

**Welfare Cost of Fluctuations: when Labor Market
Search Interacts with Financial Frictions**

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Welfare Cost of Fluctuations: when Labor Market Search Interacts with Financial Frictions

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Abstract

We provide a quantitative assessment of welfare costs of fluctuations in a search model with financial frictions. The matching process in the labor market leads positive shocks to reduce unemployment less than negative shocks increase it. We show that the magnitude of this non-linearity is magnified by financial frictions. This asymmetric effect of the business cycle leads to sizable welfare costs. The model also accounts for the responsiveness of the job finding rate to the business cycle as financial frictions endogenously generate counter-cyclical opportunity costs of opening a vacancy and wage sluggishness.

JEL Classification : E32, J64, G21

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1 Introduction

Since the provocative articles by Lucas (1987, 2003) where he shows that the welfare costs of business cycles are negligible, it seems that a large part of the literature does not challenge this result. This is surprising given the large literature on stabilizing and optimal policies using dynamic stochastic general equilibrium (DSGE) models¹: if welfare costs of the business cycle are negligible, how can we motivate the study of such policies? This lack of debate on welfare cost of fluctuations is in fact largely explained by a technical but widely-used assumption in standard DSGE models: the dynamics of the economy has a quasi-linear structure. In this context, the effect of recessions are always compensated on average by the impact of expansions. If this is the case, business cycle costs are negligible. Indeed, as in Lucas' computation, the fluctuating and the stabilized economies share the same aggregate means.² In a large majority of New-Keynesian DSGE models, this quasi-linear structure is also assumed, leading stabilizing policies to have a limited impact on welfare.³ In contrast, in presence of non-linearities, average employment, therefore, average consumption are significantly lower than their deterministic steady state levels. Welfare costs of fluctuations can be then significantly greater than those found by Lucas.⁴ Our paper proposes a reappraisal of the famous articles by Lucas (1987, 2003) on welfare costs of business cycles by identifying the link between average and standard deviation of aggregates in a DSGE model where non-linearities matter.

Our paper focuses on the interaction of two non-linearities: *(i)* the fact that the employment falls more during recessions than it increases during expansions and *(ii)* the fact that the entrepreneurs must finance with debt the creation of new jobs. We show how the interaction of *i)* and *ii)* causes symmetric shocks (expansion and recessions) to significantly reduce average employment, hence average output and consumption in the economy. A quick look at data confirms our intuition about the existence of a link between financial and labor markets⁵. Figure 1 shows how starting from the mid- 1970s, recessions have been characterized both by de-leveraging and increases in unemployment. It also shows that the episodes when jobs are created (the periods when the job finding rate rises) are also times when firms accumulate debt. Jermann & Quadrini (2012) also notice

¹In a standard New-Keynesian (NK) DSGE model, Woodford shows that the traditional arbitration between inflation and output gap can be derived from individual welfare: thus, optimal policies based on this welfare criteria implicitly focus on welfare costs of the business cycle.

²Indeed, in Lucas (1987), the stabilized economy is simply obtained by assuming that the consumption process is not hit by shocks. This counter-factual assumption implies that the process preserves its mean. Without any structural model, Lucas can not provide a link between the average of the aggregate consumption and magnitudes of these fluctuations. In DSGE models, this link can be identified.

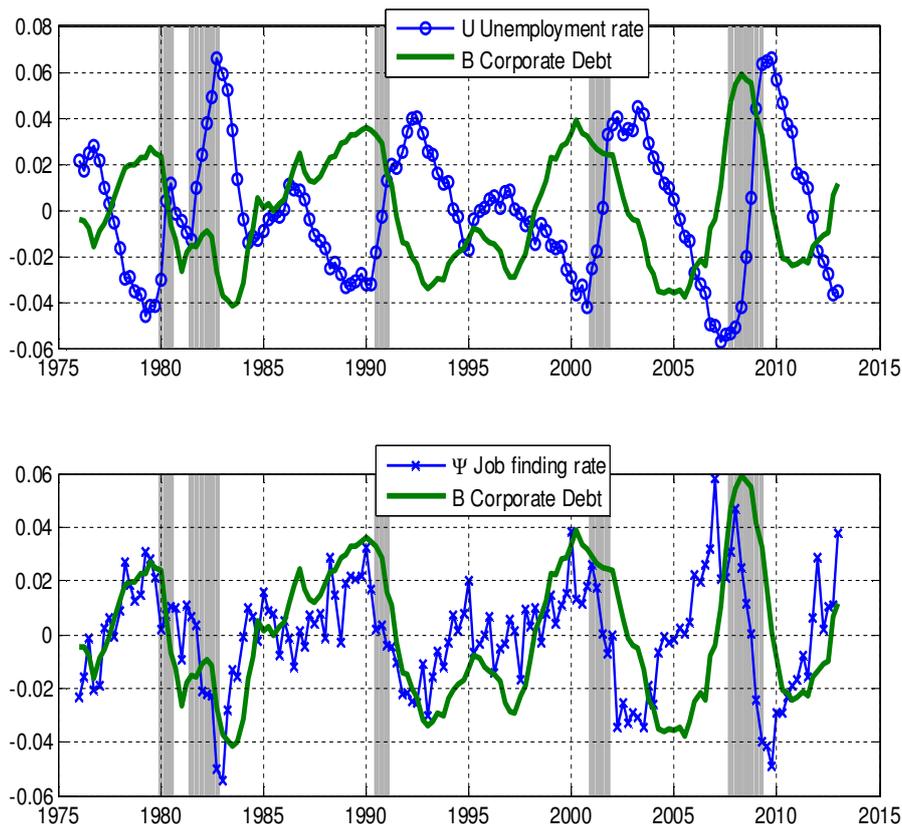
³Nevertheless, Hairault & Langot (2012) show that non-linearities matter for NK DSGE model. Indeed, risk aversion eventually prompts share-holders to set an average mark-up at a higher levels than its deterministic counterpart. Thus, imperfections on the goods market are magnified and business cycles entail large welfare costs.

⁴If they exist, such costs are on the first order magnitude, as the costs of tax distortion also evaluated by Lucas (1987).

⁵See Appendix A for a description of the data.

that debt repurchases (a reduction in outstanding debt) increase during or around recessions. In light

Figure 1: HP Filtered, logged, quarterly unemployment rate U , job finding rate Ψ and debt stock B . Recessions in shaded area (NBER dates). HP filtered data on U and Ψ have been divided by 4 for the purpose of scale consistency. Smoothing parameter 1600.



of these considerations, our paper aims at investigating the amplifying non-linear effects of financial frictions on labor market equilibrium. The first non-linearity comes from the intrinsic structure of labor market frictions. Indeed, as put forward by Hairault et al. (2010), Jung & Kuester (2011) and Petrosky-Nadeau & Zhang (2013), canonical search-and-matching labor frictions introduce a gap between the unemployment level at its deterministic steady state and its mean. They show how this generates sizable business-cycle costs.⁶ In this paper, we show that these non-linearities are amplified by the ones specific to financial frictions. On the one hand, because employment falls more during recessions than it increases during expansions, symmetric shocks reduce the average of the economy. On the other hand, financial frictions amplify this basic mechanism through two channels: tighter credit conditions *(i)* reduce the steady-state level of vacancies, making the economy more sensitive to the asymmetries of the labor market (level effect), *(ii)* generate a financial accelerator mechanism,

⁶In Hairault et al. (2010), Jung & Kuester (2011) and Petrosky-Nadeau & Zhang (2013), the computed welfare costs of the business cycle are between 0.2% and 1.2% of the permanent consumption, which is between 4 and 24 times of the Lucas estimate of 0.05% (Hairault et al. (2010) calculate a welfare cost to be 0.55%, for Jung & Kuester (2011) this cost is 0.2%, and for Petrosky-Nadeau & Zhang (2013) it is 1.2%).

which magnifies the impact of aggregate shocks and makes the economy more volatile (business cycle effect). Finally, we illustrate how financial frictions enlarge the gap between unemployment (or the jobless rate) at the deterministic steady state and its mean.

More precisely, we introduce a DSGE model where labor markets are characterized by standard search-and-matching frictions à la Mortensen & Pissarides (1994) and where entrepreneurs' access to credit is limited by a collateral constraint because of enforcement limits, à la Kiyotaki & Moore (1997). We allow entrepreneurs to finance with debt also the intra-period costs associated to hiring. Credit costs associated to financial frictions lead to a lower equilibrium level for employment though an increase in labor costs (level effect). Moreover, we recover a credit multiplier mechanism (business-cycle effect) that significantly amplifies the propagation of productivity shocks.⁷ For the sake of simplicity, in our framework we retain the assumption that there is no "productive" precautionary saving in our economy. This contrasts with Krusell et al. (2010) and Bils et al. (2011) where precautionary saving is so strong that dampens the impact of the asymmetries generated by the matching process on the labor market.⁸ These works do not provide a quantitative computation of welfare costs of the business cycle. This paper fills this gap.

The quantitative computation confirms our intuition. Indeed, the introduction of financial frictions raises welfare costs of fluctuations several times with respect to the ones obtained with labor frictions only. The business cycle cost of fluctuations with financial and labor market frictions is 2.5% of workers' permanent consumption. The welfare cost drastically falls without financial frictions (at 0.3% for workers), given the welfare costs of the labor market frictions. These costs are far larger than the estimates by Lucas (1987, 2003) who reports a welfare cost of 0.005% in the case of logarithmic utility.

Our evaluation of welfare costs can be considered as reasonable only if based on a model able to match the main characteristics of the business cycle. From this positive perspective, our work refers to a recent literature of labor economics incorporating financial frictions into an environment characterized by labor market search. These works study how evolving conditions on credit markets affect the dynamics of labor markets and can improve the ability of standard search models to match the data. We show that the mechanisms that generate both the level and the business cycle effects on the welfare can help solve the volatility puzzle emphasized by Shimer (2005). Indeed, as remarked by Shimer (2005), the textbook search and matching model cannot generate enough business-cycle-frequency fluctuations in unemployment, job vacancies, thus job finding rates (the "Shimer puzzle"). Indeed, in expansion, the increase in wages tends to lower the firms' incentive to

⁷These two features (level and business-cycle effects) are shared with papers by Acemoglu (2001), Wasmer & Weil (2004), Petrosky-Nadeau (2013) and Petrosky-Nadau & Wasmer (2013).

⁸This result was already shown in Krusell & Smith (1998), where idiosyncratic risk and credit limits lead to precautionary saving (agents "over-save"). This allows the economy to accumulate large levels of capital which is productive. As a consequence, the mean of stochastic steady states of wealth is greater than its deterministic counterpart. Fluctuations have thus negligible welfare costs.

hire more workers. In our model, in response to a productivity shock, counter-cyclical credit costs associated to financial frictions give firms an incentive to post more vacancies, independently from the expected benefit of new workers. Another channel through which financial and labor market frictions interact is the wage curve. As suggested by Chéron & Langot (2004) and Pissarides (2009), and in echo to old Keynes-Tarshis-Dunlop controversy, a solution of the puzzle must explain not only fluctuations in unemployment, but also in wages. We then show that financial frictions à la Kiyotaki & Moore (1997) allow us both to replicate volatilities of labor-market aggregates (worker flows and unemployment stock) and incorporate a calibration where a large part of the wage can fluctuate, as observed in the data. More precisely, we show that even if heterogeneous discounting induces a pro-cyclical component in the wage curve⁹, this force is always dominated by the counter-cyclical credit cost dampening the value of the highly volatile search cost. The model creates endogenous wage sluggishness along the business cycle, thereby preserving the firm's hiring incentive in expansion. This allows the model to be consistent with the large changes in job finding rates observed in the data, while accounting for the cyclicity of real wages. In this respect, our work differs from Hagendorn & Manovskii (2008) who consider a small worker's bargaining power. Their calibration is obtained by targeting the regression coefficient between the HP-filtered log of wage and the HP-filtered log of productivity. Their calibration also leads to "over-employment" at the equilibrium. In contrast, in our calibration, the elasticity of the matching function with respect to unemployment equals workers' bargaining power. Hence, there is "under-unemployment" at the equilibrium. Moreover, this calibration of the worker's bargaining power does not restrict the part of the wage that can fluctuate, which is the case in e.g. Hagendorn & Manovskii (2008). The "real wage rigidity" is thus an endogenous result in our paper. Petrosky-Nadau & Wasmer (2013), Petrosky-Nadeau (2013) or Petrosky-Nadeau & Zhang (2013) do not discuss the implications of their model with respect to the real wage dynamics. The stress on this endogenous wage rigidity – which is consistent with data – resulting from credit constraints constitutes the second contribution of our paper.

The rest of this paper is organized as follows. In section 2 we introduce the main mechanisms at work. Section 3 describes the model and section 4 focuses on equilibrium in the labor market. Our quantitative analysis is detailed in section 5. Section 6 concludes.

⁹In the Nash bargaining process, as firms are less patient than workers, their subjective cost of filling a vacancy associated to the probability to find a new worker pushes wages upward. This effect is particularly strong in expansions.

2 Significant welfare costs in a world with search frictions and financial imperfections: The economic mechanism at work

In this section, we introduce the main mechanisms at the roots of our results. Another way to understand why we obtain welfare costs of cycles, which are much greater than the ones resulting from Lucas (1987, 2003) is to focus on the fundamental difference between the two frameworks. In fact, while Lucas economy is "linear" (in the sense that the business cycle does not affect average unemployment and output), our model is characterized by non-linearities due to the combination of labor market frictions and credit imperfections. In what follows, we show how non-linearities have important implications for welfare both via level effects, which are intrinsic in the structure of the model (section 2.1) and business cycle effects – i.e., business cycle fluctuations which are amplified by the financial accelerator (section 2.2).

Notice that, with financial frictions, households can smooth their consumption using savings. We retain the simplifying assumption that saving is supplied by the households to finance entrepreneurs' projects: land investments, as well as the costs associated with working capital within the period.

2.1 Non-linearities in the labor market (level effect)

As in Lucas (1987, 2003), our exercise consists in a comparison between welfare in a stabilized economy (i.e., an economy that is always at the deterministic steady state) and welfare in a stochastic economy (i.e., its mean). As stressed by Hairault et al. (2010), non-linearities inherent to the standard search and matching model imply that the level of unemployment at its deterministic steady state is smaller than its mean. Due to congestion effects, average employment and therefore average consumption are lowered by the mere process of alternate expansions and contractions. Therefore, the welfare loss due to fluctuations in an economy characterized by labor frictions is not negligible. At the steady state, unemployment outflows equal unemployment inflows:

$$\Psi U = sN$$

with Ψ the job finding probability, U the number of unemployed workers, s the exogenous job destruction rate and $N = 1 - U$, the number of employed workers, given that the population size is normalized to 1. U is then a convex function of the job finding rate Ψ :

$$U = \frac{s}{s + \Psi}$$

Consider now a stochastic environment. Suppose for simplicity that Ψ ¹⁰ follows a Markov stochastic process defined over states i , and that unemployment converges instantaneously from one to another

¹⁰We suppose for simplicity that the separation rate is constant. Indeed, Shimer (2012) shows that, since 1948, the job finding probability has been the main driving force behind fluctuations in the unemployment rate in the United States. Fluctuations in the employment exit probability have been quantitatively irrelevant during the last two decades.

steady state, depending on the value of Ψ . Formally, conditional steady-state unemployment is $\tilde{u}_i = \frac{s}{s+\Psi_i}$. Moreover, stabilized unemployment is $\bar{u} = \frac{s}{s+\sum_i \pi_i \Psi_i}$ where π_i is the probability that Ψ takes the value Ψ_i and $\sum_i \pi_i \Psi_i$ is the mean of the job finding rate. Because of convexity,

$$\frac{s}{s + \sum_i \pi_i \Psi_i} < \sum_i \pi_i \tilde{u}_i = \tilde{u} \approx E(u)$$

i.e., as can be seen from Figure 2, unemployment is a convex function of the job finding rate. Therefore, average unemployment is greater than the structural (stabilized) unemployment. Hairault et al. (2010) show that the unemployment gap is

$$\tilde{u} - \bar{u} \approx u''(\Psi) \frac{\sigma_\Psi^2}{2} \approx \frac{s}{(s + \Psi)^3} \sigma_\Psi^2 \quad (1)$$

which increases with σ_Ψ^2 and falls with Ψ .

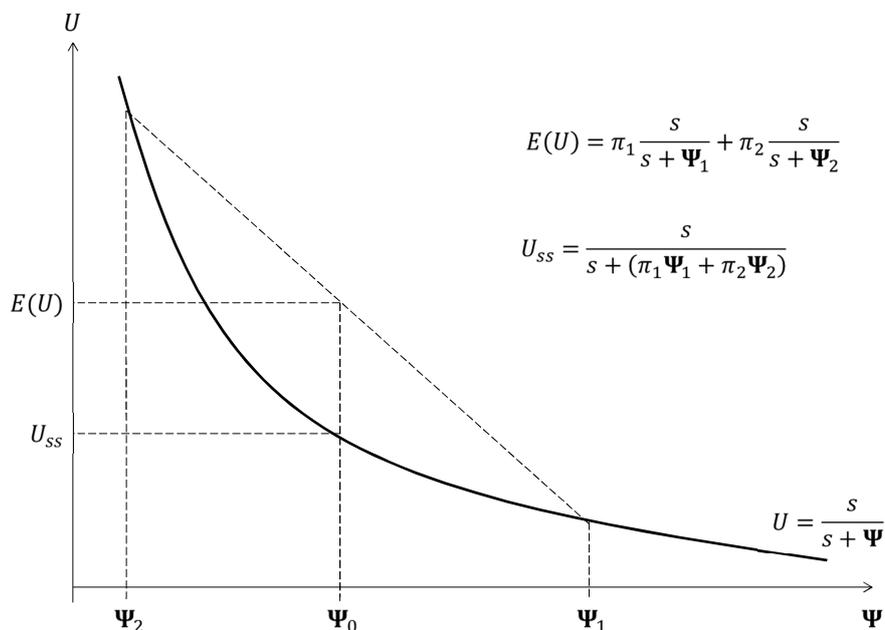
In order to understand the role of financial frictions, one has to relax the above assumption on the exogenous nature of the job finding rate. In what follows, we introduce a general equilibrium model to explain agents' behaviors in presence of credit constraints. The job finding rate, together with all other macro aggregates, is thus an endogenous result of agents' interactions. Our analysis illustrates how financial frictions imply higher costs of opening vacancies and eventually entail a lower employment level in equilibrium. As a result, the job finding rate is lower, and the economy moves leftward, towards the more convex part of Figure 2. As shown in the Figure 3 and in equation (1), the economy with financial frictions is characterized by $\Psi_2 < \Psi_1$, which raises the gap between the mean of employment and the steady-state level of unemployment. This will eventually explain one part of the welfare costs associated to financial frictions.

2.2 The credit multiplier throughout the cycle (business-cycle effect)

As shown by equation (1), a mean-preserving increase in the volatility σ_Ψ widens the gap between the stabilized and fluctuating unemployment rates. The more volatile the job finding rate Ψ , the greater the business cycle cost. We thus expect large fluctuations to increase *per se* the unemployment gap.

Introducing financial frictions destabilizes the economy and triggers an amplification of cycles (Kiyotaki & Moore (1997)). As the economy is more volatile, labor-market variables also experience greater fluctuations. This point is illustrated on Figure 3. As the volatility of the job finding rate Ψ increases, level effects associated to non-linearities are magnified. Figure 3 illustrates two economies: one characterized by labor frictions only (the one entailing $\Psi = \Psi_1$), and one with financial imperfections as well (where $\Psi = \Psi_2$). Because of financial frictions, the steady state of the job finding rate is lower (level effects), and the economy moves into the convex part of the function (see discussed in section 2.1). Moreover, because of the financial accelerator, the volatility of Ψ increases, pushing up the unemployment gap.

Figure 2: Non linearities in the labor market: the level effect



3 The model

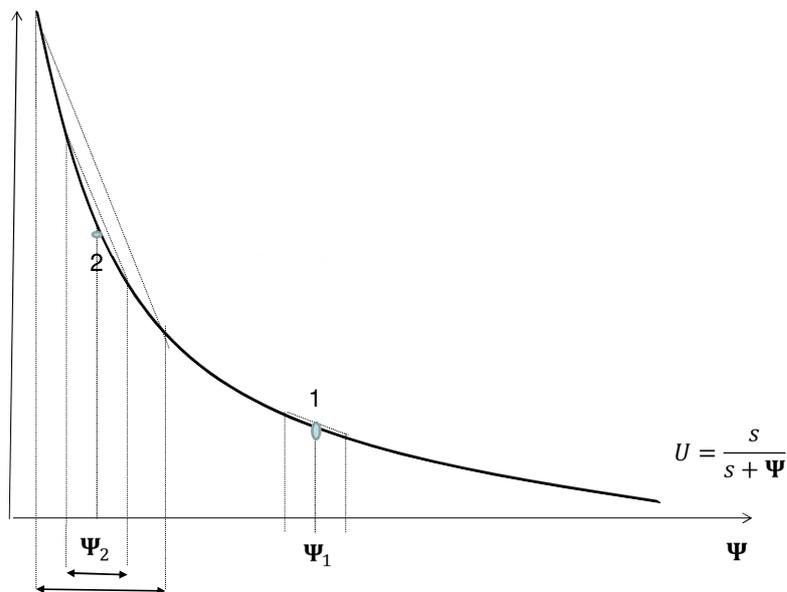
The economy is populated by two types of agents: firms and workers. The representative firm produces the final consumption good of the economy by combining labor and infrastructure (i.e., land). Firms have the possibility to finance their activity with loans funded by households. As debt contracts are not complete because of enforcement limits *à la* Kiyotaki & Moore (1997), firms are subject to a collateral constraint. Households can be either unemployed or employed workers by the firms. There is a canonical matching process *à la* Mortensen & Pissarides (1994) that allows firms to hire workers. Wages are set according to a standard Nash bargaining process.

In order to stress the economic mechanisms at work, we present a streamlined model without capital accumulation. Notice however that, even without capital, households can actually save by lending to firms. In addition, we lay stress on the extensive margin of labor, thereby discarding adjustments in hours, as in Blanchard & Gali (2010). Finally, we consider only technological shocks so as to be able to compare the model's welfare costs to those found in the literature.

3.1 Labor market flows

The economy is populated by a large number of identical households whose measure is normalized to one. Each household consists of a continuum of infinitely-lived agents. We consider a standard labor and matching model *à la* Blanchard & Gali (2010) and Mortensen & Pissarides (1994). Let N_t and M_t respectively denote the number of workers and the total number of new hires, and s the

Figure 3: The level effect is amplified by fluctuations in the job finding rate (business cycle effect)



exogenous job destruction rate. Employment evolves according to:

$$N_t = (1 - s)N_{t-1} + M_t \tag{2}$$

where M_t , the number of hiring per period, is determined by a constant returns to scale matching function:

$$M_t = \chi V_t^\psi S_t^{1-\psi}, \quad 0 < \psi < 1$$

$\chi > 0$ is a scale parameter measuring the efficiency of the matching function, and V_t the number of vacancies. Following Blanchard & Gali (2010), we suppose that at the beginning of period t , there is a pool of jobless individuals, S_t , who are available for hire. This implies that the pool of jobless agents is larger than the number of unemployed workers. Indeed, at all times, individuals are either employed or willing to work (full participation) so that S_t is given by:

$$S_t = U_{t-1} + sN_{t-1} = 1 - (1 - s)N_{t-1} \tag{3}$$

where $U_t = 1 - N_t$ is the stock of unemployed workers when the size of the population is normalized to 1 and under the assumption of full participation. U_t measures thus the fraction of the population who are left without a job after hiring takes place in period t .

Among agents looking for a job at the beginning of period t , a number M_t are hired and start working in the same period. Only workers in the unemployment pool S_t at the beginning of the

period can be hired ($M_t \leq S_t$). The ratio of aggregate hires to the unemployment pool $\Psi_t \equiv M_t/S_t$ is the rate at which the pool of jobless people find a job. Labor market tightness θ_t equals $\frac{V_t}{S_t}$.¹¹

3.2 Households

Households maximize an utility function of consumption and labor. The consumption-smoothing choice then interacts with labor-market behaviors. Each period, an agent can engage in only one of 2 activities: working or enjoying leisure. Moreover, unemployed agents are randomly matched with job vacancies. Finally, there are employment lotteries to smooth the individual idiosyncratic risk faced by each agent in his job match. Hence, the representative household's preferences are represented by:

$$E_0 \left[\sum_{t=0}^{\infty} \mu^t \{N_t U^n(C_t^n) + (1 - N_t) U^u(C_t^u + \Gamma)\} \right] \quad (4)$$

where $0 < \mu < 1$ is the discount factor and Γ the utility of leisure. C_t^z stand for the consumption of employed ($z = n$) and unemployed agents ($z = u$). We assume

$$U(C_t^n) = \frac{(C_t^n)^{1-\sigma}}{1-\sigma} \equiv \tilde{U}_t^n.$$

$$U(C_t^u + \Gamma) = \frac{(C_t^u + \Gamma)^{1-\sigma}}{1-\sigma} \equiv \tilde{U}_t^u.$$

with, $\sigma > 0$ the coefficient of relative risk aversion. With this utility function, our welfare measure indirectly depends on the employment level. Households labor opportunities evolves as follows:

$$N_t = (1 - s)N_{t-1} + \Psi_t S_t \quad (5)$$

Each household knows that the evolution of S follows (3), so that (5) can be written as:

$$N_t = (1 - s)N_{t-1} + \Psi_t (1 - (1 - s)N_{t-1}) \quad (6)$$

Households' budget constraint is:

$$[N_t C_t^n + (1 - N_t) C_t^u] + B_t \leq R_{t-1} B_{t-1} + N_t w_t + (1 - N_t) b_t + T_t \quad (7)$$

where w is the real wage and b the unemployment benefit. T is a lump-sum transfer from the government. Moreover, B are private bonds financing firms and R is the gross investment return associated to these loans. The dynamic problem of a typical household can be written as follows

$$\mathcal{W}(\Omega_t^H) = \max_{C_t^n, C_t^u, B_t} \{N_t U(C_t^n) + (1 - N_t) U(C_t^u + \Gamma) + \mu E_t \mathcal{W}(\Omega_{t+1}^H)\}$$

subject to (6) and (7), given the initial conditions on state variables (N_0, K_0, B_0) and $\Omega_t^H = \{N_{t-1}, \Psi_t, w_t, b_t, T_t, B_{t-1}\}$, the vector of variables taken as given by households. Let λ_t be the

¹¹The jobless rate is thus a convex function of the job finding rate, $S = \frac{s}{s+(1-s)\Psi}$.

shadow price of the budget constraint. The first order conditions associated with consumption choices are

$$(C_t^n)^{-\sigma} = (C_t^u + \Gamma)^{-\sigma} = \lambda_t$$

Hence $\tilde{U}_t^n = \tilde{U}_t^u$. The first order condition associated to bond holdings reads:

$$-\lambda_t + \mu E_t [R_t \lambda_{t+1}] = 0 \quad (8)$$

3.3 Firms

There are many identical firms in the economy. Entrepreneurs maximize the following sum of expected utilities:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t^F) \right] \quad (9)$$

where β denotes entrepreneurs' discount factor, and $\mu > \beta$ implying that workers are more patient than firms¹². Their budget constraint is:

$$C_t^F + R_{t-1}B_{t-1} + q_t [L_t - L_{t-1}] + w_t N_t + \bar{w} V_t \leq Y_t + B_t + \pi_t \quad (10)$$

where B is private debt, L productive land or infrastructure and q its price. Moreover, wN denotes total wages, with N the number of employees, whereas Y is the final output and π lump-sum dividends. Notice that the economy is endowed with a fixed amount of land. In practice, as land supply is vertical, each period market clearing is ensured by price adjustments.

Each firm has access to a Cobb-Douglas constant-return-to-scale production technology combining workers and infrastructure (land):

$$Y_t = A_t L_{t-1}^{1-\alpha} N_t^\alpha \quad (11)$$

where A_t represents the global productivity of factors in the economy, assumed to evolve stochastically as follows

$$\log A_t = \rho_a \log A_{t-1} + (1 - \rho_a) \log A + \varepsilon_t^a$$

with $\{\varepsilon^a\}_t$ the vector of iid innovations. Firms' activity can be financed by funds lent by households under imperfect debt contracts. Enforcement limits *à la* Kiyotaki & Moore (1997) imply that entrepreneurs are subject to collateral constraints. As Quadrini (2011) and Jermann & Quadrini (2012), we assume that, at the beginning of the period, firms can access financial markets to finance both their expenditures (i.e., entrepreneurs consumption and land investments) as well as the costs associated to the working capital within the period. Moreover, as Petrosky-Nadeau (2013) and Wasmer & Weil (2004), we suppose for simplicity that the costs associated to the working labor

¹²This assumption is needed to insure that firms are debt constrained in equilibrium. We will discuss this point in the following.

consist in hiring costs¹³. Entrepreneurs can thus borrow from agents subject to the following collateral constraint

$$B_t + \omega V_t \leq m E_t [q_{t+1} L_t] \quad (12)$$

where m is the (exogenous) loan to debt ratio.

The firms' objective is to maximize entrepreneurs utility (Equation (9)) given the technology production function (11) and subject to the constraint associated to the evolution of vacancies, i.e.:

$$N_t = (1 - s)N_{t-1} + \Phi_t V_t \quad (13)$$

where $\Phi_t \equiv M_t/V_t$ is the rate at which a vacancy is filled, considered as exogenous by the firm. The firm's program is

$$\begin{aligned} \mathcal{W}(\Omega_t^F) &= \max_{C_t^F, L_t, B_t, V_t, N_t} \{U(C_t^F) + \beta E_t [\mathcal{W}(\Omega_{t+1}^F)]\} \\ \text{s.t.} &\begin{cases} -C_t^F - R_{t-1}B_{t-1} - q_t [L_t - L_{t-1}] - w_t N_t - \bar{\omega} V_t \\ \quad + Y_t(A_t, L_{t-1}, N_t) + B_t = 0 & (\lambda_t^F) \\ -B_t - \bar{\omega} V_t + m E_t [q_{t+1} L_t] = 0 & (\lambda_t^F \varphi_t) \\ -N_t + (1 - s)N_{t-1} + \Phi_t V_t = 0 & (\xi_t) \end{cases} \end{aligned} \quad (14)$$

given the initial conditions N_0, B_0 , where $\Omega_t^F = \{N_{t-1}, \Psi_t, w_t, b_t, \pi_t, T_t, B_{t-1}, L_{t-1}\}$ is the vector of variables taken as given by firms. Letting $\lambda_t^F, \lambda_t^F \varphi_t$, and ξ_t be the Lagrange multipliers associated to (10), (12) and (13) the first order conditions of problem (14) read:

$$U'(C_t^F) = \lambda_t^F \quad (15)$$

$$\lambda_t^F q_t = \beta E_t \left[\lambda_{t+1}^F \left(q_{t+1} + \frac{\partial Y_{t+1}}{\partial L_t} \right) \right] + \lambda_t^F \varphi_t m E_t [q_{t+1}] \quad (16)$$

$$(1 - \varphi_t) \lambda_t^F = \beta E_t \lambda_{t+1}^F R_t \quad (17)$$

$$\xi_t = \lambda_t^F \bar{\omega} \frac{(1 + \varphi_t)}{\Phi_t} \quad (18)$$

$$\xi_t = \lambda_t^F \left[\left(\frac{\partial Y_t}{\partial N_t} \right) - w_t \right] + (1 - s) \beta E_t [\xi_{t+1}] \quad (19)$$

where (15) is the condition associated to consumption and (18) the one on vacancy posting¹⁴. Equation (16) is the one associated to land accumulation. It implies that, in equilibrium, the value of current marginal utility of consumption needs to equal the indirect value of utility deriving from land accumulation, i.e.: *i*) the value of future consumption utility deriving from reselling land in the next period, $\beta E_t \lambda_{t+1}^F q_{t+1}$; *ii*) the future consumption utility arising from the product of land, $\beta E_t \lambda_{t+1}^F \frac{\partial Y_{t+1}}{\partial L_t}$; *iii*) the additional utility arising from current consumption related to the effect of land in loosening the collateral constraint, $\varphi_t m \lambda_t^F E_t [q_{t+1}]$.

¹³Alternatively, we could consider also the wage bill. This would affect the analytical tractability of our results.

¹⁴Note that, entrepreneurs are not risk neutral. By letting $\lambda_t^F = 1$ we recover the canonical search model.

Equation (17) is a modified Euler equation. When the collateral constraint is not binding, φ_t is equal to zero and we recover the standard Euler equation. When the debt limit is binding, $\varphi_t > 0$ and $\varphi_t = 1 - \beta \frac{E_t \lambda_{t+1}^F R_t}{\lambda_t^F}$ implying that firms' marginal utility of current consumption is greater than their discounted marginal utility of future consumption. Impatient firms choose thus to increase consumption up to the limit imposed by (12).

3.4 The job creation and the wage curves

This section studies how financial frictions affect the mechanisms at the heart of labor market dynamics, i.e., the job creation curve (*JC*) and the wage curve (*WC*). As the relevance of our quantitative calculation of business cycle costs depends on our ability to match the volatility of data, overcoming the "Shimer puzzle" is a necessary condition for our exercise. In the Diamond-Mortensen-Pissarides (DMP) model, the opportunity cost to open a vacancy ($\bar{\omega}$ in our notation) is constant over the business cycle, and the real wage is highly pro-cyclical. Indeed, because of Nash bargaining, workers enjoy a significant share of productivity gains and of search returns. In booms, the increase in wages reduces the firms' incentives to post more vacancies. Shimer (2005) and Hall (2005*b*) hence suggest that real-wage rigidity can reconcile the DMP model with the data. This solution is retained in Hairault et al. (2010) welfare evaluation. Hagendorn & Manovskii (2008) argue that wage rigidity can be obtained by using an alternative calibration of the central parameters of the DMP model: with a very high return to non-market activity and very low bargaining power to workers, firms' incentives to post vacancies respond strongly to changes in productivity. The model then generates volatilities that are consistent with the data. However, the departure from the Hosios condition proposed by Hagendorn & Manovskii (2008) leads to very strong restrictions: the regression coefficient between the HP-filtered log of wage and the HP-filtered log of productivity can be reproduced by the model (a moment targeted in their calibration procedure), but not necessarily the real wage volatility generated by the model because it also depends on the other parameter values. We propose a solution for the volatility of the real wage. From a normative point of view, their calibration leads to an equilibrium characterized both by "under-unemployment" and negligible costs of being unemployed (i.e., a particular view of the unemployment phenomena). In this section, we argue that financial frictions can change both the view that the opportunity cost to open a vacancy is constant over the business cycle and the view that real wages increase in expansion.

3.4.1 The Job Creation curve JC .

We now focus on the (JC) condition and on the impact of financial fictions on the opportunity cost of opening a vacancy. By combining (18) with (19) we obtain the (JC) curve:

$$\bar{\omega} \frac{(1 + \varphi_t)}{\Phi_t} = \frac{\partial Y_t}{\partial N_t} - w_t + (1 - s) \beta E_t \left[\frac{\lambda_{t+1}^F}{\lambda_t^F} \frac{\bar{\omega}}{\Phi_{t+1}} (1 + \varphi_{t+1}) \right] \quad (20)$$

Term φ_t in equation (20) represents the "credit multiplier" of this model. It appears both on the LHS and RHS of equation (20). As shown in what follows, there is almost no persistence in the adjustment of φ_t : after a jump at the time of the shock, it comes back to its steady state value. We thus shift our attention on its impact on the LHS of (20). In recession, tight credit conditions (large values of φ_t) drive up the opportunity costs associated to vacancy posting, $\bar{\omega} (1 + \varphi_t)$. This introduces a counter-cyclical and time-varying wedge that has the potential to affect the propagation of productivity shocks. Indeed, as debt limits are less tight during booms, firms have an incentive to post more vacancies, independently from the expected benefit of new workers. This countercyclical wedge is a key mechanism in dampening wage fluctuations and amplifying employment volatility. In this respect, financial frictions *à la* Kiyotaki & Moore (1997) have the same implications as the ones *à la* Bernanke et al. (1999) discussed in Petrosky-Nadeau (2013). Notice however that our credit multiplier is of different nature with respect of the one in Petrosky-Nadeau (2013). In our model, the credit multiplier is the Lagrange multiplier associated to the collateral constraint: the larger φ , the tighter the constraint, and thus credit conditions. There is therefore a direct link between credit tightness and the credit multiplier. In Petrosky-Nadeau (2013), the mechanism is different. The credit multiplier is indeed associated to the productivity threshold insuring firms to default and it is amplified throughout the cycle by a time-varying and counter-cyclical monitoring cost carried by banks. In fact, for the mechanism associated to canonical financial frictions *à la* Bernanke et al. (1999) to match data, an ad-hoc counter-cyclical monitoring cost is incorporated into the financial contract.

Note finally that eliminating the vacancy-posting cost from the debt limit is not sufficient to rule out financial frictions from the model.¹⁵ Indeed, entrepreneurs are still constrained in their ability to borrow, even if they do not borrow to finance the working capital. We recover in this case a standard collateral constraint *à la* Kiyotaki & Moore (1997), where firms borrow to finance current economic activity (land purchases, debt service), including entrepreneurs consumption. In order to recover the frictionless Blanchard & Gali (2010) standard case, we need to eliminate agents impatience heterogeneity ($\beta = \mu$, and thus $\lambda_t^F = \lambda_t$) and let $\varphi_t = 0$.

¹⁵However, in this case:

$$\frac{\bar{\omega}}{\Phi_t} = \frac{\partial Y_t}{\partial N_t} - w_t + (1 - s) \beta E_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t} \right) \frac{\bar{\omega}}{\Phi_{t+1}} \right]$$

By solving this equation, it follows that the rate at which labor is hired depends on the expected discounted stream of marginal profits generated by an additional hire

3.4.2 The wage curve WC .

We suppose for simplicity that wages are determined via a generalized Nash bargaining between individual workers and firms, ie.:

$$\max_{w_t} \left(\frac{\mathcal{V}_t^F}{\lambda_t^F} \right)^\epsilon \left(\frac{\mathcal{V}_t^H}{\lambda_t} \right)^{1-\epsilon} \quad (21)$$

with $\mathcal{V}_t^F = \frac{\partial \mathcal{W}(\Omega_t^F)}{\partial N_{t-1}}$ the marginal value of a match for a firm and $\mathcal{V}_t^H = \frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_{t-1}}$ the marginal household's surplus from an established employment relationship. ϵ denotes the firm's share of a job's value, i.e., firms' bargaining power. The wage curve is ¹⁶

$$w_t = \underbrace{\epsilon b}_{(a)} + \underbrace{(1-\epsilon) \frac{\partial Y_t}{\partial N_t}}_{(b)} + \underbrace{(1-\epsilon)(1-s)\beta E_t}_{(\Sigma)} \left[\underbrace{(1+\varphi_{t+1})}_{(3)} \frac{\lambda_{t+1}^F}{\lambda_t^F} \left\{ \underbrace{\frac{\bar{\omega}}{\Phi_{t+1}} \beta \frac{\lambda_{t+1}^F}{\lambda_t^F} - \mu \frac{\lambda_{t+1}}{\lambda_t}}_{(1)} + \underbrace{\frac{\mu \frac{\lambda_{t+1}}{\lambda_t}}{\beta \frac{\lambda_{t+1}^F}{\lambda_t^F}} \bar{\omega} \theta_{t+1}}_{(2)} \right\} \right] \quad (22)$$

where (a) represents the weight of the reservation wage in total wage and (b) + (Σ) is the firms' gain from the match. This gain can be decomposed into the marginal productivity of the new employed worker, (b), and the saving on search costs if the job is not destroyed next period (Σ). When firms and workers are identical, (22) collapses to the standard Blanchard & Gali (2006)'s wage curve.¹⁷ Indeed, tightened credit conditions cause an amplification of costs associated to posting future vacancies.

Interactions between the bargained wage and the financial frictions. In presence of discounting heterogeneity, the bargaining process is influenced by the fact that workers and firms evaluate the aggregate surplus differently. As firms are less patient than workers, they underestimate the value of the intertemporal surplus. In practice, the value of the agreement after a meeting is for them lower than for workers. Thus, term (1) in (Σ) reduces the bargained wage of an amount proportional to the gap between the firm price kernel and the one of the workers. This allows wages to account for the surplus gap between workers and firm – associated to the gap in impatience rates.

In addition, during the bargaining process, the firm-worker pair shares the returns on the search process. For the worker, this is equal to the discounted time duration to find a job offer; for the

¹⁶see Appendix B.1

¹⁷Note that by eliminating financial frictions (let $\varphi = 0$, $\mu = \beta$, $\lambda_t = \lambda_t^F$) we can recover the standard wage rule of Blanchard & Gali (2010): $w_t = (1-\epsilon) \left(\frac{\partial Y_t}{\partial N_t} + \bar{\omega}(1-s)\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \theta_{t+1} \right] \right) + \epsilon b$.

firm, returns are instead equivalent to the discounted time duration to find a worker. Note however that these relative time spans cannot be proxied by the ratio of the average duration for these two search processes – as it would be the case without discounting heterogeneity. Given that firms are less patient than workers, the “subjective” duration of the search process is as well under-estimated by the firm. Thus, term (2) of (Σ) pushes up the wage through the ratio of discount rates. Indeed, entrepreneurs are impatient: this pushes them to quicker negotiations and to accept paying higher wages. Note that this second mechanism has been emphasized by Rubinstein (1982): once entered a bargaining process, the most impatient agent eventually gains less from the match.¹⁸ Moreover, the outcome of the two mechanisms is amplified by the "credit channel", $(1 + \varphi_{t+1})$. This mechanism is also at work in Petrosky-Nadeau (2013) and Wasmer & Weil (2004) and discussed in Monacelli et al. (2011).

Business cycle implications of the interactions between wages and financial frictions.

The fluctuations of the wage can be understanding using the following log linear approximation, where $y = \frac{\partial Y}{\partial N}$:

$$\begin{aligned} \hat{w}_t &= \frac{(1 - \epsilon)y}{b + (1 - \epsilon)(y - b + \Sigma)} \hat{y}_t + \frac{(1 - \epsilon)\Sigma}{b + (1 - \epsilon)(y - b + \Sigma)} \hat{\Sigma}_t \\ \text{with } \hat{\Sigma}_t &= \underbrace{\frac{\varphi}{1 + \varphi} E_t \hat{\varphi}_{t+1}}_{(3)} + \underbrace{\left(E_t \hat{\lambda}_{t+1}^F - \hat{\lambda}_t^F \right)}_{(4)} - \underbrace{\frac{\frac{\beta - \mu}{\beta} \frac{\bar{\omega}}{\Phi}}{\frac{\bar{\omega}}{\Phi} \frac{\beta - \mu}{\beta} + \bar{\omega} \theta \frac{\mu}{\beta}} \hat{\Phi}_{t+1}}_{(1a)} \\ &\quad - \underbrace{\frac{\frac{\bar{\omega}}{\Phi} \frac{\mu}{\beta}}{\frac{\bar{\omega}}{\Phi} \frac{\beta - \mu}{\beta} + \bar{\omega} \theta \frac{\mu}{\beta}} \left(E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - E_t \hat{\lambda}_{t+1}^F + \hat{\lambda}_t^F \right)}_{(1b)} \\ &\quad + \underbrace{\frac{\bar{\omega} \theta \frac{\mu}{\beta}}{\frac{\bar{\omega}}{\Phi} \frac{\beta - \mu}{\beta} + \bar{\omega} \theta \frac{\mu}{\beta}} \hat{\theta}_{t+1}}_{(2a)} + \underbrace{\frac{\bar{\omega} \theta \frac{\mu}{\beta}}{\frac{\bar{\omega}}{\Phi} \frac{\beta - \mu}{\beta} + \bar{\omega} \theta \frac{\mu}{\beta}} \left(E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - E_t \hat{\lambda}_{t+1}^F + \hat{\lambda}_t^F \right)}_{(2b)} \end{aligned}$$

where hat variables denote log deviations from the steady state. In response to a productivity shock, the wage equation share with the usual matching model the following properties: constant unemployment benefits (term (a) in (22)) lead wage to be a-cyclical whereas, when ϵ is not close to one, productivity (terms (b) in (22)) and the labor market tightness (θ_{t+1} in Σ) lead wages to be pro-cyclical.¹⁹ When the search value (Σ) matters, the financial interactions also matter because they are also in Σ . One can identify three effects of financial frictions:

¹⁸Notice however that, in Rubinstein (1982), the effect of term (1) is not present because of the eventually static approach of the game.

¹⁹Notice that Hagendorn & Manovskii (2008)'s calibration consists in generating a rigid wage by imposing ϵ close to 1: **this ensures that the movement of labor demand are very few contaminated by changes in wages.** On the other hand, the large value for b chosen by these authors magnifies the impact of the technological shocks on the labor demand.

- *Fluctuations in the relative stochastic discount rates.* These fluctuations are given by $E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t - E_t \widehat{\lambda}_{t+1}^F + \widehat{\lambda}_t^F$. Equilibrium conditions imply that this term varies in the same direction as $\widehat{\varphi}_t$ which is counter-cyclical.²⁰ In booms, the credit constraint represents for firms a smaller financial burden than at the steady state ($\widehat{\varphi}_t < 0$) and the gap in worker's and firm's discount factors falls. Term (1b) shows that the under-estimation of the match surplus by the firm decreases, implying an upward pressure on wages. However, the same force acts in the opposite direction through term (2b): entrepreneurs are more patient in expansions, leading them to delay negotiations for paying lower wages. Nevertheless, the total impact of heterogenous discounting dynamics on wages is counter-cyclical because $\frac{\bar{\omega}}{\bar{\Phi}} \frac{\mu}{\beta} - \bar{\omega} \theta \frac{\mu}{\beta} > 0$.²¹
- *Fluctuations in the time duration to fill a vacancy.* In expansion, impatient firms know that the time duration to find another worker increases ($\widehat{\Phi}_{t+1} > 0$ in (1a)). Thus, given that it is relatively more short-sighted ($\beta < \mu$), this component leads to wage moderation.
- *Fluctuations in the credit costs.* In boom, the costs to fill a vacancy are reduced through lower credit costs (term (3)), implying a wage moderation. This counter-cyclical component of the bargained wage is also present in Petrosky-Nadeau (2013).

Notice finally that one contribution of our paper is to show that the standard search model with credit frictions à la Kiyotaki & Moore (1997) can both replicate the volatility of the labor-market quantities and incorporate a calibration where a large share of the wage fluctuates as it is observed in the data. Indeed, financial frictions create a counter-cyclical force in wage dynamics allowing us to explore the ability of the model to reproduce both quantities and wage volatilities.²² This dimension is neglected in Petrosky-Nadeau (2013) and in Petrosky-Nadeau & Zhang (2013); analogously, results reported in Petrosky-Nadeau & Wasmer (2013) suggest that their type of financial frictions does not have any significant impact on the wage elasticity to aggregate shocks.

²⁰We use the following approximation of the Euler equations on consumption:

$$\left. \begin{aligned} R_t \mu E_t [\lambda_{t+1}] &= \lambda_t \\ R_t \beta E_t [\lambda_{t+1}^F] &= (1 - \varphi_t) \lambda_t^F \end{aligned} \right\} \approx \left\{ \begin{aligned} \widehat{R}_t + E_t [\widehat{\lambda}_{t+1}] &= \widehat{\lambda}_t \\ \widehat{R}_t + E_t [\widehat{\lambda}_{t+1}^F] &= -\frac{\varphi}{1-\varphi} \widehat{\varphi}_t + \widehat{\lambda}_t^F \end{aligned} \right.$$

$$\Rightarrow E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t - E_t \widehat{\lambda}_{t+1}^F + \widehat{\lambda}_t^F = \frac{\varphi}{1-\varphi} \widehat{\varphi}_t$$

²¹Indeed, we have $\text{sign} \left(\bar{\omega} \theta \frac{\mu}{\beta} - \frac{\bar{\omega}}{\bar{\Phi}} \frac{\mu}{\beta} \right) = \text{sign} \left(\theta - \frac{1}{\bar{\Phi}} \right) = \text{sign} \left(\frac{\Psi-1}{\bar{\Psi}} \right) = -$. We reduce the analysis to the numerator because the denominator is positive, around $\beta \approx \mu$ and $\mu \approx 1$: $\frac{\bar{\omega}}{\bar{\Phi}} \frac{\beta-\mu}{\beta} + \bar{\omega} \theta \frac{\mu}{\beta} = \frac{\bar{\omega} \theta}{\bar{\beta} \bar{\Psi}} (\beta - \mu + \mu \Psi) > 0$.

²²Even if the regression coefficient between the HP-filtered log of wage and the HP-filtered log of productivity is well reproduced in Hagendorn & Manovskii (2008)'s model, it is not shown that the volatility of the real wage is matched.

3.5 Markets clearing

In order to close the model, we assume that the government does not accumulate debt and pays unemployment benefits by the means of the revenues collected with the lump-sum tax, i.e.: $T_t = (1 - N_t)b_t$. It is possible to show (see Becker (1982), Becker & Foias (1982)) that in presence of standard levels of uncertainty²³, firms are collateral constrained at each period. Thus, the debt limit eventually determines the equilibrium level of corporate debt and workers savings. The private-bonds market is thus cleared. Good market equilibrium requires:

$$Y_t = N_t C_t^n + (1 - N_t) C_t^u + \bar{\omega} V_t + C_t^F$$

Finally, as discussed above, we assume that land supply is fixed and that the land market clears at each period, i.e.: $L_t = 1$.

4 From steady-state analysis to fluctuations

In what follows, we aim at showing analytically that the level effect (section 4.1) and business cycle effects (section 4.2) in Figure 3 are indeed at work in the model.

4.1 The labor market at the steady state

4.1.1 Wage curve WC

By rewriting Φ as a function of θ , WC is:

$$w = \epsilon b + (1 - \epsilon) \frac{\partial Y}{\partial N} + (1 - s)(1 - \epsilon)(1 + \varphi) \bar{\omega} \mu \left[\frac{\theta^{1-\psi}}{\chi} \left(\frac{\beta - \mu}{\mu} \right) + \theta \right]$$

At the steady state, the "credit multiplier" is summarized by relative impatience. By using the steady state relationship between β , μ and φ (equations (8) and (17)), we know that $\varphi = \left(1 - \frac{\beta}{\mu}\right)$. Hence, WC is

$$w = \epsilon b + (1 - \epsilon) \frac{\partial Y}{\partial N} + (1 - s)(1 - \epsilon) \bar{\omega} \mu \left(\underbrace{1 + 1 - \frac{\beta}{\mu}}_a \right) \left[\underbrace{\frac{\theta^{1-\psi}}{\chi} \left(\frac{\beta - \mu}{\mu} \right)}_b + \underbrace{\theta}_c \right] \quad (23)$$

As discussed above, relative impatience has two contrasting effects on WC : on the one hand, workers know that the continuing match will allow the firm to save money on future hiring costs (all the more so, as hiring costs are borrowed from patient households). As a result, workers want a share of this future gain for the firm: the larger the impatience gap (i.e. term a in (23)), the higher the wage.

²³For some discussion see, among others, Iacoviello (2005).

On the other hand, firms' gain in absence of separation is dampened by the fact that impatience reduces the intertemporal surplus of the firm, and this pushes down wages. This is represented by term b in (23) (which interacts with c). In absence of discounting heterogeneity (i.e. term $a = b = 0$) we recover the standard Blanchard & Gali (2010)'s steady-state wage curve:

$$w = \epsilon b + (1 - \epsilon) \frac{\partial Y}{\partial N} + (1 - s)(1 - \epsilon) \bar{\omega} \mu \theta \quad (24)$$

By comparing (24) with (23), we see that the WC of our model is steeper or flatter with respect to the standard Pissarides-Blanchard & Gali (2010) baseline depending on the net effect of terms a , b and c . The slope of the WC of our framework is:

$$Slope_{WC} = \frac{\partial w}{\partial \theta} \Big|_{WC} = (1 - s)(1 - \epsilon) \bar{\omega} \left(1 + 1 - \frac{\beta}{\mu}\right) \left[\frac{(1 - \psi)}{\theta^\psi \chi} (\beta - \mu) + \mu \right]$$

that is positive when $\frac{(1 - \psi)}{\theta^\psi \chi} \left(1 - \frac{\beta}{\mu}\right) < 1$.²⁴ Indeed, the larger the impatience gap, the greater the "credit multiplier". For realistic impatience gaps and our standard calibration of the labor market, this condition is always verified.

Let now focus on the effects of an increase in relative impatience. Keeping μ fixed and close to one, an increase in the heterogeneity gap entails a lower β . In this case

$$\frac{\partial Slope_{WC}}{-\partial \beta} = -(1 - s)(1 - \epsilon) \bar{\omega} \left[\frac{(1 - \psi)}{\theta^\psi \chi} \left(3 - \frac{2\beta}{\mu}\right) - 1 \right]$$

when the relative impatience increases (i.e. β decreases), the WC becomes more sloped when $\frac{(1 - \psi)}{\theta^\psi \chi} \left(3 - \frac{2\beta}{\mu}\right) > 1$. When the WC is upward sloped, this condition is always verified for any impatience gap. Thus, a decrease in β makes the WC steeper.

Note finally that relative impatience does not affect the intercept of the curve, equal to a weighted average of the outside opportunity $O \equiv b$ and marginal product of labor $MPL \equiv \frac{\partial Y}{\partial N}$.

4.1.2 Job creation curve JC

Consider now the job creation curve at the steady state (equation (20)), and rewritten as a function of θ :

$$w = \frac{\partial Y}{\partial N} + \theta^{1 - \psi} \frac{\bar{\omega}}{\chi} (1 + \varphi) [(1 - s) \beta - 1]$$

As $\varphi = \left(1 - \frac{\beta}{\mu}\right)$ (from equations (8) and (17) at the steady state), then JC can be rewritten as

$$w = \frac{\partial Y}{\partial N} + \theta^{1 - \psi} \frac{\bar{\omega}}{\chi} \left(1 + 1 - \frac{\beta}{\mu}\right) [(1 - s) \beta - 1] \quad (25)$$

²⁴More precisely, $(1 - \psi)$ and $\beta - \mu$, in absolute values, are less than 1 while $\frac{1}{\theta^\psi} > 1$ and the value of χ is determined at the steady state given the calibration on labor market facts. As at the steady state we have $\chi = \Phi \theta^{1 - \psi} < 1$ for a calibrated Φ , $\frac{1}{\chi} > 1$. In a nutshell, this condition is very likely to hold at the steady state but we still need to check it using the benchmark calibration

The slope of JC is:

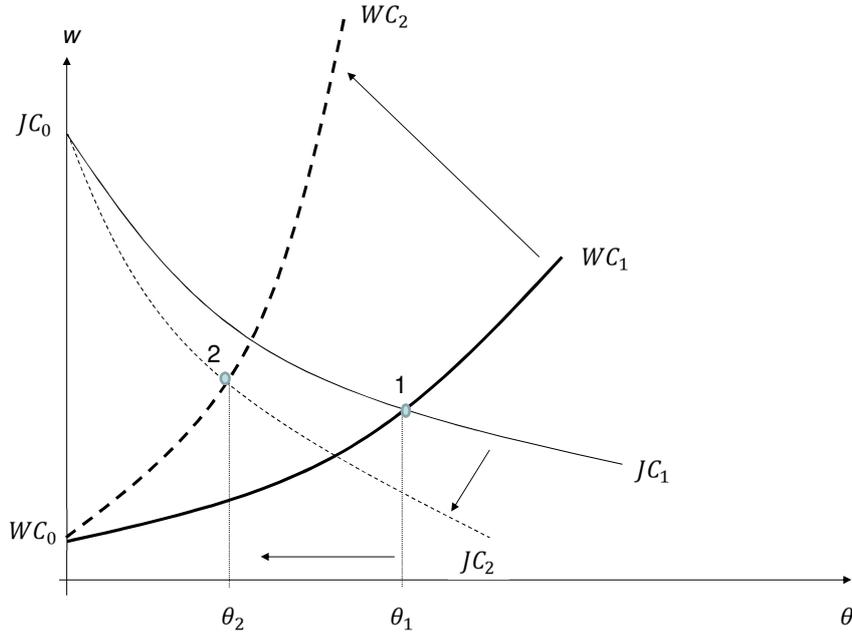
$$Slope_{JC} = \left. \frac{\partial w}{\partial \theta} \right|_{JC} = (1 - \psi)\theta^{-\psi} \frac{\bar{w}}{\chi} \left(1 + 1 - \frac{\beta}{\mu}\right) [(1 - s)\beta - 1] < 0$$

JC is unambiguously downward sloping since $(1 - s) < 1$ and $\beta < 1$. The increase in relative impatience entices the firm to post less vacancies. In addition, with a higher gap in impatience rates, the firm becomes more patient, and value more the future gain from the match. An increase in relative impatience can be proxied as a decrease in β , i.e.:

$$\frac{\partial |Slope_{JC}|}{-\partial \beta} = (1 - \psi)\theta^{-\psi} \frac{\bar{w}}{\chi} \left[\frac{1}{\mu} + (1 - s) 2 \left(1 - \frac{\beta}{\mu}\right) \right] > 0$$

It is straightforward to see that greater impatience gaps entail a steeper curve. The intercept is not affected. Notice also that relative impatience makes the JC curve unambiguously steeper than the standard JC curve.²⁵

Figure 4: Labor market equilibrium with increasing discounting heterogeneity



4.1.3 Steady state equilibrium

The above analysis has shown how discounting heterogeneity entails a wage curve whose slope (when positive) increases with the gap in impatience rates. The interplay of the above discussed

²⁵In absence of discounting heterogeneity, we recover the standard JC :

$$w = \frac{\partial Y_t}{\partial N_t} + \theta^{1-\psi} \frac{\bar{w}}{\chi} [(1 - s)\mu - 1]$$

mechanisms results in a steeper or flatter WC with respect to Blanchard & Gali (2010)'s framework, depending on the parameters involved. Thus, in steady state, wages will be greater or lower than Blanchard & Gali (2010)'s case, depending on which mechanism dominates.

For what concerns the JC curve, our results are unambiguous. Impatience heterogeneity entails a job creation equation, which is always more (negatively) sloped than the Blanchard & Gali (2010) baseline. Relative impatience entails an increase in the costs associated to vacancy posting. Because of the credit costs associated to the nature of the debt contract and the hold-up problem emphasized by Acemoglu & Shimer (1999), firms post less vacancies in equilibrium. Therefore, the steady-state level of market tightness is smaller than the baseline search and matching model. Figure 4 illustrates how the labor market equilibrium is affected by increasing heterogeneity in discount factors.

Proposition 1. *The steady-state level of market tightness in presence of discounting heterogeneity is smaller than in the baseline search and matching model.*

Proof. See Appendix B.2. This result gives theoretical foundations for the shift to the left of Ψ in Figure 3.

4.2 The impact of the business cycle

We now analyze the impact of a standard productivity shock in presence of credit and labor frictions. To this purpose, we first consider its effect on the job creation curve (equation (25)). When TFP productivity increases, labor productivity increases as well ($\frac{\partial Y}{\partial N}$ goes up). The job creation curve thus shifts up. The associated increase in output allows entrepreneurs' credit limit to be softened, so that the credit multiplier, φ decreases. The job creation curve becomes flatter.

Consider now the steady-state wage curve, equation (23). In response to a productivity shock, the outside opportunity b is constant while the marginal productivity of labor $\frac{\partial Y}{\partial N}$ jumps up. It follows that the wage rule shifts up. If $\frac{(1-\psi)}{\theta^\psi \chi} \left(1 - \frac{\beta}{\mu}\right) < 1$, WC becomes flatter as φ goes down.

Consider now the labor market at the equilibrium. To this purpose, rewrite the intersection of WC and JC (equation (32) in appendix B.2) so as to allow the credit multiplier to vary. The elasticity of the labor market to labor productivity can be rewritten as:

$$\epsilon(MPL - O) = (1 + \varphi) \bar{\omega} \left[(1 - s) (1 - \epsilon) \mu \theta + \frac{\theta^{1-\psi}}{\chi} (1 - (1 - s) \mu (1 - \varphi \epsilon)) \right] \equiv g(\theta, \varphi)$$

with MPL the Marginal Product of Labor $\frac{\partial Y}{\partial N}$. Taking the total differential we obtain

$$\epsilon \frac{MPL}{MPL - O} \widehat{MPL} = \epsilon_{g|\theta} \hat{\theta} + \epsilon_{g|\varphi} \hat{\varphi}$$

As the credit multiplier is counter-cyclical, we can write for simplicity that $\widehat{\varphi} = -\Gamma \widehat{MPL}$ with $\Gamma > 0$, then

$$\left[\epsilon \frac{MPL}{MPL - O} + \varepsilon_{g|\varphi} \Gamma \right] \widehat{MPL} = \varepsilon_{g|\theta} \widehat{\theta}$$

where $\varepsilon_{g|\varphi} \equiv \frac{\partial g(\theta, \varphi)}{\partial \varphi} \frac{\varphi}{g(\theta, \varphi)}$, with

$$\frac{\partial g(\theta, \varphi)}{\partial \varphi} = \bar{\omega} \left[(1-s)(1-\epsilon)\mu\theta + \frac{\theta^{1-\psi}}{\chi} (1 - (1-s)\mu(1-\varphi\epsilon)) \right] + (1+\varphi)\bar{\omega} \frac{\theta^{1-\psi}}{\chi} [(1-s)\mu(\epsilon)] > 0$$

As $\varepsilon_{g|\varphi} \Gamma > 0$, then the sensitivity of θ to MPL is amplified though the fluctuations in φ : the effect of a productivity shock is amplified by the credit multiplier and entails larger fluctuations of market tightness. Thus, in response to a productivity shock financial frictions amplify fluctuations in employment, while stabilizing wages. This result provides a theoretical foundation for the increase in the volatility of Ψ reported in figure 3. This confirms the intuition of the mechanism at work in the model.

5 Quantitative analysis

5.1 Calibration

The calibration is based on quarterly US data.²⁶

Preference and technology parameters : The discount factor for patient agents is consistent with a 4% annual real interest rate. For the impatient consumer, we set $\beta = 0.99$, which corresponds to a value in the range of the ones chosen by Iacoviello (2005)²⁷. The risk aversion is set to 1 for firms and 2 for workers. Both values lie within a standard interval in the literature. In addition, the firm is characterized by a lower risk aversion because, as shown by Iacoviello (2005), such a calibration ensures that, for a wide range of values for volatility shocks, impatience and loan-to-value ratio m , the borrowing constraint is binding.

Financial parameters: The corporate debt-to-GDP ratio pins down the value of m in the collateral constraint. We use as target the average 2001-2009 of corporate debt over GDP (debt outstanding, annual data, corporate sector, Flow of Funds Accounts tables of the Federal Reserve Board).

Labor market parameters: Employment level N is consistent with Hall (2005a)'s estimates of the average unemployment rate ($N = 0.88$). Hall (2005a) argues that, given the observed high

²⁶The data are described in appendix A.

²⁷Notice that it is sufficient to have a very small degree of impatience heterogeneity for debt limits to hold

transition rate from out of the labor force directly to employment suggests that some fraction of those classified as out of the labor force are nonetheless effectively job-seekers. Hall (2005a) adjusts the US unemployment rate to include individuals who are out of the labor force who are actually looking for a job.

As in Shimer (2005), the quarterly separation rate s is 0.10, so jobs last for about 2.5 years on average. Using steady-state labor-market flows, we infer Ψ given s and N . This leads to $\Psi = 0.423$. This value is lower than in the usual Mortensen-Pissarides model. Indeed, the pool of job seekers is larger in Blanchard & Gali (2010) than in the standard MP model.

The elasticity of the matching function with respect to the number of job seekers is $\psi = 0.5$, which lies within the range estimated in Petrongolo & Pissarides (2001). This value is also chosen so as to illustrate the pure effect of the non-linearities in the unemployment dynamics in the model without financial frictions (see section 2.1), as opposed to the non-linearities in the job finding rate (see Hairault et al. (2010)). The efficiency of matching, χ , is set such that firms with a vacancy find a worker with a 95% probability within a quarter, which is consistent with Andolfatto (1996).

As stressed by Hagendorn & Manovskii (2008), the parameters that determine the responsiveness of job creation to business cycles are the ratio of unemployment benefits (or home production without policy) to the wage and the firm's bargaining power. The utility of leisure parameter, Γ , is pinned down so as to match an unemployment benefit equal to 0.7 at steady state. This gives $b/w = 0.720$ (consistently with Hall & Milgrom (2008)). The cost of posting a vacancy, ω , is set to 0.17 as in Barron & Bishop (1985) and Barron et al. (1997). We obtain $\frac{\omega V}{Y} = 0.0179$, which is in the range found in the literature (0.005 in Chéron & Langot (2004), 0.01 in Hairault (2002) or 0.05 in Krause & Lubik (2007)). Notice finally that, in order to reproduce the volatility of the job finding rate, Hagendorn & Manovskii (2008) need to calibrate $b/w = 0.955$ and $\epsilon = 0.9480$, which implies that the share of wage that can fluctuate is negligible. In this paper, with $b/w = 0.720$ and $\epsilon = 0.5$, half of the wage can fluctuate. Thus, there is room for economic mechanisms to endogenously lead wages to fluctuate (which is consistent with the data). In addition, the model must endogenously generate limited fluctuations in w so as to preserve firms' incentives to hire in booms.

Shocks: The technological shock is calibrated as in Hairault et al. (2010). We choose the standard deviation of technological shock to reproduce the observed GDP standard deviation. Table 1 summarizes the calibration. ²⁸

²⁸We check that, for the benchmark calibration, the conditions on the slopes and steepness of WC hold: $\frac{(1-\psi)}{\theta\psi\chi} \left(1 - \frac{\beta}{\mu}\right) < 1$ and $\frac{(1-\psi)}{\theta\psi\chi} \left(3 - 2\frac{\beta}{\mu}\right) > 1$.

Table 1: Calibration

(a) External information			
Notation	Label	value	Reference
β	discount factor (impatient)	0.99	Iacoviello (2005)
α	production function	0.99	Iacoviello (2005)
σ_W	risk aversion, worker	2	
σ_F	risk aversion, firm	1	Iacoviello (2005)
s	Job separation rate	0.1	Shimer (2005)
N	Employment	0.88	Hall (2005 <i>a</i>)
ψ	Elasticity of the matching function	0.5	Petrongolo & Pissarides (2001)
ξ_V	financial constraint	1	Petrosky-Nadeau (2013)
ω	cost of job posting	0.17	Barron et al. (1997) and Barron & Bishop (1985)
$\frac{b}{w}$	replacement ratio	0.72	Hall and Milgrom (2008)
shock			
A	average TFP	1	Normalization
ρ_A	Persistence	0.95	Hairault et al. (2010)
(b) Empirical target			
Notation	Label	value	Empirical target
μ	discount factor (patient)	$\left(1.04^{\frac{1}{4}}\right)^{-1}$	Annual real rate of 0.04
χ	scale parameter of matching function	0.63397	Probability of filling a vacancy $\Phi = 0.95$
m	collateral constraint	0.61	corporate debt to GDP ratio $B/Y = 0.595$
σ_A	Standard deviation	0.0046	Observed σ_Y
(c) Derived parameter values			
Notation	Label	value	
Ψ	Job finding rate	0.423	
Γ	preference	0.19	

5.2 Welfare cost of fluctuations

The expected lifetime utility of a worker with uncertain consumption and employment paths is

$$\tilde{U}^w = E_0 \sum_{t=0}^{\infty} \mu^t [N_t U^n (C_t^n) + (1 - N_t) U^u (C_t^u + \Gamma)] \quad (26)$$

How much consumption at the steady state shall be reduced to make the worker indifferent between the steady state and the fluctuating economy? The welfare cost of fluctuations τ is such that

$$\sum_{t=0}^{\infty} \mu^t [\bar{N} U^n (\bar{C}^n (1 - \tau)) + (1 - \bar{N}) U^u ((\bar{C}^u + \Gamma)(1 - \tau))] = \tilde{U}^w \quad (27)$$

where variables denoted with an overbar are set at their steady state values. Expected utility \tilde{U} in the fluctuating economy can be found by simulating the economy. The mean of utility across simulations is the expected utility of the household in the fluctuating economy. Using $C_t \equiv N_t C_t^n + (1 - N_t) C_t^u$ hence $C_t = C_t^n - (1 - N_t)\Gamma$ because the FOC on consumption imply $C_t^n = C_t^u + \Gamma$, we deduce²⁹:

$$\sum_{t=0}^{\infty} \mu^t U (\bar{C}^n (1 - \tau)) = \sum_{t=0}^{\infty} \mu^t U ([\bar{C} + (1 - \bar{N})\Gamma](1 - \tau)) = \tilde{U}^w$$

and thus:

$$\tau = 1 - \left[\tilde{U}^w \frac{(1 - \mu)(1 - \sigma)}{(\bar{C}^n)^{1 - \sigma}} \right]^{\frac{1}{1 - \sigma}} = 1 - \left[\tilde{U}^w \frac{(1 - \mu)(1 - \sigma)}{(\bar{C} + (1 - \bar{N})\Gamma)^{1 - \sigma}} \right]^{\frac{1}{1 - \sigma}}$$

The result is reported in Table 2, line 1, column A. The business cycle cost of fluctuations with financial frictions is 2.5 % of workers' permanent consumption. Note that even by accounting for workers only, this number is far larger than the estimates found by Lucas (1987, 2003) who reports a welfare cost of $\tau = 0.005\%$ with log utility. It is noticeable that the welfare costs are large even though agents can save by lending to firms: workers can actually smooth business cycles with savings.

The welfare cost for workers falls drastically without financial frictions ($100 \times \tau = 0.3$, line 1, column B, table 2). This estimate is computed by using our model, once we eliminate discounting heterogeneity and collateral constraints, but keeping the parameter values reported in Table 1 (panels (b) and (c)). The rationale behind this approach is the following. The model with financial frictions is considered as the "true" model of the economy. By calibrating the model to match key financial and labor market targets, we uncover the "true" parameter values. The business cycle costs without financial frictions are then computed using these parameter values.

²⁹A similar computation is done for the firm's owner:

$$\sum_{t=0}^{\infty} \beta^t U (\bar{C}^F (1 - \tau^F)) = \tilde{U}^F = \sum_{t=0}^{\infty} \beta^t U (C_t^F)$$

When introducing financial frictions, firms' welfare cost of fluctuations can be taken into account. Aggregate welfare costs are then larger if one also consider firm's welfare cost. We choose to focus on worker's welfare cost so as to compare our results to the existing literature.

In order to decompose the business cycle cost, we use a Taylor expansion of welfare in the stabilized and volatile economies, which leads to approximate the difference in welfare in both economies as the sum of a level effect and the business cycle effect. Indeed, we have:

$$\begin{aligned}\tilde{U}^w &\equiv E_0 \left[\sum_{t=0}^{\infty} \mu^t U(C_t + (1 - N_t)\Gamma) \right] \\ &\approx \frac{1}{1 - \mu} U(E_0[C + (1 - N)\Gamma]) \left[1 - \frac{1}{2} \sigma(1 - \sigma) (\gamma_c \text{Var}(\hat{c}) + \gamma_u \text{Var}(\hat{u}) + \gamma_{cu} \text{Cov}(\hat{c}, \hat{u})) \right]\end{aligned}$$

where we denote³⁰ $\hat{x} = \frac{X_t - E_0[X]}{E_0[X]}$, for $x = C, U$ and $\gamma_c = \frac{E_0[C^2]}{E_0[(C + (1 - N)\Gamma)^2]}$, $\gamma_u = \frac{\Gamma^2 E_0[(1 - N)^2]}{E_0[(C + (1 - N)\Gamma)^2]}$ and $\gamma_{cu} = \frac{2\Gamma E_0[C(1 - N)]}{E_0[(C + (1 - N)\Gamma)^2]}$. This leads to

$$(1 - \tau) \approx \left(\frac{E_0[C + (1 - N)\Gamma]}{\bar{C} + (1 - \bar{N})\Gamma} \right) \left[1 - \frac{1}{2} \sigma(1 - \sigma) (\gamma_c \text{Var}(\hat{c}) + \gamma_u \text{Var}(\hat{u}) + \gamma_{cu} \text{Cov}(\hat{c}, \hat{u})) \right]^{\frac{1}{1 - \sigma}}$$

If we neglect the level effect, then we have $U(E_0[C + (1 - N)\Gamma]) \approx U(\bar{C} + (1 - \bar{N})\Gamma)$ because we assume that $E_0[C + (1 - N)\Gamma] \approx \bar{C} + (1 - \bar{N})\Gamma$ and then

$$1 - \tau_{BC} = \left[1 - \frac{1}{2} \sigma(1 - \sigma) (\gamma_c \text{Var}(\hat{c}) + \gamma_u \text{Var}(\hat{u}) + \gamma_{cu} \text{Cov}(\hat{c}, \hat{u})) \right]^{\frac{1}{1 - \sigma}}$$

where τ_{BC} denotes the welfare costs of the BC computing in the spirit of Lucas (1987, 2003). In contrast, if we neglect the business cycle effect ($\gamma_c \text{Var}(\hat{c}) + \gamma_u \text{Var}(\hat{u}) + \gamma_{cu} \text{Cov}(\hat{c}, \hat{u}) \approx 0$), we have

$$(1 - \tau_L) = \frac{E_0[C + (1 - N)\Gamma]}{\bar{C} + (1 - \bar{N})\Gamma}$$

where τ_L denotes the welfare costs of the business cycle effect linked to the level effect. We deduce that $(1 - \tau) = (1 - \tau_{BC})(1 - \tau_L) \Rightarrow \tau \approx \tau_{BC} + \tau_L$. Numerical computations give τ and τ_L given $E_0[C]$ and \bar{C} , $E_0[U]$ and \bar{U} . The previous formula then gives τ_{BC} .³¹

τ_{BC} captures the standard business cycle cost computed by Lucas (1987, 2003). The results reported in Table 2 are consistent with Lucas' estimates, with 0.05% of permanent consumption for workers. The most interesting result is the measure of $\tau_L = 2.45\%$ in Table 2, line 2, column A. It accounts for the large increase in business cycle cost. In Lucas (1987, 2003), $\tau_L = 0$: there is no gap between average and steady state consumption. Our model shows that this approximation is not acceptable because business cycle volatility significantly affects the gap between average and steady state employment and consumption. Thus, the business-cycle costs are sizable: they are 50 times greater than in Lucas' estimate. Without financial constraints, the magnitude of the business cycle costs is reduced to 6 times Lucas' evaluation.

³⁰Indeed, given that $\frac{\partial U}{\partial C} = U'$, $\frac{\partial U}{\partial N} = -\Gamma U'$, $\frac{\partial^2 U}{\partial C^2} = U''$, $\frac{\partial^2 U}{\partial N^2} = \Gamma^2 U''$ and $\frac{\partial^2 U}{\partial C \partial N} = -\Gamma U''$, we obtain, with the usual functional form $U(x) = \frac{x^{1 - \sigma}}{1 - \sigma}$, $U' = x^{-\sigma}$ and $U'' = -\sigma x^{-\sigma - 1} = -\sigma(1 - \sigma) \frac{x^{1 - \sigma}}{1 - \sigma} \frac{1}{x^2}$.

³¹Same considerations apply for the firm owners, using $E_0[C^F]$, \bar{C}^F and τ^F . Welfare costs are computed after simulating 30,000 periods using a 2nd-order approximation with Dynare.

Table 2: Welfare costs of business cycle: decomposition

	Worker with financial frictions A	Worker without financial frictions B
Total welfare cost		
1. $\tau \times 100$	2.50	0.30
Decomposing the welfare cost		
2. $\tau_L \times 100$	2.45	0.24
3. $\tau_{BC} \times 100$	0.05	0.06
4. $E_0[C + (1 - N)\Gamma^u]/(\bar{C} + (1 - \bar{N})\Gamma^u)$	0.975	0.997
5. $\sqrt{Var(\hat{c})} \times 100$	2.59	2.85
6. $\sqrt{Var(\hat{u})} \times 100$	7.23	0.12
7. $Cov(\hat{c}, \hat{u})$	-0.19	-0.34

line 1 = line 2 + line 3

If we take our computations to the data, the contrast with Lucas results is straightforward. Our calculations show that, with financial frictions, the mean of employment is 0.85123, while its steady state value is 0.88. This implies that the level effect associated to employment only is $100 \times \frac{\bar{N} - E(N)}{\bar{N}} = 3.2694$. If we apply this percentage to US civilian employment stock in 2013, the job losses associated to fluctuations represent more than four millions jobs. Analogous considerations apply for GDP per capita. Our results show that the level effect entails a GDP per capita loss of 1569.3 dollars a year. In absence of financial frictions, the losses are significantly reduced. Our numerical computations show that the level effect entail a loss of about 300.000 jobs and each household loses 104.03 dollars per year.

5.3 Model versus data

In this section we compare the business cycle properties of the data with the ones predicted by the model. We obviously want to gauge the empirical performances of the model. However, given our simplifying assumptions (no capital, no worked hours, productivity shocks only), we are aware that the fit cannot but be perfect. We remind the reader that we develop our model including only technological shocks, in order to compare our results to the existing literature –that considers only this source of business cycle fluctuations.

Beyond the assessment of the fit, simulations also allow to confirm the relevance of the above-discussed mechanisms at work. In particular, the exact role of relative impatience on wage dynamics. We show below that the model is indeed able to endogenously generate some wage rigidity.

5.3.1 Business cycle facts: interaction between labor and financial markets

In this section, we document the unconditional business cycles facts between financial variables and labor market adjustments. Our contribution lies in bringing together financial data (from Jermann & Quadrini (2012)) and data from the labor market literature (Shimer (2012)). All data have been re-computed and updated so that our sample covers 5 recession episodes from 1976 January through January 2013. In our view, previous papers who study the interaction between financial and real variables in DSGE models (Monacelli et al. (2011), Christiano et al. (2010)) summarize labor market adjustments using only fluctuations in employment and unemployment. In this paper, as in the labor market literature, we argue that these variables are not sufficient to summarize labor market adjustments. Indeed, workers flows (unemployment inflows and outflows) are paramount in generating stock dynamics. Shimer (2012) measures the probability that an employed worker becomes unemployed and the probability that an unemployed worker finds a job. He finds that there are substantial fluctuations in unemployed workers' job finding probability at business cycle frequencies, while the probability a worker exits employment is comparatively acyclic. Shimer (2012) insists that if one wants to understand fluctuations in unemployment, one must understand fluctuations in the transition rate from unemployment to employment, i.e., the 'outs of unemployment'. We subscribe to this view and, by updating Shimer (2012)'s job finding data, we are able to characterize worker flows fluctuations – that are otherwise missed when one looks only at the dynamics of stocks. Finally, given that several papers propose solving Shimer's volatility puzzle by introducing real wage rigidity (e.g. Hagendorn & Manovskii (2008)), it is crucial to compare the model predictions with the data, – with a particular focus on wage dynamics (see Pissarides (2009)). If the volatility of the real wage is not close to zero in the data, then the calibrations which restricts the wage to be constant over the business cycle should be rejected. This tress on wage dynamics is absent in Hagendorn & Manovskii (2008), Petrosky-Nadau & Wasmer (2013), Petrosky-Nadeau (2013) or Petrosky-Nadeau & Zhang (2013).

Data Appendix A provides further details on the data and our calculations. All data are quarterly (from 1976:Q1 through 2013:Q1), in logs, $HP(\lambda = 1600)$ filtered and multiplied by 100 in order to express them in percent deviation from steady state. Ψ is the job finding rate computed from Monthly CPS data from January 1976 to March 2013 using Shimer (2012)'s methodology. It measures the probability for an unemployed worker to find a job. As for financial data on debt and interest rate, we follow Jermann & Quadrini (2012). We finally check that our financial and labor market time series are consistent with the data available on line for Shimer (2012) and Jermann & Quadrini (2012).

Financial cycles and labor market adjustments in the US Table 5.3.1, column 1, reports business cycle properties found in the data.

Table 3: Business cycle volatility : Model versus data

	1		2	
	Data		Model	
	std(.)		std(.)	
Y	1.44	**	1.43	**
C	0.81	*	0.92	*
N	0.72	*	0.59	*
Y/N	0.54	*	0.42	*
w	0.63	*	0.68	*
U	7.90	*	4.26	*
Ψ	5.46	*	4.35	*
V	9.96		8.51	
B	1.68	*	1.52	*
R	0.92	*	0.25	*
$corr(U, \Psi)$	-0.91		-0.86	
$corr(U, V)$	-0.97		-0.71	

** std (in percentage); * relative to GDP std

The volatility of real wages is not close to zero. Moreover, this volatility is larger than the one of labor productivity. This clearly suggests that real wage rigidity (implying a zero standard deviation for fluctuations in w) is not a realistic explanation for the strong cyclicalities of labor market aggregates. $corr(U_t, V_t)$ summarizes the dynamics around the Beveridge curve. The negative covariance is consistent with the view that productivity shocks have a more important weight than reallocation shocks at business cycle frequency: any increase in job vacancies lead to more hirings, hence lower unemployment. Moreover, $corr(U_t, \Psi_t)$ captures the relationship between the probability of finding a job and business cycle changes in unemployment. As expected, it is negative. Jung & Kuester (2011) point out that mean unemployment exceeds steady-state unemployment when the job-finding rate and the unemployment rate are non-positively correlated and the average job-finding rate is lower than the steady-state job-finding rate.³²

5.3.2 Model versus data

The second moments from simulated data are reported in Table 5.3.1, column 2. Comparing columns 1 and 2 of Table 5.3.1, it can be seen that the model performs quite well in matching volatilities of

³²This can be inferred from the employment-flow equation taken at the steady state $sN_t = \Psi U_t$ where $N_t = 1 - U_t$. Hence, $sE(1_t - U_t) = cov(U_t, \Psi_t) + E(U_t)E(\Psi_t)$. Subtracting the steady-state from both sides of the above equation, leads to $E(U_t) - u_t = -\frac{1}{s+\Psi} [cov(U_t, \Psi_t) + (E(\Psi_t) - \Psi_t)E(U_t)]$. We deduce that if (i) $E(\Psi_t) - \Psi_t < 0$ and (ii) $cov(U_t, \Psi_t) < 0$, then necessarily, $E(U_t) - u_t > 0$. The correlation at the bottom of Table 5.3.1 suggests that (ii) holds in the data.

employment and vacancies. Moreover, for what concerns financial variables, the model also matches the volatility of corporate debt.

In addition, simulations confirm that financial frictions actually generates wage stickiness. Indeed, at business cycle frequencies, the decomposition of the wage equation confirms that term (1) (term (2) respectively) in equation (22) is actually counter-cyclical (procyclical). Term (1) turn our to dominate term (2). As a result, the wage increase in expansion is dampened. Firms have then a stronger incentive to create jobs, which raises the job finding rate. The wage relative standard deviation is actually a little higher than the one found in the data. This can account for the fact that the model's job finding rate is a slightly less volatile than in the data. This suggests that the endogenous wage sluggishness in the model is consistent with the wage dynamics in the data. This allows the model to generate sufficiently large movements in job finding rates. Our model solves the Shimer' volatility puzzle without introducing the counterfactual assumption of a constant real wage.

6 Conclusion

This paper provides a quantitative assessment of welfare costs of fluctuations in a search model with financial frictions à la Kiyotaki & Moore (1997). In this model, two opposite forces are at work. On the one hand, financial frictions push agents to precautionary saving. This tends to dampen business cycle costs. On the other hand, because of search frictions, fluctuations generate higher average unemployment rate with respect to its steady-state value, increasing the welfare cost of fluctuations. Financial frictions amplify this mechanism, together with the associated welfare costs. We show that this effect largely dominates the precautionary savings effect, implying significant welfare costs. Our model also allow us to obtain a large responsiveness of the job finding rate to the business cycle. Indeed, financial frictions entail wage sluggishness. In expansion, the wage increase is dampened by lower credit costs. This reduces in turn the value of the search activity. However, by preserving firms' hiring incentive, this mechanism helps the model matching the large changes in job finding rates observed in the data; at the same time, it preserves the real wage volatility observed in the data.

Given the extent of welfare costs associated to fluctuations, stabilization policies are crucial. This left for future research. Policy analysis by Kreps & Scheffel (2014) and Roulleau-Pasdeloup (2014) that explicitly take into account non-linearities explore promising avenues.

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Appendix

A Data

Aggregate data: The following quarterly time series come FRED database, from the Federal Reserve Bank of Saint Louis’ website (1976Q1-2013Q1). y is Real Gross Domestic Product from the FRED database (mnemonicsGDPC96) divided by the Civilian Non institutional Population from the FRED database (mnemonics CNP16OV). c is Real Personal Consumption Expenditures from the FRED data-base (mnemonics PCECC96) divided by the Civilian Non institutional Population from the FRED database (mnemonics CNP16OV)

Labor market data: w is Compensation of Employees: Wages & Salary Accruals from the FRED database (mnemonics WASCUR) divided by Civilian Employment (CE16OV). N is Civilian Employment (CE16OV) divided by Civilian Non institutional Population. U is FRED, Civilian Unemployment Rate (UNRATE), Percent, quarterly, Seasonally Adjusted. The previous time series are taken from the FRED database. As for the time series of job finding rate, we use monthly CPS data from January 1976 to March 2013. We follow all the steps described in Shimer (2012). As in Shimer (2012), we correct for time aggregation and take quarterly averages of monthly observations. V are vacancies Total Nonfarm, Total US Job Openings JTS00000000JOL, Seasonally Adjusted Monthly data from BLS. We take quarterly averages of this time series that is available only from December 2000 onwards.

Debt: We follow Jermann & Quadrini (2012). Financial data come from the Flow of Funds Accounts of the Federal Reserve Board. The debt stock is constructed using the cumulative sum of net new borrowing measured by the ‘Net increase in credit markets instruments of non financial business’³³. Since the constructed stock of debt is measured in nominal terms, it is deflated by the price index for business value added from NIPA. The initial (nominal) stock of debt is set to 94.12, which is the value reported in the balance sheet data from the Flow of Funds in 1952.I for the nonfarm non financial business. The cumulative sum starts in 1952, which, as in Jermann & Quadrini (2012), is not likely to affect our data starting on January 1976. R is the log of 1+ the Bank Prime Loan Rate (MPRIME) (used as a reference for short-term business loan) from the FRED database.

B Model

B.1 The wage curve

From the household’s intertemporal program, one gets:

$$\begin{aligned} \mathcal{V}_t^H &= \frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_{t-1}} = \frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_t} \frac{\partial N_t}{\partial N_{t-1}} + \mu E_t \left(\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t} \right) \frac{\partial N_t}{\partial N_{t-1}} \\ &= [U_t^n - U_t^u + \lambda_t w_t - \lambda_t b_t - \lambda_t (C_t^n - C_t^u)] \frac{\partial N_t}{\partial N_{t-1}} + \mu E_t \left(\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t} \right) \frac{\partial N_t}{\partial N_{t-1}} \end{aligned}$$

With $C_t^n = C_t^u$ and $U_t^n = U_t^u$, we have

$$\mathcal{V}_t^H = [\lambda_t w_t - \lambda_t b_t] \frac{\partial N_t}{\partial N_{t-1}} + \mu E_t \left(\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t} \right) \frac{\partial N_t}{\partial N_{t-1}}$$

³³Nonfinancial business; credit market instruments; liability; Net increase in credit markets instruments of non financial business, millions of dollars (nominal). FA144104005.Q, F.101 Line 28,

Where, from (6) $\frac{\partial N_t}{\partial N_{t-1}} = (1-s)(1-\Psi_t)$, so that

$$\frac{\mathcal{V}_t^H}{\lambda_t} = (1-s)(1-\Psi_t) \left[w_t - b_t + \mu E_t \left(\frac{1}{\lambda_t} \frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t} \right) \right] \quad (28)$$

From the firms' program $\mathcal{V}_t^F = \frac{\partial \mathcal{W}(\Omega_t^F)}{\partial N_{t-1}} = \xi_t (1-s)$ where $\xi_t = \lambda_t^F \bar{\omega} \frac{(1+\varphi_t)}{\Phi_t}$, thus:

$$\begin{aligned} \frac{\partial \mathcal{W}(\Omega_{t+1}^F)}{\partial N_t} &= (1-s) \frac{\bar{\omega}}{\Phi_{t+1}} \lambda_{t+1}^F (1+\varphi_{t+1}) \\ \frac{\mathcal{V}_t^F}{\lambda_t^F} &= (1-s) \frac{\bar{\omega}}{\Phi_t} (1+\varphi_t) \end{aligned}$$

Then, using (20) we obtain:

$$\frac{\mathcal{V}_t^F}{\lambda_t^F} = (1-s) \left[\frac{\partial Y_t}{\partial N_t} - w_t (1+\varphi_t) + \beta E_t \left(\frac{1}{\lambda_t^F} \frac{\partial \mathcal{W}(\Omega_{t+1}^F)}{\partial N_t} \right) \right]$$

Therefore, the surpluses are, respectively:

$$\frac{\mathcal{V}_t^F}{\lambda_t^F} = (1-s) \left[\frac{\partial Y_t}{\partial N_t} - w_t + \beta E_t \left(\frac{\lambda_{t+1}^F}{\lambda_t^F} \frac{\mathcal{V}_{t+1}^F}{\lambda_{t+1}^F} \right) \right] \quad (29)$$

$$\frac{\mathcal{V}_t^H}{\lambda_t} = (1-s)(1-\Psi_t) \left[(w_t - b_t) + \mu E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{\mathcal{V}_{t+1}^H}{\lambda_{t+1}} \right) \right] \quad (30)$$

By maximizing (21) with respect to the wage, we obtain $\left(\frac{\mathcal{V}_t^H}{\lambda_t} \right) = \left(\frac{\mathcal{V}_t^F}{\lambda_t^F} \right) \frac{(1-\epsilon)(1-\Psi_t)}{\epsilon}$. By substituting for (29) and (30), and rewriting it, we obtain the wage curve

B.2 Proof of proposition 1

In equilibrium, the *JC*, equation (25) and the *WC*, equation (23) need to intersect, i.e.:

$$\begin{aligned} MPL + \theta^{1-\psi} \frac{\bar{\omega}}{\chi} \left(1 + 1 - \frac{\beta}{\mu} \right) [(1-s)\beta - 1] \\ = \epsilon O + (1-\epsilon) MPL + (1-s)(1-\epsilon) \bar{\omega} \mu \left(1 + 1 - \frac{\beta}{\mu} \right) \left[\frac{\theta^{1-\psi}}{\chi} (\beta - \mu) + \mu \theta \right] \end{aligned} \quad (31)$$

where we $\frac{\partial Y}{\partial N} \equiv MPL$ and we denote by O the outside option. For simplicity, let us proxy the impatience gap with the steady-state level of the Lagrange multiplier associated to the collateral constraint, $\bar{\varphi} = 1 - \frac{\beta}{\mu}$. Indeed, $\bar{\varphi}$ is increasing in the impatience gap. Equation (31) can be rewritten as:

$$\epsilon (MPL - O) = (1 + \bar{\varphi}) \bar{\omega} \left[(1-s)(1-\epsilon) \mu \theta + \frac{\theta^{1-\psi}}{\chi} (1 - (1-s) \mu (1 - \bar{\varphi} \epsilon)) \right] \quad (32)$$

Let us also define $g(\theta, \bar{\varphi})$ as

$$g(\theta, \bar{\varphi}) = (1 + \bar{\varphi}) \bar{\omega} \left[(1-s)(1-\epsilon) \mu \theta + \frac{\theta^{1-\psi}}{\chi} (1 - (1-s) \mu (1 - \bar{\varphi} \epsilon)) \right]$$

Note that $g(\theta, \bar{\varphi})$ is increasing in θ , since

$$g'_{\theta}(\theta, \bar{\varphi}) = (1 + \bar{\varphi}) \bar{\omega} \left[(1 - s)(1 - \epsilon) \mu + (1 - \psi) \frac{\theta^{-\psi}}{\chi} (1 - (1 - s) \mu (1 - \bar{\varphi} \epsilon)) \right] > 0$$

Moreover, $g(\theta, \bar{\varphi})$ is concave in θ since

$$g''_{\theta}(\theta, \bar{\varphi}) = (1 + \bar{\varphi}) \bar{\omega} \left[-\psi (1 - \psi) \frac{\theta^{-\psi-1}}{\chi} (1 - (1 - s) \mu (1 - \bar{\varphi} \epsilon)) \right] < 0$$

In addition, $\forall \bar{\varphi}$, we have

$$\lim_{\theta \rightarrow +\infty} g(\theta, \bar{\varphi}) = +\infty$$

and $g(0, \bar{\varphi}) = 0$. Finally, $g(\theta, \bar{\varphi})$ is steeper in an economy with financial frictions, indeed:

$$\frac{\partial g'(\theta, \bar{\varphi})}{\partial \bar{\varphi}} = \bar{\omega} \left[(1 - s)(1 - \epsilon) \mu + (1 - \psi) \frac{\theta^{-\psi}}{\chi} (1 - (1 - s) \mu (1 - \bar{\varphi} \epsilon)) + (1 + \bar{\varphi}) (1 - \psi) \frac{\theta^{-\psi}}{\chi} (1 - s) \mu \epsilon \right] > 0$$

Hence, labor market tightness is lower in case of financial frictions (On figure 5, $\theta_2 < \theta_1$ with $\varphi_2 > \varphi_1$). As no restrictions on parameter values are needed, this result is not ambiguous. This means that the equilibrium level of θ is eventually driven by the steeper JC curve.

Figure 5: Steady state labor market tightness

