Search and Matching

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Introduction
Motivation

Motivations

- The recent developments of the microeconomics of the labor market
  - Rational choices in a market without frictions (hours worked, retirement, education).
  - The impacts of the search fictions on these choices (unemployment and wage dynamics, wage inequalities)
  - How do others markets affect these choice (firms dynamics, financial market, monetary policy)
- Why do microeconomics models useful ?
  - Beyond the evaluation of an existing policy
  - How to find the optimal design o a policy ?

The need of a structural approach

Motivation : The Lucas Critique (1976)

"Given that the structure of an econometric model consist of optimal decision rules of economic agents, and that optimal decision rule vary systematically with change in the structure of series relevant of the decision maker, it follows that any change in policy will systematically alter the structure of econometric models" (Lucas (1976), p.40)

Motivation : The Lucas Critique (1976)

Consider the simple representation of the log–linearized version of a forward-looking model

$$y_t = aE_t y_{t+1} + bx_t \text{ where } |a| < 1$$

where y_t and x_t respectively denote the endogenous and the forcing variables. x_t is assumed to follow a rule of the form

$$x_t = \rho x_{t-1} + \sigma \varepsilon_t \text{ with } \varepsilon_t \sim iid(0, 1)$$

The solution to this simple model is given by

$$\begin{cases} y_t = \frac{b}{1-a\rho} x_t & \text{The impact of the policy } (x) \text{ on decision } (y) \\ x_t = \rho x_{t-1} + \sigma \varepsilon_t & \text{The policy tool} \end{cases}$$

Motivation : The Lucas Critique (1976)

This reduced form may be used to estimate the parameters of the model

$$\begin{cases} y_t &= \widehat{\alpha} x_t \\ x_t &= \widehat{\rho} x_{t-1} + \widehat{\sigma}\varepsilon_t \end{cases}$$

Using this model, one can predict the impact of an innovation $\varepsilon_t = 1$ in period $t+n$:

$$y_{t+n} = \widehat{\alpha}\widehat{\rho}^n$$

This forecast is true if and only if the policy rule does not change, ie. if and only if ρ is stable.

Motivation : The Lucas Critique (1976)

At the opposite, assume that the policy rule change. ρ becomes $\rho/2$. This reduced form of the model becomes

$$\begin{cases} y_t = \widehat{\alpha} x_t \\ x_t = \left(\frac{\widehat{\rho}}{2}\right) x_{t-1} + \widehat{\sigma}\varepsilon_t \end{cases}$$

whereas the true model is

$$\begin{cases} y_t = \frac{b}{1-a\left(\frac{\rho}{2}\right)} x_t \\ x_t = \left(\frac{\rho}{2}\right) x_{t-1} + \sigma\varepsilon_t \end{cases}$$

Motivation : The Lucas Critique (1976)

Proposition

There is a gap between the naive prediction

$$y_{t+n} = \widehat{\alpha}\left(\frac{\widehat{\rho}}{2}\right)^n = \frac{b}{1-a\rho}\left(\frac{\widehat{\rho}}{2}\right)^n$$

and the "true" prediction

$$y_{t+n} = \frac{b}{1-a\left(\frac{\rho}{2}\right)}\left(\frac{\widehat{\rho}}{2}\right)^n$$

Motivation : The Lucas Critique (1976)

Proposition

We need to estimate or to calibrate the DEEP parameters $\{a, b, \rho, \sigma\}$ *using theoretical restrictions.*
$\Rightarrow$ *The reduced form model is not really useful.... except if* $a = 0$, *i.e. if agents have no expectations ! ! !*

Motivation : The Lucas Critique (1976)

The implication of the Lucas critique

1. The economist must find micro-foundations to the equilibrium in order to identify both the structural parameters and the policy parameters. This is necessary to forecast the implications of a policy change.
2. For the policy maker, the Lucas critique implies that the equilibrium of the economy is the equilibrium of a game : the private agents expect the policy changes and then the discretionary policy becomes inefficient (see Kydland and Prescott [1977] : Rules versus discretion)

Methodology : from microeconomic behaviors to policy evaluations

How to use the theory in order to evaluate a policy reform

- To propose an equilibrium model with rational expectations.
- To estimate this model before an unexpected policy reform.
- To use the model to predict the impact of the "new rules" (the reform) $\Leftrightarrow$ Policy experiments are useful to test theory.
- Theory allows to find the optimal design of the policy.

Plan

1. Allocation of Employment in a Labor Market without Frictions (1 sessions)
2. Allocation of Employment in a Frictional Labor Market (1 & 1/2 sessions)
3. Policy evaluation using a structural model (1/2 sessions)

Part I
Allocation of Employment in a Labor Market without Frictions

What are the major questions that the neoclassical model can deal with ?

- The employment (extensive margin) and the hours per worker (intensive margin) fluctuations : How do transitory shocks change the optimal allocation of these quantities ? What is the size of the gap between theory and data ?
  $\Leftrightarrow$ Business cycle analysis.
- The long-run shift in the total hours worked : How does permanent changes in taxes change the effort at work ? Is it possible to explain the contrasting experiences of different OECD countries ?
  $\Leftrightarrow$ Analysis of the structural changes
- To go beyond the simplification of the representative agent : to take into account the age and to explain the choices of retirement.
  $\Leftrightarrow$ Life-cycle features

The Neo-Classical Model : Assumptions and Notations

- The economy is populated by a large number of identical households whose measure is normalized to one.
- Each household consists of a continuum of infinitely-lived agents.
- At each period there is full employment : $N_t = 1, \forall t$ ($\Leftrightarrow$ an analysis of the intensive margin).
- Firm sales goods in a competitive market.
- The markets of input factors are competitive : the price of each input factor is equal to its marginal productivity.
- Financial market is perfect : households can borrow and save freely.

The Neo-Classical Model : Household Behaviors

The representative household's preferences are

$$\sum_{t=0}^{\infty} \beta^t U(C_t, 1 - h_t)$$

- $0 < \beta < 1$ is the discount factor.
- C_t is the per capita consumption
- $1 - h_t$ the leisure time, h_t the number of hour worked.
- The contemporaneous utility function is assumed to be increasing and concave in both arguments
- Agent have the opportunity to save : they can trade asset. The amount of asset at time t is denoted by K_t.

The Neo-Classical Model : Household Behaviors

Each household chooses $\{C_t, h_t, K_{t+1} | t \geq 0\}$ to maximize utility subject to the budget constraint

$$(1+R)K_t + (1-\tau_{w,t})w_t h_t + T_t - K_{t+1} - (1+\tau_{c,t})C_t \geq 0 \quad (\lambda_t)$$

- w_t is the real wage.
- R is real interest rate (constant over time for simplicity),
- $\tau_{c,t}$ is the consumption tax rate,
- $\tau_{w,t}$ the labor income tax rate,
- T_t is a lump-sum transfer from the government,

The Neo-Classical Model : Household Behaviors

If we use λ_t to denote the Lagrange multiplier associated to budget constraint at time t, the Lagrangian of the consumer problem is

$$\begin{aligned} & \mathcal{L} \\ = & \sum_{t=0}^{\infty} \beta^t U(C_t, 1-h_t) \\ & + \sum_{t=0}^{\infty} \lambda_t [(1+R)K_t + (1-\tau_{w,t}) w_t h_t + T_t - K_{t+1} - (1+\tau_{c,t}) C_t] \end{aligned}$$

The Neo-Classical Model : Household Behaviors

The optimality conditions of this problem lead to :

$$
\begin{aligned}
(1+\tau_{c,t})\lambda_t &= \beta^t U'_{C_t} \quad (1)\\
(1-\tau_{w,t})\lambda_t w_t &= \beta^t U'_{h_t} \quad (2)\\
\lambda_t &= \lambda_{t+1}(1+R) \quad (3)
\end{aligned}
$$

These 3 equations and the budgetary constraint allow to define the solution as follow

$$
\begin{aligned}
C_t &= C(\{w_i\}_{i=1}^{\infty}, \{\tau_{c,i}\}_{i=1}^{\infty}, \{\tau_{w,i}\}_{i=1}^{\infty}, K_0, t)\\
h_t &= h(\{w_i\}_{i=1}^{\infty}, \{\tau_{c,i}\}_{i=1}^{\infty}, \{\tau_{w,i}\}_{i=1}^{\infty}, K_0, t)
\end{aligned}
$$

Idea : the actual consumption and the instantaneous labor supply at time t are function of all the dynamics of the real wage and the taxes, for a given initial condition K_0.

The Neo-Classical Model : Household Behaviors

Assume for simplicity that the preferences are separable :

$$U(C_t, 1-h_t) = \ln C_t + \frac{\sigma}{\sigma-1}(1-h_t)^{\frac{\sigma-1}{\sigma}} \quad \sigma > 1$$

Then (1) and (2) become :

$$\begin{aligned} C_t &= \frac{1}{1+\tau_{c,t}} \frac{\beta^t}{\lambda_t} &(4) \\ 1-h_t &= \left(\frac{1}{(1-\tau_{w,t})w_t} \frac{\beta^t}{\lambda_t} \right)^{\sigma} &(5) \end{aligned}$$

Moreover, by iterating on $\lambda_t = \lambda_{t+1}(1+R)$, we obtain

$$\lambda_t = \lambda_0/(1+R)^t \qquad (6)$$

The Neo-Classical Model : Household Behaviors

Using (6), Equations (4) and (5) become :

$$C_t = \frac{1}{(1+\tau_{c,t})\lambda_0}\left[\beta(1+R)\right]^t \tag{7}$$

$$1-h_t = \left(\frac{1}{(1-\tau_{w,t})w_t\lambda_0}\left[\beta(1+R)\right]^t\right)^{\sigma} \tag{8}$$

Consumption and leisure depend on current prices and taxes, but also on the implicit value of the total wealth (forward looking behavior).

The Neo-Classical Model : Household Behaviors

Assuming $T_t = K_0 = 0$, the budgetary constraint can be rewritten as follow :

$$\begin{aligned} & \sum_{t=0}^{\infty}(1+R)^{-t}[(1+\tau_{c,t})C_t + (1-\tau_{w,t})w_t(1-h_t)] \\ = & \sum_{t=0}^{\infty}(1+R)^{-t}(1-\tau_{w,t})w_t \end{aligned}$$

The Neo-Classical Model : Household Behaviors

Then, using (7) and (8), we have :

$$\sum_{t=0}^{\infty}(1+R)^{-t}\left[\begin{array}{l}(1+\tau_{c,t})\frac{1}{(1+\tau_{c,t})\lambda_0}\left[\beta(1+R)\right]^t \\ +(1-\tau_{w,t})w_t\left(\frac{1}{(1-\tau_{w,t})w_t\lambda_0}\left[\beta(1+R)\right]^t\right)^{\sigma}\end{array}\right]$$

$$=\sum_{t=0}^{\infty}(1+R)^{-t}(1-\tau_{w,t})w_t$$

$$\Leftrightarrow \quad \sum_{t=0}^{\infty}\left[\begin{array}{l}\frac{1}{\lambda_0}\beta^t \\ +(1-\tau_{w,t})w_t\left(\frac{1}{(1-\tau_{w,t})w_t\lambda_0}\left[\beta(1+R)\right]^t\right)^{\sigma}(1+R)^{-t}\end{array}\right]$$

$$=\sum_{t=0}^{\infty}(1+R)^{-t}(1-\tau_{w,t})w_t$$

$$\Leftrightarrow \quad \sum_{t=0}^{\infty}\beta^t\left[\;1+\left((1-\tau_{w,t})w_t\lambda_0\left[\beta(1+R)\right]^{-t}\right)^{-\sigma}\left[\beta(1+R)\right]^{-t}(1-\tau_{w,t})w_t\lambda_0\;\right]$$

$$=\sum_{t=0}^{\infty}\beta^t\left\{\left[\beta(1+R)\right]^{-t}(1-\tau_{w,t})w_t\lambda_0\right\}$$

The Neo-Classical Model : Household Behaviors

Then, using (7) and (8), we can deduce the value of λ_0 :

$$0 = \sum_{t=0}^{\infty} \beta^t \left\{ 1 + \left((1-\tau_{w,t}) w_t \lambda_0 \left[\beta(1+R)\right]^{-t}\right)^{1-\sigma} - \left[\beta(1+R)\right]^{-t} (1-\tau_{w,t}) w_t \lambda_0 \right\}$$

The solution takes the implicit form $\lambda_0 = \Lambda(\{w_i\}_{i=1}^{\infty}, \{\tau_{w,i}\}_{i=1}^{\infty})$.
Lucas and Rapping (1969) propose to use this model in order to explain the short run dynamics of the employment : the impact of transitory shocks

Permanent Shocks

Assume that all wage w_t are multiplied by a single quantity $\widetilde{w}_t = \alpha w_t$. Then, with $\tau_{w,t} = 0$, $\widetilde{\lambda}_0$ is the solution of

$$\begin{aligned}
0 &= \sum_{t=0}^{\infty} \beta^t \left\{ 1 + \left(\widetilde{w}_t \widetilde{\lambda}_0 \left[\beta(1+R)\right]^{-t} \right)^{1-\sigma} - \left[\beta(1+R)\right]^{-t} \widetilde{w}_t \widetilde{\lambda}_0 \right\} \\
\Leftrightarrow 0 &= \sum_{t=0}^{\infty} \beta^t \left\{ 1 + \left(\alpha w_t \widetilde{\lambda}_0 \left[\beta(1+R)\right]^{-t} \right)^{1-\sigma} - \left[\beta(1+R)\right]^{-t} \alpha w_t \widetilde{\lambda}_0 \right\}
\end{aligned}$$

$\Rightarrow$ there is a solution iff $\widetilde{\lambda}_0 = \frac{\widehat{\lambda}_0}{\alpha}$, because in this case we have

$$\begin{aligned}
0 &= \sum_{t=0}^{\infty} \beta^t \left\{ 1 + \left(\alpha w_t \frac{\widehat{\lambda}_0}{\alpha} \left[\beta(1+R)\right]^{-t} \right)^{1-\sigma} - \left[\beta(1+R)\right]^{-t} \alpha w_t \frac{\widehat{\lambda}_0}{\alpha} \right\} \\
\Leftrightarrow 0 &= \sum_{t=0}^{\infty} \beta^t \left\{ 1 + \left(w_t \widehat{\lambda}_0 \left[\beta(1+R)\right]^{-t} \right)^{1-\sigma} - \left[\beta(1+R)\right]^{-t} w_t \widehat{\lambda}_0 \right\}
\end{aligned}$$

Permanent Shocks

Because $\widetilde{w}_t \widetilde{\lambda}_0 = w_t \widehat{\lambda}_0$**, where** $\widehat{\lambda}_0$ **is the lagrange multiplier in a economy with the initial wage, we deduce from**

$$1 - h_t = \left(\frac{1}{w_t \widehat{\lambda}_0} [\beta(1+R)]^t \right)^{\sigma} = \left(\frac{1}{\widetilde{w}_t \widetilde{\lambda}_0} [\beta(1+R)]^t \right)^{\sigma}$$

that the labor supply does not change after a permanent shock.

$$h(w_t) = h(\widetilde{w}_t) \quad \forall t$$

Transitory Shocks

In order to analyze the impact of a transitory shock, we assume that $\frac{\partial \lambda_0}{\partial w_t} \approx 0$: the impact of the change in one wage is negligible when we consider all the life cycle.
$\Rightarrow$ We can neglect the variation in λ_0, leading to

$$\begin{aligned}
\frac{\partial h_t}{\partial w_t} &= \frac{\partial \left[1 - \left(\frac{1}{(1-\tau_{w,t})w_t\lambda_0}\left[\beta(1+R)\right]^t\right)^{\sigma}\right]}{\partial w_t} \\
&= \sigma \left(w_t\right)^{-1}\left(1 - h_t\right) \\
\Rightarrow \frac{\partial h_t}{\partial w_t}\frac{w_t}{h_t} &= \sigma \frac{1-h_t}{h_t}
\end{aligned}$$

The larger σ, the larger the change in labor supply following a transitory shock. Hours are sensitive to transitory wage shocks (the business cycle)

The RBC model : the households

- Preferences

$$u(C_t, L_t) = \begin{cases} \theta \log(C_t) + (1-\theta)\log(L_t) \\ \frac{\left(C_t^{\theta} L_t^{1-\theta}\right)^{1-\sigma} - 1}{1-\sigma} \end{cases}$$

  - C_t denotes the consumption
  - L_t denotes the leisure
- The time allocation is constrained by : $T_0 = L_t + N_t$
- The budgetary constraint is given in a complete market economy :

$$(1+R_t)K_t + w_t N_t - K_{t+1} - C_t \geq 0$$

where R_t is the capital returns and w_t the unit price of labor (the wage).

The RBC model : the production function

All firms have access to the same technology :

$$Y_t = A_t F(K_t, X_{N,t} N_t) = A_t K_t^{\alpha} (X_{N,t} N_t)^{1-\alpha}$$

where

- K_t is the capital stock determined rented at time t. Its unit price is $R_t + \delta$: the financial return plus the paiement of the forgone capital during the period (δ per unit of capital used).
- N_t is the total hours determined at date t.
- A_t is the technological shock observed at date t,
- $X_{N,t}$ is the exogenous technical progress which is labor-augmenting. Its growth rate is denoted by γ_x.

Profit maximization leads to

$$A_t F_K = \alpha \frac{Y_t}{K_t} = R_t + \delta \quad \text{and} \quad A_t F_N = (1-\alpha)\frac{Y_t}{N_t} = w_t$$

The RBC model : general equilibrium

The general equilibrium is definied by the set of functions $\{C_t, K_{t+1}, N_t, Y_t\}$, solution of the following system :

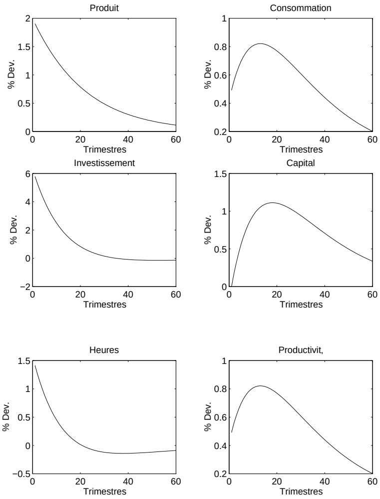
$$\begin{aligned}
u'_{C_t} &= \beta E_t \left[\left(1 - \delta + \alpha \frac{Y_{t+1}}{K_{t+1}} \right) u'_{C_{t+1}} \right] \\
\frac{u'_{L_t}}{u'_{C_t}} &= (1-\alpha) \frac{Y_t}{N_t} \\
K_{t+1} &= (1-\delta)K_t + Y_t - C_t \\
Y_t &= A_t K_t^{\alpha} (X_{N,t} N_t)^{1-\alpha} \\
\log(A_{t+1}) &= \rho \log(A_t) + (1-\rho) \log(\bar{A}) + \epsilon_t
\end{aligned}$$

This system gives the equilibrium dynamics of C_t, K_{t+1}, N_t, Y_t, given the stochastic process A_t.

The RBC model : the behaviors after a technological shock

1. After a technological shock, the labor productivity $\uparrow \Rightarrow$ labor demand $\uparrow$.
2. The production $\uparrow$ because the technological frontier shift-up and because the total hours increase.
3. At equilibrium, this adjustment is possible if the raise in labor productivity (the real wage) gives the incentive to supply more labor.
4. This is possible if the shock is transitory : intertemporal substitution $\Leftrightarrow$ large opportunities today in the labor market, leading the agents to work more today than tomorrow when these opportunities disappear.
5. The saving opportunity allows the households to smooth these additional gains $\Rightarrow \uparrow$ persistence of the shock.

The RBC model : the IRF



The RBC model : US economy versus K/P/82

Variable x	σ	Correlation between GDP and $x(t-1)$	$x(t)$	$x(t+1)$
GDP	1,8%	0,82	1,00	0,82
Model	1,79%	0,60	1,00	0,60
Consumption				
Services	0,6%	0,66	0,72	0,61
Non-durable	1,2%	0,71	0,76	0,59
Model	0,45%	0,47	0,85	0,71
Investment				
Total investment	5,3%	0,78	0,89	0,78
Model	5,49%	0,52	0,88	0,78
Employment				
Total hours	1,7%	0,57	0,85	0,89
Hour per capita	1,0%	0,76	0,85	0,61
Model	1,23%	0,52	0,95	0,55
Productivity	1,0%	0,51	0,34	-0,04
Model	0,71%	0,62	0,86	0,56

Long Run Changes : the Growing Trend in Taxation

With constant taxes, the initial lagrange multiplier of the budgetary constraint is given by :

$$0 = \sum_{t=0}^{\infty} \beta^t \left\{ 1 + \left((1-\tau_w) w_t \lambda_0^1 \left[\beta(1+R)\right]^{-t} \right)^{1-\sigma} - \left[\beta(1+R)\right]^{-t} (1-\tau_w) w_t \lambda_0^1 \right\}$$

$\Rightarrow \lambda_0^1 = \frac{\widehat{\lambda}_0}{(1-\tau_w)}$.

A constant tax has the same impact than a permanent shock : no impact.

Long Run Changes : the Growing Trend in Taxation

Assume now that the tax on wages grows.

If the tax grows, such that $\tau_{w,t+1} > \tau_{w,t}$, then

$$0 = \sum_{t=0}^{\infty} \beta^t \left\{ 1 + \left((1-\tau_{w,t}) w_t \lambda_0 \left[\beta(1+R) \right]^{-t} \right)^{1-\sigma} - \left[\beta(1+R) \right]^{-t} (1-\tau_{w,t}) w_t \lambda_0 \right\}$$

$\Rightarrow \lambda_0 \neq \frac{\widehat{\lambda}_0}{(1-\tau_{w,t})}$ because λ_0 must be a constant.
Thus $(1-\tau_{w,t}) w_t \lambda_0 \neq w_t \widehat{\lambda}_0$ implying that

$$h_t(\tau_{w,t} > 0) \neq h_t(\tau_{w,t} = 0)$$

Time varying taxes have an impact on the long run trend in hours. In which direction ?

Long Run Changes : the Growing Trend in Taxation

Assume that $w_t = w$, $t = 1, 2$, $\tau_{w,1} = 0$ and $\tau_{w,2} = \tau_w$:

$$0 = \left\{1 + \left(w\widehat{\lambda}_0\right)^{1-\sigma} - w\widehat{\lambda}_0\right\} + \beta\left\{1 + \left(\frac{w\widehat{\lambda}_0}{\beta(1+R)}\right)^{1-\sigma} - \frac{w\widehat{\lambda}_0}{\beta(1+R)}\right\}$$

$$0 = \left\{1 + (w\lambda_0)^{1-\sigma} - w\lambda_0\right\} + \beta\left\{1 + \left(\frac{(1-\tau_w)w\lambda_0}{\beta(1+R)}\right)^{1-\sigma} - \frac{(1-\tau_w)w\lambda_0}{\beta(1+R)}\right\}$$

$$0 = \left\{1 + \left((1-\tau_w)w\widetilde{\lambda}_0\right)^{1-\sigma} - (1-\tau_w)w\widetilde{\lambda}_0\right\} + \beta\left\{1 + \left(\frac{(1-\tau_w)w\widetilde{\lambda}_0}{\beta(1+R)}\right)^{1-\sigma} - \frac{(1-\tau_w)w\widetilde{\lambda}_0}{\beta(1+R)}\right\}$$

Thus, we deduce :

$$\Rightarrow \widehat{\lambda}_0 < \lambda_0 < \frac{\widehat{\lambda}_0}{1-\tau_w} = \widetilde{\lambda}_0$$

$$\Rightarrow (1-\tau_w)\lambda_0 < \widehat{\lambda}_0$$

Long Run Changes : the Growing Trend in Taxation

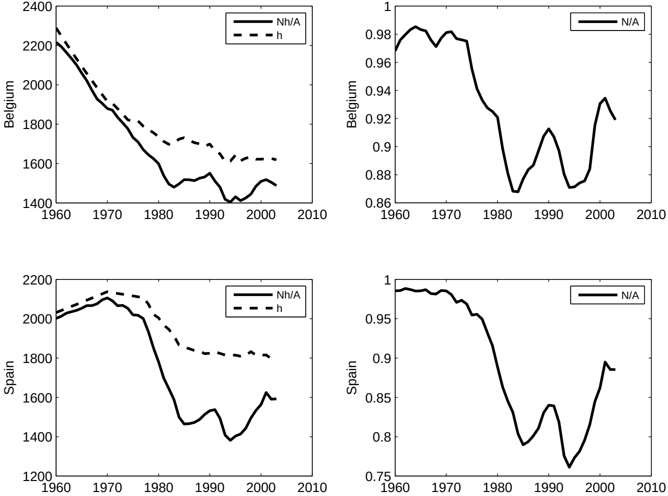
Then, by reporting the previous results in the FOC of the demand for leisure, we deduce :

$$\begin{aligned}
1-h_1=\left(\frac{1}{w\lambda_0}\left[\beta(1+R)\right]^t\right)^\sigma &< 1-\widehat{h}_1=\left(\frac{1}{w\widehat{\lambda}_0}\left[\beta(1+R)\right]^t\right)^\sigma \\
\Leftrightarrow h_1 &> \widehat{h}_1 \\
1-h_2=\left(\frac{1}{(1-\tau_w)w\lambda_0}\left[\beta(1+R)\right]^t\right)^\sigma &> 1-\widehat{h}_2=\left(\frac{1}{w\widehat{\lambda}_0}\left[\beta(1+R)\right]^t\right)^\sigma \\
\Leftrightarrow h_2 &< \widehat{h}_2
\end{aligned}$$

Labor supply declines with the trend in the tax-wage.

$\Leftrightarrow$ The best response of the agent is to work when taxes are low (intertemporal substitution).

Why do Americans work so much more than European ?



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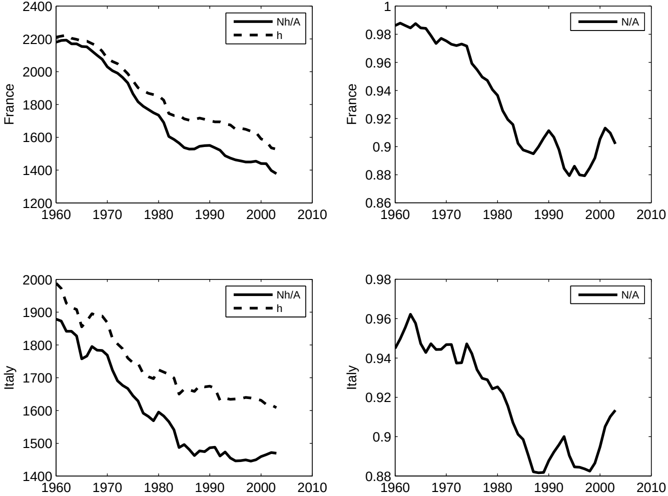

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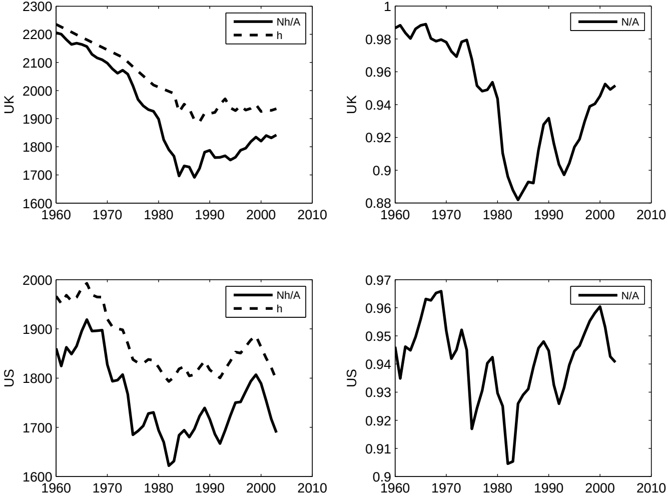

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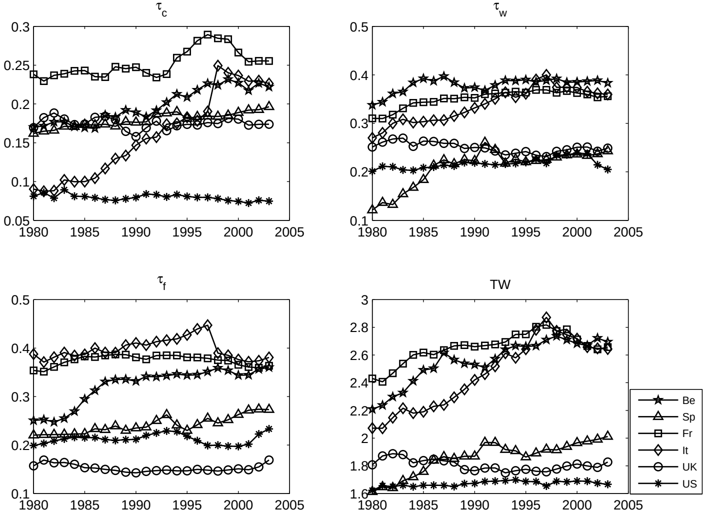


Prescott (2004) : the increase in taxes.

- The economy is populated by a large number of identical households whose measure is normalized to one.
- Each household consists of a continuum of infinitely-lived agents.
- At each period there is full employment : $N_t = 1, \forall t$.
- The utility function is assumed to be increasing and concave in both arguments and it shows conventional separability between consumption C and leisure $1-h$:

$$\sum_{t=0}^{\infty} \beta^t U(C_t, 1-h_t) = \ln C_t + \sigma \ln(1-h_t) \quad \sigma > 0 \quad \beta \in]0;1[$$

- The capital stock K_t is rented to firms at net price $(r_t + \delta)$, where $0 < \delta < 1$ is the depreciation rate of capital.

Prescott (2004) : the tax increase in Europe.

- Each household chooses $\{C_t, h_t, K_{t+1} | t \geq 0\}$ to maximize her intertemporal utility subject to the budget constraint

$$(1+R_t)K_t + (1-\tau_{w,t})w_t h_t + T_t - K_{t+1} - (1+\tau_{c,t})C_t \geq 0 \quad (\lambda_t)$$

$R_t = (1-\tau_k)r$: effective interest rate
τ_k, τ_c, τ_w : capital, consumption, labor income tax rates
T : is a lump-sum transfer from the government.
w : real wage.

- Each firm has access to a Cobb-Douglas production technology to produce output and seeks to maximize the following profit flow :

$$\pi_t = A_t K_t^{\alpha} (h_t)^{1-\alpha} - (1+\tau_{f,t}) w_t h_t - (r_t + \delta) K_t \quad 0 < \alpha < 1$$

τ_f : payroll taxes.

Prescott (2004) : the tax increase in Europe.

Therefore, the labor market equilibrium is then determined by :

$$\begin{aligned}(1-\alpha)\frac{Y_t}{h_t} &= \left\{\frac{(1+\tau_{f,t})(1+\tau_{c,t})}{1-\tau_{w,t}}\right\}\sigma(1-h_t)^{-1}C_t \\ \Leftrightarrow MPH_t &= (1+TW_t)\times MRS(H/C)_t\end{aligned}$$

where MPH_t and $MRS(H/C)$ denote respectively

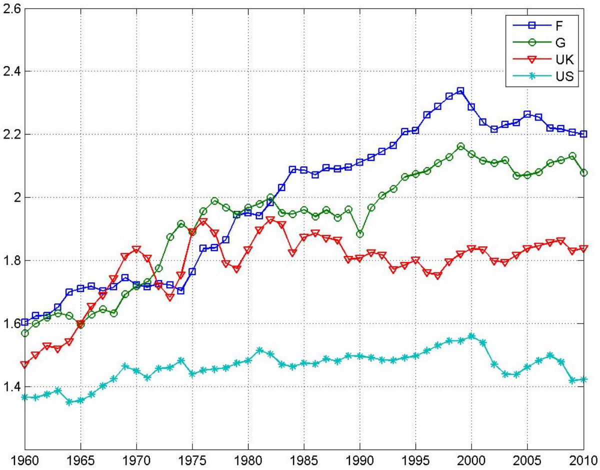
- the marginal product of an hour worked
- the marginal rate of substitution between hours worked and consumption

The tax wedge is defined by :

$$1+TW_t = \frac{(1+\tau_{f,t})(1+\tau_{c,t})}{1-\tau_{w,t}}$$

In a economy without taxes, we have $TW_t = 0$

Prescott (2004) : the tax wedge $1 + TW_t = \frac{(1+\tau_{f,t})(1+\tau_{c,t})}{1-\tau_{w,t}}$



Prescott (2004) : the tax increase in Europe.

How to quantify the disincentive impact of growing taxes on labor supply ?

- Under the assumption that the model is a good approximation of the household behaviors, the observed data can be generated by this model.
  $\Rightarrow$ the data integrate the optimal reactions of the agent following the increase in taxes.
- Let Δ_t defined by

$$MRS(H/C)_t = (1 - \Delta_t)MPH_t$$

- From the point of view of the econometrician, Δ_t is the residual of the FOC estimated with an omitted variable, TW_t.
- If $\Delta_t < 0$, agents react as if there is employment subsidies $\uparrow h$.
- If $\Delta_t > 0$, agents react as if there is employment taxes : $\downarrow h$.

Prescott (2004) : the tax increase in Europe.

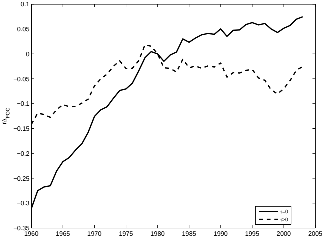
- Using observed data, we can compute Δ_t. If the correlation between Δ_t and observed taxes is large, then the increase of taxes explain the decline of the labor supply.
- The time series of Δ_t are computed for the six countries of our sample. Theory gives the restriction

$$\Delta_t = 1 - \frac{\sigma(1-h_t)^{-1}C_t}{(1-\alpha)\frac{Y_t}{h_t}}$$

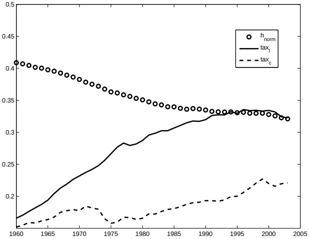
  - Y_t is measured by the aggregate production per capita,
  - C_t by the aggregate consumption per capita
  - h_t by the average number of hours worked per employee,
  - $\alpha = .4$ and $\sigma = 2$ are calibrated.

Prescott (2004) : the tax increase in Europe.

Mean wedges



Hours and Taxes



Left : We take 1980 as normalization year. — : $\Delta_{t,TW=0}$. – – : $\Delta_{t,TW>0}$.
Right : — : τ_w ; – – : τ_c and o : h.

Prescott (2004) : the tax increase in Europe.

Various factors can explain the labor wedges (Δ) :

- The role of changes in taxes is remarkable over 1960-1985 $\Rightarrow$ their incorporation reduces the size of the wedge over time and across the countries.
- On the other side, the negative impact of the LMI, like the worker's bargaining power ($Barg$), and the average unemployment benefits (ARR), on the performance of European labor markets after 1980 is well documented (Blanchard and Wolfer (1999)).

The specification of the panel regression is :

$$\ln(1-\Delta^{h,w}_{i,t,TW=0}) = a_i + b\ln(TW_{i,t}-1) + \gamma Barg_{i,t} + \beta ARR_{i,t} + \epsilon_{i,t}$$

Prescott (2004) : the tax increase in Europe.

The long-run decline in the hours worked per employee is mainly due to the increase in the taxes.

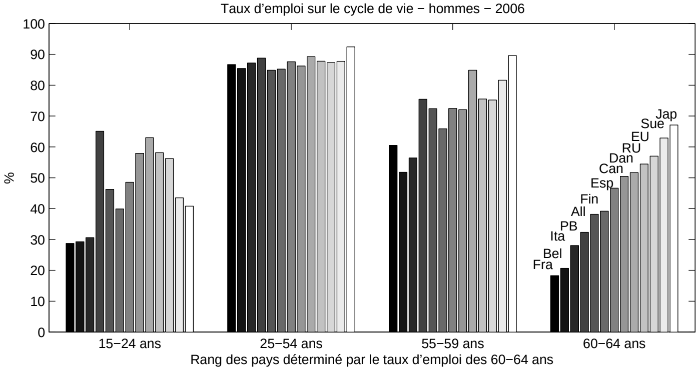
	Reg (1)	Reg (2)	Reg (3)	Reg (4)	Reg (5)	Reg (6)	Reg (7)
a_{be}	.2055	.0370	.2690	.0047	.0348	.0119	.3177
	[-.08 ;.49]	[.00 ;.07]	[-.001 ; .53]	[-.05 ; .05]	[-.30 ; .37]	[-.05 ; .07]	[-.03 ; .66]
a_{sp}	.0773	.0054	.1364	-.0427	.1367	.1518	.3300
	[-.10 ;.26]	[-.01 ;.02]	[-.016 ; .28]	[-.10 ; .02]	[-.05 ; .33]	[.09 ; .20]	[.12 ; .53]
a_{fr}	.1383	.0236	.1829	.0002	-.0427	-.0359	.1755
	[-.06 ;.34]	[-.01 ;.062]	[-.00 ; .37]	[-.04 ; .05]	[-.27 ; .18]	[-.09 ; .02]	[-.06 ; .42]
a_{it}	.1510	-.0094	.1834	-.0100	-.0115	-.0904	.1568
	[-.08 ;.38]	[-.04 ;.02]	[-.04 ; .40]	[-.04 ; .02]	[-.29 ; .26]	[-.12 ;-.06]	[-.12 ; .43]
a_{uk}	.2237	.1353	.2712	.1032	.2872	.2576	.4376
	[.04 ;.40]	[.11 ;.15]	[.11 ; .43]	[.05 ; .15]	[.08 ; .48]	[.22 ; .29]	[.23 ; .64]
a_{us}	.1045	.0843	.1387	.0519	.2462	.2484	.3239
	[.01 ;.19]	[.05 ;.11]	[.06 ; .20]	[-.00 ; .10]	[.16 ; .33]	[.21 ; .27]	[.23 ; .41]
TW	-.3571	-.3209	-.3254	-.3627			
	[-.44 ;-.26]	[-.39 ;-.25]	[-.39 ; -.25]	[-.45 ; -.27]			
Barg	-.2666		-.3196		-.1921		-.4062
	[-.64 ;.11]		[-.68 ; .05]		[-.66 ; .27]		[-.86 ; .05]
ARR	.0949			.1222		-.2785	-.3107
	[-.06 ;.25]			[-.03 ; .28]		[-.43 ;-.12]	[-.46 ; -.15]
N	144	144	144	144	144	144	144
R^2	.9248	.9222	.9224	.9237	.8764	.8867	.8892

OLS regression. "*TW*" is the tax wedge, "*Barg*" the bargaining power of the workers and "*ARR*" the average replacement rate. The confidence intervals (in brackets) are at the 95% level.

Conclusion : beyond the intensive margin

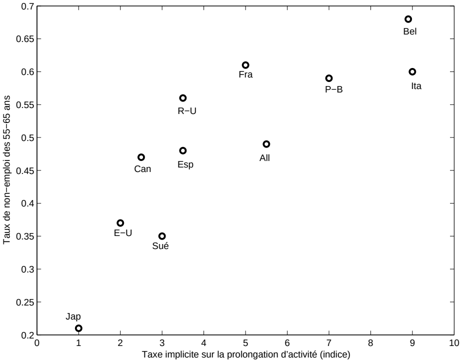
- In this first part, we have focussed on the intensive margin : the number of hour worked by worker.
- There is another margin : the extensive margin, which give the number of labor market participant in the population of the 15-65 years old.
- Two important decisions drive this extensive margin :
  1. The retirement decision which determines the employment rate of the older workers.
  2. The education decision which determines the employment rate of the young workers.

Conclusion : beyond the intensive margin



Retirement Choice

Taxes on labor incomes have also an impact on the retirement age decision



Retirement Choice

Assumptions and notations :

- Let T and R denote respectively the age of the death and of the retirement age,
- Let w denotes the real wage
- Let r and δ denote respectively the interest and subjective discount rate.

We assume that there is no uncertainty, and that agents can save and borrow in perfect financial markets.
At this stage, we assume that Social Security does not exist $\Leftrightarrow$ There is no distortions.

Retirement Choice

Preferences :

$$U = \begin{cases} U(c(t)) & \text{if} \quad t \in [0, R) \\ U(c(t)) + v & \text{if} \quad t \in [R, T] \end{cases}$$

where v denotes the leisure when the individual is a retiree.
The problem of the agent is :

$$\max_{c(t), R} \left\{ \int_0^T e^{-\delta t} U(c(t)) dt + \int_R^T e^{-\delta t} v dt \right\}$$

$$s.t. \quad \int_0^T e^{-rt} c(t) dt = \int_0^R e^{-rt} w dt$$

Retirement Choice

The FOC are :

$$\left.\begin{array}{rcl} U'(c(t)) & = & \lambda e^{-(r-\delta)t} \\ ve^{-\delta R} & = & \lambda we^{-rR} \end{array}\right\} \Rightarrow \underbrace{v}_{\substack{\text{Opportunity} \\ \text{Cost}}} = \underbrace{U'(c(R))w}_{\substack{\text{Marginal} \\ \text{return}}}$$

In order to determine R, it is necessary to determine $c(R)$. Assuming, for simplicity, that $r = \delta$, implying $c(t) = c$, $\forall t$, the budgetary constrain of the agent leads to :

$$c(R) = w\frac{\int_0^R e^{-rt}dt}{\int_0^T e^{-rt}dt} = w\frac{1-e^{-rR}}{1-e^{-rT}}$$

The optimal age of retirement is then given by :

$$v = U'\left(w\frac{1-e^{-rR}}{1-e^{-rT}}\right)w$$

Retirement Choice

Assume now that there is a Social Security system. The agent budgetary constraint becomes

$$\int_0^T e^{-rt}c(t)dt = \int_0^R e^{-rt}(1-\tau)wdt + \int_R^T e^{-rt}p(R)dt$$

The FOC are :

$$\begin{aligned} U'(c(t)) &= \lambda e^{-(r-\delta)t} \\ ve^{-\delta R} &= \lambda\left[(1-\tau)we^{-rR} - p(R)e^{-rR} + \int_R^T e^{-rt}p'(R)dt\right] \end{aligned}$$

$$\Rightarrow \underbrace{v}_{\text{Opportunity Cost}} = \underbrace{U'(c(R))\left[(1-\tau)w - p(R) + \int_0^{T-R} e^{-rt}p'(R)dt\right]}_{\text{For a given } U'(c(R))\text{, the marginal return can be taxed}}$$

Retirement Choice

We can then define this tax on the continued activity tax :

$$TI = \underbrace{\tau w}_{\substack{\text{Effective}\\ \text{Tax}}} + \underbrace{p(R)}_{\substack{\text{Implicit}\\ \text{Tax}}} - \underbrace{\int_0^{T-R} e^{-rt} p'(R) dt}_{\text{Bonus}}$$

The SS system is actuarially fair if and only if

$$\tau w + p(R) = \int_0^{T-R} e^{-rt} p'(R) dt$$

implying no distortion on the labor supply

$$v = U'(c(R^{SS\star}))w \;\; \Rightarrow R^{SS\star} = R$$

Retirement Choice

Because for all SS system, we have

$$c = w\frac{1-e^{-rR}}{1-e^{-rT}} \text{ because } \tau w\frac{1-e^{-rR}}{1-e^{-rT}} = p(R)\frac{e^{-rR}-e^{-rT}}{1-e^{-rT}}$$

- If the SS system is actuarially fair, the optimal age of retirement is the same than in a economy without SS
- At the opposite, if the SS system is not actuarially fair, but under the constrain that the SS system has no deficit, the optimal age of retirement is given by :

$$p+\frac{v}{U'(c)} = (1-\tau)w \Leftrightarrow v = U'\left(w\frac{1-e^{-rR^{SS}}}{1-e^{-rT}}\right)[(1-\tau)w-p]$$

If there is no bonus when the agent delays her retirement age $\Rightarrow R^{SS} < R$

Part II
Allocation of Employment in a Frictional Labor Market
Job Search and Matching

Job Search Theory

Consider an unemployed worker.

- Finding a good job is an uncertain process which requires both time and financial resources.
- This assumption stands in contrast to the classical model, in which workers and firms are assumed to have full information at no cost about job opportunities and workers.
- References : McCall's (1970),

Intertemporal Choices and Job Search Strategy

- Consider an unemployed worker who is searching for a job by visiting area firms :
  1. Although the worker likely has many job opportunities, she has incomplete information as to the location of her best opportunities.
  2. Hence, she must spend time and resources searching, and she must hope she has luck finding one of her better opportunities quickly.
- The intertemporal arbitration :
  1. In any given week the worker may receive a job offer at some wage w.
  2. The decision she faces is whether to accept that offer and forego the possibility of finding a better job, or to continue searching and hope that she is fortunate enough to get a better offer in the near future.

Assumptions

- Each week the worker receives one wage offer.
- In order to capture the uncertainty of job offers, we assume that this offer is drawn at random from an urn containing wage offers between $\underline{w}$ and $\overline{w}$.
- Draws from this urn are independent from week to week, so the size of next week's offer is not influenced by the size of this week's offer.
- While I will interpret draws as weekly wage rates, they can be thought of more generally as capturing the total desirability of a job, which could depend on hours, location, prestige, and so on.
- For simplicity, assume that all jobs require the same number of hours and are of the same overall quality, so that jobs differ only in terms of the wage.

Preferences

We assume that individuals derive utility from consumption. There is no saving. Intertemporal preferences are given by :

$$E_0 \int_0^{\infty} e^{-rt} y(t) dt$$

where E_0 is the expectation operator conditional at time 0, $r \in [0, 1]$ the subjective discount rate and $y(t)$ the after-tax income from employment or unemployment compensation :

$$y(t) = \begin{cases} w(t) & \text{employed worker} \\ b(t) & \text{unemployed worker} \end{cases}$$

Employment Opportunities

- An unemployed worker, in each period t, receives an unemployment benefit b.
- The probability of getting a job offer is denoted λdt
- This offer is drawn from a wage offer distribution $F(w)$, which denotes the probability of receiving a wage offer between the lower wage of the distribution $\underline{w}$ and w_t

$$F(w) = \text{Prob}(w(t) \leq w)$$

- Let w^r denote the reservation wage. The worker accepts the wage offer $w(t)$ if $w(t) > w^r$, which implies that she earns that wage in period t and thereafter for each period she has not been laid off.
- The probability of being laid off at the beginning of the period is sdt.

The Decision Problem (1)

In making her decision, the unemployed worker must compare the expected lifetime incomes of accepting or rejecting a particular offer.

$$
\begin{aligned}
rv^{accept}(w) &= w - s\left[v^{accept}(w) - v^{reject}\right] \\
rv^{reject} &= b + \lambda \int_{w^r}^{\overline{w}} [v^{accept}(x) - v^{reject}] dF(x)
\end{aligned}
$$

- $v^{reject}(w)$ is the expected present value of lifetime income if she rejects a wage offer w and waits for a better offer,
- $v^{accept}(w)$ is the expected present value of lifetime income if she accepts w,

The Decision Problem (2)

The value function of an acceptable offer can be rewritten as follow

$$v^{accept}(w) = \frac{w + sv^{reject}}{r+s}$$

$v^{accept}(w)$ increases linearly with w, whereas v^{reject} is constant.

- For values of w less than w^r, v^{reject} is greater than $v^{accept}(w)$, so the worker is better off rejecting the offer.
- For w greater than w^r, v^{reject} is less than $v^{accept}(w)$, so the worker is better off accepting the offer.

The Reservation Wage

The reservation wage w^r is the value of w which satisfies

$$v^{accept}(w^r) \quad = \quad v^{reject}$$

This implies that

$$rv^{accept}(w^r) = w^r$$

and then

$$\begin{aligned} w^r &= b + \lambda \int_{w^r}^{\overline{w}} [v^{accept}(x) - v^{reject}] dF(x) \\ &= b + \frac{\lambda}{r+s} \int_{w^r}^{\overline{w}} [1 - F(x)] dx \end{aligned}$$

The Reservation Wage : computational details

$$\begin{aligned}\frac{d}{dx}\{[v^{accept}(x) - v^{reject}][1 - F(x)]\} &= dv^{accept}(x)[1 - F(\tilde{x})] \\ &\quad -[v^{accept}(x) - v^{reject}]dF(x)\end{aligned}$$

$$\left[[v^{accept}(x) - v^{reject}][1 - F(x)]\right]_{w^r}^{\overline{w}} = 0$$

$$\Rightarrow \int_{w^r}^{\overline{w}} dv^{accept}(x)[1 - F(x)]dx = \int_{w^r}^{\overline{w}} [v^{accept}(x) - v^{reject}]dF(x)$$

Because $dv^{accept}(x) = \frac{1}{r+s}$, we deduce :

$$w^r = b + \frac{\lambda}{r+s}\int_{w^r}^{\overline{w}} [1 - F(x)]dx$$

Implications (1) : the reservation wage

1. Because a higher interest rate implies discounting future earnings more rapidly, an increase in the real interest rate lowers the benefits of waiting for a higher wage. This suggests that the reservation wage will decrease.
2. An increase in the firing rate reduces the expected length of time at a given job, and thus reduces the benefit of waiting for a relatively high wage offer : the reservation wage decreases.
3. If the maximum of the wage offer distribution increases, then incentive to wait for a better job raises : w^r increases.
4. Since unemployment compensation acts as a subsidy to search activity, the worker is willing to wait longer for a high-paying job and thus increases her reservation wage.

Implications (2) : the unemployment

The exist rate of unemployment is given by $\lambda[1 - F(w^r)]$. The average duration of unemployment is then given by

$$UD = \frac{1}{\lambda[1 - F(w^r)]}$$

when w^r increases, UD increases.
The unemployment rate generated by the model is :

$$\begin{aligned} \underbrace{s(1-u)}_{\text{exists from employment}} &= \underbrace{u\lambda[1 - F(w^r)]}_{\text{exists from unemployment}} \\ \Rightarrow u &= \frac{s}{s + \lambda[1 - F(w^r)]} \end{aligned}$$

Extension 1 : On-the-job search

Now assume that worker can search on-the-job. The value to be employed is now

$$\begin{aligned} rV(w) &= w + \lambda_1 \int_{w}^{\overline{w}} [V(x) - V(w)] dF(x) - s[V(w) - \mathcal{U}] \\ r\mathcal{U} &= b + \lambda_0 \int_{w^r}^{\overline{w}} [V(x) - \mathcal{U}] dF(x) \end{aligned}$$

where λ_0 and λ_1 denote the two exogenous arrival rates of wage offers, for the unemployed and the employed, respectively.

On-the-job search : the reservation wage

The lowest acceptable offer for unemployed workers is then defined by $\mathcal{U} = V(w^r)$.
The unemployed worker's reservation wage is given by :

$$\begin{aligned} w^r &= b + (\lambda_0 - \lambda_1) \int_{w^r}^{\overline{w}} [V(x) - V(w^r)] dF(x) \\ &= b + (\lambda_0 - \lambda_1) \int_{w^r}^{\overline{w}} \frac{1 - F(x)}{r + s + \lambda_1 [1 - F(x)]} dx \end{aligned}$$

- if $\lambda_0 > \lambda_1$ the unemployed worker have a competitive advantage in the search activity : the reservation wage is greater than unemployment benefits.
- if $\lambda_0 = \lambda_1$, then $w^r = b$: the option to search a better job has the same value for an employed worker than for a unemployed. It is optimal to take the first offer and to jump on-the-job.

The Diamond Paradox

Assume that the distribution of wage offers is not exogenous : how firms can set optimally the wages ?
The objective is now to endogenize $F(w)$.

- If there is no on-the-job search, every unemployed workers accept to work for w^r.
- Because all workers have the same characteristics, all firms propose $w = w^r$,
- We then have

$$dF(w) = 0, \quad \forall w > w^r \text{ and } F(w^r) = 1$$

- In this case, the option to wait has zero value and $w^r = b$.
- There is no equilibrium with wage dispersion.

A solution to the Diamond Paradox

If there is on-the-job search, the story is different.

- An unemployed worker still accepts to work for b.
- But, now, employers have two reasons for offering a wage greater than b.
  1. First, the firm's acceptance rate increases with the wage offer, since a higher wage is more attractive.
  2. Second, the firm's retention rate increases with the wage paid by limiting voluntary quits that lead to an increase the firm value.
- $\Rightarrow$ in this case, an job search equilibrium exists with non-degenerate dispersion of wage offers.

The job-search equilibrium : steady state job flows

- the equilibrium rate of unemployment is deduce from

$$s(1-u) = \lambda_0[1 - F(w^r)]u$$

- We denote the steady-state number of employed workers being paid a wage no greater than w by $G(w)(1-u)$, where $G(w)$ is the distribution of wage earnings across employed workers and u the overall unemployment rate.
- At steady-state the flow of workers leaving employers offering a wage no greater than w equals the flow of workers hired with a wage no greater than w :

$$\underbrace{\lambda_0 \max\{F(w) - F(w^r); 0\}u}_{\text{hirings}} = \underbrace{s(1-u)G(w)}_{\text{firings}} + \underbrace{\lambda_1[1 - F(w)](1-u)G(w)}_{\text{volontary quits}}$$

The Wage-Posting Game (1)

Each employer commits to a wage offer.
The expected discounted present value of the employer's future flows of quasi-rents once a worker has been hired at wage w is

$$J(w) = \frac{y - w}{r + s + \lambda_1[1 - F(w)]}$$

where y represents the productivity of the match.
The *ex ante* asset value associated with a job is given by :

$$\max_{w \geq \underline{w}} \left\{ h(w) \left(\frac{y - w}{r + s + \lambda_1[1 - F(w)]} \right) \right\}$$

where $h(w)$ stands for the probability that a job with wage offer w is accepted by the worker.

The Wage-Posting Game (2)

The probability that a job with wage offer w is accepted by the worker is defined by :

$$\begin{aligned} h(w) &= (1-u)\lambda_1 G(w) + \lambda_0 u \\ &= \frac{s\lambda_0 \left(\frac{s+\lambda_1}{s+\lambda_0}\right)}{s+\lambda_1[1-F(w)]} \end{aligned}$$

This function increase with the posted wage w.
This leads to a arbitration for the firm :

1. High wages make firm more attractive ($\uparrow h(w)$) and then leads to reduce its search costs.
2. But high wage reduces the profits ($\downarrow J(w)$).

The Wage-Posting Game (3)

- All jobs are *ex ante* identically productive.
- firm can retain the worker if the wage is high, making a lower instantaneous profit,
- At the opposite, a another firm can decide to have a large instantaneous profit but large turnover

$\Rightarrow$ There exists a mixed-strategy equilibrium where different wage policies lead to the same profit for each firm.
In this case, the equilibrium is stable.

The Wage-Posting Game (4)

The solution is such that :

1. $\underline{w} = w^r$ because nobody accept to work for a lower wage.
2. Every wage in the support of the equilibrium wage distribution must yield the same profit. Then, wa have :

$$\begin{aligned} h(\underline{w})\left[\frac{y-\underline{w}}{r+s+\lambda_1}\right] &= h(w)\left[\frac{y-w}{r+s+\lambda_1[1-F(w)]}\right] \\ \Leftrightarrow_{r\to 0} \frac{y-\underline{w}}{(s+\lambda_1)^2} &= \frac{y-w}{(s+\lambda_1[1-F(w)])^2} \\ \Rightarrow F(w) &= \left(\frac{s+\lambda_1}{\lambda_1}\right)\left(1-\sqrt{\frac{y-w}{y-w^r}}\right) \end{aligned}$$

3. Because $F(\underline{w}) = 0$ and $F(\overline{w}) = 1$,

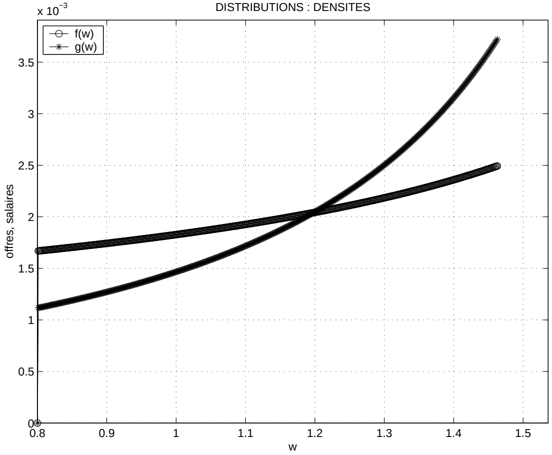
$$\overline{w} = w^r + (y-w^r)\left[1-\left(\frac{s}{s+\lambda_1}\right)^2\right]$$

Calibration

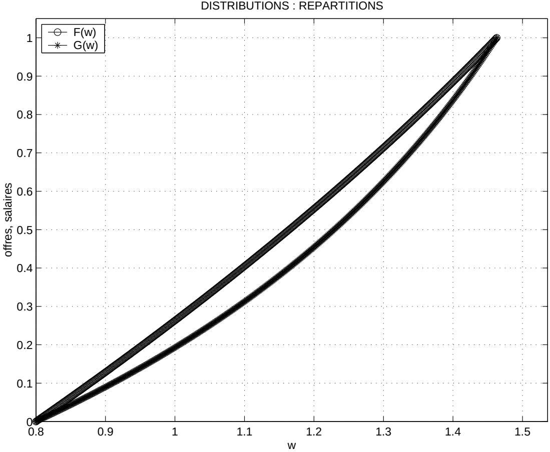
Table – Parameters – Mortensen 2002

b	s	$\lambda_0 = \lambda_1$	y
0.8	0.287	0.142	2

Wage densities

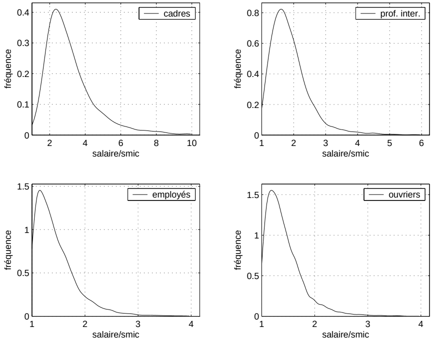


Cumulative distribution functions



Wage dispersion measured in the data

FIGURE – Executives, non-managerial workers, white-color workers, blue-color workers - France - 2002



Human Capital in a frictional labor market

- Acemoglu and Shimer (1999) suggest that search equilibrium is consistent with *ex-post* endogenous distribution of productivities.
- The logic is the following
  - Firms are ex ante homogenous : equilibrium is characterized by a wage dispersion.
  - Higher-paying firm has an incentive to adopt a more capital-intensive technology.
  - As a consequence, worker employed by firms with high capital are more productive.
  - Higher wages can be paid in these firms.
  - The cost to open a highly productive is large : these positions become rare.

Human Capital in a frictional labor market : empirical motivation

Empirical relevance of the like between productivity and wage distributions.

1. Bontemps, Robin and van den Berg (1999) : it is necessary to allow for productive heterogeneity across employers in order to reproduce the observed wage distribution.
2. Posel-Vinay and Robin (2002) : the productivity differentials across firms explain the half of the French low-skilled wage variance, while the remaining part is due to the search frictions.
   This emphasizes the importance of **specific human capital** in the low skilled workers wage distribution.

Specific Human Capital : assumptions

- Firm invests in specific-match capital. This amount of capital is denoted k.
- The price of one unit of this capital is p_k.
- The production of a job is $f(k)$ with $f' > 0$ and $f'' < 0$.
- After a separation, a destruction or a job-to-job transition, this capital is dissolved.

$\Rightarrow$ this investment is match-specific and can be viewed as a training paid by firms.

Specific Human Capital : the model (1)

Wages and training investments are set to maximize the expected return to the posting of a vacancy :

$$\max_{w \geq \underline{w}, k \geq 0} \{h(w)\left[J(w,k) - p_k k\right]\}$$

$$h(w) = \frac{s\lambda_0 \left(\frac{s+\lambda_1}{s+\lambda_0}\right)}{s + \lambda_1[1 - F(w)]}$$

with $J(w,k)$ the expected present value of the employer's future flow of quasi-rent once a worker is hired at wage w :

$$J(w,k) \quad = \quad \frac{f(k) - w}{r + s + \lambda_1[1 - F(w)]}$$

Each employer pre-commits to both the wage offered and the extend of the specific capital investment in the match.

Specific Human Capital : the model (2)

For each wage offer w, the optimal investment is given by :

$$f'(k) = p_k(r + s + \lambda_1[1 - F(w)]) \implies k = k(w) \quad \forall w$$

$\implies$ **Trade-off between low training and low rotation costs :** When the wage is high, the expected duration of the match is longer and the period during which the firm can recoup its investment increases. Therefore, firm specific productivity increases with wages.

Specific Human Capital : the model (3)

The wage distribution is then given by (assuming that $r \to 0$) :

$$F(w) = \left[\frac{s+\lambda_1}{\lambda_1}\right]\left[1 - \sqrt{\frac{f(k(w)) - w - k(w)(s+\lambda_1[1-F(w)])}{f(k(\underline{w})) - \underline{w} - k(\underline{w})(s+\lambda_1)}}\right]$$

whereas the support of the wag distribution is bounded by

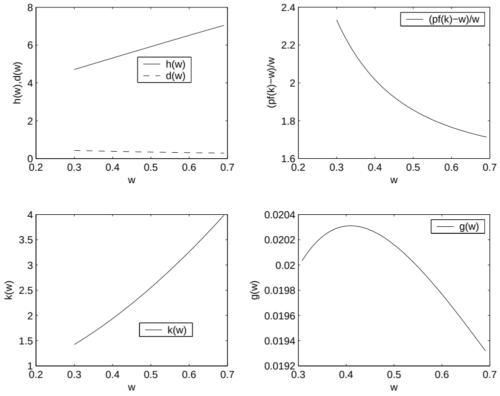
$$\begin{aligned}
\underline{w} &= w^r \\
\overline{w} &= w^r + (\overline{y} - \underline{y}) + (\underline{y} - w^r)\left(1 - \left(\frac{s}{s+\lambda_1}\right)^2\right) \\
\overline{y} &= f(k(\overline{w})) - p_k s k(\overline{w}) \\
\underline{y} &= f(k(\underline{w})) - p_k(s+\lambda_1)k(\underline{w})
\end{aligned}$$

Quantitative evaluation : Calibration

TABLE – Parameters – Mortensen 2002

b	α	s	$\lambda_0 = \lambda_1$	p_k
0.3	0.6	0.287	0.142	1

Quantitative evaluation : Labor market equilibrium



Equilibrium search $\Leftrightarrow$ Partial equilibrium analysis

- The main default of this theory is that **the unemployment rate is exogenous.**

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$\Rightarrow$ We choose to endogenize the number of firms and thus λ_0 and λ_1.

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The objective : a theory allowing to endogenize the probability to meet a worker for a firm/a job for a worker.
$\Rightarrow$ **The matching model**.

In this model, market failures are caused by search externalities :

- the job-acquisition rate is positively related to v and negatively related to u
- whereas the job-filling rate has exactly the opposite sign.

There is two types of externalities :

1. *negative intra-group externalities*, ie. more searching workers reduces the job-acquisition rate.
2. *positive inter-group externalities*, ie. more searching firms increases the job-acquisition rate.

The matching technology

- We assume that the total population L is normalized to 1, $L = 1$, so that the number of workers is equal to $u + e = 1$.
- There is on-the-job search so that employed workers can search for a job in order to improve their wage.
- The matching function with on-the-job search can now be written as :

$$m = m(u + e, v)$$

with $m'_i > 0$, $m''_{ii} < 0$ for $i = 1, 2$, and $m(0, v) = m(u + e, 0) = 0$.

From the matching function to the labor market transition rates

The rate at which firms fill a vacant job is given by :

$$q(\theta) = \frac{m}{v} = m\left(\frac{1}{\theta}, 1\right)$$

where the labor market tightness θ is given by $v/(u+e) = v$.
Because of the Poisson process, the average duration of a vacant job is thus $1/q(\theta)$.
Similarly, the arrival rate of wage offers is equal to :

$$\theta q(\theta) = \frac{m}{u+e} = m\left(1, \theta\right)$$

The average duration time before getting a wage offer is thus $1/\theta q(\theta)$. Because e and u are perfect substitute, unemployed and employed have the same probability to meet a firm.

Equilibrium labor market flows

In steady state, the number of workers leaving unemployment is equal to the number of workers leaving employment. We obtain :

$$\theta q(\theta) u = s(1-u)$$

There exists flows from employment to employment for all possible wage offers. Exists and entries of workers with a wage less or equal than w is :

$$(1-u)\left[s+\theta q(\theta)\left(1-F(w)\right)\right] G(w) = \theta q(\theta) u F(w)$$

This equation leads to the steady-state equilibrium wage distribution :

$$G(w) = \frac{\theta q(\theta) u F(w)}{(1-u)\left[s+\theta q(\theta)\left(1-F(w)\right)\right]} = \frac{sF(w)}{s+\theta q(\theta)\left(1-F(w)\right)}$$

Firm and workers behaviors

Firms can hire workers or “poach” workers by proposing them a higher wage than the current one. In steady-state, the value functions $J(w)$ and V are respectively given by :

$$\begin{aligned} rJ(w) &= y - w - \theta q(\theta)[1 - F(w)](J(w) - V) - s(J(w) - V) \\ rV &= \max_{w \geq w^r} \{-c + q(\theta)h(w)(J(w) - V)\} \end{aligned}$$

where

$$h(w) = u + (1 - u)G(w)] = \frac{s}{s + \theta q(\theta)[1 - F(w)]}$$

Workers. Because $\theta q(\theta) = \lambda_0 = \lambda_1$, the reservation wage with on-the-job search is $w^r = b$.

Equilibrium

As long as there exit strictly positive profits in the economy, firms can enter the labor market. This free-entry condition implies that $V = 0$. Then we have

$$\frac{c}{q(\theta)} = \max_{w \geq b} \left\{ \left(\frac{s}{s + \theta q(\theta)[1 - F(w)]} \right) \left(\frac{y - w}{r + s + \theta q(\theta)[1 - F(w)]} \right) \right\}$$

The left-hand-side of this equation gives the expected cost of a vacant job, which has to be equal in equilibrium to the expected value of the future profits.

Equilibrium solution

Determining the steady-state labor market equilibrium consists in finding $\{\theta, F(:)\}$, i.e. the labor market tightness and the wage offer distribution. Assume $r \to 0$.

- The solution for θ is provide by

$$\frac{c}{q(\theta)} = \left(\frac{s}{s+\theta q(\theta)}\right)\left(\frac{(y-b)}{s+\theta q(\theta)}\right)$$

- The solution for $F(:)$ come from

$$\frac{y-b}{(s+\theta q(\theta))^2} = \frac{y-w}{(s+\theta q(\theta)[1-F(w)])^2}$$

and

$$\overline{w} = b + \left[1 - \left(\frac{s}{(s+\theta q(\theta))}\right)^2\right](y-b)$$

Quantitative implications : calibration

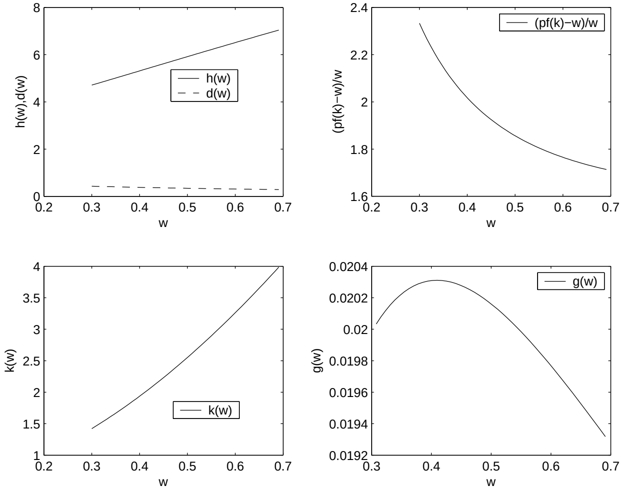
In order to generate an hump-shaped wage distribution, we assume that firms invest in training. Then we have $y = f(k)$. We assume that $m(u+e, v) = (u+e)^{1-\psi} v^{\psi}$.

TABLE – Parameters – Mortensen 2002

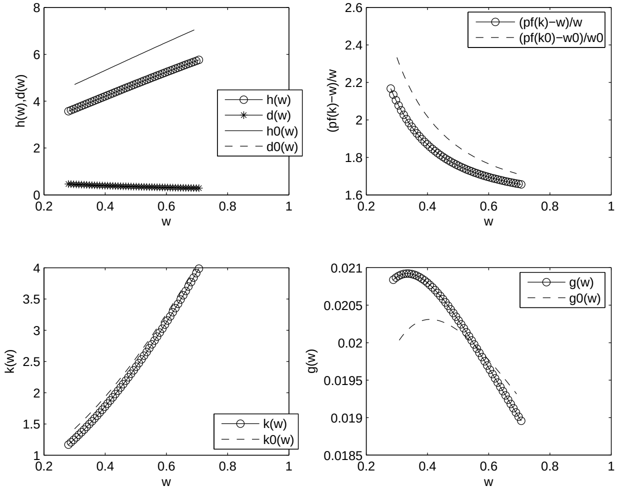
b	α	s	$\theta q(\theta) = \lambda_0 = \lambda_1$	p_k	ψ
0.3	0.6	0.287	0.142	1	0.6

Given this initial calibration, we can evaluate the impact of a change in b, because this change in the labor cost modifies the meeting probabilities on the labor market.

Quantitative implications : benchmark



Quantitative implications : a decrease in b equal to 7%

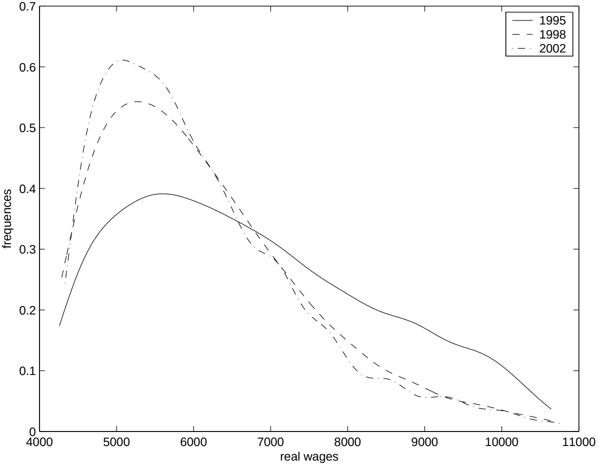


Part IV
Policy Evaluation using Structural Model

Evaluation 1 : The payroll tax reform in France

- Phelps (1994) proposed "a system of low-wage employment subsidies be introduced, a subsidy to every qualifying firm based on the stock of low-wage workers on its roll".
- Since the mid-1990s, France has experimented with this strategy of maintaining a high minimum wage level with large, permanent subsidies to employer payroll taxes on low wages.
- The subsidy increased dramatically in October 1995 and September 1996.
- In its current state, it corresponds to a linear reduction spanning from 1 to 1.33 times the minimum wage and ranging from 18.6 points at the minimum wage to roughly 0 points at the end point of the exemption interval.

Before and after the reform : Manual Worker Wages



Evaluation 1 : The payroll tax reform in France

- The subsidies tend to introduce a bias in favor of low-wage jobs and a potentially large decrease in aggregate productivity.
- Hence, a balance sheet drawn out in terms of output is particularly interesting compared to one based solely on employment (Malinvaud (1998)).
- Chéron , Hairault and Langot (2008) therefore propose a structural model of the French low-skilled workers labor market

Evaluation 1 : The payroll tax reform in France

- The French experience offers the opportunity of evaluating the low-wage subsidy policy suggested Phelps (1994).
- Previous works Kramarz and Philippon (2001), Crépon and Desplatz (2002) assessed this policy based only on its implication for employment.
- In contrast, our analysis incorporates both employments effects and productivity effects owing to endogenous human capital formation.
- Using our structural model, we can give the design of the optimal policy and its quantitative implications.

Strategy

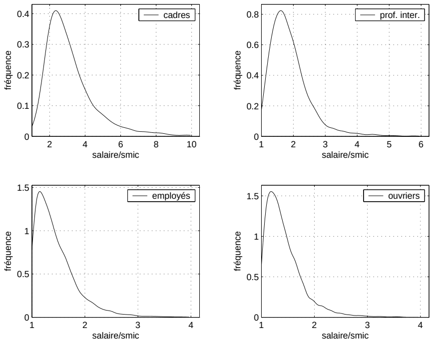
- In contrast to the equilibrium search literature which seeks to match the existing stationary wage distribution (Ridder and Van den Berg (1998), Bontemps, Robin, and van den Berg (1999) and Postel-Vinay and Robin (2002)), we propose an equilibrium search model that is also suitable for policy analysis : impact of the policy on employment and inequalities.
- Estimation of the model parameters before the reform.
- Predictions of the wage distribution after the reform
  $\Rightarrow$ Test of the model assumptions.
- Determination of the optimal design of the policy.

The limits of the previous models

- the BM model predicts an increasing and convex wage distribution : not in accordance with the data.
- the literature : an exogenous productivity distribution allows to correct this gap between theory and data
- But productivity is endogenous (investment choice) : this assumption is not useful for a policy experiments.
- the BM model focus on wage inequalities, for a given unemployment rate which is exogenous...
- This short-cut is not possible for a policy evaluation.
- There exist solutions :
  - human capital investment and endogenous productivity
  - matching process and endogenous unemployment rate.

Wage dispersion measured in the data

FIGURE – Executives, non-managerial workers, white-color workers, blue-color workers - France - 2002



Human Capital in a frictional labor market

- Acemoglu and Shimer (1999) suggest that search equilibrium is consistent with *ex-post* endogenous distribution of productivities.
- The logic is the following
    - Firms are ex ante homogenous : equilibrium is characterized by a wage dispersion.
    - Higher-paying firm has an incentive to adopt a more capital-intensive technology.
    - As a consequence, worker employed by firms with high capital are more productive.
    - Higher wages can be paid in these firms.
    - The cost to open a highly productive is large : these positions become rare.

Human Capital in a frictional labor market : empirical motivation

Empirical relevance of the like between productivity and wage distributions.

1. Bontemps, Robin and van den Berg (1999) : it is necessary to allow for productive heterogeneity across employers in order to reproduce the observed wage distribution.
2. Posel-Vinay and Robin (2002) : the productivity differentials across firms explain the half of the French low-skilled wage variance, while the remaining part is due to the search frictions.
   This emphasizes the importance of **specific human capital** in the low skilled workers wage distribution.

Specific Human Capital : assumptions

- Firm invests in specific-match capital. This amount of capital is denoted k.
- The price of one unit of this capital is p_k.
- The production of a job is $f(k)$ with $f' > 0$ and $f'' < 0$.
- After a separation, a destruction or a job-to-job transition, this capital is dissolved.

$\Rightarrow$ this investment is match-specific and can be viewed as a training paid by firms.

Specific Human Capital : the model (0)

- the equilibrium rate of unemployment is deduce from

$$s(1-u) = \lambda_0[1-F(w^r)]u$$

- We denote the steady-state number of employed workers being paid a wage no greater than w by $G(w)(1-u)$, where $G(w)$ is the distribution of wage earnings across employed workers and u the overall unemployment rate.
- At steady-state the flow of workers leaving employers offering a wage no greater than w equals the flow of workers hired with a wage no greater than w :

$$\underbrace{\lambda_0 \max\{F(w)-F(w^r);0\}u}_{\text{hirings}} = \underbrace{s(1-u)G(w)}_{\text{firings}} + \underbrace{\lambda_1[1-F(w)](1-u)G(w)}_{\text{volontary quits}}$$

Specific Human Capital : the model (00)

The probability that a job with wage offer w is accepted by the worker is defined by :

$$\begin{aligned} h(w) &= (1-u)\lambda_1 G(w) + \lambda_0 u \\ &= \frac{s\lambda_0 \left(\frac{s+\lambda_1}{s+\lambda_0}\right)}{s+\lambda_1[1-F(w)]} \end{aligned}$$

This function increase with the posted wage w.
This leads to a arbitration for the firm :

1. High wages make firm more attractive ($\uparrow h(w)$) and then leads to reduce its search costs.
2. But high wage reduces the profits ($\downarrow J(w)$).

Specific Human Capital : the model (1)

Wages and training investments are set to maximize the expected return to the posting of a vacancy :

$$\max_{w \geq \underline{w}, k \geq 0} \{h(w) [J(w,k) - p_k k]\}$$

$$h(w) = \frac{s\lambda_0 \left(\frac{s+\lambda_1}{s+\lambda_0}\right)}{s + \lambda_1[1 - F(w)]}$$

with $J(w,k)$ the expected present value of the employer's future flow of quasi-rent once a worker is hired at wage w :

$$J(w,k) = \frac{f(k) - w}{r + s + \lambda_1[1 - F(w)]}$$

Each employer pre-commits to both the wage offered and the extend of the specific capital investment in the match.

Specific Human Capital : the model (2)

For each wage offer w, the optimal investment is given by :

$$f'(k) = p_k(r + s + \lambda_1[1 - F(w)]) \implies k = k(w) \quad \forall w$$

$\implies$ **Trade-off between low training and low rotation costs :** When the wage is high, the expected duration of the match is longer and the period during which the firm can recoup its investment increases. Therefore, firm specific productivity increases with wages.

Specific Human Capital : the model (3)

The wage distribution is then given by (assuming that $r \to 0$) :

$$F(w) = \left[\frac{s+\lambda_1}{\lambda_1}\right]\left[1 - \sqrt{\frac{f(k(w)) - w - k(w)(s+\lambda_1[1-F(w)])}{f(k(\underline{w})) - \underline{w} - k(\underline{w})(s+\lambda_1)}}\right]$$

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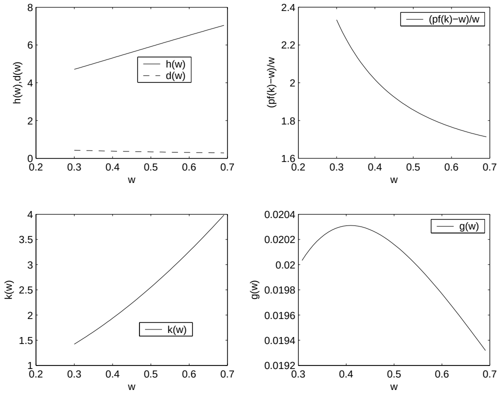

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$\Rightarrow$ At least, we need to endogenize the number of participants in this labor market.

$\Rightarrow$ **We choose to endogenize the number of firms and thus λ_0 and λ_1 $\Rightarrow$ Matching à la Pissarides**

Search and Matching : assumptions

The objective : a theory allowing to endogenize the probability to meet a worker for a firm/a job for a worker.
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The average duration time before getting a wage offer is thus $1/\theta q(\theta)$. Because e and u are perfect substitute, unemployed and employed have the same probability to meet a firm.

Key features for the model

- Endogenous wage distribution : the taxation depends on the level of the wage and is non-linear.
- Endogenous productivity in each job : by changing the labor costs the reform can change the distribution of the quality of job, ie. the productivity associated to each wage level.
- Endogenous number of jobs in the economy : by changing the labor cost, the reform modifies the incentive to open new job in the economy.

$\Rightarrow$ Theoretical Approach :

1. Wage posting (monopsony power for the firm).
2. Investment in specific human capital paid by firms.
3. Number of jobs : free entry condition.
4. Matching process with heterogenous search efficiency.

Model Assumptions

- Labor force :

$$\underbrace{e}_{\text{Employees}} + \underbrace{u^s}_{\substack{\text{unemployed workers} \\ \text{entitled to UB}}} + \underbrace{u^l}_{\substack{\text{unemployed workers} \\ \text{entitled to MSI}}} = 1$$

- Job-to-Job transitions $\Rightarrow$ On-the-job search.

Matching function

Constant returns to scale matching technology :

$$H = h(v, h^e e + h^s u^s + h^l u^l)$$

where

- v is the number of vacancies,
- $h^e \geq 0, h^s \geq 0, h^l \geq 0$ are the exogenous search efficiencies (intensities) for employed workers and short-term and long-term unemployed workers

We denote :

$$\begin{aligned} u &\equiv u^s + u^l \quad \text{aggregate unemployment rate} \\ \overline{h} &\equiv h^e e + h^s u^s + h^l u^l \quad \text{search in efficiency units} \\ \theta &\equiv \frac{v}{\overline{h}} \quad \text{labor market tightness} \end{aligned}$$

Meeting Probabilities

- for the employees

$$h^e\lambda(\theta) \equiv \frac{h^e}{\overline{h}}\frac{H}{e+u^s+u^l} = h^e\frac{H}{\overline{h}}$$

- for the short-term unemployed

$$h^s\lambda(\theta) \equiv \frac{h^s}{\overline{h}}\frac{H}{e+u^s+u^l} = h^s\frac{H}{\overline{h}}$$

- for the long-term unemployed

$$h^l\lambda(\theta) \equiv \frac{h^l}{\overline{h}}\frac{H}{e+u^s+u^l} = h^l\frac{H}{\overline{h}}$$

- The transition rate at which vacant jobs are filled is :

$$q(\theta) = \frac{H}{v} = h\left(1, \frac{\overline{h}}{v}\right)$$

Entries and Exits from Unemployment

Let $F(w)$ and x denote respectively the distribution of wage offers and x the reservation wage.

$$\underbrace{s(1-u)}_{\text{firings}} = \underbrace{h^s \lambda(\theta)\left[1 - F(x_s)\right] u^s}_{\text{hirings}} + \underbrace{\delta u^s}_{\substack{\text{flow into}\\ \text{long-term}\\ \text{unemployment}}}$$

$$\underbrace{\delta u^s}_{\substack{\text{flow out of}\\ \text{short-term}\\ \text{unemployment}}} = \underbrace{h^l \lambda(\theta)\left[1 - F(x_l)\right] u^l}_{\text{hirings}}$$

Entries/Exits from Employment at or Less than Wage w

Let $G(w)$ denote the fraction of workers employed at or less than wage w.

- If $x_l \leq w < x_s$,

$$\underbrace{(1-u)G(w)h^e\lambda(\theta)\left[1-F(w)\right]}_{\text{volontary quits}}+\underbrace{s(1-u)G(w)}_{\text{firings}}=\underbrace{h^l\lambda(\theta)F(w)u^l}_{\text{hirings}}$$

- If $w \geq x_s$,

$$\begin{aligned} & \underbrace{h^e\lambda(\theta)\left[1-F(w)\right](1-u)G(w)}_{\text{volontary quits}} && + && \underbrace{s(1-u)G(w)}_{\text{firings}} \\ = & \underbrace{u^s h^s\lambda(\theta)F(w)+u^l h^l\lambda(\theta)F(w)}_{\substack{\text{potential}\\ \text{hirings}}} && - && \underbrace{u^s h^s\lambda(\theta)F(x_s)}_{\text{rejections}} \end{aligned}$$

Worker Behaviors

The Value functions are :

$$
\begin{aligned}
rV^n(w) &= u((1-t_w)w+\mathcal{T}) + h^e\lambda(\theta)\int_w \left[V^n(\tilde{w}) - V^n(w)\right] dF(\tilde{w}) \\
&\quad - s\left[V^n(w) - V^{us}\right] \\
rV^{us} &= u(b+\mathcal{T}) + h^s\lambda(\theta)\int_{x_s} \left[V^n(\tilde{w}) - V^{us}\right] dF(\tilde{w}) \\
&\quad - \delta\left[V^{us} - V^{ul}\right] \\
rV^{ul} &= u(msi+\mathcal{T}) + h^l\lambda(\theta)\int_{x_l} \left[V^n(\tilde{w}) - V^{ul}\right] dF(\tilde{w})
\end{aligned}
$$

The reservation wage policies x_s and x_l are derived from the two conditions

$$V^n(x_s) = V^{us} \quad \text{and} \quad V^n(x_l) = V^{ul}$$

Firm Behaviors (1)

For an optimal wage w and an optimal investment choice k, the expected present value of the employer solves :

$$rJ(w,k) = f(k) - (1+t_f(w))w - h^e\lambda(\theta)[1-F(w)][J(w,k)-V] - sJ(w,k)$$

where $t_f(w) \geq 0$ is the employer's payroll taxes.

This tax can be a function of the wage when employment subsidies are introduced.

Firm Behaviors (2)

The asset value of a vacant job solves :

$$rV = \max_{w \geq x_l, k \geq 0} \{\eta(w) [J(w,k) - p_k k - V] - \gamma\}$$

$\eta(w)$ is the probability that a vacancy with posted wage w is filled :

$$\begin{aligned}
\eta(w) &= \frac{H}{v}\left[\frac{h^l}{\overline{h}}u^l + \frac{h^e}{\overline{h}}(1-u)G(w)\right] \qquad \forall\ w \in [x_l, x_s] \\
\eta(w) &= \frac{H}{v}\left[\frac{h^l}{\overline{h}}u^l + \frac{h^s}{\overline{h}}u^s + \frac{h^e}{\overline{h}}(1-u)G(w)\right] \qquad \forall\ w \in [x_s, \overline{w}]
\end{aligned}$$

where $H/v = \lambda(\theta)/\theta$ gives the probability of having a contact with a firm.

Firm Behaviors (3)

- Free entry conditions at each wage level imply that $V = 0$
- Expected intertemporal profits are identical for $w \geq \underline{w}$
- $x_l \leq \underline{w}$, because it is not in the firms' interest to offer a wage rejected by all workers.

θ, $F(w)$ and $k(w)$ can be derived from the system of equations defined by :

$$\gamma = \eta(w)\left[\max_{k\geq 0}\{J(w,k) - p_k k\}\right] \quad \forall w \geq \underline{w} \;\; \text{with } F(\underline{w}) = 0$$

The optimal HK investment solves :

$$f'(k) = p_k(r + s + h^e\lambda(\theta)[1 - F(w)]) \implies k = k(w,\theta) \quad \forall w \geq \underline{w}$$

Sources of inefficiency

What are the sources of inefficiency in our economy ?

1. the **monopsony power** of the firms which leads them to propose too low wages (the rate of vacancy is too high : congestion externality)
2. **On-the-job search** leads to underinvestment as the job duration can be shortened by voluntary quits. This deters firms from investing a lot in human capital. Moreover, job-to-job transitions lead to human capital destructions as firms invest in specific-firm training.

Measures of Inefficiency (1)

The steady state aggregate output flow net of the recruiting costs :

$$
\begin{aligned}
\mathcal{Y} \; = \; & \underbrace{\underbrace{(1-u)}\underbrace{\int_{\underline{w}}^{\overline{w}} f(k(w))dG(w)}}_{\text{Employment (E)} \times \text{Productivity (P)}} \; - \; \underbrace{\gamma v}_{\text{Hiring costs}} \\
& \text{gross output} \\
- & \underbrace{p_k h^s \lambda(\theta) u^s \int_{\underline{w}}^{\overline{w}} k(w)dF(w)}_{\substack{\text{training costs} \\ \text{short-term unemployed}}} - \underbrace{p_k h^l \lambda(\theta) u^l \int_{\underline{w}}^{\overline{w}} k(w)dF(w)}_{\substack{\text{training costs} \\ \text{long-term unemployed}}} \\
- & \underbrace{p_k h^e \lambda(\theta)(1-u) \int_{\underline{w}}^{\overline{w}} \left(\int_{\underline{w}}^{w} k(w)dF(w) \right) dG(w)}_{\substack{\text{training costs} \\ \text{job-to-job mobility}}}
\end{aligned}
$$

Measures of Inefficiency (2)

We also compute aggregate welfare, which takes into account the risk aversion of workers and the distributive implications of different reforms :

$$\mathcal{W} = (1-u)\left(\int_{\underline{w}}^{\overline{w}} V^n(w)dG(w)\right) + u^s V^{us} + u^l V^{ul}$$

The budget surplus is defined by :

$$\mathcal{B} = (1-u)\left(\int_{\underline{w}}^{\overline{w}} [t_f(w) + t_w]wdG(w)\right) - (u^s \times b + u^l \times msi)$$

Aggregate firms' profits are defined by :

$$\Pi = \mathcal{Y} - (1-u)\left(\int_{\underline{w}}^{\overline{w}} [1 + t_f(w)]wdG(w)\right)$$

Total transfers perceived by all worker are then :

$$\mathcal{T} = \Pi + \mathcal{B}$$

Estimation and Tests of the Model

- Using pre-reform data, we estimate the structural parameters of the model. We use Simulated Method of Moment. It is then possible to propose a first test of the model : Does the model allows to reproduce more statistics than the number of estimated parameters ?
- Does the model predict the observed impact of the implemented policy ?
- Is it possible to predict the same results with a “less structural model” ?

Estimation method : the SMM (1)

Functional forms :

$$
\begin{aligned}
f(k) &= A_1 + (k+A)^{\alpha} \\
u(x) &= \frac{x^{1-\sigma}}{1-\sigma} \\
H &= v^{\zeta}(h^e e + h^s u^s + h^l u^l)^{1-\zeta}
\end{aligned}
$$

The 17 parameters of the model :

$$\Phi = \{h^e, h^s, h^l, \zeta, \gamma, s, \delta, \sigma, b, msi, r, t_w, t_f, p_k, \alpha, A, A_1\}$$

Estimation method : the SMM (2)

- We restrict the size of the vector of unknown structural parameters :

$$\Theta = \{\alpha, p_k, h^e\}$$

- A first vector Φ_1, with $\dim(\Phi_1) = 7$, defined by

$$\Phi_1 = \{s, \delta, \sigma, msi, r, t_w, t_f\}$$

  is fixed on the basis of external information.
- The second vector Φ_2, with $\dim(\Phi_2) = 6$, defined by

$$\Phi_2 = \{b, h^s, h^l, A_1, \gamma, \zeta\}$$

  is calibrated, using the model restrictions, to reproduce some stylized facts and assumptions.

Estimation method : the SMM (3)

Step 1

The vector of moments ψ is estimated by minimizing the following loss function :

$$Q_N = h_N(w_i; \psi_N)' \Omega_N h_N(w_i; \psi_N)$$

where Ω_N is a positive definite weighting matrix and N denotes the size of the sample. $\{w_i\}'$ represents the s-dimensional set of wages. $h(.)$ takes the form

$$h(w_i; \psi) = E \begin{bmatrix} w_i - \mu \\ (\mathbb{I}_{[w_i < D1]} w_i - \mu_1) \\ (\mathbb{I}_{[Dn \leq w_i < Dn+1]} w_i - \mu_{n+1}) \\ (\mathbb{I}_{[D8 \leq w_i < D9]} w_i - \mu_9) \end{bmatrix} \quad \text{for } n = 1, .., 7$$

Estimation method : the SMM (4)

Step 2

Given the vector of structural parameters Θ, the simulated wage density is computed from the set of equations defining the theoretical model.

Step 3 :

An estimate $\widehat{\Theta}$ for Θ minimizes the quadratic form :

$$J(\Theta) = g_N' W_N \, g_N \quad \text{with } g_N = \left(\widehat{\psi}_N - \widetilde{\psi}_N(\Theta)\right)$$

W_N is a symmetric non-negative matrix defining the metric,
$\widehat{\psi}_N = \{\mu, \mu_1, ..., \mu_9\}$ and
$\widetilde{\psi}_N(\Theta)$ denotes the moments implied by the model.

Steps 2 and 3 are conducted until convergence *i.e.* **until a value of Θ minimizing the objective function is obtained.**

Estimation method : the data (1)

- Full-time manual workers, who are affected by the minimum wage and the probability of being excluded from the unemployment benefit system
- Enquete Emploi : wages paid to each manual worker in the 1995. $N = 14202$ individuals.
- Wages, minimum income and unemployment benefits are expressed in 1990 French Francs (FF).
- In 1995, the payroll taxes on labor t_f and t_w for firms and workers are set at, respectively, 40% and 20%

Estimation method : the data (2)

TABLE – Observable parameters Φ_1

s	δ	σ	msi	r	t_w	t_f
0.0185	1/30	2.5	FF 2,500	4%	20%	40%

TABLE – Calibrated parameters Φ_2

	b	h^s	h^l	A_1	γ	ζ
such that	$b/E(w)$	u	u^l/u	V	$\gamma\theta/\lambda(\theta)$	$\epsilon_{N\|\underline{w}}$
equals	0.6	15.51	45.75%	0	0.3	14,000

Estimation results (1)

TABLE – Parameter Estimates

Θ	$\widehat{\Theta}$	$\widehat{\sigma}(\Theta)$	$t-stat$
α	0.7158	0.0310	23.0790
p_k	18.2866	1.2814	14.2706
h^e	0.5265	0.0147	35.7266
$J-stat$	1.6519	P-value	97.65%

- The simulated moments are also estimated with precision : the diagnostic test (Diag. Test) does not lead to rejection of the model in terms of its ability to reproduce each moment.
- The *mw* is above the highest estimated x^s : It is a binding minimum wage $\Rightarrow F(x^s) = 0$.

Estimation results (2) : Benchmark Equilibrium

Labor market stocks and flows					
u	u^l	u^s	$h^e\lambda$	$h^s\lambda$	$h^l\lambda$
0.1551	0.071	0.0841	0.0801	0.1520	0.0395
Employment and Unemployment Durations					
model	data	model	data		
32.22	34.00	14.50	17.00		
Productivity, Output and Welfare					
P	$\mathcal{Y}$	$\mathcal{W}$			
18575	10006	-106.2556			

Production is expressed in 1990 French francs,
duration in months and stocks and flows in percentages

Does the model predict the observed impact of the implemented policy ?

- We estimate the model, using the 1998 sample, by imposing the same structural parameters as in 1995 in order to test the restriction

$$\widehat{\Theta}_{1998} = \widehat{\Theta}_{1995}$$

- this gives the $J - stat$ of the constrained models (J_c).
- the parametric restrictions are not rejected at 5% level
- All the predicted moments are not significantly different from their empirical counterparts.

A comparison with an alternative model (1)

Is it possible to predict the same results with a "less structural model" ?

An alternative model : a search equilibrium model with an exogenous productivity distribution :

(H1) $f(k)$ becomes $\Psi + p$. The matching-specific productivity shock p is distributed according to the function $\Phi(p)$.

(H2) This shock is observed when the worker meets the firm. It is an unknown parameter for unemployed workers and for vacant jobs.

(H3) Wage posting is assumed.

(H4) There are no counteroffers.

A comparison with an alternative model (2)

We follow exactly the same econometric strategy used to estimate the model with endogenous productivity.

- We estimate three parameters $\{\nu; \kappa; h^e\}$ because we retain a Gamma law :

$$d\Phi(p) = \frac{\nu}{\Gamma(\kappa)} e^{-\nu \times p} (\nu \times p)^{\kappa - 1} \quad \text{for any } p > 0$$

  ν and κ are the parameters of this law, with $E[p] = \frac{\kappa}{\nu}$ and $V[p] = \frac{\kappa}{\nu^2}$.
- all the other parameters in Φ_1 and Φ_2 are calibrated according to the same restrictions.

The estimation results : we cannot reject the model at 10% level, the model is able to match all the historical moments.

A comparison with an alternative model (3)

- We have now two models able to explain the wage distribution observed in 1995.
- A test is then to compare their predictions of the impact of the subsidy policy on the wage data in 1998.
- $\Rightarrow$ the shock is the policy change, the response is the implied moments

The predictions of the model with exogenous productivity are rejected : the average wage in the sample and the average wages in the deciles higher than the median are largely over-estimated by this simplest model.
$\Rightarrow$ Endogenous behaviors imply a better fit of the post-reform wage distribution $\Leftrightarrow$ The necessity of our theoretical assumptions : our model is "better" than the Robin and Postel-Vinay (2002) or Flinn (2006) models.

Optimizing the policy though our structural model

Policy Evaluation :

- Why to introduce a policy ?
  $\Rightarrow$ A measure of the pre-reform distortions.
- How to improve the implemented policy ?
  $\Rightarrow$ The optimal design of the policy.

The optimal minimum wage in France (1)

- During the 1990s, tax exemptions on employer-paid payroll taxes were introduced in France to lower labor costs.
- This policy aimed to counteract the negative impact of minimum wage legislation on employment without lowering wages earned by employees.
- In order to evaluate the welfare cost of the binding minimum wage, we determine its optimal level with and without the productivity channels.
- We then investigate the efficiency implications of the recent payroll tax subsidy policy aimed at reducing the negative employment effect caused by minimum wage legislation.

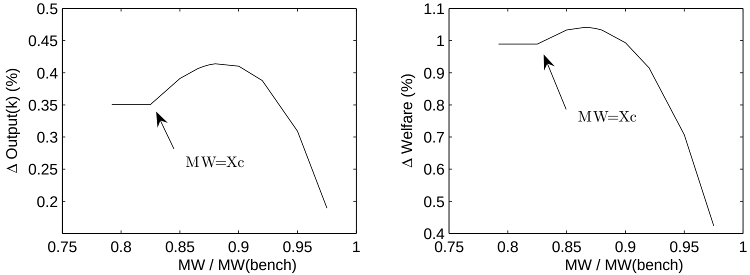
The optimal minimum wage in France (2)

- As in any matching model, a decrease in the minimum wage leads to a higher vacancy rate and hence to a higher employment level
- But, in general equilibrium, it can be offset by an increase in vacancy costs (congestion effect and prohibitive hiring costs).
- In addition, it can lead to an underinvestment in human capital due to the increased probability of finding a better job and the reduction of the expected job duration.

$\Rightarrow$ There is a trade-off and then an optimal level of the MW.

The optimal minimum wage in France (3)

FIGURE – Optimal Minimum Wage



The optimal minimum wage in France (4)

TABLE – The Optimal Minimum Wage Level

mw	$\mathcal{Y}$	E	$E(k)$	P	$\mathcal{W}$	$\mathcal{B}$
-12%	0.4140	2.1698	-2.0186	-1.0981	1.0330	2.0761
-14%	0.4020	2.4133	-2.2440	-1.2214	1.0402	2.2408

Variations in % relative to the benchmark calibration

$\Delta^{-}MW$ is welfare improving because the gains in employment dominate the loss in productivity.

The PTE Reform (1)

- The analysis of the *mw* confirms that there are two distinct productivity channels in our setup : an investment one and a job-to-job transition one.
  1. more vacancy $\Rightarrow$ more job-to-job transitions $\Rightarrow \downarrow$ employment duration $\Rightarrow$ disincentives to invest.
  2. more job-to-job transitions allow to the workers to find a better job.
- Based on our optimal minimum wage analysis, the labor cost reduction (**the PTE reform**) must be relatively weak to preserve high productivity levels.

The PTE Reform (2)

Labor market stocks and flows

u	u^l	u^s	$h^e\lambda$	$h^s\lambda$	$h^l\lambda$
0.1358	0.0581	0.0778	0.0904	0.1718	0.0446
0.1551	*0.071*	*0.0841*	*0.0801*	*0.1520*	*0.0395*

Employment and Unemployment Durations

model	Bench.	model	Bench.
31.75	32.22	12.36	14.50
32.22	*34.00*	*14.50*	*17.00*

Variations (in %)

$\mathcal{Y}$	E	P	$\mathcal{W}$	$\mathcal{B}$
0.4080	2.2768	-1.1469	1.1793	-0.9302

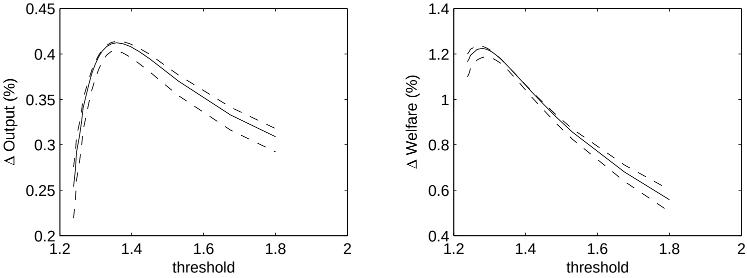
Duration is expressed in months and stocks and flows in percentages

The optimal design of PTE reform (1)

- Given the same *ex ante* (direct) budget cost, the question remains whether we should further increase the subsidy at the minimum wage level or, on the contrary, spread out the subsidy over a wider range.
- However, the policymaker faces uncertainty about the model parameters. We use the estimated covariance of the model parameters as a measure of this uncertainty.
- We compute the confidence interval of the output and welfare changes. Let us note that the confidence interval is not very large which demonstrates the robustness of our policy results.

The optimal design of PTE reform (2)

FIGURE – The Optimal Subsidy Scheme



The optimal design of PTE reform (3)

- Limiting the analysis to the employment side only would suggest using concentrated exemptions around the minimum wage level.
- Spreading out exemptions over a wider distribution range appears more efficient.
- By adding the welfare criterion, the analysis considers the concavity of the utility function and grants more importance to the decline in unemployment.
- Accordingly, the optimal scheme ranges up to 1.27 times the minimum wage, allowing more exemptions at the minimum wage level.
- This analysis lends strong support to the PTE reform implemented in France in the 1990s.

Part VI
Optimal unemployment insurance scheme and Job search

Optimal unemployment insurance : motivations

1. UI programs provide first of all insurance to risk averse workers against income fluctuations.
2. Workers face search frictions in the labor market (the problem has a dynamic nature)
3. Workers can affect their labor market outcomes, e.g. the probability of getting a job offer $\pi(a)$ through effort (here is where incentives enter into the picture)
4. Governments face a moral hazard problem (e.g., search intensity a is unobservable).

$\Leftrightarrow$ Governments face a trade-off between insurance and incentives.

Optimal unemployment insurance : motivations

Since the seminal work of Shavell and Weiss (1979), it has been recognized that the optimal unemployment benefits should be such that the replacement ratio decreases with the unemployment spell.

- Assumptions :
  1. the search intensity made by the agent (the unemployed worker) cannot be observed by the principal (the unemployment insurance agency) $\Rightarrow$ punishments if the worker cheats.
  2. the worker is risk-adverse $\Rightarrow$ principal must smooth her consumption.

$\Rightarrow$ there is a trade-off between incentive and smoothing motives.

Optimal unemployment insurance : motivations

- Solution : an unemployment insurance contract

$\Leftrightarrow$ a sequence of transfers between the principal and the agent which aims to cope optimally with moral hazard.

- It minimizes the expected discounted value of net transfers provided by the principal for a given ex-ante utility.

Optimal unemployment insurance : assumptions

- If she finds a job in period τ, she is employed from $\tau+1$.
- t denotes the length of the last unemployment episode.
- Once employed, the workers perceived a wage w each period. The jobs are never destroyed.
- The agent's preferences are given by :

$$E_0 \sum_{\tau=0}^{\infty} \beta^{\tau}[u(c_\tau) - a_\tau] \quad \text{with } u(c_\tau) = \frac{c_\tau^{1-\sigma}}{1-\sigma}$$

- The probability to find a job is given by an exponential distribution :

$$\pi(a) = 1 - \exp(-\psi a) \quad \text{with } \psi > 0 \text{ and } \pi'(0) = \psi < +\infty$$

For some workers, the marginal cost can dominate the marginal return to search activity.

Optimal unemployment insurance : agent behaviors

- Employed workers

$$V^e = u(w) + \beta V^e \Rightarrow V^e = \frac{u(w)}{1-\beta}$$

- Unemployed workers

$$V^u(t) = \max_{a(t)} \left\{ u(b(t)) - a(t) + \beta \left[\pi(a(t)) V^e + (1 - \pi(a(t))) V^u(t+1) \right] \right\}$$

Remark that this value function is not stationary.

- The optimal search intensity is then given by :

$$1 \geq \beta \pi'(a(t)) \left[V^e - V^u(t+1) \right] \quad \text{with equality if } a(t) > 0$$

Optimal unemployment insurance : the principal

- We consider a risk-neutral planner.
- The principal cannot observe the search intensity $a(t)$, but knows the economic environment, in particular the hazard function $\pi(a)$.
- The contract is a vectors $B = \{(b(1), b(2), ..., b(T))\}$ where $b(t)$ is the benefit level after t periods of unemployment.
- Given this vector, the agent maximizes her intertemporal utility by choosing a vector of search intensity $A^i = \{(a(1), a(2), ..., a(T))\}$ where $a(t)$ is the search intensity after t periods of unemployment.

Optimal unemployment insurance : the principal

The objective of the principal is to minimize its total expenditures, under two constraints : *(i)* a given expected utility $V^u(1)$ for a newly unemployed worker (the promise-keeping constraint), and *(ii)* an incentive-compatibility constraint :

$$C(V^u(t)) = \min_{\mathcal{C}} \{b(t) + \beta \left[(1-\pi(a(t)))C(V^u(t+1))\right]\}$$

subject to

$$\begin{aligned} V^u(t) &= u(b(t)) - a(t) + \beta[\pi(a(t))V^e + (1-\pi(a(t)))V^u(t+1)] \\ & \text{and} \\ 1 &= \beta\pi'(a(t))[V^e - V^u(t+1)] \end{aligned}$$

where $\mathcal{C} \equiv \{b(t), a(t), V^u(t+1)\}$.

Optimal unemployment insurance : the full information

Assume that the government can perfectly observe the agent's search intensity $a(t)$ and therefore can fully enforce it. The optimal insurance contract solves

$$C(V^u(t)) = \min_{\mathcal{C}} \{b(t) + \beta [(1 - \pi(a(t)))C(V^u(t+1))]\}$$

subject to

$$V^u(t) = u(b(t)) - a(t) + \beta[\pi(a(t))V^e + (1 - \pi(a(t)))V^u(t+1)]$$

The incentive-compatibility constraint is not taken into account because for sure the agents participate.

Optimal unemployment insurance : the full information

The FOC of this program are :

$$\begin{aligned} u'(b_t) &= \frac{1}{\mu} \\ C(V^u(t+1)) &= \mu\left[\frac{1}{\beta\pi'(a_t)} - (V^e - V^u(t+1))\right] \\ C'(V^u(t+1)) &= \mu \end{aligned}$$

And the envelope condition leads to :

$$C'(V^u(t)) = \mu \Rightarrow \begin{cases} C'(V^u(t)) = C'(V^u(t+1)) \\ u'(b_t) = u'(b_{t+1}) \end{cases}$$

Given that $C(.)$ is strictly convex $\Rightarrow V^u(t) = V^u(t+1)$, whereas the concavity of $u(.)$ implies that $b_t = b_{t+1}$.

Optimal unemployment insurance under moral hazard

The FOC of the principal problem with respect to $b(t)$, $V^u(t+1)$ and $a(t)$, the envelope condition and the constraints are respectively given by :

$$
\begin{aligned}
1 &= \mu u'(b(t)) \\
C'(V^u(t+1)) &= \mu - \nu \frac{\pi'(a(t))}{1-\pi(a(t))} \\
C(V^u(t+1)) &= \frac{\mu\{1-\pi'(a(t))\beta[V^e - V^u(t+1)]\}}{\beta\pi'(a(t))} \\
&\quad -\nu \frac{\pi''(a(t))}{\pi'(a(t))}[V^e - V^u(t+1)] \\
0 &= \mu\left\{V^u(t) - u(b(t)) - a(t)\right. \\
&\quad \left. -\beta\left[\pi(a(t))V^e + (1-\pi(a(t)))V^u(t+1)\right]\right\} \\
0 &= \nu\{1-\beta\pi'(a(t))[V^e - V^u(t+1)]\}
\end{aligned}
$$

Optimal unemployment insurance under moral hazard

If the incentive-compatibility constraint is bounded, then

$$1-\pi'(a(t))\beta[V^e - V^u(t+1)] = 0 \quad \Rightarrow 0 \leq \nu$$

implying that the FOC are

$$\begin{aligned}1 &= \mu u'(b(t)) \\ C'(V^u(t+1)) &= \mu - \nu\frac{\pi'(a(t))}{1-\pi(a(t))} \\ C(V^u(t+1)) &= -\nu\frac{\pi''(a(t))}{\pi'(a(t))}[V^e - V^u(t+1)]\end{aligned}$$

And the envelope condition leads to :

$$C'(V^u(t)) = \mu \Rightarrow \left\{ \begin{array}{ccc} C'(V^u(t)) & \geq & C'(V^u(t+1)) \\ u'(b_t) & \leq & u'(b_{t+1}) \end{array} \right.$$

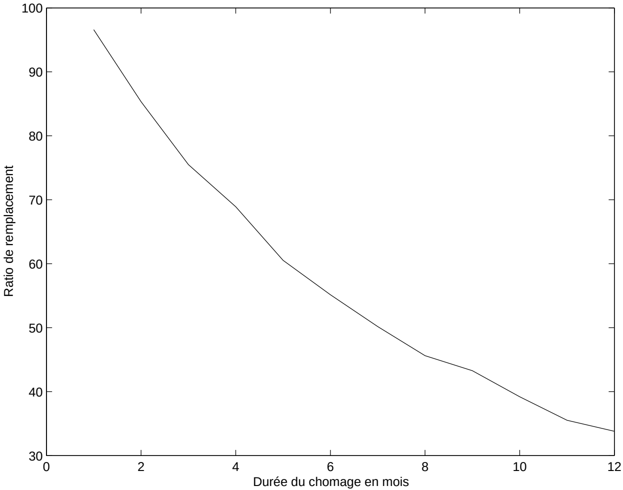
We then deduce that $b_t \geq b_{t+1}$.

Optimal unemployment insurance under moral hazard

Calibration on Franch data.

- monthly data
- discount rate $\beta = 0.995$ (5% by year)
- risk aversion $\sigma = 0.5$, idem that Hopenhayn and Nicolini [JPE,97]
- wage $w = 100$
- La promise $V(1)$ is calibrated such that the expected utility is the one obtained in a non-digressive system with a replacement ratio equal to 50%.
- $\psi = 0.0015$ is chosen in order to match the average unemployment duration, 13 mouths in the French economy.

Optimal unemployment insurance under moral hazard



Optimal unemployment insurance under moral hazard

TABLE – Cost of the UI programs

Level of the replacement ratio in the non-digressive system	$\bar{b} = 45$	$\bar{b} = 50$	$\bar{b} = 55$
Non-digressive system	486.66	609.18	780.13
Digressive system	415.26 (-14.67%)	493.58 (-18.98%)	586.54 (-24.82%)

Optimal unemployment insurance under moral hazard

TABLE – Average unemployment duration (in mouths)

Level of the replacement ratio in the non-digressive system	$\bar{b} = 45$	$\bar{b} = 50$	$\bar{b} = 55$
Non-digressive system	11.37	12.9	15.19
Digressive system	7.15 (-37.13%)	7.4 (-42.64%)	7.66 (-49.57%)