Can animal spirits explain the dynamics of European unemployment?

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Introduction

Since the middle of the seventies, European economies have experienced a long period of high unemployment. This fact challenges our understanding of the dynamics of these labor markets. Consequently, the macroeconomist must provide new insights about the persistence of unemployment and the shocks responsible for its fluctuations.

Looking for the business cycle impulsions is one of the most active areas in macroeconomic research. If we follow real business cycle (RBC) theorists(1), aggregate fluctuations are the results of optimal responses of a rational agent to stochastic changes in technology. In such an economy, the allocation is Pareto optimal: there is no unemployment. These models can be viewed as theoretical curiosities if we focus on European labor markets. Non-Walrasian models seem to be more relevant in explaining unemployment fluctuations during the business cycle. For example, search models, first proposed by Mortensen [1990], can explain both the presistence and the short-run dynamics of unemployment. Some extensions of this model, such as the one proposed by Fève and Langot [1996], show the importance of demand and foreign shocks in the mimicking of co-movements between aggregate time series. Moreover, Langot [1995] and Mortensen [1994] show that the labor

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(1) See Kydland and Prescott [1982] for example.

market dynamics of Western countries can be essentially attributed to macroeconomic disturbances on goods markets[2].

These works consider that such shocks are sufficient to explain aggregate fluctuations. Nevertheless, in the recent debate about the causes of the 1990-1991 recession in the U.S., some macroeconomists, such as Blanchard [1993] or Cochrane [1994], argued that traditional shocks are not sufficient to induce such a recession. Blanchard [1993] suggests that business cycles could be explained by an exogenous wave of pessimism: after being considered as a curiosity, can the *animal spirits* hypothesis allow us to improve our understanding of the business cycle?

> "A long tradition of attempts to explain recurrent cyclical fluctuations in business activity assigned a central role to shifts in the 'degree of optimism' or 'confidence' on the part of economic actors – changes in outlook that were not required by any objective change in economic circumstances"[3].

Indeed, with the publication of Keynes's *General Theory*, the view that the business cycle is driven by the investors' beliefs became popular. Nevertheless, the way of endogenizing beliefs proposed by the traditional rational expectations model suggests that any changes in expectations result from changes in economic fundamentals. Cass and Shell [1983] showed that the rational expectations equilibrium allocations could depend on the realization of a *sunspot*, that is, an event that does not affect the fundamentals of the economy: the fact that people modify their expectations and hence their optimal decisions in response to an extrinsic random event can make it rational to change one's forecast when this event occurs. So, this literature[4], which formalizes Keynes' idea, gives a leading role to expectations' revisions in the explanation of the business cycle. When cast in a dynamic model, Woodford [1986] showed that *sunspots* can affect the equilibrium allocation when the latter is indeterminate.Contrary to the usual representative agent model where there exists a unique trajectory, in the sense that there exists a unique belief consistent with the convergence to the steady state, models with *sunspots* display indeterminate equilibrium paths. There are many sets of self-fulfilling beliefs, each of them consistent with a dynamic equilibrium which converges to the same steady state,

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(2) These results confirm the ones obtained by Blanchard and Diamond [1989] and Jacques and Langot [1993], using the VAR methodology.

(3) Woodford [1991], p.77.

(4) See also the seminal paper of Azariadis [1981]. See Benhabib and Rustichini [1994] for a recent survey of the literature, or Farmer [1994] for an introduction. See Woodford [1986] and Chiappori and Guesnerie [1991] for a complete discussion on stationary *sunspot* equilibria.

but not for the same endogenous variable levels. In these circumstances technology, preference and endowments are not sufficient to determine the economic equilibrium. In order to give some predictive power to such models, one must explain how equilibria are selected. One may assume that extrinsic uncertainty, corresponding to political, institutional or expectational considerations, coordinates the agents on one sequence of economic variables converging to the steady state. Following the proposition in Woodford [1991], to test the empirical relevance of models exhibiting indeterminacy and *sunspots*, Farmer and Guo [1994] and Gali [1994] show that calibrated models, driven solely by extrinsic uncertainty, can match some moments of the aggregate U.S. data as well as standard RBC models. Nevertheless, in these two papers, the models do not consider unemployment, which constitutes one of the main components of the European business cycle.

In this paper, we propose to develop a simple matching model of the labor market. Following Mortensen [1989] and Pissarides [1990], we assume that trade in the labor market is costly and uncoordinated, and modelled by a matching function. In such a framework, the real wage divides the rent generated by employment between employees and employers. This wage-setting rule induces involuntary aspects in aggregate unemployment. As shown in Mortensen [1989], the wage-setting rule obtained in a bilateral bargain is not sufficient to generate indeterminacy. Indeterminacy can be obtained if and only if the matching function has increasing returns to scale. If the steady state is stable, all the trajectories satisfying the first-order conditions in the neighborhood of the steady state converge back to it. In such a case, there will be a continuum of equilibrium paths. We focus on such a dynamics around the steady state because it implies that unemployment fluctuations can be generated by the self-fulfilling beliefs of the agents.

Then, the quantitative evaluation of the *animal spirits* hypothesis on the labor market could be achieved along two statistical dimensions. First, one needs to know if the sufficient conditions for indeterminacy are accepted by the data. Such a test could be performed by estimating the scale parameters(5) of the matching function. Second, following Eichenbaum [1991](6), we use a formal statistical methodology in order to test the capacity of the model to match a set of stylized facts on the

(5) Pissarides [1986] and Burda and Wyplosz [1994] give some empirical evidence about the returns to scale of the matching function. Nevertheless, these tests are not conditional on the complete structure of the labor market, and more particularly do not take into account the information about the wage setting rule.

(6) In view of Eichenbaum [1991]'s criticism, the calibration-simulation method used in the RBC literature to test a model does not seem to be a relevant quan-

labor market. We use the Generalized Method of Moments proposed by Hansen [1982] to estimate the parameters of the model and to test its ability to reproduce the historical cyclical properties of the economy [(7)]. This statistical method allows us to estimate the "true" standard error of the *sunspot* variable conditional on the structure of the theoretical model, and to evaluate the empirical relevance of the *animal spirits* hypothesis, using time series analysis. Moreover, in order to test the robustness of our results, we propose to evaluate our model using an international data set including the French, German and U.K. economies.

The remainder of the paper is organized as follows. A model of the process by which matches form, and wages are bargained, is sketched in section 1. We present the dynamics around the steady state and discuss the conditions under which a stationary *sunspots* equilibrium exists. The econometric methodology is described in section 2. Empirical results and the improvements related to our specific assumptions are discussed in section 3. The last section concludes.

1 A Search Model of the Labor Market

In this section, we describe the behavior of the job market participants and the allocative process governing trade on this market. We also present the dynamic proprieties of the equilibrium.

1.1 Trade on the Labor Market

Following Mortensen [1986] or Pissarides [1990], we assume that trade is a costly and uncoordinated economic activity on the labor market. We suppose a complete specialization in either trade or production. A firm can have both filled and unfilled jobs, but only the unfilled ones are engaged in the trade process. Symmetrically, only unemployed workers search for jobs. Hence, without a Walrasian labor market structure, the allocation of resources is driven by a search process: the size of the population being normalized to unity, we assume that there exists a *matching function* linking the rate of hiring M_t to the rate of vacancies V_t and the unemployment rate U_t:

$$M_t = h(V_t, U_t) \ .$$

titative validation exercise. See also Fève and Langot [1994] for a statistical evaluation of RBC models on French data.

(7) In order to facilitate the comparison of our results with the existing ones, all the second-order moments are computed on the cyclical component of the data extracted by the Hodrick and Prescott filter.

This function has the following properties[8]:

$$h(0, U_t) = h(V_t, 0) = 0 \tag{1}$$

$$\frac{\partial h(V_t, U_t)}{\partial V_t} > 0 \text{ and } \frac{\partial h(V_t, U_t)}{\partial U_t} > 0 \tag{2}$$

$$\frac{\partial [h(V_t, U_t)/V_t]}{\partial V_t} < 0 \text{ and } \frac{\partial [h(V_t, U_t)/U_t]}{\partial U_t} < 0 \tag{3}$$

$$h(V_t, U_t) = U_t^k h(V_t/U_t, 1). \tag{4}$$

Restriction (3) does not rule out increasing returns to scale in the transition technology. The function $h(V_t, U_t) = V_t U_t$ is on the boundary: the scale parameter must satisfy $0 < k < 2$. At each period, vacant jobs and unemployed workers are matched and move from trade to production. In matching models, the search process implies that moving from vacancy to occupancy or from unemployment to employment is an uncertain event for any agent. The number of job vacancies and unemployed workers matched at time t are randomly selected from the sets V_t and U_t. The rates at which job vacancies are filled and the transition rate at which unemployed workers move to employment are given by $h(V_t, U_t)/V_t$ and $h(V_t, U_t)/U_t$. There are two types of trade externalities (see equation (3)):

1. If V_t increases, the probability of rationing firms increases. Symmetrically, if U_t increases, the probability of rationing unemployed workers increases. These trade externalities are *congestion* externalities.
2. If U_t increases, the probability of rationing firms decreases. Symmetrically, if V_t increases, the probability of rationing unemployed workers decreases: there is some *complementarity* between traders.

In equilibrium, unemployment persists because at each time some existing jobs disappear at a constant rate s, resulting in a flow of new unemployed workers. Thus, at each time t, there are two flows on the labor market: the hiring M_t and the quits sN_t. The resulting employment in period $t+1$ is given by:

$$\begin{aligned} N_{t+1} &= (1-s)N_t + h(V_t, U_t) \\ (1-U_{t+1}) &= (1-s)(1-U_t) + h(V_t, U_t). \end{aligned} \tag{5}$$

[8] The Cobb-Douglas matching function, used in the empirical part of the paper and given by $M_t = \overline{H} V_t^\alpha U_t^\beta$, satisfies condition (1). Conditions (2) and (3)-(4) imply respectively $\alpha > 0$, $\beta > 0$, and $\alpha < 1$, $\beta < 1$ ($k = \alpha + \beta \in]0; 2[$).

The only control variable is the number of vacancies whereas the number of unemployed workers is an adjustment variable.

1.2 Wage Bargaining

Firms and workers are engaged in a costly trading process on the labor market. A successful job match yields an economic rent equals to the sum of the firms' and workers' search costs. This rent is shared according to the Nash solution to a bargaining problem. Following Pissarides [1990], individual firms and workers are assumed too small to influence the market. The bargaining output is thus computed by taking the behaviors of the other participants as given. We assume that there is a wage bargain at each period.

Each firm has only one job, which can be filled or vacant. If the firm has a worker, its value is given by $\Upsilon^F_{j,t}$. If the firm searches for a worker, its value is given by $\Upsilon^V_{j,t}$. These values are given by the following equations:

$$\begin{aligned}
\Upsilon^F_{j,t} &= y_t - w_{i,j,t} + \frac{1}{(1+r)} E_t\left[(1-s)\Upsilon^F_{j,t+1} + s\Upsilon^V_{j,t+1}\right] \\
\Upsilon^V_{j,t} &= -\omega + \frac{1}{(1+r)} E_t\left[\frac{h(V_t,U_t)}{V_t}\Upsilon^F_{j,t+1} + \left(1 - \frac{h(V_t,U_t)}{V_t}\right)\Upsilon^V_{j,t+1}\right],
\end{aligned}$$

where y_t is the marginal product of an employee. We assume a "free entry" condition in the labor market: at each time t, new firms post vacancies until the value of a vacancy is equal to zero, *i.e.* $\Upsilon^V_{j,t} = 0$, $\forall t$. Thus, the expected value of employment is equal to the cost of vacancy:

$$\frac{1}{1+r} E_t\left[\Upsilon^F_{j,t+1}\right] = \frac{\omega}{h(V_t,U_t)/V_t} \cdot \tag{6}$$

Hence, during the bargaining process, firm j wants to maximize its employment surplus, which is given by:

$$\Omega^F_{j,t} = \Upsilon^F_{j,t+1} - \Upsilon^V_{j,t+1} = y_{j,t} - w_{i,j,t} + (1-s)\frac{\omega}{h(V_t,U_t)/V_t} \cdot \tag{7}$$

The workers' objective is to maximize the expected discounted earning in the labor market. This value depends on the state in which the finds himself. If, at time t, the worker i is employed (n) or unemployed (u) its value is given by:

$$\begin{aligned}
\Upsilon^n_{i,t} &= w_{i,j,t} + \frac{1}{(1+r)} E_t\left[(1-s)\Upsilon^n_{i,t+1} + s\Upsilon^u_{t+1}\right] \\
\Upsilon^u_{i,t} &= b_t + \frac{1}{(1+r)} E_t\left[\frac{h(V_t,U_t)}{U_t}\Upsilon^n_{t+1} + \left(1 - \frac{h(V_t,U_t)}{U_t}\right)\Upsilon^u_{t+1}\right],
\end{aligned}$$

where $w_{i,j,t}$ is the real wage and b_t is the unemployment benefits, which is given at the individual level. Then, the individual objective of a worker in the bargaining process is to maximize the employment surplus:

$$\Omega_{i,t}^{M} = \Upsilon_{i,t}^{n} - \Upsilon_{i,t}^{u} = w_{i,j,t} - b_t + \frac{1-s-[h(V_t,U_t)/U_t]}{1+r} E_t\left[\Omega_{i,t+1}^{M}\right].$$

The wage solution is derived from the following Nash bargaining criterion:

$$\max_{w_{i,j,t}} \left(\Omega_{j,t}^{F}\right)^{1-\xi} \left(\Omega_{i,t}^{M}\right)^{\xi},$$

where ξ can be interpreted as a measure of workers' relative bargaining power. At the steady state, this wage bargaining process implies some inefficient unemployment. At a symmetric equilibrium, $w_{i,j,t} = w_t \; \forall \, i,j$. Thus, the solution to this problem is:

$$w_t = \xi \left\{ y_t + \frac{\omega}{(1-s)} \frac{h(V_t,U_t)/U_t}{h(V_t,U_t)/V_t} \right\} + (1-\xi) b_t .$$

Finally, under the assumption of unemployment benefits proportional to the symmetric equilibrium real wage, *i.e.* $b_t = b w_t$, we obtain:

$$w_t = \frac{\xi}{1-(1-\xi)b} \left\{ y_t + \omega \frac{V_t}{U_t} \right\}. \tag{8}$$

This wage-setting rule implies a decreasing link between the real wage and unemployment.

1.3 Equilibrium Dynamics

The symmetric general equilibrium is defined by the set of functions $\{w(\cdot), V(\cdot), U(\cdot)\}$ depending on the state variables $(\mathcal{I}_t)$ which solve the following system[9]:

1. $w(\mathcal{I}_t) = w(\mathcal{I}_{i,t}, \mathcal{I}_{j,t}) \; \forall \, i,j,t$ solves the wage bargaining for the wage rate.
2. $V_t = V(\mathcal{I}_t)$ and $U_{t+1} = U(\mathcal{I}_t)$ solves the set of aggregate firms hiring policy rules for the vector of prices.

Equations (5), (6), (7) and (8) describe the equilibrium dynamics of the economy. We focus now on the joint dynamics of unemployment and vacancies. At the aggregate level, we assume that the marginal product

[9] We denote $\mathcal{I}_t = \{U_t, e_t\}$, with $U_t = 1 - N_t$ and where e_t is the sunspot variable, to be defined below.

of a job is an increasing function of employment[10]: $y_t = F(N_t) = F(1-U_t)$. We thus obtain a system of two dynamic equations in V_t and U_t:

$$\frac{\omega}{h(V_t,U_t)/V_t} = \frac{1}{1+r}E_t\left[(1-a_1)F(1-U_{t+1}) - a_1\omega\frac{V_{t+1}}{U_{t+1}} + \frac{(1-s)\omega}{h(V_{t+1},U_{t+1})/V_{t+1}}\right] \tag{9}$$

$$(1-U_{t+1}) = (1-s)(1-U_t) + h(V_t,U_t)\,, \tag{10}$$

with $a_1 = \xi/[1-(1-\xi)b]$. The system of (9) and (10) describes the equilibrium dynamics of the economy. Unemployment is predetermined, and vacancies are a free choice which depends of the expectations (or the beliefs) about the future values of employment costs.

Pissarides [1990] shows that if the matching function has constant returns to scale and marginal labor productivity is constant or decreasing[11], the equilibrium of this labor market is *locally unique or determinate*: for any given initial value of unemployment U_0 in the neighborhood of the unique stationary equilibrium there is a locally unique V_0, associated with this sequences, that converges to it. This solution characterizes a *saddle point*.

Nevertheless, as Mortensen [1989] suggests, the system (9)-(10) can display multiple stationary equilibria. For a set of restrictions, one can show that one of these equilibria is *locally stable*. In such a case, for any given U_0, there exists a continuum of initial values of vacancies V_0, each of which characterizes a particular sequence $\{U_t, V_t\}_{t=1}^{\infty}$ consistent with the labor market equilibrium and convergent to its locally unique steady state. One can show that if the production function has *decreasing* returns to scales, the matching function has *increasing* returns to scales and the wage-setting rule is *not competitive*, the equilibrium is locally stable.

We cannot obtain an analytical solution to the system including equations (9) and (10). Thus, following Farmer [1994], we log-linearize the system of the efficiency conditions around the deterministic steady

(10) In the empirical part of the paper, the marginal productivity of employment is specified as follows: $y_t = \gamma A N_t^{\gamma-1}$.

(11) We also require the following transversality condition:

$$\lim_{t\to\infty}(1+r)^{-t}E_0[\Omega^F_{t+1}] = 0.$$

state $\{\overline{U}, \overline{V}\}$. Thus, we obtain:

$$\widehat{v}_t + \pi^1_{vu}\widehat{u}_t = E_t\left[\pi^2_{vu}\widehat{u}_{t+1} + \pi^2_{vv}\widehat{v}_{t+1}\right] \quad (11)$$

$$\widehat{u}_{t+1} = \pi^1_{uu}\widehat{u}_t + \pi^1_{uv}\widehat{v}_t \,. \quad (12)$$

where $\widehat{z}_t = (Z_t - \overline{Z})/\overline{Z}$ for $Z = U, V$. This system can be transformed into a first-order recurrent matricial equation:

$$\begin{bmatrix} \pi^1_{uu} & \pi^1_{uv} \\ \pi^1_{vu} & 1 \end{bmatrix}\begin{bmatrix} \widehat{u}_t \\ \widehat{v}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \pi^2_{vu} & \pi^2_{vv} \end{bmatrix}\begin{bmatrix} \widehat{u}_{t+1} \\ \widehat{v}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \pi^3_{vu} & \pi^3_{vv} \end{bmatrix}\begin{bmatrix} \widehat{w}^u_{t+1} \\ \widehat{w}^v_{t+1} \end{bmatrix} \quad (13)$$

where $\widehat{w}^x_{t+1} = E_t[x_{t+1}] - x_{t+1}$, for $x = u, v$, is the gap between the expected and the realized value of the endogenous variables. This system can be rewritten as follows:

$$\begin{bmatrix} \widehat{u}_t \\ \widehat{v}_t \end{bmatrix} = \widetilde{J}\begin{bmatrix} \widehat{u}_{t+1} \\ \widehat{v}_{t+1} \end{bmatrix} + \widetilde{R}\begin{bmatrix} \widehat{w}^u_{t+1} \\ \widehat{w}^v_{t+1} \end{bmatrix} \quad (14)$$

where $\dim(\widetilde{J}) = 2 \times 2$ and $dim(\widetilde{R}) = 2 \times 2$. If the two eigenvalues of the matrix $\widetilde{J}$ are greater that unity, the dynamics is locally stable.

How do the sunspot realizations disturb the economy?

If the economy is not affected by random events (so that the expected operator E_t becomes unnecessary), equation (14) describes the perfect foresight dynamics of the labor market with predetermined unemployment. This implies that $E_t[x_{t+1}] = x_{t+1}$, and thus $w^x_{t+1} = 0$, for $x = u, v$. In this case, the system (14) is:

$$S_t = \widetilde{J}S_{t+1}$$

where $S_t = [\widehat{u}_t, \widehat{v}_t]'$. A perfect foresight equilibrium is a sequence of unemployment levels and vacancies $\{S_1, S_2, \cdots\}$. To define the perfect foresight equilibrium, it is not necessary that the initial level of vacancies render optimal the initial unemployment stock: an arbitrary choice of $\widehat{v}_0$ selects a path among the continuum of equilibrium paths which converge to the steady state.

Now, assume that the agents believe that the expected profitability of employment is affected by random events whose probability function is given. Moreover, assume that these stochastic events characterize an *extrinsic uncertainty*. For simplicity, we assume that $E_{t-1}\epsilon_t = 0$,

$E_{t-1}\varepsilon_t\varepsilon_{t-1} = 0$ and $E_{t-1}\varepsilon_t^2 = \sigma_\varepsilon$. In this case, one can introduce this shock, denoted ε_{t+1}, in the equation (14) as follows:

$$\begin{aligned} S_t &= \widetilde{J}S_{t+1} + \widetilde{R}w_{t+1} + \varepsilon_{t+1} \\ \Longleftrightarrow S_{t+1} &= \widetilde{J}^{-1}S_t - \widetilde{J}^{-1}\widetilde{R}w_{t+1} + \widetilde{J}^{-1}\varepsilon_{t+1} \end{aligned} \tag{15}$$

where $w_t = [\widehat{w}_t^u, \widehat{w}_t^v]'$. The introduction of ε_{t+1} implies that $E_t[x_{t+1}]$ is not equal to x_{t+1}. The introduction of this random event models the fact that the agents believe that non-economic events can affect the equilibrium of the economy. One can interpret these events as *sunspot* realizations. Can this extrinsic uncertainty affect the rational expectations equilibrium of the model? To give an answer, one can show how this uncertainty affects the contemporaneous choices *via* the expectations of the agents. Equation (15) allows us to determine these expectations: $E_t[S_{t+1}] = \widetilde{J}^{-1}S_t$. This implies: $w_{t+1} = E_t[S_{t+1}] - S_{t+1} = \widetilde{J}^{-1}S_t - S_{t+1}$. Plugging these results into equation (15), we obtain:

$$\begin{aligned} \left[I - \widetilde{J}^{-1}\widetilde{R}\right]\left[S_{t+1} - \widetilde{J}^{-1}S_t\right] &= \widetilde{J}^{-1}\varepsilon_{t+1} \\ \Longleftrightarrow \quad S_{t+1} &= \widetilde{J}^{-1}S_t + \left[I - \widetilde{J}^{-1}\widetilde{R}\right]^{-1}\widetilde{J}^{-1}\varepsilon_{t+1} \\ \Longleftrightarrow \quad S_{t+1} &= \widetilde{J}^{-1}S_t + \mathcal{E}_{t+1} \end{aligned}$$

where it is easy to show that $\mathcal{E}_t = [0, e_t]'$. If the system is locally stable, it is possible, given an arbitrary choice of the initial conditions $\{\widehat{u}_0, \widehat{v}_0\}$, to generate a stochastic sequence $\{\widehat{u}_t, \widehat{v}_t\}_{t=0}^{\infty}$ conditionally on the realizations of the random events $\{e_t\}_{t=1}^{\infty}$. In other words, starting from the stationary equilibrium, the random events drive the equilibrium on to a particular path among the continuum of equilibrium paths which satisfy the optimality condition $E_t[S_{t+1}] = \widetilde{J}^{-1}S_t$.

This result indicates how the realization of the shock, interpreted as the investors' beliefs, can be a *self-fulfilling prophecy* explaining unemployment fluctuations. Thus, the system

$$\begin{bmatrix} \widehat{u}_{t+1} \\ \widehat{v}_{t+1} \end{bmatrix} = \widetilde{J}^{-1}\begin{bmatrix} \widehat{u}_t \\ \widehat{v}_t \end{bmatrix} + \begin{bmatrix} 0 \\ e_{t+1} \end{bmatrix} \tag{16}$$

shows that current unemployment and current vacancies depend on whole the history of the *sunspot* variable:

$$\begin{aligned} V_t &= H(H(V_{t-2}, e_{t-1}), e_t) = \cdots = H^*(e_t, e_{t-1}, e_{t-2}, \ldots) \\ U_{t+1} &= G(G(U_{t-1}, e_{t-1}), e_t) = \cdots = G^*(e_t, e_{t-1}, e_{t-2}, \ldots) \end{aligned}$$

where the function $H(\cdot)$ is the transformed policy rule of vacancies ($V_t = H(V_{t-1}, e_t)$) and the function $G(\cdot)$ is the optimal law of motion

($U_{t+1} = H(U_t, e_t)$) of unemployment[12].

> "The infinite regress of expectations creates an indeterminacy of the same sort associated by Keynes in his discussion of the long-term rate of interest" (Woodford [1991], p.91).

Under the assumptions of increasing returns to scale on the matching function and a bargained wage, it is possible to display belief-driven cycles without any fundamental uncertainty.

Table 1: Eigenvalues in terms of returns to scale

$k(*)$	Root 1	Root 2
0.95	1.17	0.74
0.98	1.07	0.76
1.00	1.03	0.80
1.02	$0.88 + 0.06i$	$0.88 - 0.06i$
1.05	$0.84 + 0.15i$	$0.84 - 0.15i$

(*) The parameter k of the condition (4) is equal to the returns to scale of the matching function. $k=1$ corresponds to the case of constant returns to scale.

Sunspots equilibria arise if the two roots of matrix $\widetilde{J}^{-1}$ lie inside the unit circle. In Table 1, we report the theoretical values of the two eigenvalues of a perfect foresight model for different values of the scale parameters[13].

In the case of constant or decreasing returns to scale, the model admits a *saddle point* solution, *i.e.* one root inside the unit circle and one root outside. Conversely, increasing returns to scale give a stable solution, in the sense that all the roots lie inside the unit circle. In this experiment, we obtain complex roots for returns to scale equal to 1.014. In Figure 1, we graph the real parts of the eigenvalues as a function of returns to scale. For increasing returns to scale, there exist two real roots inside the unit circle or a pair of complex roots with positive real part when we select higher increasing returns.

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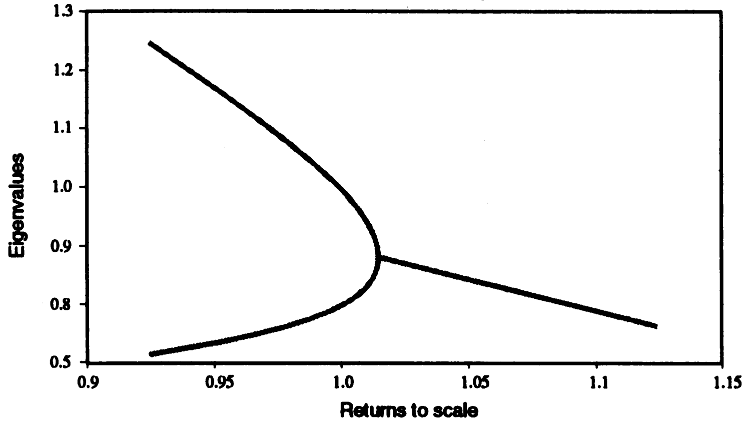
(12) We assume that σ_ε is sufficiently small to ensure that $H(\cdot)$ and $G(\cdot)$ are defined of all V_{t-1} and U_t in some interval containing the unemployment-vacancies deterministic steady state ($\overline{V}, \overline{U}$).

(13) For these examples, we calibrate our model as follows: r (the discount factor) is equal to 4% per year, γ (the labor share) is equal to 1, s (the quit rate) is equal to 10% per year, b (the compensation rate) is equal to 0.5 and ξ (the measure of workers' relative bargaining power) is equal to 0.5.

Figure 1: Real Parts of the Eigenvalues



2 Econometric Methodology

In order to estimate and test this intertemporal optimization model, we use GMM. Following the evaluation strategy suggested by Christiano and Eichenbaum [1992], we do not systematically impose overidentifying restrictions on conditional moments and we do not test only the theoretical model on this basis. The statistical test is also about the model's ability to reproduce some stylized facts of the European labor market. Nevertheless, following the first application of GMM (see for example Hansen and Singleton [1982]), the estimation of the structural parameters allows us to test some theoretical assumptions, using an overidentified system. Then, the assumption of orthogonality between the *sunspot* variable and its lagged values is tested. In this section, we proceed as follows. First, we present the estimation method and the pre-setting values for some structural parameters. Second, the method used to test the model's hypothesis is discussed. Finally, we explain the method used to test the model's implications.

2.1 Estimation Method

We are interested in the statistical evaluation of the theoretical model for three European countries during the eighties. We use monthly, seasonnally adjusted data on vacancies and unemployment for France, the United Kingdom and Germany. The starting period is January 1980 and the ending period is January 1990 for France, August 1989 for the United Kingdom and February 1989 for Germany. All the data come from the "Main Economic Indicators" OECD dataset. The vacancies and unemployment rates are expressed in terms of the total labor force. We choose to restrict our empirical investigation to the eighties because the inflow rate in unemployment displays strong instability during the last two decades. This parameter, which is assumed constant over the sample in our model, does not display an upward trend during the eighties (especially for Germany and the U.K.) despite a significant instability and volatility (see Burda and Wyplosz [1994]). We can also impose an average value without introducing strong biases in our estimations (see the paragraph below for a discussion for the calibrated value of s).

The model developped in the previous section admit the linear approximation of its dynamics as given by system (13). For all assumptions concerning the returns to scale of the matching function, this system determines the fluctuations around the steady state.

After substitution of the two equations of system (13), one can write this dynamics as follows[14]:

$$\widehat{v}_t = \alpha_1 \widehat{u}_{t-1} + \alpha_2 \widehat{v}_{t-1} + e_t \,. \tag{17}$$

Equation (17) thus gives the relation between vacancies, unemployment and the structural shock, which takes into account all the cross-equation restrictions.

Using (17), one can estimate α_1, α_2 and σ_e, assuming that

$$E_{t-1}[e_t] = 0, \quad E_{t-1}[e_t e_{t-1}] = 0 \quad \text{and} \quad E_{t-1}[e_t^2] = \sigma_e \,.$$

As the error term e_t corresponds to a linear combination of the expectation errors, this term is an *iid* disturbance, uncorrelated with its past terms, as in the theoretical model. We must obviously identify the parameter σ_e which characterizes the volatility of the *sunspot* variable. This parameter corresponds to the standard-error of the residuals of equation (17). The assumption that e_t is an *iid* process will be tested, using over-identifying restrictions.

The identifying condition implies that the number of structural parameters must be less or equal than the number of overidentifying restrictions. Thus, the estimation of α_1 and α_2 allows us to identify at most two structural parameters. We choose to estimate the parameters of the matching function, because the dynamics of the theoretical model

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(14) Where

$$\alpha_1 = \frac{\pi^1_{uv} - \pi^2_{vu}\pi^1_{uu}}{\pi^2_{vv}} \text{ and } \alpha_2 = \frac{1 - \pi^2_{vu}\pi^1_{uv}}{\pi^2_{vv}} \,.$$

The linear combination of expectation errors e_t is given by:

$$e_t = \frac{-\pi^3_{vu}}{\pi^2_{vv}} \widehat{w}^u_t + \frac{-\pi^3_{vv}}{\pi^2_{vv}} \widehat{w}^v_t \,.$$

Note that the term $\widehat{w}^u_t$ is equal to zero, because this term is associated with a pre-determined variable.

depends crucially on the values of the scale parameters, *i.e.* the existence of increasing returns to scale of the matching function. Then, the exactly-identified model allows us to estimate α and β, which are the parameters of the following matching function:

$$M_t = \bar{H} V_t^{\alpha} U_t^{\beta} .$$

The other parameters of the model take pre-set values on the basis of empirical studies (see table 2). Concerning the parameter of the production function, we choose to impose a value for each country equal to the relative share of labor income in the national product during the eighties (INSEE and OECD sources)[15]. The values for the quit rate parameter s are founded on Burda and Wyplosz [1994]. The parameter b is computed using the ratio of the public wage compensation to total labor income (INSEE and OECD sources). Since our model does not consider the possibility of insurance in a complete financial market, we keep here only a public wage compensation.

Table 2: Calibrations of Structural Parameters

	France	United Kingdom	Germany
γ	0.689	0.684	0.666
s	1.77%	0.55%	0.85%
ξ	0.5	0.5	0.5
b	0.49	0.16	0.47
$\bar{u}$	9.35%	9.73%	7.92%
$\bar{v}$	0.28%	0.59%	0.63%
r	4%	4%	4%

The measure of workers' relative bargaining power, ξ, is not estimated and we suppose arbitrarily that it is equal to 0.5 for each country, which implies a symmetric Nash bargaining criterion. The scale parameters of the matching function $\overline{H}$ and of the production function A are set in order to have steady state values of vacancies and unemployment which match respectively the sample means of vacancies rate ($\overline{v}$ in table 2) and unemployment rate ($\overline{u}$ in table 2) during the eighties. Finally, the discount factor of the expected sum of profit flows is set to 4% per year for each country.

[15] As pointed out by a referee, γ does not correspond to the relative share of labor income in the national product. Nevertheless, these calibrated values of γ are close to the estimated value of labor returns in a general equilibrium matching model (see Fève and Langot [1996]).

So the parameters of interest that we estimate using our model, are:

$$\Psi_1 = \{\alpha, \beta, \sigma_e\}.$$

The parameters of equation (17) are non-linear functions of α and β and linear functions of σ_e. We assume that α_1 and α_2 are a twice continuously differentiable function of the structural parameters. The conditional density of the t-th observation of the innovation e_t is given by $f(e_t/I_{t-1};\Psi_1)$, where $I_{t-1} = \{v_{t-1}, u_{t-1}, \cdots\}$. If the data were effectively generated by the conditional density $f(e_t/I_{t-1};\Psi_1)$, then the conditional expectation of the score should be zero:

$$E\left[h(e_t, I_{t-1};\Psi_1)/I_{t-1}\right] = 0.$$

We denote by $h(e_t, I_{t-1};\Psi_1)$ the derivative of the log of the conditional density of the t-th observation. The size of the vector $h(e_t, I_{t-1};\Psi_1)$ is equal to the size of Ψ_1, for $t = 1,\ldots,T$. This vector is known as the *score* of the t-th observation. The *score* vectors $\{h(e_t, I_{t-1};\Psi_1)\}_{t=1}^{\infty}$ form a martingale difference sequence[16]. In our simple linear rational expectations model, this last condition could be written:

$$E_0\left[e_t(\Psi_1^0)/I_{t-1}\right] = 0, \tag{18}$$

with

$$e_t(\Psi_1^0) = \widehat{v}_t - \alpha_1^0\widehat{u}_{t-1} - \alpha_2^0\widehat{v}_{t-1}.$$

This relation is a conditional moment restriction implied by the theoretical model. To study the econometric implications, it is convenient to replace the conditional moment restriction with a corresponding unconditional moment restriction. We denote z_{t-1} be a matrix of instrumental variables such that $z_{t-1} \in \widetilde{I}_{t-1} \subset I_{t-1}$. Under the hypothesis that $e_t z_{t-1}$ has a finite first moment, by the law of iterated expectations, restriction (18) implies:

$$E[e_t z_{t-1}] = 0.$$

We assume that the vector $X_t = \{\widehat{v}_t, \widehat{u}_{t-1}, \widehat{v}_{t-1}\}$ forms a strictly stationary stochastic process and that the process $\{z_t\}_{t=1}^{\infty}$ is jointly stationary with $\{X_t\}_{t=1}^{\infty}$. This assumption is constistent with our data set, because for each variable, cyclical features are isolated by the application of the Hodrick and Prescott [1980] filter.

(16) The unconditional expectation of the *score* implies also that the *score* has an unconditional expectation of zero, provided that the unconditional first moment exists. This last condition represents a set of orthogonality conditions, which size equal to the size of the parameter Ψ_1.

We can summarize all the orthogonality conditions on the variable e_t by the following form:

$$E\, H^1[X_t, z_{t-1}; \Psi_1] = 0 \quad \text{for} \quad t = 1 \ldots T. \tag{19}$$

The function $H^1(.;.)$ is a vector of dimension $s = \dim \Psi_1$. We choose to estimate the Euler equation on vacancies using GMM because this methodology allows us to estimate the structural parameters of a dynamic model without solving it. Indeed, there are two alternative methods to select the optimal path at each time:

1. If the returns to scale of the matching function are constant or decreasing, the model admits a *saddle point* solution. In this case, when we solve the model, the selection method of the saddle path is taken into account in the cross-equation restrictions.
2. Conversely, if the matching function admits increasing returns to scale, the model displays indeterminacy and *sunspot* equilibria. In this case, all the paths converge to the steady state: we do not need the same cross-equation restrictions to solve the model.

Only the estimated values of the scale parameters of the matching function provide insight into the method to solve the equilibrium. Hence, the econometric methodology must be independent of the solving method. As the GMM uses the Euler conditions which are only the first-order conditions of the problem, this econometric methodology allows us to find the estimators of the model without solving it.

If the model produces a good description of agents' behavior, its cyclical properties must be consistent with those of the French, U.K and German economies. Following the Kydland and Prescott [1982] methodology, the cyclical features are isolated by the application of the Hodrick-Prescott filter. We are interested in a set of second-order moments of the cyclical components of the data generated by the model, under the null hypothesis that the model is "true". Hence, the test of the second-order moments bears on a vector of five parameters Ψ_2 defined by:

$$\Psi_2 = \{\sigma_v, \sigma_u, \mathrm{corr}\,(u, v), \rho_v, \rho_u\}$$

where σ_x is the standard deviation of the cyclical component of the variable x for $x \in \{v, u\}$, $corr(u, v)$ is the instantaneous unemployment-vacancies correlation, and ρ_x is the autocorelation of odrer one of the variable x for $x \in \{v, u\}$. We determine five unconditional moments from which it is possible to estimate the five second order moments of the vector Ψ_2. We summarize the orthogonality conditions in the following form, where $x_t = \{\widehat{v}_t, \widehat{u}_t, \widehat{v}_{t-1}, \widehat{u}_{t-1}\}$:

$$E\, H^2[x_t; \Psi_2] = 0 \quad \text{for} \quad t = 1 \ldots T. \tag{20}$$

The function $H^2(.;.)$ is a five-dimensional vector of restrictions that the unconditional second order moments must satisfy:

$$\begin{aligned} E\left[v_t^2 - \sigma_v^2\right] &= 0 \\ E\left[u_t^2 - \sigma_u^2\right] &= 0 \\ E\left[\left(\frac{\sigma_u^2}{\sigma_u/\sigma_v}\right)\operatorname{corr}(u,v) - u_t v_t\right] &= 0 \\ E\left[(v_t - \rho_v v_{t-1})v_{t-1}\right] &= 0 \\ E\left[(u_t - \rho_u u_{t-1})u_{t-1}\right] &= 0 \,. \end{aligned}$$

Let $\Psi = [\Psi_1, \Psi_2]$ be a p-dimensional vector ($p = s + 5$), representing all the estimated moments. The function H, representing all the restrictions that the unconditional moments must satisfy, includes H^1 and H^2:

$$E\, H([X_t, z_{t-1}, x_t]'; \Psi) = 0 \,. \tag{21}$$

The joint writing of these conditional and unconditional moments involves the simultaneous estimation of the structural parameters (Ψ_1) and of the empirical moments of the cyclical components (Ψ_2). We define the function $g_T(\Psi)$ of dimension p. This function represents the empirical counterpart of (21):

$$g_T(\Psi) = \frac{1}{T}\sum_{t=0}^{T} H\left([X_t, z_{t-1}, x_t]'; \Psi\right) \,. \tag{22}$$

If $\dim \Psi = \dim g_T(\Psi)$, the model is just-identified and an estimator of Ψ, denoted $\widehat{\Psi}_T$, is a value of Ψ which satisfies: $g_T(\widehat{\Psi}_T) = 0$. We can introduce additional orthogonality conditions. In this case, the previous condition is not satisfied. Hence, we can estimate Ψ by choosing a value of Ψ which minimizes the following objective function:

$$J_T = g_T(\Psi)' W_T g_T(\Psi) \,,$$

where W_T is a symmetric positive definite distance matrix that may depend on sample information. Hansen [1982] shows that W_T converges almost surely to $W_0 = S_0^{-1}$, the limit of the covariance matrix of $g_T(\Psi)$. W_T is a consistent estimator of the inverse of the matrix S_0 defined by:

$$S_0 = \sum_{i=-q}^{q} E\Big[H\left([X_{t+i}, z_{t+i-1}, x_{t+i}]'; \Psi\right), \\ H\left([X_{t+q-i}, z_{t+q-1-i}, x_{t+q-i}]'; \Psi\right)' \Big] \,,$$

where q is the order of the MA part of the errors of equation (22). The main difficulty concerning this estimation method is to compute the matrix W_T[17]. Contrary to Christiano and Eichenbaum [1992] or Burnside, Eichenbaum and Rebelo [1993], we do not arbitrarily impose the order of the moving average q. Instead, we compute the weighting matrix with a Bartlett kernel (Newey and West [1987]), using the formula proposed by Andrews [1991] for selecting the lag windows, as in Fève and Langot [1994] and [1996].

2.2 Testing the Model's hypothesis

Most empirical studies using GMM were mostly concerned with intertemporal choices in partial equilibrium: the issue was to test a model's specification from the conditional moments, through overidentifying restrictions. Theory provides the econometrician with a set of overidentifying restrictions, which can be tested by GMM: the global specification test is a joint test of the functional forms of production, matching and wage functions, the rational expectations hypothesis and the choice of the instrumental variables. This global specification test allows us to test if the process followed by the innovation variable e_t is orthogonal to the past values of vacancies and unemployment. Thus, for each country, we check whether if our assumptions concerning e_t are satisfied. In this case, one can interpreted e_t as the realization at time t of a *sunspot*. So we use two subsets of the information set, represented by the variables $z_{1,t-1}$ and $z_{2,t-1}$:

$$E[e_t z_{1,t-1}] = 0 \quad \text{with} \quad z_{1,t-1} = \{u_{t-1}, v_{t-1}, u_{t-2}, v_{t-2}\}$$
$$E[e_t z_{2,t-1}] = 0 \quad \text{with} \quad z_{2,t-1} = \{u_{t-1}, v_{t-1}, u_{t-2}, v_{t-2}, u_{t-3}, v_{t-3}\}.$$

These restrictions could also be tested using the Hansen's J statistic, which has an asymptotic Chi-square distribution with a degree of freedom equal to the number of overidentifying restrictions. The introduction of these additional orthogonality conditions allows to evaluate the sensitivity of our empirical results to various econometric restrictions.

2.3 Testing the Model's Implications

Given the estimated values of Ψ_1, an $s \times 1$ vector, our model implies particular values for Γ, an $m \times 1$ vector[18] of second-order moments obtained by simulating the model. We represent this relationship *via*

[17] See Fève and Langot [1995] for a discussion.

[18] As Ψ_2 is a 5×1 vector, $m \in [1, 5]$.

the function $\Phi(.)$ that maps $\mathbb{R}^s$ into $\mathbb{R}^m$:

$$\Phi(\widehat{\Psi}_1) = \Gamma .$$

In the vector Ψ_2, we select the moments we want to test. Let A be an $m \times p$ selection matrix:

$$A\widehat{\Psi} = \widehat{\Gamma} .$$

Under the null hypothesis, $f(\Psi^0) = \Phi(\widehat{\Psi}_1^0) - A\widehat{\Psi}_2^0 = 0$. If the sample is large, the test of this hypothesis corresponds to the test of non-linear restrictions discussed in Ogaki [1993]. Nevertheless, as suggested by Christiano and Eichenbaum [1992], $f(\widehat{\Psi})$ need not be zero in a small sample, because of sampling uncertainty in $\widehat{\Psi}$. Therefore, following these authors, we take a first order expansion of $f(\widehat{\Psi})$ around the true value Ψ^0:

$$f(\widehat{\Psi}) = f(\Psi^0) + \left(\frac{\partial f(\Psi)}{\partial \Psi}\right)\left[f(\widehat{\Psi}) - f(\Psi^0)\right] .$$

It follows that a consistent estimator of the variance-covariance matrix of $f(\widehat{\Psi})$ is given by:

$$\mathrm{Var}\left[f(\widehat{\Psi})\right] = \left(\frac{\partial f(\Psi)}{\partial \Psi}\right)\mathrm{Var}\left[\widehat{\Psi}\right]\left(\frac{\partial f(\Psi)}{\partial \Psi}\right)' .$$

Let J_c be the restricted objective function, defined by:

$$J = f\left(\widehat{\Psi}\right)' \mathrm{Var}\left[f\left(\widehat{\Psi}\right)\right]^{-1} f\left(\widehat{\Psi}\right) ,$$

with $J_c(\Psi^0) = 0$, under the null hypothesis. This statistic is distributed as a Chi-square with degrees of freedom equal to the number of restrictions. As the distribution of $f(\widehat{\Psi})$ is conditional on the non-linear transformation of the distribution of $\widehat{\Psi}_1$ and on the distribution of $\widehat{\Psi}_2$, Fève and Langot [1994] remark that we can therefore decompose the uncertainty in $f(\widehat{\Psi})$ as coming from two sources: the first concerns the sample uncertainty associated with the historical moments and the second is related to the uncertainty about the deep parameters. Thus, in the standard calibration-estimation exercises, we consider implicitly that there is no uncertainty on structural parameters. The statistical test of the model could thus be achieved only by taking into account the sample uncertainty on historical moments. When structural parameters are estimated using the same information set as the one used for the historical moments, we can consider an additional source of uncertainty via the estimated variance-covariance matrix of the structural parameters[19].

[19] See Fève and Langot [1994] for more details concerning this decomposition.

This testing method allows us to evaluate the sensitivity of the theoretical moments to the various parameterizations which are statistically acceptable.

3 Empirical Results

In this section, we present the empirical results obtained with a European data set. First, the estimation of the structural parameters is performed and second, the cyclical implications of the model are tested.

3.1 The Estimated Structural Parameters

Estimates for the model were obtained using also the following orthogonality conditions:

$$\begin{aligned} E_0\, H^1[X_t, \Psi_1] &= 0 \text{ exactly identified} \\ E_0\, H^1[X_t, \Psi_1/z_{1,t-1}] &= 0 \text{ over-identified (2 restrictions)} \\ E_0\, H^1[X_t, \Psi_1/z_{2,t-1}] &= 0 \text{ over-identified (4 restrictions)}. \end{aligned}$$

We report in Table 3 the parameter estimates for the three economies with (Overidentified) or without (Exactly identified) imposing the nullity of the correlation between the *sunspot* variable and the variables belonging to the information set at period $t-1$. The GMM estimators of conditional and unconditional moments ($\widehat{\Psi}_T$) are obtained using a weighting matrix computed with a Bartlett kernel: we fit an ARMA(1,1) on each orthogonality condition and then apply Andrews' formula in order to obtain an automatic bandwidth parameter.

The two scale parameters (α and β) are economically meaningful. Their standard error are very low and implies precise measurement of the matching function. The estimation results indicate that the increasing returns to exchange on the labor market cannot be rejected for any country. More precisely, the sum of the two parameters α et β is close to 1.5 for each country. Moreover, these two parameters display strong stability among countries during the eighties. The parameter β (between 0.4 and 0.6) is smaller than α (around 0.98) for all the performed estimations. These two estimated values of α and β satisfy the constraint of conditions (3) and (4).

Our results depart from findings of previous studies, which find evidence of constant or decreasing returns to scale (Pissarides [1986] and Layard, Nickell and Jackman [1991] for the U.K., Burda and Wyplosz [1994] for France, Germany and the U.K.). This discrepancy could be partly explained by four differences between our estimation and the preceding ones:

Table 3: Empirical Results

	Exactly Identified	Over Identified$^{(*)}$	Over Identified$^{(**)}$
	France ($\bar{u} = 9.35\%; \bar{v} = 0.28\%$)		
α	0.9737 (0.0025)	0.9738 (0.0024)	0.9739 (0.0024)
β	0.5837 (0.0362)	0.5902 (0.0358)	0.5911 (0.0358)
σ_e	0.0318 (0.0020)	0.0313 (0.0020)	0.0309 (0.0019)
$J-$stat	...	2.0193 [0.364]	3.7183 [0.445]
Root 1	$0.919 + 0.302i$	$0.926 + 0.301i$	$0.926 + 0.304i$
Root 2	$0.919 - 0.302i$	$0.926 - 0.301i$	$0.926 - 0.304i$
$\widetilde{J}^{-1}$ matrix	$\begin{pmatrix} 0.955 & 0.553 \\ -0.167 & 0.882 \end{pmatrix}$	$\begin{pmatrix} 0.972 & 0.554 \\ -0.167 & 0.881 \end{pmatrix}$	$\begin{pmatrix} 0.970 & 0.565 \\ -0.167 & 0.880 \end{pmatrix}$
	United Kingdom ($\bar{u} = 9.73\%; \bar{v} = 0.59\%$)		
α	0.9886 (0.0005)	0.9883 (0.0004)	0.9881 (0.0005)
β	0.4405 (0.0277)	0.4173 (0.0187)	0.4230 (0.0177)
σ_ε	0.0254 (0.0029)	0.0200 (0.0023)	0.0181 (0.0022)
$J-$stat	...	4.2344 [0.120]	7.4731 [0.113]
Root 1	$0.864 + 0.097i$	0.794	0.797
Root 2	$0.864 - 0.097i$	0.856	0.875
$\widetilde{J}^{-1}$ matrix	$\begin{pmatrix} 0.759 & 0.374 \\ -0.055 & 0.969 \end{pmatrix}$	$\begin{pmatrix} 0.679 & 0.368 \\ -0.055 & 0.971 \end{pmatrix}$	$\begin{pmatrix} 0.702 & 0.301 \\ -0.055 & 0.970 \end{pmatrix}$
	Germany ($\bar{u} = 7.92\%; \bar{v} = 0.63\%$)		
α	0.9798 (0.0005)	0.9796 (0.0006)	0.9797 (0.0006)
β	0.5671 (0.0225)	0.5523 (0.0216)	0.5659 (0.0209)
σ_e	0.0172 (0.0014)	0.0163 (0.0014)	0.0148 (0.0014)
$J-$stat	...	5.0094 [0.082]	10.0280 [0.040]
Root 1	$0.932 + 0.069i$	$0.916 + 0.038i$	$0.933 + 0.038i$
Root 2	$0.932 - 0.069i$	$0.916 - 0.038i$	$0.933 - 0.038i$
$\widetilde{J}^{-1}$ matrix	$\begin{pmatrix} 0.932 & 0.046 \\ -0.103 & 0.932 \end{pmatrix}$	$\begin{pmatrix} 0.899 & 0.017 \\ -0.103 & 0.933 \end{pmatrix}$	$\begin{pmatrix} 0.933 & 0.014 \\ -0.103 & 0.932 \end{pmatrix}$

(*) 2 restrictions; (**) 4 restrictions.

Standard error robust to heteroskedasticity in parentheses. P-Value in Brackets

1. Previous studies provide estimates of the matching function without specifying the whole dynamic structure of the labor market. More particularly, our estimations are based on a wage-setting rule

and the resulting estimates are obtained through the joint dynamics of vacancies and unemployment.

2. From a statistical point of view, our estimated values are obtained for a sample period covering only the eighties and it cannot be in accordance with the empirical finding of Burda and Wyplosz, who provide estimates of the matching function on a larger sample period for France (1971:1-1993:1) and Germany (1968:3-1991:2) and a shorter sample for U.K. (1985:1-1993:1).
3. Another explanation comes from the use for estimation of cyclical data, from which we have suppressed all non-stationary components by application of the Hodrick-Prescott filter. On the contrary, Burda and Wyplosz do not use this filter and study the sensitivity of the results to the choice of other filters (deterministic trend, first-difference).
4. Finally, we use for estimation only the unemployment and the vacancies data. We do not use flow data, contrary to Burda and Wyplosz. Nevertheless, some recent estimations using flow and stock data do not reject the hypothesis of increasing returns to scale of the matching function. With US data, Anderson and Burgess [1995] show that the returns to scale are equal to 1.24. This result shows the great sensitivity of the matching function estimations to the data set and the sample size.

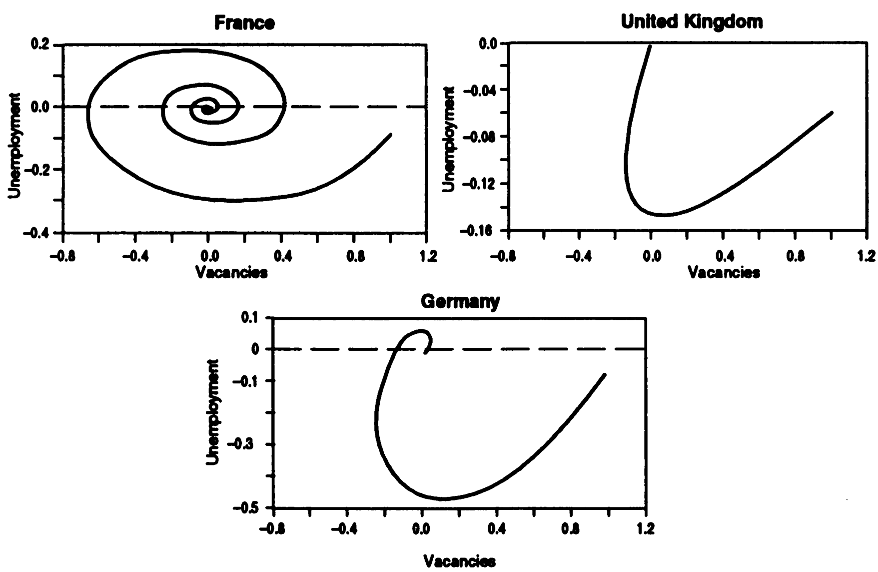
The parameter estimates are not affected by the two sets of over-identifying restrictions. These results show that the estimations are robust. Only the standard-error of the *sunspot* variable decreases when we impose additional orthogonality conditions, especially for the U.K.

The model is always accepted when we impose these two restrictions (see the $J-statistic$) for France and the U.K. Conversely, the model cannot be accepted when we use the subset $z_{2,t-1}$ for Germany. There are of course many possible explanations for this finding, including misspecification of the firms' objective function, measurement errors on unemployment and vacancies, misspecification of the timing decisions and the problems associated with temporal aggregation, and omitted variables. From a statistical viewpoint, we perform several specification tests on the *sunspot* variable. First, we find significant conditional heteroskedasticity but the Jarque and Bera statistic indicates that normality cannot be rejected. The misspecification comes essentially from temporal dependence in the *sunspot* variable, which invalidates the basic assumption of our rational expectations model.

The roots of the $\widetilde{J}^{-1}$ matrix of equation (16) lies inside the unit circle for each estimation and country. For the French and German economies, we always obtain complex roots, but the imaginary part in the French case is larger (around 0.30). For the U.K., we find evidence of two real roots less than 0.9. Contrary to standard dynamic rational expectations models, the modulus of these roots implies that the transitional dynamics of the labor market has a large persistence. Indeed, with these roots, one can expect a large unemployment and vacancies persistence without imposing *a priori* a great persistence in the exogenous shocks process.

From these estimation results, we can compute the estimated impulse response functions (IRF) of vacancies and unemployment. This gives the adjustment dynamics of unemployment and vacancies around the French, U.K. and German Beveridge Curves (see figure 2).

Figure 2: Adjustment Dynamics around the Beveridge Curve



A shift in the degree of optimism (+1% in the *sunspot* process) increases the expected labour returns, because the marginal value of employment responds positively to this shock. Hence, firms have an incentive to choose a higher search intensity and so vacancies increase. Investments which keep these labour market opportunities are immediately engaged. In the short run and for all countries, the investment in vacancies is larger that the induced decrease in unemployment because the hiring process is time consuming. Hence job availability, measured by $\theta_t = V_t/U_t$, increases. During the transition, the instantaneous jump of vacancies induces a positive exchange externality (increasing returns to scale of matching function), which increases the probability, for each worker of finding a job. Hence, the unemployed workers find a job more easily: the mean duration of vacancy and unemployment are negatively correlated. Hence, the adjustment path in $(u - v)$ space describes a conterclockwise loop along the Beveridge curve. The IRF for all the countries are consistent with previous empirical studies (Jacques and Langot [1993]). Figure 2 shows the implications of the increasing returns to scale of the matching function: this leads to a model with the dynamics of a sink which may have either real or complex roots. The panels on the France and German economies depict a situation where the two roots of the matrix $\widetilde{J}^{-1}$ are complex conjugates, whereas the panel on the United Kingdom economy depicts a case where the roots are both real.

3.2 Testing the Model's Implications

On the basis of the global specification test, we can assert that the model is an acceptable representation of the firm decision rules. Nevertheless, the main difficulty in testing the overidentifying restrictions is that it is clearly impossible to discriminate between various assumptions of the model. Therefore, testing the model's implications seems to be an advance in order to disentangle the crucial hypotheses of the theoretical models. With the method of quantitative analysis used in this paper, we can measure the sensitivity of parameter uncertainty to the model's implications[20], *i.e.* the statistical properties of various moments of interest.

Before discussing the statistical test of the model, let us briefly review the main stylized facts of the European labor market on the basis of our few selected moments (see Table 4). First, unemployment volatility is smaller than the volatility of vacancies for each country. The gap between these two variables is very large for the French economy. Second, vacancies and unemployment display an instantaneous negative correlation, but this correlation is close to zero for the U.K. Finally, unemployment is relatively more persistent in France than in the U.K. and Germany.

From a global viewpoint, our statistical inferences on cyclical moments are in accordance with the specification test on the conditional moments. The global test, *i.e.* the joint test of the five moments, shows that the theoretical model is pretty occurate for the French labor market. On the contrary, our theoretical specification is globally rejected by the German data. The U.K. economy provides intermediate results: we cannot reject the model when we use a small set of restrictions during

(20) This econometric approach allows us to show the limits of the validation procedure by calibration and simulations.

Table 4: Second-order Moments

	Historical	**Theoretical** (i)	**Theoretical** (ii)	**Theoretical** (iii)
		France		
σ_v	0.0842	0.1160	0.0129	0.0129
		[0.00] **[0.02]**	[0.00] **[0.00]**	[0.00] **[0.00]**
σ_u	0.0179	0.0607	0.0677	0.0671
		[0.00] **[0.00]**	[0.00] **[0.00]**	[0.00] **[0.00]**
corr(u, v)	-0.3139	-0.2223	-0.2318	-0.2274
		[0.29] **[0.42]**	[0.35] **[0.51]**	[0.69] **[0.80]**
ρ_v	0.9160	0.9168	0.9241	0.9223
		[0.98] **[0.98]**	[0.76] **[0.79]**	[0.80] **[0.82]**
ρ_u	0.9442	0.9503	0.9507	0.9498
		[0.71] **[0.81]**	[0.69] **[0.80]**	[0.72] **[0.81]**
Global Test		[0.00] **[0.25]**	[0.00] **[0.20]**	[0.00] **[0.21]**
		United Kingdom		
σ_v	0.0444	0.0447	0.0442	0.0434
		[0.93] **[0.94]**	[0.93] **[0.95]**	[0.73] **[0.89]**
σ_u	0.0196	0.0120	0.0115	0.0112
		[0.00] **[0.14]**	[0.00] **[0.06]**	[0.00] **[0.00]**
corr(u, v)	-0.0533	0.1203	0.1416	0.1469
		[0.21] **[0.27]**	[0.16] **[0.22]**	[0.15] **[0.17]**
ρ_v	0.7187	0.7974	0.7919	0.7866
		[0.15] **[0.22]**	[0.18] **[0.24]**	[0.22] **[0.26]**
ρ_u	0.9094	0.9764	0.9748	0.9744
		[0.00] **[0.15]**	[0.00] **[0.09]**	[.00] **[0.01]**
Global Test		[0.00] **[0.47]**	[0.00] **[0.26]**	[0.00] **[0.03]**
		Germany		
σ_v	0.0471	0.0529	0.0488	0.0493
		[0.44] **[0.50]**	[0.82] **[0.83]**	[0.78] **[0.80]**
σ_u	0.0238	0.0387	0.0358	0.0361
		[0.00] **[0.01]**	[0.00] **[0.02]**	[0.00] **[0.02]**
corr(u, v)	-0.2953	-0.3672	-0.3538	-0.3640
		[0.53] **[0.60]**	[0.61] **[0.67]**	[0.55] **[0.61]**
ρ_v	0.9271	0.9391	0.9304	0.9313
		[0.65] **[0.67]**	[0.90] **[0.91]**	[0.87] **[0.88]**
ρ_u	0.8609	0.9915	0.9914	0.9915
		[0.00] **[0.00]**	[0.00] **[0.00]**	[0.00] **[0.00]**
Global Test		[0.00] **[0.00]**	[0.00] **[0.00]**	[0.00] **[0.00]**

(i) exactly identified; (ii) Over-identified (2 restictions); (iii) Over-identified (4 restictions)
First brackets: P-value without parameters uncertainty. Second brackets: P-value with parameters uncertainty.

the estimation, but the model is rejected with a larger set of restrictions on Euler equations.

Let us now look at the statistical test of the model, for each moment. The unemployment volatility reproduced by the model for France and

Germany is larger than the historical one, but we cannot reject the equality of the theoretical and historical moments. Concerning this test, parameter uncertainty, especially for the German economy, increases the P-value and allows us not to reject the model's implications. Because there is sample uncertainty on the scale parameters of the matching function, *i.e.* the structural parameters are unknown for the econometrician, and because the model's implications are very sensitive to parameter choice, our statistical test indicates that the estimated theoretical model is able to explain unemployment volatility for each country during the eighties.

The restrictions imposed on the *sunspot* processes provide better results for the U.K. and Germany, because the theoretical standard-error of the *sunspot* processes decreases in the over-identified case. This implies that the volatility decreases for each estimated model.

Concerning the volatility of vacancies, the estimated models provides good results for each country. Once more, when we impose restrictions, we do not affect significantly the quantitative implications of the model.

Concerning the unemployment-vacancies correlation, the model provides negative correlation, as in the historical data. For France, in the just-identified case, the theoretical correlation (−0.22) is not significantly different from the theoretical one (−0.31). For the U.K., the historical correlation and the model's prediction do not have the same sign but the two moments are not significantly different. Finally, for the German economy, the theoretical correlation is negative but its value is too large. These results show that the dynamics implied by the sunspots in this model reproduce the dynamics around the Beveridge Curve, but not the shifts of this curve.

The autocorrelation parameter of vacancies is well reproduced for each country and we cannot reject the equality between the historical and the theoretical moments. Finally, concerning the autocorrelation parameter of unemployment, the model provides good results, except for Germany where the theoretical value is too large. This last result shows that the real rigidities introduced in this labor market model are sufficient in order to match unemployment persistence. Indeed, this result is obtained without any exogenous shocks on fundamentals with large persistence.

4 Concluding Remarks

Most developments in business cycle theory have focused on the relative contribution of various fundamentals shocks. However, these models have to impose *ad-hoc* specification of the forcing variables in order to obtain persistence in the theoretical paths of endogenous variables. In this paper, we evaluate statistically the predictive accuracy for the European labor market of a simple matching model. We use the GMM in order to estimate and test this theoretical model with French, U.K. and German data and we find that the model works well along various dimensions. Indeed, we find that the matching functions have increasing returns to scale in each country, and so, with no shocks to fundamentals in the economy, simulations of the model provide good empirical results concerning the unemployment-vacancies dynamics. Moreover, the model displays large persistence in aggregate unemployment and vacancies with a strictly *iid* disturbance.

We have thus shown that there is a promise of resolving some problems of the European labor market using a very simple model with increasing return in the matching function. However, this study only constitutes a first stage of a more intensive theoretical and empirical investigation into labor market dynamics. We are thinking in particular about a general equilibrium perspective, including capital accumulation and intertemporal choice in consumption.

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