a composition effect.

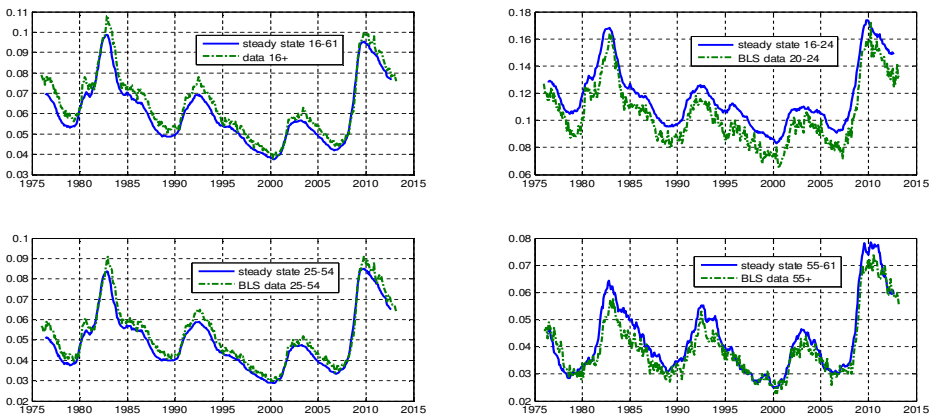
Table 18: Standard deviation. Employment and Unemployment, Monthly CPS data, 1976Q1 - 2013Q1, Men, HP-filtered, smoothing parameter 10^5, Authors' calculations.

	High school and less			High school and more		
	All: 16+	16-54	Old: 55+	All: 16+	16-54	Old: 55+
JSR	0.13485	0.14002	0.24422	0.14218	0.14512	0.27525
		1	1.7441		1	1.8967
JFR	0.16708	0.16673	0.27954	0.17743	0.17878	0.28459
		1	1.6766		1	1.5919
u	0.23679	0.23546	0.36584	0.2777	0.27982	0.38679
		1	1.5537		1	1.3823

A.8 Steady state unemployment rate and actual unemployment rate

Figure 6 displays the steady state unemployment found when using monthly CPS data for men and women in a 3 state labor market à la Shimer (2012) and the corresponding BLS unemployment rates. Notice that the age groups are not exactly the same. Fluctuations in steady state unemployment closely match those found in the data.

Figure 6: Steady state unemployment by age and BLS unemployment



B Computing male real hourly wage by age

Questions on wages are asked only of about one-quarter of the entire sample (Outgoing Rotation Group). We use usual weekly earnings of wage and salary workers. All self-employed persons are excluded, regardless of whether their businesses are incorporated. Data represent earnings before taxes and other deductions and include any overtime pay, commissions, or tips usually received. The hourly wage is computed as the ratio of usual weekly earnings to usual weekly hours. We delete outliers (hourly wage larger than 250 US dollars, hourly wage smaller than half

the net minimum wage, young workers working more than 45 hours per week). We divide the time series of nominal hourly wages by a trend derived from the aggregate wage time-series. This trend captures long-term increases in inflation, technology and general human capital. After correcting for seasonal movements using x12, we consider quarterly averages of monthly observations then look at logged-HP filtered real hourly wages using 10^5 as smoothing parameters. We check that levels of real hourly wages are consistent with findings in Heathcote et al. (2010) as well as BLS data on weekly earnings by age. We also check that business cycle features are not too far from Jaimovich & Siu (2009)'s statistics on annual wage data.

C Model

C.1 Stock-flow dynamics

C.1.1 Levels of unemployment and employment

The mass of age-i workers employed during the month t in a firm such that $\tau \in [R_{i,t}, x]$, is given by

$$n_i(z,x) = \int_{R_i(z)}^{x} \mu(\tau) d\tau$$

where $\mu(\tau)$ the mass of firms with a productivity z. This stock of jobs evolves as follows:

$$\begin{aligned} & \text{If } i = Y \\ n_Y(z',x) &= \pi_Y \left[\begin{array}{l} [(1-s_e)\lambda(m_Y - u_Y(z)) + e_Y(z)p(\theta_Y(z))u_Y(z)][G(x) - G(R_Y(z'))] \\ +(1-s_e)(1-\lambda)[n_Y(z,x) - n_Y(z, R_Y(z'))] \end{array} \right] \qquad (23) \\ & \text{If } i \neq Y \\ n_i(z',x) &= \pi_i \left[\begin{array}{l} [(1-s_e)\lambda(m_i - u_i(z)) + e_i(z)p(\theta_i(z))u_i(z)][G(x) - G(R_i(z'))] \\ +(1-s_e)(1-\lambda)[n_i(z,x) - n_i(z, R_i(z'))] \end{array} \right] \\ & \quad +(1-\pi_{i-1}) \left[\begin{array}{l} [(1-s_e)\lambda(m_{i-1} - u_{i-1}(z)) + e_i(z)p(\theta_i(z))u_{i-1}(z)][G(x) - G(R_i(z'))] \\ +(1-s_e)(1-\lambda)[n_{i-1}(z,x) - n_{i-1}(z, R_i(z'))] \end{array} \right] \qquad (24) \end{aligned}$$

where, as in Hairault et al. (2010), we assume that when worker ages (from $i-1$ to i), his job contact probability $(e_i(z)p(\theta_i(z)))$, and his reservation productivity $R_i(z)$ are those of a worker of the age i.

C.1.2 Unemployment and employment rates

The dynamics of the unemployment rates by age are given by:

$$u_i(z) = m_i - n_i(z,1) \Leftrightarrow \underbrace{\frac{u_i(z)}{m_i}}_{\equiv u_i^r(z)} = 1 - \underbrace{\frac{n_i(z,1)}{m_i}}_{\equiv n_i^r(z,1)} \qquad \forall i, z$$

The dynamics of the employment rate is given by

$$
\begin{aligned}
& \text{If } i = Y \\
n_Y^r(z',x) &= \pi_Y \left[\begin{array}{l} [(1-s_e)\lambda(1-u_Y^r(z)) + e_Y(z)p(\theta_Y(z))u_Y^r(z)][G(x) - G(R_Y(z'))] \\ +(1-s_e)(1-\lambda)[n_Y^r(z,x) - n_Y^r(z,R_Y(z'))] \end{array} \right] \qquad (25) \\
& \text{If } i \neq y \\
n_i^r(z',x) &= \pi_i \left[\begin{array}{l} [(1-s_e)\lambda(1-u_i^r(z)) + e_i(z)p(\theta_i(z))u_i^r(z)][G(x) - G(R_i(z'))] \\ +(1-s_e)(1-\lambda)[n_i^r(z,x) - n_i^r(z,R_i(z'))] \end{array} \right] \\
& \quad +(1-\pi_{i-1})\frac{m_{i-1}}{m_i} \left[\begin{array}{l} [(1-s_e)\lambda(1-u_{i-1}^r(z)) + e_i(z)p(\theta_i(z))u_{i-1}^r(z)][G(x) - G(R_i(z'))] \\ +(1-s_e)(1-\lambda)[n_{i-1}^r(z,x) - n_{i-1}^r(z,R_i(z'))] \end{array} \right] \qquad (26)
\end{aligned}
$$

Thus, given the definition of $n_i^r(z,x)$ (equations (25) and (26)), $G(1) = 1$ and $u_i^r(z) = 1 - n_i^r(z,1)$, we obtain

$$
\begin{aligned}
& \text{If } i = Y \\
u_Y^r(z') &= \pi_Y \left[\begin{array}{l} [1 - e_Y(z)p(\theta_Y(z))(1 - G(R_Y(z')))]u_Y^r(z) \\ +(1-s_e)(1-\lambda)n_Y^r(z,R_1(z')) \\ +[s_e + (1-s_e)\lambda G(R_Y(z'))](1-u_Y(z)) \end{array} \right] \\
& \quad +(1-\pi_{O_7O_7})\frac{m_{O_7}}{m_Y} \qquad (27) \\
& \text{If } i \neq Y \\
u_i^r(z') &= \pi_i \left[\begin{array}{l} [1 - e_i(z)p(\theta_i(z))(1 - G(R_i(z')))]u_i^r(z) \\ +(1-s_e)(1-\lambda)n_i^r(z,R_i(z')) \\ +[s_e + (1-s_e)\lambda G(R_i(z'))](1-u_i^r(z)) \end{array} \right] \\
& \quad +(1-\pi_{i-1})\frac{m_{i-1}}{m_i} \left[\begin{array}{l} [1 - e_i(z)p(\theta_i(z))(1 - G(R_i(z')))]u_{i-1}^r(z) \\ +(1-s_e)(1-\lambda)n_{i-1}^r(z,R_i(z')) \\ +[s_e + (1-s_e)\lambda G(R_i(z))](1-u_{i-1}^r(z)) \end{array} \right] \qquad (28)
\end{aligned}
$$

Unemployed workers of age i in month $t+1$ are those of age i in month t who do not age, and

- who do not find a job (first term of the first line of the right-hand side of equations (27) and (28)),
- employed workers of age i who loose their job in week $t+1$ due to a change in aggregate productivity leading to a change in the reservation productivity[26] (second term of the first line),
- the age-i employed workers who loose their jobs due to a separation, which can result from an exogenous reason with a probability s_e and from endogenous decisions with a probability $(1-s_e)\lambda G(R_i(z'))$ (first term of the second line),

[26] When $R_i(z) < R_i(z')$, the mass of obsolete jobs depends on the job creations over the past. Obviously, if $R_i(z) > R_i(z')$, these jobs do not exist.

- and the new participants (last term of the second line).

Due to the aging, in these age-i unemployed, there is the mass of the age-$i-1$ unemployed who age without finding a job (the two last lines of (28), which are composed by the same flows than the two first, except that the age of the agents is not the same). Finally, the second line of the equation (27) shows that newly born agents enter in the labor market as unemployed workers. Note that the unemployment dynamics is a function of $n_i(z, R_i(z'))$ and $n_{i-1}(z, R_i(z'))$, which are themselves function of the past values if the unemployment.[27] This underlines the interdependence of the age-i unemployment stock to the unemployment level at previous age. The average unemployment rate is $u_t^r = \sum_{i=1}^{T} u_{i,t}$.

C.1.3 Transition rates

The job finding rate (JFR) and the job separation rate (JSR) are respectively:

$$\begin{aligned}
JFR_i(z) &= \frac{e_i(z)p(\theta_i(z))(1-G(R_i(z'))\left[\pi_i u_i^r(z) + (1-\pi_{i-1})\frac{m_{i-1}}{m_i}u_{i-1}^r(z)\right]}{u_i^r(z)} \\
JSR_i(z) &= \frac{(1-s_e)(1-\lambda)\left[\pi_i n_i^r(z, R_i(z')) + (1-\pi_{i-1})\frac{m_{i-1}}{m_i}n_{i-1}^r(z, R_i(z'))\right]}{n_i^r(z,1)} \\
&\quad + \frac{[s_e + (1-s_e)\lambda G(R_i(z'))]\left[\pi_i(1-u_i^r(z)) + (1-\pi_{i-1})\frac{m_{i-1}}{m_i}(1-u_{i-1}^r(z))\right]}{n_i^r(z,1)}
\end{aligned}$$

In the basic infinite horizon model, we have $\pi_i = 1, \forall i$, $m_i = 1$, and $n_i(z, R_i(z')) = n_{i-1}(z, R_i(z')) = 0$ leading to $JFR(z) = e(z)p(\theta(z))(1-G(R(z'))$ and $JSR(z) = s_e + (1-s_e)\lambda G(R(z'))$. These definitions of worker flows have an empirical counterpart and are used by Fujita & Ramey (2012) to test the ability of the MP model to match labor market features. In the data, it is only possible to detect the worker age before a transition. Thus, we compute the transition rate conditionally on being of a given age prior to the transition. In this case, all workers have "the same" age in our measure of the transition rates by age. The counterparts in the model are:

$$\begin{aligned}
JFR_i(z) &= e_i(z)p(\theta_i(z))[1-G(R_i(z'))] \\
JSR_i(z) &= \frac{(1-s_e)(1-\lambda)n_i^r(z, R_i(z')) + [s_e + (1-s_e)\lambda G(R_i(z'))]n_i^r(z,1)}{n_i^r(z,1)}
\end{aligned}$$

[27] Indeed, given the equation (26), we deduce $n_i^r(z, R_i(z'))$, where z^- denotes the realization of the aggregate shock at the previous period:

$$\begin{aligned}
n_i^r(z, R_i(z')) &= \pi_i \left[\begin{array}{l} [(1-s_e)\lambda(1-u_i^r(z^-)) + e_i(z^-)p(\theta_i(z^-))u_i^r(z^-)][G(R_i(z')) - G(R_i(z))] \\ +(1-s_e)(1-\lambda)[n_i^r(z^-, R_i(z')) - n_i^r(z^-, R_i(z))] \end{array} \right] \\
&\quad +(1-\pi_{i-1})\frac{m_{i-1}}{m_i} \left[\begin{array}{l} [(1-s_e)\lambda(1-u_{i-1}^r(z^-)) + e_i(z^-)p(\theta_i(z^-))u_{i-1}^r(z^-)][G(R_i(z')) - G(R_i(z))] \\ +(1-s_e)(1-\lambda)[n_{i-1}^r(z^-, R_i(z')) - n_{i-1}^r(z^-, R_i(z))] \end{array} \right]
\end{aligned}$$

and we have the same expression for $n_{i-1}^r(z, R_i(z'))$.

where $n_i^r(z,1) = 1 - u_i^r(z)$. We use this usual approximation of the worker flows per age in order to measure the ability of the theory to explain the observed data, computed using the same formula.

C.2 Steady state surplus

The surplus function is defined by

$$S_i(z,\epsilon) = \max\left\{ \begin{array}{l} z(\epsilon - R_i(z)) + \beta\pi_i(1-\lambda_i)(1-s_e)E_z[S_i(z',\epsilon) - S_i(z',R_i(z))] \\ +\beta(1-\pi_i)(1-\lambda_{i+1})(1-s_e)E_z[S_{i+1}(z',\epsilon) - S_{i+1}(z',R_i(z))] \end{array} ;0 \right\}$$

Thus, at age $i+1$ and for $\epsilon = R_i(z)$, we have, at the conditional steady state

$$S_{i+1}(z,R_i(z)) = \max\left\{ \begin{array}{l} z(R_i(z) - R_{i+1}(z)) + \beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)S_{i+1}(z,R_i(z)) \\ +\beta(1-\pi_{i+1})(1-\lambda_{i+2})(1-s_e)[S_{i+2}(z,R_i(z)) - S_{i+2}(z,R_{i+1}(z))] \end{array} ;0 \right\}$$

Assuming that $S_{i+1}(z,R_i(z)) > 0$, we obtain:

$$\begin{aligned} S_{i+1}(z,R_i(z)) &= \frac{z(R_i(z) - R_{i+1}(z))}{1-\beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)} \\ &+ \frac{\beta(1-\pi_{i+1})(1-\lambda_{i+2})(1-s_e)}{1-\beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)}[S_{i+2}(z,R_i(z)) - S_{i+2}(z,R_{i+1}(z))] \end{aligned}$$

For age $i+2$, we then have

$$\begin{aligned} S_{i+2}(z,R_i(z)) &= \frac{z(R_i(z) - R_{i+2}(z))}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)} \\ &+ \frac{\beta(1-\pi_{i+2})(1-\lambda_{i+3})(1-s_e)}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)}[S_{i+3}(z,R_i(z)) - S_{i+3}(z,R_{i+2}(z))] \\ S_{i+2}(z,R_{i+1}(z)) &= \frac{z(R_{i+1}(z) - R_{i+2}(z))}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)} \\ &+ \frac{\beta(1-\pi_{i+2})(1-\lambda_{i+3})(1-s_e)}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)}[S_{i+3}(z,R_{i+1}(z)) - S_{i+3}(z,R_{i+2}(z))] \end{aligned}$$

We deduce the value for $S_{i+2}(z,R_i(z)) - S_{i+2}(z,R_{i+1}(z))$, which is

$$\begin{aligned} S_{i+2}(z,R_i(z)) - S_{i+2}(z,R_{i+1}(z)) &= \frac{z(R_i(z) - R_{i+1}(z))}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)} \\ &+ \frac{\beta(1-\pi_{i+2})(1-\lambda_{i+3})(1-s_e)}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)}[S_{i+3}(z,R_i(z)) - S_{i+3}(z,R_{i+1}(z))] \end{aligned}$$

Introducing this result in the expression of $S_{i+1}(z,R_i(z))$, we obtain,

$$\begin{aligned} S_{i+1}(z,R_i(z)) &= \frac{z(R_i(z) - R_{i+1}(z))}{1-\beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)} \\ &+ \frac{\beta(1-\pi_{i+1})(1-\lambda_{i+2})(1-s_e)}{1-\beta\pi_{i+1}(1-\lambda_{i+1})(1-s_e)}\left[\frac{z(R_i(z) - R_{i+1}(z))}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)}\right. \\ &\left.+ \frac{\beta(1-\pi_{i+2})(1-\lambda_{i+3})(1-s_e)}{1-\beta\pi_{i+2}(1-\lambda_{i+2})(1-s_e)}[S_{i+3}(z,R_i(z)) - S_{i+3}(z,R_{i+1}(z))]\right] \end{aligned}$$

This leads to an expression for $S_{i+1}(z, R_i(z))$ such that

$$S_{i+1}(z, R_i(z)) = \Omega_{i+1} z (R_i(z) - R_{i+1}(z))$$

More generally, the surplus is given by, $\forall \epsilon \geq R_i(z)$

$$S_i(z, \epsilon) = \Omega_i z (\epsilon - R_i(z))$$

where $\Omega_i = a_i \{1 + a_{i+1} b_{i+1} [1 + a_{i+2} b_{i+2}(\ldots)]\}$ with $a_i = \frac{1}{1-\beta\pi_i(1-\lambda_i)(1-s_e)}$ and $b_i = \beta(1 - \pi_i)(1 - \lambda_{i+1})(1 - s_e)$ and until $i + n \leq O_7$. Thus, we have, e.g., $\Omega_{O_7} = \frac{1}{1-\beta\pi_{O_7}(1-\lambda_{O_7})(1-s_e)}$ and $\Omega_{O_6} = \frac{1}{1-\beta\pi_{O_6}(1-\lambda_{O_6})(1-s_e)} \left[1 + \frac{\beta(1-\pi_{O_6})(1-\lambda_{O_7})(1-s_e)}{1-\beta\pi_{O_7}(1-\lambda_{O_7})(1-s_e)}\right] \ldots$

C.3 Employment distribution

The employment distribution, assuming that $1 - \pi_i \approx 0$ and the z is the permanent level of productivity:

$$dn_i(z, \epsilon) \approx \frac{e_i(z)p(\theta_i(z)) + [(1 - s_e)\lambda_i - e_i(z)p(\theta_i(z))] \int_{R_i(z)}^1 dn_i(z, x)dx}{1 - (1 - s_e)(1 - \lambda_i)} dG(\epsilon)$$

where $dn_i(z, x)$ is the mass of age-i workers employed on a x-productivity job. The impact of the change in $R_i(z)$ is given by

$$\frac{d}{dR_i(z)} dn_i(z, \epsilon) = -\frac{[(1 - s_e)\lambda_i - e_i(z)p(\theta_i(z))]\, dn_i(z, R_i(z))}{1 - (1 - s_e)(1 - \lambda_i)} dG(\epsilon) > 0$$

iff $(1 - s_e)\lambda_i < e_i(z)p(\theta_i(z))$, which is always satisfied to have $R_i < R_{i+1}$.

C.4 The derivation of the model elasticity to the business cycle

The decision rule on θ leads to

$$p(\theta_i(z)) \int_{R_i(z)}^1 S_i(z, x) dG(x) = \frac{1}{(1 - \gamma)\beta\pi_i} c\theta_i(z)$$

The decision rule on e leads to

$$\phi'(e_i(z)) = \frac{\gamma}{1 - \gamma} c\theta_i(z)$$

Using the functional form, we obtain

$$\frac{e_i(z)^{1+\phi}}{1 + \phi} = \frac{1}{1 + \phi} \frac{\gamma}{1 - \gamma} c e_i(z)\theta_i(z) \Rightarrow \widehat{e}_i(z) = \frac{1}{\phi} \widehat{\theta}_i(z)$$

The surplus function, defined by

$$S_i(z, \epsilon) = \max \left\{ \begin{array}{l} z(\epsilon - R_i(z)) + \beta\pi_i(1 - \lambda_i)(1 - s_e)E_z[S_i(z', \epsilon) - S_i(z', R_i(z))] \\ +\beta(1 - \pi_i)(1 - \lambda_{i+1})(1 - s_e)E_z[S_{i+1}(z', \epsilon) - S_{i+1}(z', R_i(z))] \end{array} ; 0 \right\}$$

leads to

$$S_{i+1}(z, R_i(z)) = \Omega_{i+1} z (R_i(z) - R_{i+1}(z))$$

More generally, the surplus is given by, $\forall \epsilon \geq R_i(z)$

$$S_i(z, \epsilon) = \Omega_i z (\epsilon - R_i(z))$$

implying

$$\int_{R_i(z)}^{1} S_i(z, x) dG(x) = \Omega_i z \int_{R_i(z)}^{1} (x - R_i(z)) dG(x) = \Omega_i z \int_{R_i(z)}^{1} (1 - G(x)) dx$$

If we denote $I(R_i(z)) = \int_{R_i(z)}^{1} (1 - G(x)) dx$, we have

$$\widehat{\overline{S}}_i(z) = \widehat{z} - \varepsilon_{I|R} \widehat{R}_i(z)$$

where $\varepsilon_{I|R} = \left| \frac{I'R}{I} \right|$.

Given the free entry condition, the FOC w.r.t. e and the solution for the surplus, the impled solution for $\Sigma_i(z)$, $\Lambda_i(z)$ and $\Gamma_i^r(z)$ are

$$\begin{aligned}
\Sigma_i(z) &= \frac{\gamma}{1-\gamma} c \left[\frac{\phi}{1+\phi} e_i(z) \theta_i(z) + \frac{1-\pi_i}{\pi_{i+1}} e_{i+1}(z) \theta_{i+1}(z) \right] \\
\Lambda_i(z) &= (1 - s_e) \frac{c}{1-\gamma} \left[\lambda_i \theta_i(z)^\eta + \lambda_{i+1} \frac{1-\pi_i}{\pi_{i+1}} \theta_{i+1}(z)^\eta \right] \\
\Gamma_i^r(z) &= \beta (1 - s_e)(1 - \pi_i)(1 - \lambda_{i+1}) \Omega_{i+1} z (R_i(z) - R_{i+1}(z))
\end{aligned}$$

The Log-linear approximations of the free entry condition, the FOC w.r.t e and the separation decision rule are:

$$\begin{aligned}
\widehat{\theta}_i &\approx \frac{1}{\eta} \left[\widehat{z} - \varepsilon_{I|R} \widehat{R}_i \right] \\
\widehat{e}_i &\approx \frac{1}{\phi} \widehat{\theta}_i \\
\widehat{R}_i &\approx -\frac{b + \Sigma_i - \Lambda_i - \Gamma_i}{b + \Sigma_i - \Lambda_i} \widehat{z} + \frac{\Sigma_i \frac{1+\phi}{\phi} - \Lambda_i \eta}{b + \Sigma_i - \Lambda_i} \widehat{\theta}_i
\end{aligned}$$

By combining these equation, we obtain (19), (20) and (18).

The elasticity of the average productivity. In order to compute the elasticities of the average wage $\mathcal{W}$, it is necessary to use the properties of the employment distribution. Indeed, we have

$$\widehat{\mathcal{W}}_i = \gamma \frac{z \mathcal{G}_i}{\mathcal{W}_i} (\widehat{z} + \widehat{\mathcal{G}}_i) + (1 - \gamma) \frac{\Sigma_i}{\mathcal{W}_i} \widehat{\Sigma}_i$$

where $\mathcal{G}(R_i(z)) = \frac{1}{n_i(z)} \int_{R_i(z)}^{1} x dn_i(z, x)$. The Log-linear approximation of this expression is

$$\widehat{\mathcal{G}}_i = -\widehat{n}_i + \frac{1}{\int_{R_i}^{1} x dn_i(x)} \left(\kappa_i^r \widehat{R}_i + \kappa_i^e \widehat{e}_i + \kappa_i^\theta \widehat{\theta}_i \right)$$

where $\kappa_i^e = e_i \int_{R_i}^1 x \frac{\partial dn_i(x)}{\partial e_i} > 0$, $\kappa_i^\theta = \theta_i \int_{R_i}^1 x \frac{\partial dn_i(x)}{\partial \theta_i} > 0$ and $\kappa_i^r = R_i \left[-R_i dn_i(R_i) + \int_{R_i}^1 x \frac{\partial dn_i(x)}{\partial R_i} \right] \lessgtr 0$. Using the elasticity of the search efforts ($\widehat{e}_i$ and $\widehat{\theta}_i$), which are $\widehat{e}_i = \frac{1}{\phi}\widehat{\theta}_i$ and $\widehat{\theta}_i = \frac{1}{1-\eta}\widehat{J}_i$, and the log-linear approximation the age-i employment rate [28]

$$\widehat{n}_i = \vartheta_i^e \widehat{e}_i + \vartheta_i^\theta \widehat{\theta}_i + \vartheta_i^r \widehat{R}_i$$

where $\vartheta_i^e = \frac{1}{1-\eta}\vartheta_i^\theta > 0$ and $\vartheta_i^r < 0$, we obtain

$$\widehat{\mathcal{G}}_i = \left(-\vartheta_i^r + \frac{\kappa_i^r}{\int_{R_i}^1 x dn_i(x)} \right) \widehat{R}_i + \frac{-\left(\vartheta_i^e \frac{1}{\phi} + \vartheta_i^\theta\right) + \frac{\kappa_i^e \frac{1}{\phi} + \kappa_i^\theta}{\int_{R_i}^1 x dn_i(x)}}{1-\eta} \left\{ \begin{array}{c} \frac{ze(R_i)}{ze(R_i(z)) - (b+\Sigma_i)(1-G(R_i))} \widehat{z} \\ -\frac{\Sigma_i (1-G(R_i))}{ze(R_i(z)) - (b+\Sigma_i)(1-G(R_i))} \widehat{\Sigma}_i \end{array} \right\}$$

We then deduce that

$$\widehat{\mathcal{W}}_i = \gamma \frac{z\mathcal{G}(R_i)}{\mathcal{W}_i} \left((1+\Gamma_i^z)\widehat{z} + \Gamma_i^r \widehat{R}_i + \Gamma_i^s \widehat{\Sigma}_i \right) + (1-\gamma) \frac{\Sigma_i}{\mathcal{W}_i} \widehat{\Sigma}_i$$

where

$$\begin{aligned}
\Gamma_i^z &= \frac{-\left(\vartheta_i^e \frac{1}{\phi} + \vartheta_i^\theta\right) + \frac{\kappa_i^e \frac{1}{\phi} + \kappa_i^\theta}{\int_{R_i}^1 x dn_i(x)}}{1-\eta} \frac{ze(R_i)}{ze(R_i(z)) - (b+\Sigma_i)(1-G(R_i))} \\
\Gamma_i^r &= -\vartheta_i^r + \frac{\kappa_i^r}{\int_{R_i}^1 x dn_i(x)} \\
\Gamma_i^s &= -\frac{-\left(\vartheta_i^e \frac{1}{\phi} + \vartheta_i^\theta\right) + \frac{\kappa_i^e \frac{1}{\phi} + \kappa_i^\theta}{\int_{R_i}^1 x dn_i(x)}}{1-\eta} \frac{\Sigma_i (1-G(R_i))}{ze(R_i(z)) - (b+\Sigma_i)(1-G(R_i))}
\end{aligned}$$

C.5 Numerical algorithm

The model has three exogenous state variables: the worker's age i, the match-specific productivity ϵ and the aggregate productivity z. For the grid of the match-specific productivity ϵ, we don't follow Fujita & Ramey (2012): its highest value x^h is set to sufficient large value to generate mean match productivity of 1, given that $G(\epsilon)$ is approximated by a discrete distribution with support $X = \{x_1, ..., x_M\}$, satisfying $x_1 = 1/M$, $x_m - x_{m-1} = x_M/M$. The associated probabilities

[28] At this stage, we consider that the employment rate at the steady state can be approximated by

$$n_i = \frac{1 - \pi_i + \pi_i e_i p(\theta_i)(1 - G(R_i))}{1 - \pi_i + \pi_i \{e_i p(\theta_i)(1 - G(R_i)) + [s_e + (1-s_e)\lambda G(R_i)]\}}$$

which means that we consider that the impact of the fluctuations of the people that change age negligible with respect to their mass. Thus, we have:

$$\frac{\partial n_i}{\partial e_i} = \frac{\pi_i p(\theta_i)(1 - G(R_i))\pi_i [s_e + (1-s_e)\lambda G(R_i)]}{[1 - \pi_i + \pi_i \{e_i p(\theta_i)(1 - G(R_i)) + [s_e + (1-s_e)\lambda G(R_i)]\}]^2} \Rightarrow \vartheta_i^e > 0$$

$$\frac{\partial n_i}{\partial \theta_i} = \frac{\pi_i e_i p'(\theta_i)(1 - G(R_i))\pi_i [s_e + (1-s_e)\lambda G(R_i)]}{[1 - \pi_i + \pi_i \{e_i p(\theta_i)(1 - G(R_i)) + [s_e + (1-s_e)\lambda G(R_i)]\}]^2} \Rightarrow \vartheta_i^\theta > 0$$

$$\frac{\partial n_i}{\partial R_i} = -\frac{\pi_i G'(R_i)\{\pi_i e_i p(\theta_i)[1 + s_e(1-\lambda)] + (1-\pi_i)(1-s_e)\}}{[1 - \pi_i + \pi_i \{e_i p(\theta_i)(1 - G(R_i)) + [s_e + (1-s_e)\lambda G(R_i)]\}]^2} \Rightarrow \vartheta_i^r < 0$$

$\{\gamma_1, ..., \gamma_M\}$ are $\gamma_m = g(x_m)/M$ for $m = 1, ..., M-1$, where $g(x)$ is the Log-normal density, and $\gamma_M = 1 - \sum_{i=1}^{M-1} \gamma_i$. For the aggregate shock, we also follow Fujita & Ramey (2012): we choose the method presented in Tauchen (1986) in order to represent the process z_t as a Markov chain with a state space $Z = \{z_1, ..., z_I\}$. The transition matrix of this process is $\Pi_z = [\pi_{ij}^z]$, where $\pi_{ij}^z = Pr(z_{t+1} = z_j | z_t = z_i)$. We then form two transition matrix: firstly, the matrix $\Pi_{z,\epsilon} = [\pi_{ij}^{z,\epsilon}]$ where $\pi_{ij}^{z,m} = Pr(z_{t+1} = z_j | z_t = z_i)\gamma_m$, which gives the joint probability when both aggregate and match-specific shocks can change simultaneously, and secondly, the matrix $\overline{\Pi}_z = [\overline{\pi}_{ij}^z]$, where $\overline{\pi}_{ij}^z = Pr(z_{t+1} = z_j | z_t = z_i)\mathbb{I}_m$, which gives the probability when only aggregate shock can change, for each level of match-specific productivity.

Let $\mathcal{S}_{O_7}$ the vector $[S(x_1, z_1), \cdots, S(x_M, z_1), \cdots, S(x_1, z_I), \cdots, S(x_M, z_I)]$, and $\mathcal{R}$ be the vector $Z \bigotimes X$. Then, for an initial guess for $e_{O_7}(z)$ and $\theta_{O_7}(z)$, we find the fix point of

$$\mathcal{S}_{O_7} \quad = \quad \max\left\{\mathcal{R} - z + \pi_{O_7}\beta\left[\lambda\Pi_{z,\epsilon}\mathcal{S}_{O_7} + (1-\lambda)\Pi_z\mathcal{S}_{O_7} - \Pi_{z,\epsilon}^{e,\theta,O_7}\mathcal{S}_{O_7}\right]; 0\right\}$$

where $\Pi_{z,\epsilon}^{e,\theta,O_7}\mathcal{S}_{O_7}$ is deduced from the definition of the opposite of the search value, which is $\phi(e_{O_7}) - \gamma e_{O_7} p(\theta_{O_7})\pi_{O_7}\beta\Pi_{z,\epsilon}\mathcal{S}_{O_7}$. At each iteration, we use the FOC w.r.t. e[29] to substitute $\phi(e_{O_7})$ by $\frac{1}{1+\phi}\gamma e_{O_7} p(\theta_{O_7})\pi_{O_7}\beta\Pi_{z,\epsilon}\mathcal{S}_{O_7}$ We then have $\Pi_{z,\epsilon}^{e,\theta,O_7}\mathcal{S}_{O_7} = \frac{\phi}{1+\phi}\gamma e_{O_7} p(\theta_{O_7})\Pi_{z,\epsilon}\mathcal{S}_{O_7}$. When convergence criteria are satisfied, we obtain the decision rules $\theta_{O_7}^\star(z)$, $e_{O_7}^\star(z)$ and $R_{O_7}^\star(z)$, and the optimal value for the surplus $S_{O_7}^\star(x, z)$, $\forall z$ and $\forall x$.

For $i = O_6$, we solve the same problem, except that we integrate the solution for the age $i = O_7$ in the agents' expectations. Then, we find the fix point of

$$\mathcal{S}_{O_6} \quad = \quad \max\left\{\begin{array}{l} \mathcal{R} - z + \pi_{O_6}\beta\left[\lambda\Pi_{z,\epsilon}\mathcal{S}_{O_6} + (1-\lambda)\Pi_z\mathcal{S}_{O_6} - \Pi_{z,\epsilon}^{e,\theta,O_6}\mathcal{S}_{O_6}\right] \\ (1-\pi_{O_6})\beta\left[\lambda\Pi_{z,\epsilon}\mathcal{S}_{O_7}^\star + (1-\lambda)\Pi_z\mathcal{S}_{O_7}^\star - \Pi_{z,\epsilon}^{e^\star,\theta^\star,O_7}\mathcal{S}_{O_7}^\star\right] \end{array}; 0\right\}$$

which gives, when the convergence criteria are satisfied, $\{\theta_{O_6}^\star(z), e_{O_6}^\star(z), R_{O_6}^\star(z)\}$, and $S_{O_6}^\star(x, z)$, $\forall z$ and $\forall x$. we repeat the procedure until $i = Y$. Given this complete set of decision rules, we can simulate the Markov chain for JFR and JSR and construct the theoretical distribution of the employment per age, using equations (3), (5) and (4).

[29] This FOC is $\phi'(e_{O_7}) = \gamma p(\theta_{O_7})\pi_{O_7}\beta\Pi_{z,\epsilon}\mathcal{S}_{O_7}$