Life-Cycle Equilibrium Unemployment

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Abstract

This paper extends the job creation-job destruction approach to the labor market to take into account a deterministic finite horizon. As hirings and separations depend on the time over which investment costs can be recouped, the life-cycle setting implies age-differentiated labor-market flows. While search by the unemployed falls with age, the separation rate is rather U-shaped over the life cycle. Worker heterogeneity in the context of undirected search implies an intergenerational externality, which is not eliminated by the Hosios condition. We show that age-specific policies are required to attain the first-best allocation.

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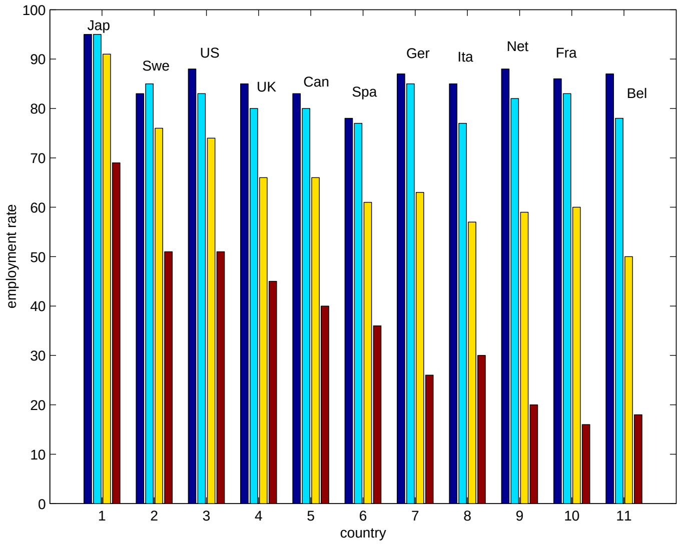
1 Introduction

It is now well-known that the low employment rate of older workers accounts for half of the European employment gap (see OECD [2006]). Moreover, lower older-worker employment rates, especially prior to the early-retirement age, are a matter of concern in many countries in the context of aging populations. A first strand of the empirical literature which has attempted to explain these features emphasizes the negative role played by labor-market institutions (particular insurance programs) on the job-search decisions of the older unemployed (see for instance Blondal and Scarpetta [1998]). A second strand puts greater emphasis on skill obsolescence, arguing that older workers suffer from technological progress. With wage stickiness, this gives firms incentives to send older workers into early retirement (see for instance Hellerstein, Newmark and Troske [1999]).[1]

Figure 1: Employment rates from age 30 to 64 in OECD Countries



Ranking of effective retirement age
(from the highest to the lowest)

For each country, the bars refers to the employment rates of age groups: 30-49 (first bar on the left), 50-54, 55-59 and 60-64 (last bar on the right).
Source: OECD data for 1995 (authors' calculations).

However, there is something missing in this picture. Figure 1 shows that the fall in the employment rate of older workers is steeper closer to retirement, whatever the country considered. Two country groups emerge clearly in the mid-1990s: those with high employment rates for 55-59 year-olds (Canada, Great Britain, Japan, the United States and Sweden) and those which exhibit

[1]This point has already been put forward by Lazear [1979], from a theoretical standpoint.

a huge drop in employment rates at this age, of around 25 points with respect to 50-54 year-olds (Belgium, France, Italy and the Netherlands). As noted by Gruber and Wise [1999], the second group of countries was characterized by a normal retirement age of 60 (versus 65 in the first group). This suggests that the retirement age could affect the employment rate of older workers prior to this age: the later the retirement age, the higher the employment of older workers under 60.

This paper investigates the influence of the distance to retirement on labor-market job flows. We propose a canonical labor-market equilibrium model with matching frictions and Nash wage bargaining, along the lines of Mortensen and Pissarides [1994], characterized by a deterministic exogenous age at which workers exit the labor market, typically an exogenous retirement age. We aim to show that this canonical theoretical setting naturally yields an influence of impending retirement on both job creation and job destruction, as endogenous hirings and separations depend on the expected duration of jobs. Surprisingly enough, the implications of a deterministic finite horizon in this extensively-used framework have not to date been addressed. Although this horizon effect has already received empirical support and some theoretical foundation based on job-search theory (Seater [1977], Ljungqvist and Sargent [2008] and Hairault, Langot and Sopraseuth [2010]), our paper is the first to propose a life-cycle equilibrium unemployment theory including both endogenous separations and unemployment search effort in a matching model *à la* Mortensen-Pissarides. It is a significant generalization of Cheron, Hairault and Langot [2008], which is restricted to the analysis of age-dependent employment protection in a simplified economy without either unemployment search effort or persistent idiosyncratic productivity shocks. The framework retained in this paper allows us to propose a quite general characterization of the equilibrium and optimal age-profiles for job creation and job destruction over the life cycle.

In our model, worker heterogeneity only comes from the distance to retirement: the expected sum of production flows generated by older workers is lower, due to their shorter career duration. This horizon effect explains why unemployed older workers search less: as is the case for any investment, the shorter the payoff period, the smaller is the return. This horizon effect has an ambiguous impact on job separations. If new opportunities are easier to attain inside the firm, via labor hoarding, then the job surplus of older workers is penalized by their shorter horizon, leading to higher separation rates for older workers. However, if new opportunities are more easily attained outside the firm, through job search (we do not consider on-the-job search here), as older workers have a lower return to unemployment search, their job surplus is higher and separation rates are lower for older workers. We show that the age-profile of job separations ultimately depends on the persistence of the idiosyncratic productivity shocks and the search intensity of unemployed workers. We moreover emphasize, as an intrinsic feature of the life-cycle matching model with persistent idiosyncratic shocks, that there is job destruction induced by the mere process of aging in the case of reservation productivity that rises with age.[2]

The endogenous age-dynamics of search intensity is then key for the age-profile of both job creation and job separations. As search falls with age, the natural prediction of our model is a separation rate that is U-shaped over the life-cycle. Provided that search effort is high enough to yield reservation productivity that falls with age at the beginning of the life cycle, the equilibrium path is then characterized by an inversion of the age-profile of reservation productivity once search effort is low enough to make labor hoarding a better strategy to obtain new opportunities: search decisions explain why separation rates are relatively high at the two ends of the life cycle, whereas older workers experience particularly low hiring rates. This is partially at odds with Menzio, Telyukova and Visschers [2010], who have recently shown in US data that the flows from employment to unemployment fall with age. Using some basic descriptive statistics, we argue in Section 3.3 that the older-worker labor market intrinsically generates a particular unemployment status characterized by low search intensity, consistent with our model's predictions, which explains unemployed older workers' claims to be retired. Once this population of workers is included in the unemployment pool, the proportion of individuals flowing from employment to unemployment continuously increases up to retirement. Overall, these predictions for older workers imply that the life-cycle matching model may help to explain why countries experience a drop in the employment rate at the end of the working life, and why this occurs at different ages in Figure 1. Moreover, at the beginning of the working life, the high returns to unemployment search render younger workers more choosy in their job acceptance and thus generates some persistence in the high level of youth unemployment.

We assume here that search is not age-directed. This assumption is crucial for the normative implications of the horizon effect, whereas the positive implications discussed above are independent of this assumption.[3] Although firms could implement age-directed recruitment policies, as age is observable, legislation prohibits age discrimination in the US and Europe as far as vacancies are concerned. On the other hand, as job productivity is assumed to be non age-specific,[4] a search equilibrium with separate markets does not exist in our economy. We therefore assume that firms cannot *ex-ante* age-direct their search: vacancies cannot be targeted at a specific age group.[5] However, as we observe ex-post discrimination against older workers (Neumark [2001]), we assume that once contact has been made with a worker, according to the observed productivity of the job-worker pair, the firm can make the choice not to recruit the worker.[6]

[2]This is a clear generalization of the analysis proposed in Cheron, Hairault and Langot [2008].

[3]See Chéron, Hairault and Langot [2009] for an analysis with directed search.

[4]Technology diffusion and standardization, training programmes and worker experience are likely to make age requirements irrelevant to most jobs, except perhaps for jobs requiring the use of the most recent wave of technology.

[5]Directed search is not incentive-compatible from the viewpoint of the bad-type workers, the older workers in our model, as they would suffer from a lower contact probability for the same expected productivity draws if they specialized their search towards older-worker specific jobs.

[6]From our point of view, there is a crucial difference between advertising for a job vacancy, on the one hand,

Following Davis [2001], it is well-known that heterogeneity can have significant implications for the efficiency of the matching allocation when there is only one matching function. The non-directed search assumption in our life-cycle framework implies the existence of a specific externality in the matching model, called hereafter an intergenerational externality: the impact of each generation of unemployed workers on the average return to vacancies renders the internalization of the hiring costs for the other generations imperfect through ex-post Nash wage bargaining. In this case, the Hosios condition does not suffice to restore the social optimality of the life-cycle equilibrium. The job surplus is over-estimated during wage bargaining for older workers and under-estimated for younger workers in equilibrium: there are too many separations and too high a search intensity for older workers, whereas the opposite is true for younger workers. We moreover emphasize that the horizon heterogeneity across workers implies that there is a life-cycle particularity of the efficiency gap created by non-directed search, leading to intrinsically suboptimal labor-market tightness.

We then derive age-specific policies to restore the social optimality of the decentralized life-cycle equilibrium. These share the objective of keeping older workers away from search, by either subsidizing their jobs and/or taxing their search effort. The latter policy suffices to make both the separation and job-search margins efficient, as the tax on job search acts on the root of the inefficiency centered on the return to unemployment search. This unveils the crucial role played by the search effort of the unemployed in setting equilibrium job separations at their optimal level.[7]

All of these results confirm our belief that the life-cycle equilibrium unemployment model is a natural starting point for a life-cycle approach to labor-market flows, even without any additional features. Adopting a more quantitative approach, some papers have used a simplified version of Mortensen and Pissarides [1994], but incorporate additional heterogeneity by age, especially in human capital and/or labor-market institutions. Along these lines, Ljungvist and Sargent [2008] and Cheron, Hairault and Langot [2009] explain the difference between the US and French labor-market performance. More recently, Menzio, Telyukova and Visschers [2010] add on-the-job search to human-capital accumulation in order to better match the observed wage and labor-market transition age-profiles in US data. On the other hand, Bettendorf and Broer [2003] take into account capital accumulation. By contrast, our paper relies on the canonical matching model and adopts a more analytical approach in order to derive the basic horizon effect. Moreover, we show that the Mortensen-Pissarides model, modified only by the existence of a retirement age, generates labor-market flows that can be considered, at least at the qualitative level, as

and hiring and wage contracting on the other. The former decision is made public ex ante, before meeting the worker. The latter decisions are taken on the basis of private information once the worker has been met: other individual characteristics can be put forward to justify age-differences, which are difficult to prove to be age discrimination.

[7]Overall, we generalize the welfare analysis presented in Cheron, Hairault and Langot [2008] by underlining not only the crucial role of search-effort optimality, but also the particularity of the intergenerational externality.

consistent with reality.

From a normative point of view, our analysis is related to the literature incorporating ex-ante heterogeneity into undirected matching models (Davis [2001], Acemoglu [2001], Bertola and Caballero [1994]). In these papers, heterogenous jobs are allocated via a non-directed matching function to homogeneous workers. In this context, even if the Hosios condition is satisfied, there is an excessive relative supply of bad jobs, as firms obtain only a fraction of the extra surplus associated with upgrading job quality, but incur all of the additional costs. The result is symmetric (an excessive supply of low-skilled workers) if workers are heterogeneous and jobs homogenous.[8] We thus share with these papers the failure of the Hosios condition to achieve efficiency, although we focus on the congestion effects between age-differentiated workers in an economy with no job-assignment problems. This in particular contrasts with Teulings and Gautier [2004], who rule out congestion effects by imposing an increasing returns to scale matching function, allowing them to focus on the inefficiencies due to suboptimal job assignments.[9] The shortcoming of the Hosios condition in the case of heterogeneity also relates to workers' search and participation decisions. This point has already been discussed by Shimer and Smith [2001] in a search model with a fixed number of participants. Albrecht, Navarro and Vroman [2010] have recently extended this result to an endogenous participation margin.

Section 2 presents the benchmark model, and Section 3 shows the age-dynamic properties of the equilibrium. Section 4 deals with the social efficiency of the equilibrium and presents some optimal age-policies. The final section concludes.

2 Job creation and job destruction when the horizon is finite

Consider an economy *à la* Mortensen - Pissarides [1994] (MP hereafter). Labor-market frictions imply that there is a costly delay in the process of filling vacancies, and endogenous job destruction closely interacts with job creation. Unlike the large literature following MP, we consider a life-cycle setting characterized by a deterministic age at which workers exit the labor market, interpreted as the retirement age.

2.1 Model environment

We consider a discrete-time model and assume that at each period the older worker generation retiring from the labor market is replaced by a younger worker generation of the same size

[8] See Albrecht and Vroman [2002] and Blazquez and Jansen [2008] for an analysis with two-sided heterogeneity in a search-matching model. These extend the seminal paper of Sattinger [1995], in which the meeting rate is fixed.

[9] On the other hand, Teulings and Gautier [2004] offer a fairly general methodology to analyze and measure the distortions with two-sided heterogeneity in a search-frictions environment

(normalized to unity) so that there is no labor-force growth in the economy. We denote the worker's age by i and T is the exogenous age at which workers exit the labor market: these are both perfectly known by employers. There is no other heterogeneity across workers. The economy is at the steady-state, and we do not allow for any aggregate uncertainty. We assume that each worker of the new generation enters the labor market as unemployed.

2.1.1 Shocks

Firms are small and each has one job. The destruction flows derive from idiosyncratic productivity shocks that hit the jobs at random. At the beginning of each age, a new productivity level is drawn with probability $\lambda \leq 1$ from the distribution $G(\epsilon)$, with $\epsilon \in [0, 1]$: the higher λ, the lower the persistence of the current productivity draw. Once a shock arrives, the firm has no choice but to either continue production or destroy the job. For age $i \in [2, T-1]$, employed workers are faced with layoffs when their job becomes unprofitable. The firms decide to close down any jobs whose productivity is below a productivity threshold (the reservation productivity) denoted by R_i. Workers, contacted when they were $i-1$ years old, whose productivity is drawn below the reservation productivity R_i, are not hired,[10] as the productivity value is revealed after the firm and worker meet.

2.1.2 Worker flows with non age-directed search

We assume that firms cannot *ex-ante* age-direct their search, and that the matching function includes all unemployed workers. Let v be the number of vacancies, u the number of unemployed workers, and e_i the endogenous search effort for a worker of age i. We define the effective unemployment rate as $\tilde{u} = \sum_{i=1}^{T-1} e_i u_i$. The matching function gives the number of contacts, $M(v, \tilde{u})$, where M is increasing and concave in both its arguments, and is constant returns to scale. From the perspective of a firm, the contact probability is $q(\theta) \equiv \frac{M(v,\tilde{u})}{v} = M(1, \theta^{-1})$ with $\theta = \frac{v}{\tilde{u}}$ being labor-market tightness. The probability that an unemployed worker of age i be employed at age $i+1$ is then defined by $jc_i \equiv e_i p(\theta)[1 - G(R_{i+1})]$ with $p(\theta) \equiv \frac{M(v,\tilde{u})}{\tilde{u}} = M(\theta, 1)$ the contact probability of the effective unemployed worker.[11] Note that the hiring process is then age-differentiated even though firms are not allowed to age-direct their search.

[10] Contrary to MP, new jobs are not opened at the highest productivity: their productivity level is drawn from the distribution $G(\epsilon)$.

[11] For simplicity, we assume that a vacancy can at most receive one application and that an unemployed worker can send at most one application every period. Introducing the multiple application case using the "urn-ball" model would not change our main conclusions when one unemployed worker among multiple applicants for one vacancy is chosen at random (see Petrongolo and Pissarides [2001] for a survey). It is far beyond the scope of this paper to analyze the case where workers could apply to more than one job in our theoretical setting. It is left for further research.

In our life-cycle setting, the mere process of aging may induce some job separations. Even if the job productivity was above the reservation productivity level at age i, it may drop below it at age $i+1$ if reservation productivity rises with age. We denote by $n_i(R_{i+1})$ the number of jobs filled by workers of age i with productivity below R_{i+1}. This mass of jobs is defined as follows:

$$n_i(R_{i+1}) = \begin{cases} (1-\lambda)[n_{i-1}(R_{i+1}) - n_{i-1}(R_i)] \\ +[\lambda(1-u_i) + u_i e_i p(\theta)][G(R_{i+1}) - G(R_i)] & \text{if } R_i \leq R_{i+1} \\ 0 & \text{if } R_i > R_{i+1} \end{cases} \quad (1)$$

When $R_i \leq R_{i+1}$, the mass of obsolete jobs depends on past job creation. Obviously, if $R_i > R_{i+1}$, there are no obsolete jobs. The age-dynamic of unemployment is then given by:

$$u_{i+1} = [1 - e_i p(\theta)(1 - G(R_{i+1}))]u_i + \lambda G(R_{i+1})(1-u_i) + (1-\lambda)n_i(R_{i+1}) \quad (2)$$

for a given initial condition $u_1 = 1$. Unemployed workers of age $i+1$ are those of age i who do not find a job (the first term on the right-hand side of (2)), employed workers of age i who lose their job due to a bad productivity shock (the second term) and employed workers whose job becomes obsolete due to aging (the third term). The overall level of unemployment is $u = \sum_{i=1}^{T-1} u_i$, so that the average unemployment rate is $u/[T-1]$.

2.2 The intertemporal values of firms and workers

Firms. Any firm is free to open a job vacancy and engage in hiring. Let c denote the flow cost of hiring a worker and $\beta \in [0,1]$ the discount factor. Let V be the expected value of a vacant position and $J_i(\epsilon)$ the value of a job filled by a worker of age i with productivity ϵ:

$$V = -c + q(\theta)\beta \sum_{i=1}^{T-1} \left(\frac{e_i u_i}{\widetilde{u}} \int_0^1 J_{i+1}(x) dG(x) \right) + (1 - q(\theta))\beta V$$

We will assume hereafter the standard free-entry condition, i.e. $V = 0$. Vacancies are determined according to the expected value of a contact with an unemployed worker. This value is the average of the expected job-worker pair values (hiring values) over the age distribution of unemployed workers. The expected value of a contact for the firm then depends on the age distribution of the unemployed, as uncertainty in the hiring process arises not only from productivity, but also from the age of workers. We will show in Section 4 that heterogeneity across ages in hiring values will imply the existence of intergenerational externalities in the search process.

For a bargained wage $w_i(\epsilon)$, the expected value $J_i(\epsilon)$ of a job filled by a worker of age i, $\forall i \in [1, T-1]$, is given by:

$$J_i(\epsilon) = \max \left\{ \epsilon - w_i(\epsilon) + \beta \left[\lambda \int_0^1 J_{i+1}(x) dG(x) + (1-\lambda) J_{i+1}(\epsilon) \right]; 0 \right\} \quad (3)$$

Workers. The values of employed (on a job of productivity ϵ) and unemployed workers of age i, $\forall i \in [1, T-1]$, are respectively given by:

$$\mathcal{W}_i(\epsilon) = \max\left\{ w_i(\epsilon) + \beta\left[\lambda \int_0^1 \mathcal{W}_{i+1}(x)dG(x) + (1-\lambda)\mathcal{W}_{i+1}(\epsilon)\right]; \mathcal{U}_i \right\} \quad (4)$$

$$\mathcal{U}_i = \max_{e_i}\left\{ b - \phi(e_i) + \beta\left[e_i p(\theta) \int_0^1 \mathcal{W}_{i+1}(x)dG(x) + (1 - e_i p(\theta))\mathcal{U}_{i+1}\right]\right\} \quad (5)$$

with $b \geq 0$ denoting the instantaneous opportunity cost of employment and $\phi(.)$ a convex function capturing the disutility of the search effort e_i. The optimal search decision of the worker then satisfies the following condition:

$$\phi'(e_i) = \beta p(\theta)\left[\int_0^1 \mathcal{W}_{i+1}(x)dG(x) - \mathcal{U}_{i+1}\right] \quad (6)$$

The marginal cost of search at age i is equal to its expected marginal return.

2.3 Job surplus and wage bargaining

The surplus $S_i(\epsilon)$ generated by a job of productivity ϵ is the sum of the worker and firm surpluses: $S_i(\epsilon) \equiv (\mathcal{W}_i(\epsilon) - \mathcal{U}_i) + J_i(\epsilon)$. The surplus associated with a job is divided between the employer and the employee according to a wage rule. Following the most common specification, wages are determined by the Nash solution to a bargaining problem:

$$w_i(\epsilon) = \arg\max_w (\mathcal{W}_i(\epsilon) - \mathcal{U}_i)^\gamma J_i(\epsilon)^{1-\gamma}$$

For given worker bargaining power γ, considered to be constant by age, the surplus generated by a job $S_i(\epsilon)$, is divided according to the following sharing rule:

$$\mathcal{W}_i(\epsilon) - \mathcal{U}_i = \gamma S_i(\epsilon) \quad \text{and} \quad J_i(\epsilon) = (1-\gamma)S_i(\epsilon) \quad (7)$$

The reservation productivity R_i can then be defined by the condition $S_i(R_i) = 0$. As in MP, a crucial implication of this rule is that the job destruction is optimal, not only from the firm's point of view but also from that of the worker: $S_i(R_i) = 0$ indeed entails $J_i(R_i) = 0$ and $\mathcal{W}_i(R_i) = \mathcal{U}_i$. Note that the lower bound of any integral over $S_i(\epsilon)$ is actually the reservation productivity, as no productivity levels below the reservation productivity yield a positive job surplus. The equilibrium wage rule then solves:

$$w_i(\epsilon) = \gamma\epsilon + (1-\gamma)\left[b - \phi(e_i) + e_i p(\theta)\gamma\beta \int_{R_{i+1}}^1 S_{i+1}(x)dG(x)\right] \quad (8)$$

The reservation wage depends not only on the instantaneous opportunity cost of employment b but also on the return to unemployment search, which is equal to the probability that the unemployed worker contacts a firm $e_i p(\theta)$ (the job-finding rate) times the share γ of the expected job surplus $\int_{R_{i+1}}^{1} S_{i+1}(x)dG(x)$, net of the search cost $\phi(e_i)$. The higher the net return to unemployment search, the higher the reservation wage, and the higher the bargained wage. On the other hand, it is possible to provide another expression for the wage in terms of the hiring costs saved by the firms.

$$w_i(\epsilon) = \gamma\left[\epsilon + c\theta e_i \tau_i\right] + (1-\gamma)[b - \phi(e_i)] \tag{9}$$

where τ_i is defined as follows:

$$\tau_i \equiv \frac{\int_{R_{i+1}}^{1} S_{i+1}(x)dG(x)}{\sum_{i=1}^{T-1}\left(\frac{e_i u_i}{\widetilde{u}}\int_{R_{i+1}}^{1} S_{i+1}(x)dG(x)\right)} \tag{10}$$

As in the MP model, market tightness enters the wage equation via the asymmetry between firms and workers in the search process. If the worker has a competitive advantage in search, implying $p(\theta)e > q(\theta)$, i.e. $\theta e > 1$, then the evaluation of the hiring cost is larger than c in the wage equation. In our life-cycle model, there is an additional asymmetry due to undirected search. For workers, the hiring cost saved when the match is formed is proportional to their individual return to unemployment search. On the other hand, for the firms, this cost is proportional to the average return to unemployment search, as a result of non-directed search. This is why the bargained wage depends on τ_i, which is the relative value of the expected job surplus for a worker of age i in the population of unemployed workers. Workers with a higher than average expected job surplus can capture a larger fraction of the hiring costs than can workers with a lower expected job surplus. Despite the assumption of undirected search, implying a homogenous hiring cost for each match, wages are age-specific.

2.4 Labor-market equilibrium

Definition 1. *The labor-market equilibrium with undirected search in a finite-horizon environment is defined by:*

$$ES_{i+1} \equiv \int_{R_{i+1}}^{1} S_{i+1}(x)dG(x) \tag{11}$$

$$\frac{c}{q(\theta)} = (1-\gamma)\beta\sum_{i=1}^{T-1}\left(\frac{e_i u_i}{\widetilde{u}} ES_{i+1}\right) \tag{12}$$

$$R_i = b - \phi(e_i) - [\lambda - \gamma e_i p(\theta)]\beta ES_{i+1} - (1-\lambda)\beta S_{i+1}(R_i) \tag{13}$$

$$S_i(\epsilon) = \max\{\epsilon - R_i + (1-\lambda)\beta[S_{i+1}(\epsilon) - S_{i+1}(R_i)]; 0\} \tag{14}$$

$$\phi'(e_i) = \gamma p(\theta)\beta ES_{i+1} \tag{15}$$

The stock-flow dynamics on the labor market are given by equations (1) and (2). The terminal conditions are $e_{T-1} = 0$ and $R_{T-1} = b$. The initial condition is given by u_1.

Equation (11) defines the notation ES_{i+1} for the expected job surplus at age $i+1$. Equation (12) is derived from the zero-profit condition $V = 0$, and shows that labor-market tightness depends on the expected job surplus averaged over the unemployed of different ages. Equation (13) defines the reservation productivity R_i from the condition $S_i(R_i) = 0$. Equation (14) is the job surplus definition using the expression for reservation productivity. Equation (15) is the first-order condition on search by the unemployed using the definition of job surplus. At the end of the life cycle, we have $e_{T-1} = 0$ because at age T agents are retired and so there is no expected job surplus at age $T-1$. For the same reason, $R_{T-1} = b$.

3 Age-dynamics of job creations and job destructions

The labor-market equilibrium cannot be solved analytically, as it is highly non-linear.[12] We then characterize the age-dynamics of the productivity thresholds R_i and search intensity e_i, given homogenous labor-market tightness θ. This age pattern can be solved by considering the set of equations (13)-(15) independently of equations (1), (2) and (12).

3.1 Job-surplus heterogeneity over the life cycle

We first investigate the key mechanisms underlying the age-dynamics of the job surplus, which is essential for the understanding of the age-profile of both R_i and e_i. Combining equations (3), (4) and (5), job surplus can be written in labor-market equilibrium as follows:

$$S_i(\epsilon) = \max \left\{ \begin{array}{l} \underbrace{\epsilon - b}_{\text{Current surplus}} + \underbrace{(1-\lambda)\beta S_{i+1}(\epsilon)}_{\text{Continuation}} \\ \underbrace{+\underbrace{\lambda \int_{R_{i+1}}^{1} S_{i+1}(x)dG(x)}_{\text{labor hoarding}} - \underbrace{\gamma e_i p(\theta) \int_{R_{i+1}}^{1} S_{i+1}(x)dG(x) - \phi(e_i)}_{\text{unemployment search}}}_{\text{New opportunities}} ; 0 \end{array} \right\} \quad \forall i < T-1$$

with the terminal condition $S_{T-1}(\epsilon) = \max\{\epsilon - b; 0\}$.

Job surplus is formed from the value generated by the current level of productivity (the current surplus plus its continuation value) and the net value of new opportunities. The latter value is the difference between the option value of the existing job, arising from labor hoarding, and the net return to unemployment search. We should first emphasize that staying on the job[13] or

[12] Another route would have been to propose an approximation of the equilibrium along the line of Teulings and Gautier [2004]. But the life-cycle setup and the non-stationarity of the heterogeneity across workers makes this method difficult to apply in our framework. This is left for further research.

[13] An alternative view would be to interpret λ as the probability of obtaining new opportunities from searching on the job. However, this probability is not the promise of up-grading the existing job: when a job-worker pair is

searching while unemployed provides access to the same distribution of job offers characterized by the same expected job surplus. This implies that the expected job surplus affects current job surplus according to the probability of receiving a new job offer. The expected job surplus can be obtained within the firm with probability λ, implying a certain value to labor hoarding. A share γ of the expected job surplus can also be obtained outside the firm by an unemployed worker with probability $e_i p(\theta)$, giving the value of search. There is a key difference here: the probability of new opportunities from searching depends on costly search, whereas the probability of new opportunities on the job is given exogenously by the persistence in job productivity. If $\lambda > \gamma e_i p(\theta)$, search is too slow relative to the probability of receiving a new opportunity within the firm: here the higher is the expected job surplus, the higher is the job surplus. However, search effort can also be sufficiently high to make search a better strategy than labor hoarding, leading job surplus to fall with the expected job surplus. The value of λ relative to $\gamma e_i p(\theta)$ can be interpreted as the "efficiency" of labor hoarding relative to unemployment search.

Job surplus differs by age, although there is no heterogeneity across workers apart from their distance to retirement. The fundamental feature of older workers is that their remaining time on the labor market is shorter: their horizon is reduced by the proximity to retirement. A key implication is then the greater sensitivity of job surplus to current surplus for older workers. For the oldest workers, just before retirement, it is obvious that only the current surplus matters. Both the continuation value of their current productivity and the value of new opportunities are lower due to this shorter horizon (horizon effect): the sum of any given productivity draw on the expected remaining career time is particularly low at the end of the working life[14] The fall in the value of new opportunities then affects the range of profitable productivities over the life cycle. When these opportunities are easier to attain via unemployment search than on the job, aging reduces the reservation productivity via a reservation-wage effect; when they are higher inside the firm through labor hoarding, aging increases the reservation productivity. Beyond these implications for reservation productivity, the age-profile of the expected job surplus also affects that of search effort (see equation (15)), which in turn makes the relative efficiency of unemployment search age-dependent.

The horizon effect creates age-heterogeneity across the job surplus and thus across reservation productivity. Once reservation productivity is age-dependent, aging can lead to more or less strictness in the selection of profitable jobs according to the age-dynamics of the productivity thresholds. The expected number of profitable jobs, and so the expected job surplus, changes over the life cycle. If reservation productivity rises with age, there are fewer and fewer profitable jobs with age: this effect lowers the expected job surplus for older workers, exacerbating the horizon effect. On the other hand, when reservation productivity falls with age via a reservation-

hit by a shock, with probability λ, it can be the case that the productivity draw is lower than the previous one. This is why we prefer not to refer to on-the-job *search*, but to labor hoarding.

[14]Note that this is all the more true when the shock is highly persistent (λ is low).

wage effect, there are more and more profitable jobs when aging, counteracting the horizon effect. The age-profile of the expected job surplus is in this case *a priori* indeterminate.

Overall, the life-cycle framework creates age-heterogeneity in the job surplus for a given age-heterogeneity in reservation productivity and search. The latter itself depends on the former, as the age-profile of the expected surplus determines that of search and reservation productivity. It remains to solve for the equilibrium paths in order to check the internal consistency of all of these effects.

3.2 The equilibrium age-dynamics of search and reservation productivity

As the solution for the job surplus depends on the age-profile of reservation productivity, solving for the age-dynamics of job creation and job destruction implies postulating the existence of a given age-profile for reservation productivity in order to derive the implications for job surplus. We then characterize the conditions under which this age-profile exists. As benchmark cases, we will first analyze the two monotonic dynamics for R over the life cycle, either age-increasing or age-decreasing; we will then present the hybrid case where R falls then rises (i.e. is U-shaped).

3.2.1 Age-increasing reservation productivity and age-decreasing search effort

We first consider the case with monotonic age-increasing reservation productivity.

Proposition 1. *If $R_i < R_{i+1}, \forall i < T-1$, then the job surplus, the expected job surplus and the reservation productivity are the solutions to:*

$$S_i(\epsilon) = \sum_{j=0}^{T-1-i} \beta^j (1-\lambda)^j \max\{\epsilon - R_{i+j}; 0\} \quad (16)$$

$$ES_{i+1} = \sum_{j=0}^{T-1-i} \beta^j (1-\lambda)^j \int_{R_{i+j}}^{1} [1-G(x)]dx \quad (17)$$

$$R_i = b - \phi(e_i) - [\lambda - \gamma e_i p(\theta)]\beta ES_{i+1} \quad (18)$$

with the terminal conditions $R_{T-1} = b$ and $e_{T-1} = 0$.

Proof. See Appendix A □

When reservation productivity rises with age, all jobs with the current reservation productivity are unprofitable at the next age ($S_{i+1}(R_i) = 0$), as are those with productivity below R_{i+1}.

Proposition 2. *If $\lambda > \gamma p(\theta) e_i$, $\forall i < T-1$, the equilibrium path is characterized by a monotonic age-decreasing profile for search effort e_i and a monotonic age-increasing profile for the reservation productivity R_i. A sufficient condition ensuring the existence of this equilibrium path is then $\lambda > \gamma$.*

Proof. First we show that $R_i < R_{i+1}$ $\forall i$ implies that $ES_i > ES_{i+1}$ and then $e_i > e_{i+1}$ $\forall i$. If $R_i < R_{i+1}$, we deduce from equation (17) that $ES_i - ES_{i+1} = \int_{R_i}^{R_{i+1}}(1-G(x))dx + \beta(1-\lambda)(ES_{i+1} - ES_{i+2})$. As $ES_{T-2} > ES_{T-1}$, it is then straightforward that $ES_i > ES_{i+1}, \forall i$. Then, using equation (15), we obtain $e_i > e_{i+1}$, given that $\phi''(\cdot) > 0$.

Second we show that $e_i > e_{i+1}$ implies that $R_i < R_{i+1}$ if $\lambda > \gamma p(\theta)e_i$, $\forall i < T-1$. Consistent with equations (15) and (18), the solution for reservation productivity can be written as follows:

$$\gamma p(\theta)R_i = \gamma p(\theta)b - \underbrace{[\gamma p(\theta)\phi(e_i) + (\lambda - \gamma p(\theta)e_i)\phi'(e_i)]}_{\equiv \Upsilon(e_i)}$$

R_i is age-increasing if and only if:

$$\begin{array}{rclcll} \gamma p(\theta)(R_{i+1} - R_i) & = & \Upsilon(e_i) - \Upsilon(e_{i+1}) & > & 0 & \forall i < T-2 \\ \gamma p(\theta)(R_{T-1} - R_{T-2}) & = & \Upsilon(e_{T-2}) & > & 0 & \end{array}$$

If $e_i > e_{i+1}$ $\forall i < T-2$, $R_i < R_{i+1}$ $\forall i < T-2$ if $\Upsilon'(e_i) > 0$, $\forall e_i \in [e_{T-2}, e_1]$. As $\Upsilon'(e_i) = (\lambda - \gamma p(\theta)e_i)\phi''(e_i)$, the condition $\lambda > \gamma p(\theta)e_i$, $\forall e_i \in [e_{T-2}, e_1]$ implies that $\Upsilon'(e_i) > 0$. $\lambda > \gamma p(\theta)e_i$, $\forall i \leq T-2$, then implies $R_i < R_{i+1}$. On the other hand, for $i = T-2$, the terminal restriction is given by $\Upsilon(e_{T-2}) > 0 \Leftrightarrow \lambda > \left(\frac{\omega(e_{T-2})-1}{\omega(e_{T-2})}\right)\gamma p(\theta)e_{T-2}$ with $\omega(e_{T-2}) \equiv \phi'(e_{T-2})\frac{e_{T-2}}{\phi(e_{T-2})}$ the elasticity of the search disutility valued at age $T-2$. $\omega(e_{T-2}) > 1$ as $\phi(\cdot)$ is a convex function. This terminal condition is then a less restrictive condition than $\lambda > \gamma p(\theta)e_{T-2}$. This is why, $\forall i < T-1$, $\lambda > \gamma p(\theta)e_i$ implies that $R_i < R_{i+1}$. Finally, as $e_i p(\theta) < 1$, a sufficient condition for $R_i < R_{i+1}, \forall i < T-1$, is then $\lambda > \gamma$. □

Proposition 2 presents the conditions under which there exists an equilibrium path with an age-increasing productivity threshold and age-decreasing search effort. When the shock persistence and the job-finding rate (for a given bargaining power of γ) are low enough to make the value of future opportunities unambiguously better inside the firm than outside ($\lambda > \gamma p(\theta)e_i$), labor hoarding is a dominant strategy: firms keep some currently unprofitable jobs in order to be able to start production at the new productivity without searching for new jobs. Labor hoarding is all the more valuable in that the employees can compensate for the current losses with high expected surpluses in the future.

In this case, age-increasing reservation productivity and age-decreasing expected job surplus and search are an equilibrium path. First, when reservation productivity rises with age, the expected job surplus unambiguously falls with age: the expected number of profitable jobs falls over the life cycle. This reinforces the horizon effect to make the expected job value lower for older workers. The age-dynamics of search then mirrors that of the expected job surplus. The return to search is lower for older unemployed workers: as retirement becomes closer, the return to job search falls as the horizon over which workers can recoup their investment is reduced on the one hand, and fewer jobs are profitable on the other. Second, when the expected job surplus for older workers is lower, labor hoarding on their jobs is less valuable: their reservation productivity is then higher than that of younger workers, who expect to up-grade their job during their longer remaining career. At the limit, for employees who will soon retire, who have no future, keeping their jobs with current losses is not rational and reservation productivity converges to the domestic production b. This helps to generate age-decreasing hiring rates, whereas job separations are age-increasing.

Corollary 1. *When the condition $\gamma e_i p(\theta) < \lambda$ holds for a particular age $\widehat{i}$, reservation productivity is age-increasing from this age $\widehat{i}$ until retirement. The equilibrium path is characterized by a monotonic age-increasing profile for the reservation productivity R_i over the life cycle if and only if $\lambda > \gamma p(\theta) e_1$.*

Proof. Straightforward from Proposition 2 and the age-decreasing profile of search effort. □

Once the condition $\gamma e_i p(\theta) < \lambda$ holds for a particular age $\widehat{i}$, as search e_i falls over the life cycle, the inequality $\gamma e_i p(\theta) < \lambda$ will continue to hold for $i > \widehat{i}$. The age-increasing reservation productivity is then stable in the sense that the labor-market equilibrium never moves away from these dynamics. A particular case is shown in Figure 2 when the condition holds at age 1: starting from an initial condition e_1 on search such that $\gamma e_1 p(\theta) < \lambda$ implies that R rises over the whole life cycle while e falls. In this case, the age-dynamics of search never crosses the vertical line defined by the equation $e^s = \frac{\lambda}{\gamma p(\theta)}$ as the initial condition is such that $e_1 < e^s$. Note that R and e are respectively bounded by R_∞ and e_∞, the equilibrium values of reservation productivity and search intensity as T goes to infinity.

3.2.2 Age-decreasing reservation productivity and age-decreasing search effort

We now turn to the opposite case characterized by a continuous age-decreasing reservation productivity.

Proposition 3. *If $R_i > R_{i+1}, \forall i < T-1$, then the job surplus, the expected job surplus and the productivity reservation are the solutions to:*

$$S_i(\epsilon) = P_i(T)\max\{\epsilon - R_i; 0\} \quad \textit{with } P_i(T) = \textstyle\sum_{j=0}^{T-1-i} \beta^j (1-\lambda)^j \tag{19}$$

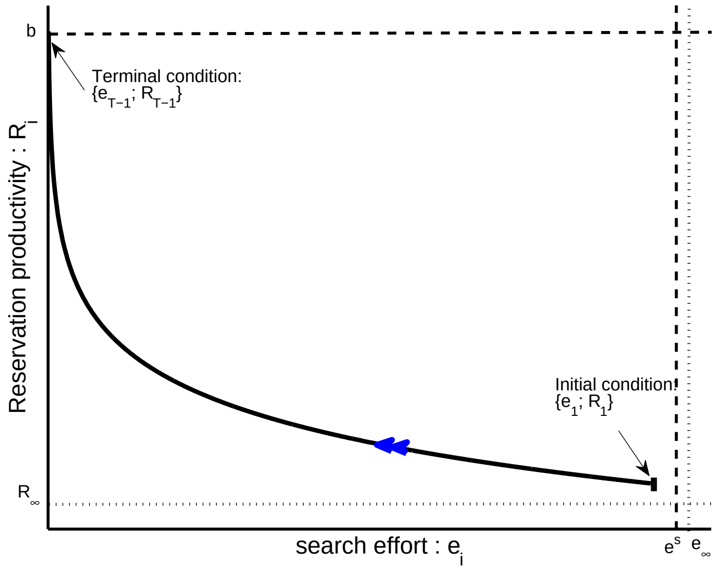
$$ES_{i+1} = P_{i+1}(T)\int_{R_{i+1}}^{1} [1 - G(x)]dx \tag{20}$$

$$R_i = b - \phi(e_i) - [\lambda - \gamma e_i p(\theta)]\beta ES_{i+1} - (1-\lambda)\beta P_{i+1}(T)(R_i - R_{i+1}) \tag{21}$$

with the terminal conditions $R_{T-1} = b$ and $e_{T-1} = 0$.

Proof. Straightforward using equations (13) and (14) and noting that $\int_{R_{i+1}}^{1}[x - R_{i+j}]dG(x) = \int_{R_{i+1}}^{1}[1 - G(x)]dx$. □

Figure 2: R and e age-dynamics (Proposition 2)



When reservation productivity falls with age, a job opened at a given age remains profitable at older ages until retirement, as long as the level of productivity is unchanged. This implies that the continuation value of a job depends on the degree of persistence, as in MP, but also, more interestingly, on the horizon of the workers until retirement through the variable $P_i(T)$.

Older workers have a lower job surplus due to their shorter horizon and can expect a lower value of new opportunities ($P_i(T) > P_{i+1}(T)$). However, when the productivity thresholds are age-decreasing due to a reservation-wage effect, more jobs become profitable when aging. The expected job surplus from new opportunities is then not necessarily lower for older workers, as the horizon and reservation wage effects move in opposite directions, as can be seen from equation (20):

$$ES_i - ES_{i+1} = \underbrace{[P_i(T) - P_{i+1}(T)] \int_{R_i}^{1} [1 - G(x)]dx}_{\text{horizon effect}} - \underbrace{P_{i+1}(T) \int_{R_{i+1}}^{R_i} [1 - G(x)]dx}_{\text{reservation wage effect}} \qquad (22)$$

Given that $P_i(T) > P_{i+1}(T)$ and $R_i > R_{i+1}$, $ES_i \lesseqgtr ES_{i+1}$. The expected surplus is age-decreasing when the horizon effect dominates the reservation-wage effect. Intuitively, though, age-decreasing reservation productivity means that new jobs become profitable when aging, implying that their job surplus is age-increasing. There is however nothing inconsistent here. Consider two successive ages i and $i + 1$: the surplus from jobs that are already profitable at age i unambiguously falls with age due to the horizon effect, whereas job surplus rises with age for relatively low-productivity jobs which become profitable at age $i+1$. The domination of the horizon effect actually means that the former age-profile dominates the latter in the determination of expected job surplus. This fundamentally depends on the number of jobs that become profitable when ageing due to the reservation-wage effect relative to the number that were already profitable at younger ages. The former jobs are actually in a *small* interval whose boundaries are given by the two successive reservation productivities R_i and R_{i+1}, whereas the latter are in a *large* interval between the highest productivity and, at least, the highest reservation productivity.[15]

Proposition 4. *If $\lambda < \gamma p(\theta)e_i$, $\forall i < T-2$, and if the horizon effect dominates the reservation-wage effect in the age-dynamics of the expected job surplus, the equilibrium path is characterized by a monotonic age-decreasing profile for search effort e_i and a monotonic age-increasing profile for reservation productivity R_i, provided that the terminal condition $R_{T-2} > R_{T-1}$ holds.*

Proof. First, if $R_i > R_{i+1}$ $\forall i$, using equation (15) and (22), it is straightforward to show that $e_i > e_{i+1}$ $\forall i$ if the horizon effect dominates the reservation-wage effect. Second, combining equations (15) and (21) leads to:

$$\gamma p(\theta)R_i = \gamma p(\theta)b - \Upsilon(e_i) + \gamma p(\theta)\beta(1-\lambda)P_{i+1}(T)(R_{i+1} - R_i)$$

R_i is then age-decreasing if and only if

$$\begin{array}{rcll} R_{i+1} - R_i & = & \frac{\Upsilon(e_i) - \Upsilon(e_{i+1}) + \gamma p(\theta)\beta(1-\lambda)P_{i+2}(T)(R_{i+2}-R_{i+1})}{\gamma p(\theta)[1+\beta(1-\lambda)P_{i+1}(T)]} & < \ 0 \qquad \forall i < T-2 \\ R_{T-1} - R_{T-2} & = & \frac{\Upsilon(e_{T-2})}{\gamma p(\theta)[1+\beta(1-\lambda)]} & < \ 0 \end{array}$$

If the terminal restriction is satisfied ($R_{T-1} - R_{T-2} < 0 \Leftrightarrow \Upsilon(e_{T-2}) < 0$), by backward iteration it suffices to determine the restriction which ensures that $R_{i+1} - R_i < 0$, given that $R_{i+2} - R_{i+1}$ is negative. The terminal condition is $\lambda < \left(\frac{\omega(e_{T-2})-1}{\omega(e_{T-2})}\right)\gamma p(\theta)e_{T-2}$, with $\omega(e_{T-2})$ the elasticity of search disutility evaluated at age $T-2$. This terminal condition is more restrictive than the condition $\lambda < \gamma p(\theta)e_{T-2}$. If e_i is age-decreasing and $\Upsilon'(e_i) < 0$, $\forall e_i \in [e_{T-2}, e_1]$, i.e. $\lambda < \gamma p(\theta)e_i, \forall i \leq T-2$, then $\Upsilon(e_i) - \Upsilon(e_{i+1}) < 0$ and $R_{i+1} - R_i < 0, \forall i < T-2$. □

Proposition 4 presents the conditions under which there exists an equilibrium path along which both reservation productivity and search fall with age. If the job-finding rate and productivity persistence are sufficiently high (for a given bargaining power γ) to imply that unemployment search provides more chances of new opportunities than labor hoarding, reservation productivity is lower when the expected job surplus is smaller. This is why an age-decreasing profile in the expected job surplus ensures that reservation productivity falls with age. Older workers are less

[15] In our model, with a smooth decreasing profile for reservation productivity, although no analytical proofs can be provided, numerical simulations for the usual calibrations of the structural parameters show that the reservation-wage effect is at least of order 1 relative to the horizon effect. On the other hand, it can be shown that the reservation-wage effect is of the same order as the horizon effect only if reservation productivity is artificially forced to converge to b in one step (age), rather than smoothly over the life cycle. This analysis is available upon request.

likely to leave their jobs, simply because there is less time for them to reap the benefit of finding a better job when searching, making a shorter horizon synonymous with a lower reservation wage.

Proposition 4 shows that an age-decreasing profile in expected job surplus is consistent with an age-decreasing profile in reservation productivity if the horizon effect dominates the reservation-wage effect. In this case, search by older workers is lower. Overall, job separations fall with age, whereas the age-profile of job creation is indeterminate.

Corollary 2. *Falling reservation productivity at young ages may switch to a rising profile for older workers. The equilibrium path is characterized by monotonic age-decreasing profile for reservation productivity R_i only if e_{T-2} is high enough to ensure that $R_{T-2} > R_{T-1}$, i.e. $\lambda < \left(\frac{\omega(e_{T-2})-1}{\omega(e_{T-2})}\right)\gamma p(\theta)e_{T-2}$.*

Proof. Consistent with e_i falling with age, the condition $\lambda < \gamma p(\theta)e_i$ is the most restrictive for $i = T-2$, implying that the terminal condition $\lambda < \left(\frac{\omega(e_{T-2})-1}{\omega(e_{T-2})}\right)\gamma p(\theta)e_{T-2}$ must hold to ensure that $R_i > R_{i+1}$ $\forall i < T-1$. □

The age-decreasing profile for R is potentially unstable, in the sense that search falling with age makes the necessary condition $\lambda < \gamma p(\theta)e_i$ more difficult to satisfy for older workers. If the condition $\gamma e_i p(\theta) < \lambda$ holds for the youngest worker e_1, nothing ensures that it will continue to hold with age. Figure 3 shows the case consistent with Proposition 4, where the age-decreasing path of search never crosses the vertical line defined by the equation $e^s = \frac{\lambda}{\left(\frac{\omega(e^s)-1}{\omega(e^s)}\right)\gamma p(\theta)}$, as the terminal condition is such that $e_{T-2} > e^s$.

3.2.3 A U-shaped pattern in the productivity threshold and age-decreasing search effort

As search falls with age, rising reservation productivity with age may switch sign for older workers, leading to U-shaped reservation productivity over the life cycle.

Proposition 5. *There may exist a given age $\widehat{i}$ such that $R_i > R_{i+1}$ for $i < \widehat{i}$ and $R_i < R_{i+1}$ for $i \geq \widehat{i}$. The age $\widehat{i}$ is defined by the condition $\lambda = \gamma p(\theta)e_{\widehat{i}}$. A sufficient condition for the existence of this equilibrium is $\lambda < \gamma p(\theta)e_i$ for $i < \widehat{i}$ and $\lambda \geq \gamma p(\theta)e_i$ for $i \geq \widehat{i}$, provided that search e_i is age-decreasing.*

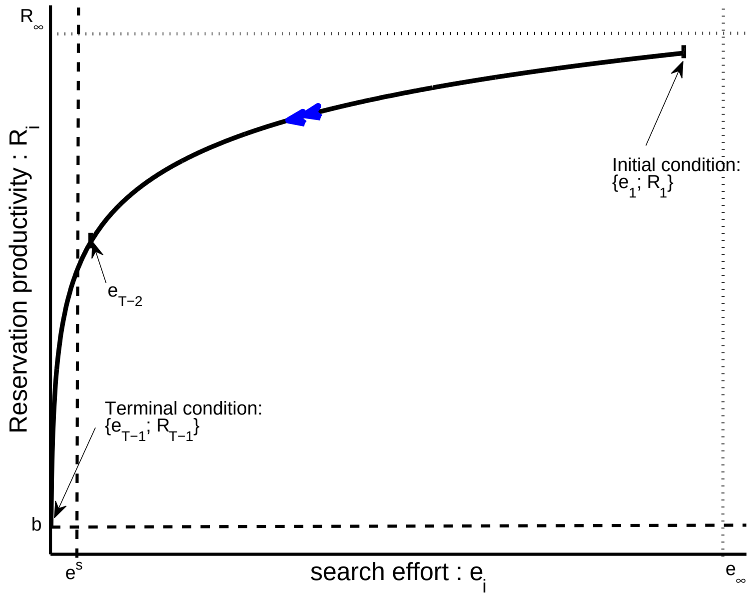
Proof. See Appendix D. □

The intuition for Proposition 5 is straightforward.[16] For simplicity,[17] we assume in Proposition

[16]Though intuitive, Proposition 5 is complicated to prove due to the inversion of the age-dynamics after a given age $\widehat{i}$. See Appendix D.

[17]More generally, the age-profile of R changes sign for age $\widehat{i}$ such that λ becomes greater or equal to $\gamma p(\theta)e_{\widehat{i}}$. As

Figure 3: R and e age-dynamics (Proposition 4)



5 that there exists an integer $\widehat{i}$ such that $\lambda = \gamma p(\theta)e_{\widehat{i}}$. While reservation productivity and search effort fall with age for workers younger than $\widehat{i}$, reservation productivity may rise for workers from age $\widehat{i}$ onwards up to retirement, as a consequence of the age-dynamics of search (Figure 4). Below age $\widehat{i}$, job search dominates labor hoarding ($\lambda < \gamma p(\theta)e_i$); above this age ($i > \widehat{i}$), the opposite is true ($\lambda > \gamma p(\theta)e_i$). Over the life-cycle, reservation productivity is then U-shaped if the search intensity of older workers is low enough to invert the age-decreasing profile featuring younger ages. In this case, job creation unambiguously falls with age after $\widehat{i}$, whereas the age-profile is indeterminate up to $\widehat{i}$, as result of the two opposite effects from R_i and e_i. In Figure 4, search effort crosses (at age $e_{\widehat{i}}$) the vertical line of equation $e^s = \frac{\lambda}{\gamma p(\theta)}$ that separates the life cycle into two opposite age-profiles for reservation productivity, contrary to the two cases with a monotonic age-profile (Figures 2 and 3).[18]

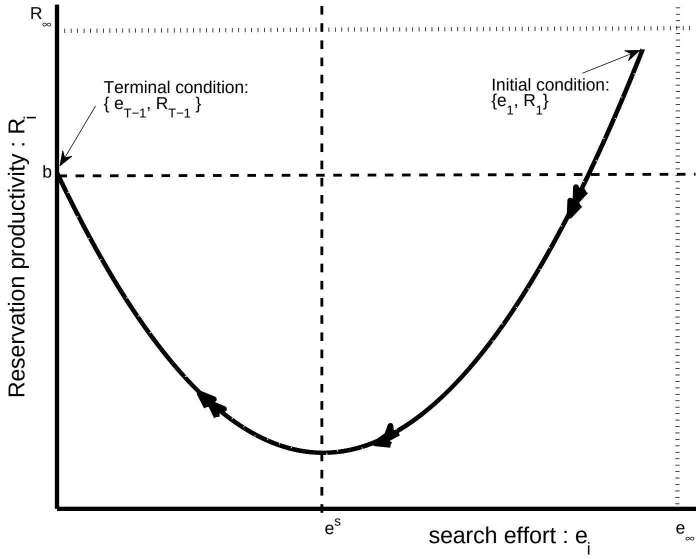
3.3 Discussion

Model predictions. The previous section characterized the age-profiles of job creation and job separation under different scenarios. Whatever the case under consideration, the horizon effect is at the root of the different flows by age independently of any other factors. The shorter

the proof in Appendix D relies on an approximation around the equation $\lambda = \gamma p(\theta)e_{\widehat{i}}$, we assume strict equality.

[18]In Figure 4, R is assumed to start above b for younger workers: $\widehat{i}$ is far enough away to render the influence of the age-increasing profile insignificant.

Figure 4: R and e age-dynamics (Proposition 5)



horizon of older workers unambiguously reduces the continuation value of profitable jobs. It also leads the value of new opportunities they can expect to fall with age, either from unemployment search or labor hoarding. This unambiguously implies an age-decreasing search effort. On the other hand, the age-profile of reservation productivity depends on the efficiency of labor hoarding relative to unemployment search, i.e. the value of shock persistence relative to the job-finding rate.

As shown in Proposition 2, the condition $\lambda > \gamma$ ensures continuous age-increasing reservation productivity over the life cycle. For instance, for a realistic value of γ of 0.5, the expected duration of a shock must be less than 2 years. Such persistence values are not necessarily unrealistic. Indeed, it is assumed that shocks uniformly hit workers, whatever their age, in order to focus on the horizon effect. In that sense, these shocks drive worker flows, and not job flows as in Mortensen and Pissarides [1994]. The persistence of these job-worker pair shocks is less than the persistence of the job-technology pair shocks. There is some empirical evidence of low persistence in the shocks that hit job-worker pairs.[19] For instance, in Ljungqvist and Sargent [2008] earnings are the product of the wage by unit of human capital and the level of the human capital. In this context, they consider that the average time between wage draws is 1.9 years within a skill occupation, whereas the average duration for a skill occupation is 1.435 years.

On the other hand, if $\lambda < \gamma$, reservation productivity may be still continuously age-increasing

[19] The average duration of a job for a given worker is lower than the average duration of capital equipment in the firm.

over the life cycle, provided that the search effort of the youngest workers is low enough to ensure that $\lambda > \gamma e_1 p(\theta)$: in this case, the age-profile is a stable path until retirement as search effort is age-decreasing. On the contrary, search effort can remain high enough at the end of the life cycle to ensure that reservation productivity is continuously age-decreasing over the life cycle. Beyond these two monotonic age-profiles, it is likely more realistic to consider reservation productivity with a U-shaped age-profile (Proposition 5). As younger workers are likely to search intensively for a job, unemployment search is certainly more efficient than labor hoarding at the beginning of the working life up to some age $\widehat{i}$. At this age, the relative efficiency switches up to retirement, as search effort will continue to fall. This leads to a U-shape in separation rates, i.e. higher values at the two ends of the life cycle, for younger and older workers. Up to age $\widehat{i}$, the age-dynamics of the hiring rate is the result of two opposing forces arising from falling search effort and reservation productivity. On the other hand, hiring rates are unambiguously lower for older workers, as the result of lower search effort and higher productivity thresholds.

Overall, these predictions for older workers imply that the life-cycle matching model may help to explain why countries experience falling employment rates at the end of the working life in Figure 1, and why this occurs at different ages. The important dimension of the model is indeed the retirement age. Only the distance between current age and the retirement age matters according to the horizon effect: biological age does not matter in itself. These results suggest that the observed low employment rate of those close to retirement cannot be considered as a reason not to postpone the retirement age. The reasoning is actually the opposite: retirement postponement will actually likely increase the employment rate of these workers, thereby contradicting the widespread view that the lower employment rate of older workers makes any extension of the retirement age pointless.[20] For younger workers, the high return to unemployment search makes them more selective in their job acceptance, which explains the persistence in the high level of unemployment at the beginning of the life cycle.

What are the facts? Menzio, Telyukova and Visschers [2010] have recently shown in US data that both the flows from employment to unemployment and those from unemployment to employment are age-decreasing. More precisely, the job creation rate is fairly flat over the life cycle, with a large decline after age 50, which seems consistent with our theoretical predictions, whereas the age-profile of the job separation rate has a more pronounced downward slope over the life-cycle. These results on job separations are partially at odds with those shown in a previous paper (Chéron, Hairault and Langot [2009]), which are U-shaped over the life cycle in US and French data (Labor Force Survey): the job separation rate increases at the end of the working life before retirement. The opposite result for separation flows at the end of the working life relies on the difference in the pool of unemployed workers considered. Even more than for other ages, we believe that the definition of unemployment is too restrictive when based on the

[20]See Hairault, Langot and Sopraseuth [2010] for such policy experiments.

self-declaration of looking for a job:[21] The older-worker labor market then generates a specific unemployment status which is typified by such low search intensity that the unemployed claim to be retired, as the probability of becoming employed again is very low. This is in line with our model's predictions. These individuals are actually unemployed with particularly low search intensity, and not retired as they claim to be.

This point can be illustrated by focusing on the status of older workers in US data from the Health and Retirement Survey (HRS). The HRS data clearly distinguishes between the unemployed (U), and the partially (PR) and fully retired (FR). The partially retired are looking for a job, and so can unambiguously be associated with the unemployed, whereas the fully retired declare themselves as not searching. For those between 50 and 60, before the early age of eligibility for Social Security, "fully retired" predominates among the non-employed. In particular, it is striking in Figure 5 that the structure of job separations is highly biased toward full retirement. As people age, the proportion retired increases as that unemployed falls. The proportion who leave work to become partly retired remains stable at a low level. For instance, 56% of people aged 50 leaving employment declare themselves to be fully retired. This proportion jumps to 85% at around 60, when workers are closer to the age at which it is possible to claim SS benefits.

Are these workers still participating despite their claim to be fully retired? We believe that they may be considered to be unemployed with a particularly low search intensity. Figure 5 shows the probability of working for individuals who are "retired" at different ages. What emerges is that, although the fully retired declare that they are not looking for work, and certainly despite their low search effort, a significant fraction will work in the future. Once the "fully retired" are included in the pool of workers searching for a job, Figure 5 shows that the separation rates from employment to unemployment change dramatically:[22] they increase almost continuously from age 50 to age 60. In this case, our model can simultaneously replicate the age-decline in job creation and the age-increase in the separation rate as retirement approaches.[23].

This interpretation of the fully-retired status for workers in their Fifties is debatable, and a more detailed empirical analysis is required for a definite conclusion.[24] We acknowledge that retirement and unemployment with low search effort are hard to distinguish from each other. However, it is beyond doubt that the end of the working life intrinsically generates this mixed status between retirement and unemployment. From our point of view, a key point is that the job separations leading to this mixed status are strongly affected by the distance to retirement.

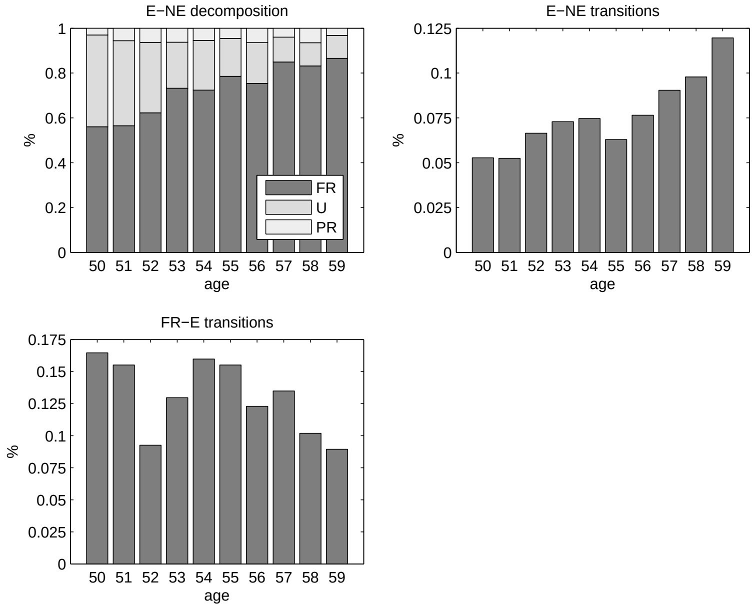
[21] Blanchard and Diamond [1990] suggest being less restrictive concerning the definition of unemployment: one half of new matches does not come from this set of unemployed, showing that workers not reporting "looking for a job" do in fact search, as some of them do obtain jobs.

[22] Leaving this population to one side leads to an age-decreasing separation rate, as shown by Menzio, Telyukova and Visschers [2010].

[23] Menzio, Telyukova and Visschers [2010] explain the fall with age that they observe by the greater human capital of older workers that compensates for the horizon effect.

[24] It is beyond the scope of this paper to provide this empirical analysis.

Figure 5: Full retirement called into question



Source: HRS data for 1995 (authors' calculations). The E-NE decomposition is the structure of separations; E-NE transitions refer to separations from employment to (unemployment+fully retired); FR-E transitions are job-finding rates from full retirement.

In other words, an increase in the "real" retirement age would eliminate most of these separations and increase the search intensity for a given age,[25] as retirement would be further away.

4 Optimal allocation and age policy

The previous sections have highlighted the economic rationales behind the age heterogeneity in job separations and creation. It remains to be shown that this behavior is not socially optimal. The existence of an additional externality, such as an intergenerational externality, would lead us to reconsider the efficiency result obtained with an infinite horizon when the Hosios condition holds. Specific policies, in particular by age, should then be implemented to restore social optimality.

4.1 The efficient allocation

The problem of the planner is to determine the optimal allocation of each worker between the production and search sectors, and optimal search. Following Pissarides (2000), we suppose the planner maximizes the sum of the flows of market and domestic productions net of the search

[25] See Hairault, Langot and Sopraseuth [2010] for such counterfactual experiments.

costs implied by search effort and vacancies. In the overlapping-generations framework, these flows must be considered over all ages i and different time periods: there are T overlapping generations of different ages i living in a given period $t+i$, where t is the date of birth of a given generation. Social choices are constrained by the dynamics of both unemployment and market production. We start from the dynamic problem in order to derive the conditions for social efficiency, and then impose a steady state.

The planner's decision is the solution to:

$$\max_{\mathcal{C}} \sum_{t=0}^{\infty} \sum_{i=1}^{T-1} \beta^{t+i} \left[y_{i,t+i} + (b - \phi(e_{i,t+i}) - c e_{i,t+i} \theta_{t+i}) u_{i,t+i} \right] \tag{23}$$

subject to the following constraints (for $i = 1, ..., T-2$ and for $t = 1, ..., \infty$):

$$\begin{aligned} u_{i+1,t+i+1} &= [1 - e_{i,t+i} p(\theta_{t+i})(1 - G(R_{i+1,t+i+1}))] u_{i,t+i} \qquad (24) \\ &\quad + (1-\lambda) n_{i,t+i}(R_{i+1,t+i+1}) + \lambda G(R_{i+1,t+i+1})(1 - u_{i,t+i}) \\ y_{i+1,t+i+1} &= (1-\lambda)[y_{i,t+i} - y_{i,t+i}(R_{i+1,t+i+1})] \qquad (25) \\ &\quad + [\lambda(1 - u_{i,t+i}) + u_{i,t+i} e_{i,t+i} p(\theta_{t+i})] \int_{R_{i+1,t+i+1}}^{1} x dG(x) \end{aligned}$$

Here $\mathcal{C} = \{R_{i,t+i}, u_{i,t+i}, y_{i,t+i}, e_{i,t+i}, \theta_{t+i}\}$ is the vector of control variables. Equation (24) presents the unemployment dynamics between the ages of i and $i+1$ for the generation born in period t. As in equation (2), this takes into account that the jobs of workers of age i can be destroyed by the mere process of aging under age-increasing reservation productivity. This characteristic is shared by the production dynamics in equation (25): $y_{i,t}(R_{i+1,t+1})$ is the production that may possibly be made unprofitable by aging.[26]

The control variables in this optimization program are all indexed by both age i and birth date t, except for labor-market tightness which is not particular to any given age, as a result of our assumption of non-directed search. This latter variable covers all of the generations of workers equally, whereas the other variables are specific to a given generation over the life cycle. This already-complex dynamic problem is made even more difficult to deal with by the possible obsolescence due to aging. Appendix E shows the derivation of the first-order conditions of the planner's problem.

[26] See Appendix E for a detailed presentation of these constraints.

Definition 2. *Let $\eta(\theta^\star) = 1 - \frac{\theta^\star p'(\theta^\star)}{p(\theta^\star)}$, the first best allocation at the steady-state is defined by:*

$$ES^\star_{i+1} \equiv \int_{R^\star_{i+1}}^{1} S^\star_{i+1}(x)dG(x) \tag{26}$$

$$\frac{c}{q(\theta^\star)} = [1-\eta(\theta^\star)]\beta \sum_{i=1}^{T-1} \left(\frac{e^\star_i u^\star_i}{\widetilde{u}^\star} ES^\star_{i+1} \right) \tag{27}$$

$$R^\star_i = b - \phi(e^\star_i) - c e^\star_i \theta^\star - [\lambda - e^\star_i p(\theta^\star)]\beta ES^\star_{i+1} - (1-\lambda)\beta S^\star_{i+1}(R^\star_i) \tag{28}$$

$$S^\star_i(\epsilon) = \max\{\epsilon - R^\star_i + (1-\lambda)\beta[S^\star_{i+1}(\epsilon) - S^\star_{i+1}(R^\star_i)]; 0\} \tag{29}$$

$$\phi'(e^\star_i) = -c\theta^\star + p(\theta^\star)\beta ES^\star_{i+1} \tag{30}$$

$$u^\star_{i+1} = [1 - e^\star_i p(\theta^\star)(1 - G(R^\star_{i+1})]u^\star_i + (1-\lambda)n^\star_i(R^\star_{i+1}) + \lambda G(R^\star_{i+1})(1-u^\star_i) \tag{31}$$

$$n^\star_i(R^\star_{i+1}) = \begin{cases} (1-\lambda)[n^\star_{i-1}(R^\star_{i+1}) - n^\star_{i-1}(R^\star_i)] \\ +[\lambda(1-u^\star_i) + u^\star_i e^\star_i p(\theta^\star)][G(R^\star_{i+1}) - G(R^\star_i)] & \text{if } R^\star_i \leq R^\star_{i+1} \\ 0 & \text{if } R^\star_i > R^\star_{i+1} \end{cases} \tag{32}$$

with the terminal conditions $e^\star_{T-1} = 0$ and $R^\star_{T-1} = b$.

Equation (27) is similar to equation (12) from the decentralized equilibrium, with the workers' share of the employment surplus (γ) being replaced by the elasticity with respect to unemployment in the matching function ($\eta(\theta^\star)$). Equation (28) shows that a job is destroyed when the expected profit from the marginal job (the current product plus the option value for expected productivity shocks) fails to cover the social return to the unemployed worker. For the social planner, the return from an additional age-i unemployed worker is reduced by the cost of vacancy per age-i "efficient" unemployed worker, which equals $c\theta e_i$ (the social hiring cost). Equation (29) is the definition of job surplus using the expression for reservation productivity at the social optimum. The stock-flow dynamics on the labor market at the social optimum are given by equations (31) and (32).

Finally, equation (30) shows that the marginal cost of search effort is set equal to its social marginal return. The positive return is the expected surplus associated with the transition from unemployment to employment for a worker aged i ($p(\theta^\star)\beta ES^\star_{i+1}$). However, the planner takes into account the fact that greater effort devoted to search from a worker aged i reduces the likelihood that the other unemployed will find a job. In order to keep job opportunities per unemployed worker constant, the planner must raise the number of vacancies: this cost is measured by $c\theta^\star$ in equation (30). This hence implies that an unemployed worker should search only if the expected value of the transition from unemployment to employment is greater than this congestion cost. More formally, we have:

$$e^\star_i \geq 0 \quad \text{if and only if} \quad p(\theta^\star)\beta ES^\star_{i+1} \geq c\theta^\star \qquad (C)$$

We note that condition (C) always holds in the MP model:[27] hiring costs are lower than the

[27] In the MP model with homogenous agents, the free-entry condition is simply $\frac{c}{q(\theta)} = (1-\eta(\theta))\int_R^1 [1-G(x)]dx \Leftrightarrow$

expected job surplus. In our model, whereas hiring costs are shared by all of the unemployed, the expected job surplus for a specific age can be lower than its average value over all ages. The efficient allocation can then be such that optimal search is zero up to retirement if condition (C) does not hold. In this case, the social planner simply decides to reallocate these unemployed workers to home production. In the following, we assume for simplicity, and without loss of generality, that condition (C) is satisfied, $\forall i < T-1$.

Following the same reasoning as in the equilibrium allocation, we characterize the age-dynamics of the productivity thresholds $R_i^\star$ and search intensity $e_i^\star$, given the homogenous labor-market tightness of $\theta^\star$, using equations (28)-(30) independently of the other equations defining the social optimum.

Proposition 6. *If $\lambda > p(\theta^\star)e_i^\star$, $\forall i < T-1$, the optimal path is characterized by monotonic age-decreasing search effort $e_i^\star$ and monotonic age-increasing reservation productivity $R_i^\star$.*

Proof. See Appendix B. □

Proposition 6 restates, at the social optimum, the condition under which job separations are age-increasing, and the resulting implications for search intensity over the life cycle. This proposition emphasizes that higher (lower) job-destruction (creation) rates for older workers are not only an equilibrium outcome but also efficient when persistence and search effort are low enough to make labor hoarding more efficient than unemployment search for the social planner. Due to their shorter horizons, older workers should be fired more often and hired less often than younger workers. On the other hand, for the same reasons as in the market equilibrium, it is optimal for reservation productivity to fall with age when persistence and search are high enough (Proposition 7). We should emphasize that the age-i value of the expected job surplus amongst the unemployed, $\tau_i^\star$, defined by the counterpart of equation (10) at the social optimum, falls with age when the horizon effect dominates the reservation-wage effect.

Proposition 7. *If $\lambda < \gamma p(\theta)e_i^\star$, $\forall i < T-2$, and the horizon effect dominates the reservation-wage effect in the age-dynamics of expected job surplus, the optimal path is characterized by a monotonic age-decreasing profile for search effort $e_i^\star$ and a monotonic age-decreasing profile for reservation productivity $R_i^\star$, provided that the terminal condition $R_{T-2}^\star > R_{T-1}^\star$ holds.*

Proof. See Appendix C. □

Although the socially-optimal and equilibrium labor-market flows share the same age profile, this does not however imply that the equilibrium flow values are at the efficient level.

$c\theta = (1-\eta(\theta))p(\theta)\int_R^1[1-G(x)]dx$. For $\eta(\theta) \in]0;1[$, we deduce that in the MP model $c\theta < p(\theta)\int_R^1[1-G(x)]dx$.

4.2 The specific inefficiency of life-cycle equilibrium unemployment: the intergenerational externality

Traditionally, the equilibrium unemployment framework is known to generate search externalities which push the decentralized equilibrium away from the efficient allocation. However, when the elasticity relative to vacancies in the matching function is equal to the bargaining power of workers (the Hosios condition), social optimality can result. We now ask whether this result continues to hold when life-cycle features are taken into consideration.

It is well-known since Davis [2001] that a single matching function when there is heterogeneity across workers or jobs can create additional inefficiency. Traditionally, it is assumed that there are heterogenous jobs with different productivities, whereas workers are homogenous.[28] This case is exactly symmetric to ours, where jobs are identical but workers differ by age. Beyond this difference, it should be emphasized that there is a specific life-cycle efficiency gap created by the horizon heterogeneity: the individual efficiency gaps do not cancel out when averaged out over the different generations of unemployed workers due to job-duration heterogeneity. This particularity implies that job creation is intrinsically inefficient. To unveil this property, assume that search effort is constant by age and normalized to unity ($e_i = 1, \forall i$) and that the destruction rate is exogenous, and denoted by s. Then, the equilibrium and first-best allocations are defined as follows:

Equilibrium

$$\begin{aligned}
S_i &= y - b - \tfrac{1}{1-\gamma} c\theta\tau_i + c\theta\tau_i + \beta(1-s)S_{i+1} \\
\tfrac{c}{q(\theta)} &= \beta(1-\gamma)\textstyle\sum_{i=1}^{T-1}\left(\tfrac{u_i}{\widetilde{u}}S_{i+1}\right) \\
u_{i+1} &= u_i\left[1-p(\theta)\right] + s(1-u_i) \\
\tau_i &= \frac{S_{i+1}}{\sum_{i=1}^{T-1}\left(\frac{u_i}{\widetilde{u}}S_{i+1}\right)}
\end{aligned}$$

First best

$$\begin{aligned}
S_i^\star &= y - b - \tfrac{1}{1-\eta(\theta^\star)} c\theta^\star\tau_i^\star + c\theta^\star + \beta(1-s)S_{i+1}^\star \\
\tfrac{c}{q(\theta^\star)} &= \beta[1-\eta(\theta^\star)]\textstyle\sum_{i=1}^{T-1}\left(\tfrac{u_i^\star}{\widetilde{u}^\star}S_{i+1}^\star\right) \\
u_{i+1}^\star &= u_i^\star\left[1-p(\theta^\star)\right] + s(1-u_i^\star) \\
\tau_i^\star &= \frac{S_{i+1}^\star}{\sum_{i=1}^{T-1}\left(\frac{u_i^\star}{\widetilde{u}^\star}S_{i+1}^\star\right)}
\end{aligned}$$

The Hosios condition $\gamma = \eta(\theta^\star)$ allows the market and the planner to agree on the return to unemployment search for any age-i worker ($\frac{1}{1-\gamma}c\theta\tau_i = \frac{1}{1-\eta(\theta^\star)}c\theta^\star\tau_i^\star$). But they disagree on the valuation of the hiring costs ($c\theta\tau_i$ for the market versus $c\theta^\star$ for the planner), and thus on the job surplus. The ex-ante hiring costs are sunk for the market, and only the ex-post hiring costs saved by an age-specific hire are accounted for in the wage-bargaining process. On the contrary, for the planner, only the ex-ante hiring costs, independent of the worker's age, matter. As each individual return to unemployment search differs from the average value ($\tau_i \neq 1$), social hiring costs are not properly internalized in equilibrium. If the matched worker is young (old), the market over(under)-estimates the hiring costs. Nevertheless, these age-specific gaps could lead to efficient vacancies, as the latter depend on average job surplus. Given the expression for the

[28] See for example Acemoglu [2001], Bertola and Caballero [1994] and Ljunqvist and Sargent [2000].

age-specific job surplus in the simplified model, this is the case if and only if:

$$\sum_{i=1}^{T-1} \frac{u_i}{\widetilde{u}} \left[\tau_i + \sum_{j=1}^{T-i-1} \tau_{i+j} \beta^j (1-s)^j \right] c\theta \;\; = \;\; \sum_{i=1}^{T-1} \frac{u_i^\star}{\widetilde{u}^\star} \left[1 + \sum_{j=1}^{T-i-1} \beta^j (1-s)^j \right] c\theta^\star \qquad (33)$$

As long as workers are heterogenous in terms of distance to retirement, condition (33) never holds and job creation is not socially efficient. The distortion does not vanish because the summation over ages is truncated by the retirement of older workers, making job duration different by age. By way of contrast, this is why condition (33) holds when all jobs exogenously last for only one period ($s \to 1$). This is also the case in a model with some job persistence but without any heterogeneity in the expected duration of the match, typically in an infinite-life environment (see Appendix F). In our life-cycle framework, only the initial distortions at the hiring stage of all the jobs compensate for each other, because, by the definition of τ_i, we have $\sum_{i=1}^{T-1} \frac{u_i}{\widetilde{u}} \tau_i = 1$: the overestimation of the hiring costs induced by the hiring of younger workers is fully compensated by the underestimation of these costs implied by the hiring of older workers. This is no longer the case over all job durations, the future of younger workers not being compensated for. Moreover, as this future is aging, the average job surplus is underestimated in equilibrium, due to the age-decreasing profile of τ_i. There is thus less incentive to post vacancies at the market equilibrium than for the planner ($\theta < \theta^\star$). In a nutshell, the intergenerational externality results from endogenous heterogeneity due to the distance to retirement, and also implies inefficient job-creation decisions *per se*.

The comparison between the equilibrium and the first-best allocations in the simplified model shows that they differ in terms of the valuation of the job surplus. This difference identically exists in the complete model with endogenous separation and search effort. Moreover, as these latter decisions are based on the individual job surplus, they are necessarily distorted, even in the case of undistorted job creation.[29] To show this result, it is more convenient to rewrite equations (28) and (30) as follows:

$$R_i^\star + \lambda\beta E S_{i+1}^\star + (1-\lambda)\beta S_{i+1}^\star(R_i^\star) \;\; = \;\; b - \phi(e_i^\star) + \frac{1}{1-\eta(\theta^\star)} c e_i^\star \theta^\star \tau_i^\star - c e_i^\star \theta^\star \qquad (34)$$

$$\phi'(e_i^\star) \;\; = \;\; \frac{1}{1-\eta(\theta^\star)} c\theta^\star \tau_i^\star - c\theta^\star \qquad (35)$$

and to compare them to their equilibrium counterparts:

$$R_i + \lambda\beta E S_{i+1} + (1-\lambda)\beta S_{i+1}(R_i) \;\; = \;\; b - \phi(e_i) + \frac{1}{1-\gamma} c\theta e_i \tau_i - c\theta e_i \tau_i \qquad (36)$$

$$\phi'(e_i) \;\; = \;\; \frac{1}{1-\gamma} c\theta\tau_i - c\theta\tau_i \qquad (37)$$

As the evaluation of the hiring costs is not the same for the market ($c\theta\tau_i$ in equations (36) and (37)) as for the planner ($c\theta^\star$ in equations (34) and (35)), the Hosios condition no longer yields

[29] In the particular case of purely idiosyncratic shocks ($\lambda = 1$), as jobs last only one period, the vacancy decision is not distorted *per se*.

efficiency in the separation and search effort decisions. The key point is again that the individual return to unemployment search through wage bargaining cannot capture the age-independent size of hiring costs. When $\tau_i < 1$, for older workers, the search cost is under-estimated: reservation productivity and search effort are then both too high in equilibrium. The opposite is true for younger workers. It is worth re-emphasizing this efficiency gap by considering the equilibrium wage:

$$w_i(\epsilon) = \gamma\left(\epsilon + c\theta e_i\right) + (1-\gamma)\left(b - \phi(e_i)\right) + \gamma c\theta e_i\left(\tau_i - 1\right) \tag{38}$$

whereas the optimal wage would be:

$$w_i^\star(\epsilon) = \eta(\theta^\star)\left(\epsilon + c\theta^\star e_i^\star\right) + (1-\eta(\theta^\star))\left(b - \phi(e_i^\star)\right) + c\theta^\star e_i^\star\left(\tau_i^\star - 1\right) \tag{39}$$

If $\gamma = \eta(\theta^\star)$, the gap between the equilibrium and optimal wage is:

$$w_i^\star(\epsilon) - w_i(\epsilon) = (1-\gamma)c\theta^\star e_i^\star(\tau_i^\star - 1)$$

For younger (older) workers $\tau_i^\star > 1$ ($\tau_i^\star < 1$), implementing the optimal wage would lead to a higher (lower) wage. As vacancy investments are made as a function of the representative worker, when the contact is a younger (older) worker, the job surplus is higher (lower) than expected. The optimal evaluation of this ex post gain (loss) is $c\theta^\star e_i^\star\left(\tau_i - 1\right)$ (see equation (39)). In equilibrium, only a fraction γ of this value is captured by the worker (see equation (38)), leading to an incorrect evaluation of individual job surplus. Were the bargaining power of workers γ to equal one, this inefficiency would be eliminated. This result is in line with Davis [2001]: there exists a tension between the condition for an efficient mix of jobs and the condition for an efficient total number of jobs.

4.3 The optimal policy

Labor-market policies designed by age may allow firms and workers to internalize the intergenerational externality. In order to specifically deal with this externality, we assume that the Hosios condition is satisfied ($\eta(\theta^\star) = \gamma$). To restore optimality, we propose introducing age-specific taxes or subsidies. As different decisions are distorted, from job creation to job separation passing via search effort, different instruments may be necessary. We first assume that search effort is exogenous in order to show that it is possible to restore optimality in job creation and separation through a unique tax on filled jobs. Second, we reintroduce endogenous search effort: we then show that a unique tax on search effort suffices to restore optimality for all decisions.

4.3.1 Exogenous search effort and employment policy

When search effort is exogenous, the age-specific externality cannot be partially compensated for by age-specific search intensity: an older worker has the same probability of meeting a firm as a younger worker. The equilibrium job surplus can be affected only via a change in the surplus of a filled job. We consider an age-specific tax/subsidy (a_i) on jobs once they have been filled. This policy, which aims to eliminate the gap between the equilibrium and optimal job surpluses, or equivalently between the equilibrium and optimal reservation productivity, can be derived by comparing equations (40) and (41):

$$\underbrace{R_i + \lambda\beta E S_{i+1} + (1-\lambda)\beta S_{i+1}(R_i)}_{\substack{\text{Lowest worker value inside}\\ \text{the production sector}}} = \underbrace{b - a_i + \frac{1}{1-\gamma} c\theta\tau_i - c\theta\tau_i}_{\substack{\text{Worker value outside}\\ \text{the production sector}}} \tag{40}$$

$$\overbrace{R_i^\star + \lambda\beta E S_{i+1}^\star + (1-\lambda)\beta S_{i+1}^\star(R_i^\star)} = \overbrace{b + \frac{1}{1-\eta(\theta^\star)} c\theta^\star\tau_i^\star - c\theta^\star} \tag{41}$$

Proposition 8. *An optimal age-sequence for employment subsidies, denoted by* $\{a_i\}_{i=1}^{T-1}$, *is* $a_i = c\theta^\star(1-\tau_i^\star)$, *where* $\theta^\star$ *and* $R_i^\star$ *are defined by the efficient allocation.*

Proof. Straightforward by comparing equations (40) and (41). □

Note that the optimal policy rule is independent of the age-profile of reservation productivity, as the relative value of the expected job surplus for the unemployed of age i, $\tau_i^\star$, always falls with age under the conditions of Propositions 6 and 7.

Corollary 3. *Optimal employment subsidies increase with age,* $a_i^\star < a_{i+1}^\star \;\; \forall i \in [0, T-1]$, *and there exists a threshold age* $\tilde{i}$ *such that* $a_i \leq 0 \;\; \forall i \in [0, \tilde{i}]$ *and* $a_i \geq 0 \;\; \forall i \in [\tilde{i}, T-1]$.

Proof. Whatever the relationship between the productivity threshold and age, expected job surplus falls with age under the conditions of Propositions 6 and 7. We necessarily have $\tau_i^\star > \tau_{i+1}^\star$, implying that $a_i^\star < a_{i+1}^\star$. Because $\tau_1^\star > 1$ and $\tau_{T-1}^\star < 1$, there exists $a_{\tilde{i}}^\star = c\theta^\star(1-\tau_{\tilde{i}}^\star) = 0$ □

This implies a greater subsidy for the employment of older workers, or even a tax on the employment of younger workers (for $i \leq \tilde{i}$). By setting the equilibrium job surplus at its optimal level for each age, the expected job surplus averaged over all of the unemployed is at its optimal value, leading to efficient job creation. The investment in vacancies taken without any information concerning the future match is now the same as the planner's, as there is a compensatory age-dependent subsidy/tax once the job is filled by a worker at a given distance to retirement.

4.3.2 Endogenous search effort and unemployment policy

Consider the general case with endogenous search effort. It is reasonably intuitive that making this decision optimal, which implies correcting for the value of the search cost considered by the unemployed, is also of benefit for job-separation and job-creation decisions. We consider a subsidy which is conditional on search effort, $s_i e_i$. The instantaneous utility of an unemployed worker is then $b - \phi(e_i) + s_i e_i$. Consider again the employment subsidy of a_i. In this case, the equilibrium conditions are:

$$R_i + \lambda\beta E S_{i+1} + (1-\lambda)\beta S_{i+1}(R_i) = b - a_i - \phi(e_i) + s_i e_i + \frac{1}{1-\gamma} c\theta e_i \tau_i - c\theta e_i \tau_i \quad (42)$$

$$\phi'(e_i) = s_i + \frac{1}{1-\gamma} c\theta \tau_i - c\theta \tau_i \quad (43)$$

whereas the optimal allocation satisfies equations (34) and (35). Equation (43) shows that the marginal cost of the search effort is equal to its marginal return net of the subsidy s_i.

Proposition 9. *An optimal age-sequence for employment and search effort subsidies, denoted* $\{a_i^\star, s_i^\star\}_{i=1}^{T-1}$*, solves* $a_i^\star = 0$ *and* $s_i^\star = -c\theta^\star(1-\tau_i^\star)$.

Proof. Straightforward from the comparison of equations (42) and (43) with equations (34) and (35). □

Proposition 9 shows that only one instrument is required to attain the optimal allocation. This result is intuitive: search policy intervenes upstream from employment policy, at the root of the inefficiency centered on the return to unemployment search. Restoring optimality in search implies that the return to unemployment search be at its optimal level, which then makes the separation and vacancy decisions optimal.

Corollary 4. *The optimal age-dynamics of search effort subsidies is characterized by* $s_i^\star > s_{i+1}^\star \;\; \forall i \in [0, T-1]$*, and there exists a threshold age* $\tilde{i}$ *such that* $s_i \geq 0 \;\; \forall i \in [0, \tilde{i}]$ *and* $s_i \leq 0 \;\; \forall i \in [\tilde{i}, T-1]$.

Proof. See the proof of Corollary 3. □

The above suggests that it is optimal to tax the search effort of older workers (and to subsidize that of younger workers): it is optimal to discourage older workers from job search. As was the case for employment policy, the objective is to keep older workers away from search. This result provides some theoretical arguments for the existence of pre-retirement schemes in some European countries.[30] In France, for example, unemployed workers aged over 57 receive generous benefits if they do not search for a job. It should be emphasized that our result supports this policy and its particular age threshold only because these workers are close to retirement (at age 60 in France).

On the other hand, the size of the tax/subsidy correcting for the efficiency gap yields some insights into the quantitative importance of the intergenerational externality. The expected job surplus

[30] See Hairault et al. [2009] for a similar conclusion in a principal-agent framework.

is close to zero for older workers ($\tau_o^\star = 0$), implying that the tax is of the same magnitude as the hiring costs ($s_o = -c\theta^\star$). A back-of-the envelope calculation allows us to determine the subsidy that should be provided for younger workers: assume that the population consists of three age groups of the same size, and that the expected surplus of each population is proportional to their horizon, leading to $\tau_y^\star = 2$ for younger workers, with prime-age workers being representative of the average job surplus.[31] The optimal policy for younger workers is then $s_y = c\theta^\star$. Overall, the correction induced by the policy is then of the same order as the labor-market frictions, implying that the intergenerational externality is of quite significant size in the search framework.[32]

5 Conclusion

Older workers have shorter horizons: as such firms and workers invest less in job search and labor hoarding at the end of the life cycle. The retirement age is then a key factor governing the employment rate of older workers. Countries with lower retirement ages will also suffer from lower employment rates for older workers at a relatively early age. This may explain why countries with a retirement age of around 60, such as France and Belgium, also have lower employment rates for workers aged between 55 and 59 than those with a retirement age of 65, such as Sweden and the United States (see Figure 1).

Age policies are required to attain the optimal level of output when search is not age-directed. In equilibrium, due to an intergenerational externality, there are not enough (too many) job destructions for younger (older) workers. This is why it is optimal to subsidize the employment of older workers. Regarding labor supply, we show that it could be optimal to discourage search by older workers.

It should be emphasized that all of these results are independent of the age profile of human capital. Traditionally, this has been at the heart of the life-cycle approach, especially when the aim is to provide a quantitative analysis of the age profile of wages. This paper leaves aside this dimension to focus on the basics of life-cycle equilibrium unemployment theory. On the one hand, in the case of specific human capital, which is not transferable to a new job, it is intuitive that the horizon effect is the only factor determining the relative return to unemployment search by age. On the other hand, the return to unemployment search of older workers may be greater than that of younger workers if their greater general human capital dominates their shorter horizons.

[31] Note that these numbers depend on the population structure, which determines the relative scarcity of younger workers. More younger workers in the population implies that they are closer to the average group. This would lead to a subsidy less than proportional to hiring costs, whereas the tax on older workers would remain unchanged.

[32] Given the optimal choices of the unemployed, implying $e_y > 0$ and $e_o = 0$, and using a quadratic cost-search function, the total subsidies received by the younger unemployed are $s_y^\star e_y^\star = 2\left(0.5 + \frac{\gamma}{1-\gamma}\right)(c\theta^\star)^2$. Using the values in Pissarides [2009] ($c = .356$, $\theta = .72$, $\gamma = \eta = .5$ and $b = .7$), we then deduce that $s_y e_y = 0.196$, corresponding to a 28.1% increase in unemployment income ($b + s_y e_y$).

In this case, the age-dependent policy could be reversed. However, this case is fairly unrealistic as general human-capital accumulation is typically considered to be hump-shaped with a peak at about age forty (Kotlikoff and Gokhale [1992]). Our normative results are then robust to the introduction of human capital.[33] However, we acknowledge that taking into account human capital along the lines of Menzio, Telyukova and Visschers [2010] could well be useful for a more quantitative analysis of life-cycle equilibrium unemployment. This is left for further research.

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A Proof of Proposition 1

When $R_i < R_{i+1}$, the job surplus is defined as follows:

$$S_i(\epsilon) = \max\{\epsilon - R_i + (1-\lambda)\beta S_{i+1}(\epsilon); 0\}$$

1. If $\epsilon > R_{T-1}$, then $S_{i+1}(\epsilon) > 0, \forall i$. It is then possible by forward iteration to derive the following expression for the job surplus:

$$S_i(\epsilon) = \sum_{j=0}^{T-i-1} \beta^j (1-\lambda)^j (\epsilon - R_{i+j})$$

2. If $R_{T-2} < \epsilon < R_{T-1}$, then $S_{i+1}(\epsilon) > 0, \forall i \leq T-2$ and $S_{T-1}(\epsilon) = 0$. We then obtain:

$$S_i(\epsilon) = \sum_{j=0}^{T-i-2} \beta^j (1-\lambda)^j (\epsilon - R_{i+j})$$

3. If $R_{T-3} < \epsilon < R_{T-2}$, then $S_{i+1}(\epsilon) > 0, \forall i \leq T-3$ and $S_{i+1}(\epsilon) = 0, \forall i > T-3$ and then:

$$S_i(\epsilon) = \sum_{j=0}^{T-i-3} \beta^j (1-\lambda)^j (\epsilon - R_{i+j})$$

4. ...

For $R_{T-2} < \epsilon$, whatever the sign of $\epsilon - R_{T-1}$, note that it is possible to write $S_i(\epsilon)$, as follows:

$$S_i(\epsilon) = \sum_{j=0}^{T-i-2} \beta^j (1-\lambda)^j (\epsilon - R_{i+j}) + \beta^{T-i-1}(1-\lambda)^{T-i-1} \max[(\epsilon - R_{T-1}), 0]$$

For $R_{T-3} < \epsilon$, whatever the sign of $\epsilon - R_{T-2}$ and $\epsilon - R_{T-1}$, we have:

$$\begin{aligned} S_i(\epsilon) &= \sum_{j=0}^{T-i-3} \beta^j (1-\lambda)^j (\epsilon - R_{i+j}) + \beta^{T-i-2}(1-\lambda)^{T-i-2} \max[(\epsilon - R_{T-2}), 0] \\ &+ \beta^{T-i-1}(1-\lambda)^{T-i-1} \max[(\epsilon - R_{T-1}), 0] \end{aligned}$$

Finally, whatever the value of ϵ with respect to the different productivity thresholds, we obtain equation (16). As Proposition 1 is based on a continuous age-increasing profile in reservation productivity, it requires that $R_i < R_{i+1}, \forall i$. Noting that $\int_{R_{i+j}}^{1} [x - R_{i+j}] dG(x) = \int_{R_{i+j}}^{1} [1 - G(x)] dx$, we then obtain equation (17).

B Proof of Proposition 6

If $R_i^\star < R_{i+1}^\star$, $\forall i < T-1$, then the solutions for the surplus, reservation productivity and search effort are:

$$\begin{aligned} S_i^*(\epsilon) &= \sum_{j=0}^{T-1-i} \beta^j (1-\lambda)^j \max\{\epsilon - R_{i+j}^\star; 0\} && (44) \\ R_i^\star &= b - \phi(e_i^\star) - ce_i^\star\theta^\star - [\lambda - e_i^\star p(\theta^\star)]\beta ES_{i+1}^\star && (45) \\ \phi'(e_i^\star) &= -c\theta^\star + p(\theta^\star)\beta ES_{i+1}^\star && (46) \end{aligned}$$

We denote $I(y) = \int_y^1 (x-y)dG(x) = \int_y^1 (1-G(x))dx$, with $I'(x) < 0$, and $ES_i^\star = \int_0^1 S_i^*(x)dG(x) = \sum_{j=0}^{T-1-i} \beta^j (1-\lambda)^j I(R_{i+j}^\star)$.

We start by showing that $R_{i+1}^\star > R_i^\star$ implies $e_{i+1}^\star < e_i^\star$. As $ES_i^\star - ES_{i+1}^\star = [I(R_i^\star) - I(R_{i+1}^\star)] + \beta(1-\lambda)(ES_{i+1}^\star - ES_{i+2}^\star)$, and given that $I'(x) < 0$ and $ES_{T-2}^\star > ES_{T-1}^\star$, it is then straightforward that $ES_i^\star > ES_{i+1}^\star$, and so that $\tau_i^\star > \tau_{i+1}^\star$. If condition (C) is satisfied and using equation (46), we then obtain $e_{i+1}^\star < e_i^\star$, given that $\phi''(\cdot) > 0$.

It remains to show that $R_i^\star < R_{i+1}^\star$, $\forall i < T-1$, if $\lambda > p(\theta^\star)e_i^\star$. Consistent with equations (45) and (46), the solution for reservation productivity can be written as follows:

$$p(\theta^\star)R_i^\star = p(\theta^\star)b - \lambda c\theta^\star - \underbrace{[p(\theta^\star)\phi(e_i^\star) + (\lambda - p(\theta^\star)e_i^\star)\phi'(e_i^\star)]}_{\equiv \mathcal{Z}(e_i)}$$

If condition (C) is satisfied, $R_i^\star$ is then age-increasing if and only if

$$\begin{aligned} p(\theta^\star)(R_{i+1}^\star - R_i^\star) &= \mathcal{Z}(e_i^\star) - \mathcal{Z}(e_{i+1}^\star) > 0 \qquad \forall i < T-2 \\ p(\theta^\star)(R_{T-1}^\star - R_{T-2}^\star) &= \lambda c\theta^\star + \mathcal{Z}(e_{T-2}^\star) > 0 \end{aligned}$$

We have $R_i^\star < R_{i+1}^\star$ $\forall i < T-2$, implying that $e_i^\star > e_{i+1}^\star$ if and only if $\mathcal{Z}'(x) > 0$, $\forall x \in [e_{T-2}^\star, e_1^\star]$. As $\mathcal{Z}'(x) = (\lambda - p(\theta^\star)x)\phi''(x)$, the condition $\lambda > p(\theta^\star)e_i^\star$, $\forall i \leq T-2$, is sufficient to ensure that $R_i^\star < R_{i+1}^\star$, $\forall i < T-2$. On the other hand, for $i = T-2$, the terminal restriction is given by $\lambda c\theta^\star + \mathcal{Z}(e_{T-2}^\star) > 0 \Leftrightarrow \lambda > \left(\frac{\omega(e_{T-2}^\star)-1}{\omega(e_{T-2}^\star)\left(1+\frac{c\theta^\star}{\phi'(e_{T-2}^\star)}\right)}\right) p(\theta^\star)e_{T-2}^\star$, where $\omega(e_{T-2}^\star)$ is the elasticity of the search cost function at age $T-2$. This inequality is less restrictive than $\lambda > p(\theta^\star)e_{T-2}^\star$. This is why, $\forall i < T-1$, $\lambda > p(\theta^\star)e_i^\star$ implies that $R_i^\star < R_{i+1}^\star$.

C Proof of Proposition 7

If $R_i^\star > R_{i+1}^\star$, $\forall i < T-1$, then the solutions for the surplus, reservation productivity and search are:

$$S_i^*(\epsilon) = P_i(T)\max\{\epsilon - R_i^\star; 0\} \quad \text{with } P_i(T) = \sum_{j=0}^{T-1-i} \beta^j (1-\lambda)^j \tag{47}$$

$$R_i^\star = b - ce_i^\star\theta^\star - \phi(e_i^\star) - [\lambda - e_i^\star p(\theta)]\beta ES_{i+1}^\star - (1-\lambda)\beta P_{i+1}(T)(R_i^\star - R_{i+1}^\star) \tag{48}$$

$$\phi'(e_i^\star) = -c\theta^\star + p(\theta^\star)\beta ES_{i+1}^\star \tag{49}$$

where $ES_i^\star = \int_0^1 S_i^*(x)dG(x) = P_i(T)I(R_i^\star)$, with $I(y) = \int_y^1 (1-G(x))dx$ and then $I'(x) < 0$.

We start by showing that $R_{i+1}^\star < R_i^\star$ implies that $e_{i+1}^\star < e_i^\star$ only if the horizon effect dominates the reservation-wage effect. As $ES_i^\star - ES_{i+1}^\star = [P_i(T) - P_{i+1}(T)]I(R_i^\star) - P_{i+1}(T)\int_{R_{i+1}^\star}^{R_i^\star}[1 - G(x)]dx$, given that $[P_i(T) - P_{i+1}(T)] > 0$ and $R_{i+1}^\star < R_i^\star$, we obtain $ES_i^\star \lesseqgtr ES_{i+1}^\star$. Using equation (49), we then deduce that $e_i^\star \lesseqgtr e_{i+1}^\star$, given that $\phi(\cdot) > 0$, $\phi'(\cdot) > 0$ and $\phi''(\cdot) > 0$. If the horizon effect dominates the reservation-wage effect, i.e. $[P_i(T) - P_{i+1}(T)]I(R_i^\star) > P_{i+1}(T)\int_{R_{i+1}^\star}^{R_i^\star}[1 - G(x)]$, then $ES_i^\star > ES_{i+1}^\star$, implying that $e_i^\star > e_{i+1}^\star$ and $\tau_i^\star > \tau_{i+1}^\star$.

Equations (48) and (49) yield:

$$p(\theta)R_i = p(\theta)b - \lambda c\theta^\star - \mathcal{Z}(e_i^\star) + p(\theta^\star)\beta(1-\lambda)P_{i+1}(T)(R_{i+1}^\star - R_i^\star)$$

If condition (C) is satisfied, $R_i^\star$ is age-decreasing if and only if

$$\begin{aligned} R_{i+1}^\star - R_i^\star &= \frac{\mathcal{Z}(e_i^\star) - \mathcal{Z}(e_{i+1}^\star) + p(\theta^\star)\beta(1-\lambda)P_{i+2}(T)(R_{i+2}^\star - R_{i+1}^\star)}{p(\theta^\star)[1+\beta(1-\lambda)P_{i+1}(T)]} < 0 \qquad \forall i < T-2 \\ R_{T-1}^\star - R_{T-2}^\star &= \frac{\lambda c\theta^\star + \mathcal{Z}(e_{T-2}^\star)}{p(\theta^\star)[1+\beta(1-\lambda)]} < 0 \end{aligned}$$

If the terminal restriction is satisfied ($R_{T-1}^\star - R_{T-2}^\star < 0 \Leftrightarrow \lambda c\theta^\star + \mathcal{Z}(e_{T-2}^\star) < 0$), it is sufficient by backward induction to determine the restriction which ensures that $R_{i+1}^\star - R_i^\star < 0$, given that $R_{i+2}^\star - R_{i+1}^\star$ is negative. The terminal condition is $\lambda < \frac{\omega(e_{T-2}^\star) - 1}{\omega(e_{T-2}^\star)\left(1 + \frac{c\theta^\star}{\phi'(e_{T-2}^\star)}\right)} p(\theta^\star)e_{T-2}^\star$, which is more restrictive than the condition $\lambda < p(\theta)e_{T-2}^\star$. If $e_i^\star$ is age-decreasing and $\mathcal{Z}'(x) < 0$, $\forall x \in [e_{T-2}^\star, e_1^\star]$, i.e. $\lambda < p(\theta^\star)e_i^\star$, $\forall i \leq T-2$, then $\mathcal{Z}(e_i^\star) - \mathcal{Z}(e_{i+1}^\star) < 0$ and $R_{i+1}^\star - R_i^\star < 0$, $\forall i < T-2$. Consistent with $e_i^\star$ falling with age, the condition $\lambda < p(\theta^\star)e_i^\star$ is the most restrictive for $e_{T-2}^\star$, implying that the terminal condition suffices to ensure that $R_i^\star > R_{i+1}^\star$.

D U-shaped reservation productivity

D.1 The match surplus

If there exists an age $\widehat{i}$ such that $R_i > R_{i+1}$ for $i < \widehat{i}$ and $R_i < R_{i+1}$ for $i \geq \widehat{i}$, the job surplus is given by:

$$S_i(\epsilon) = \begin{cases} \max\left\{(\epsilon - R_i) + \beta(1-\lambda)\left[S_{i+1}(\epsilon) - S_{i+1}(R_i)\right];0\right\} & \text{for} \quad i < \widehat{i} \\ \max\left\{(\epsilon - R_i) + \beta(1-\lambda)S_{i+1}(\epsilon);0\right\} & \text{for} \quad i \geq \widehat{i} \end{cases}$$

Forward iteration leads to the following expression for the surplus, for $i \in [\widehat{i}; T-1]$

$$S_i(\epsilon) = \sum_{j=0}^{T-1-i} [\beta(1-\lambda)]^j \max\left\{\epsilon - R_{i+j};0\right\}$$

whereas for $i < \widehat{i}$, we have

$$\begin{aligned} S_i(\epsilon) &= \max\left\{\left(\sum_{j=0}^{\widehat{i}-i-1}[\beta(1-\lambda)]^j\right)(\epsilon - R_i) + [\beta(1-\lambda)]^{\widehat{i}-i}\left[S_{\widehat{i}}(\epsilon) - S_{\widehat{i}}(R_i)\right];0\right\} \\ &= P_i(\widehat{i})\max\left\{\epsilon - R_i;0\right\} + \mathcal{V}_i(\widehat{i},\epsilon) \end{aligned} \tag{50}$$

The U-shape of R_i implies the presence of the additional term $\mathcal{V}_i(\widehat{i},\epsilon)$ when considering the negative relationship up to $\widehat{i}$. On the other hand, the analysis is unchanged after $\widehat{i}$.

D.2 The age-dynamics of reservation productivity after age $\widehat{i}$

For $i \geq \widehat{i}$ we have

$$R_{i+1} - R_i = \frac{-\Upsilon(e_{i+1}) + \Upsilon(e_i)}{\gamma p(\theta)}$$

As e_i is age-decreasing (see Proposition 1), and if $\Upsilon'(x) > 0, \forall x \in [e_{T-2}, e_{\widehat{i}}[$, i.e. $\lambda > \gamma p(\theta)e_i, \forall i > \widehat{i}$, then $-\Upsilon(e_{i+1}) + \Upsilon(e_i) > 0$, which ensures that $R_i < R_{i+1}, \forall i \geq \widehat{i}$.

D.3 The age-dynamics of reservation productivity before age $\widehat{i}$

The dynamics of the reservation productivity is given by:

$$\begin{aligned} R_{i+1} - R_i &= \frac{-\Upsilon(e_{i+1}) + \Upsilon(e_i)}{\gamma p(\theta)[1 + \beta(1-\lambda)P_{i+1}(\widehat{i})]} \\ &\quad + \frac{\gamma p(\theta)(1-\lambda)\beta[P_{i+2}(\widehat{i})(R_{i+2} - R_{i+1}) + (\mathcal{V}_{i+1}(\widehat{i},R_i) - \mathcal{V}_{i+2}(\widehat{i},R_{i+1}))]}{\gamma p(\theta)[1 + \beta(1-\lambda)P_{i+1}(\widehat{i})]} \\ &= \frac{-\Upsilon(e_{i+1}) + \Upsilon(e_i)}{\gamma p(\theta)[1 + \beta(1-\lambda)P_{i+1}(\widehat{i})]} \\ &\quad + \Phi_{1,i}(R_{i+2} - R_{i+1}) + \Phi_{2,i}(\mathcal{V}_{i+1}(\widehat{i},R_i) - \mathcal{V}_{i+2}(\widehat{i},R_{i+1})) \end{aligned} \tag{51}$$

with $\Phi_{j,i} > 0$ for $i = 1, 2$. In the case where reservation productivity is continuously age-decreasing, a sufficient condition to ensure the existence of this sequence is $-\Upsilon(e_{i+1}) + \Upsilon(e_i) < 0$. In the U-shaped case, it is necessary to solve the continuation values after $\widehat{i}$, i.e. $\mathcal{V}_{i+1}(\widehat{i}, R_i)$ and $\mathcal{V}_{i+2}(\widehat{i}, R_{i+1})$, in order to determine the sufficient condition for $R_{i+1} < R_i$. Using the definition of $\mathcal{V}_{i+1}(\widehat{i}, R_i)$, we obtain:

$$\begin{aligned}
\mathcal{V}_{i+2}(\widehat{i}, R_{i+1}) &= [\beta(1-\lambda)]^{\widehat{i}-i-2}\left[S_{\widehat{i}}(R_{i+1}) - S_{\widehat{i}}(R_{i+2})\right] \\
\mathcal{V}_{i+1}(\widehat{i}, R_i) &= [\beta(1-\lambda)]^{\widehat{i}-i-1}\left[S_{\widehat{i}}(R_i) - S_{\widehat{i}}(R_{i+1})\right] \\
\Rightarrow \mathcal{V}_{i+1}(\widehat{i}, R_i) - \mathcal{V}_{i+2}(\widehat{i}, R_{i+1}) &= [\beta(1-\lambda)]^{\widehat{i}-i-2}\left\{[\beta(1-\lambda)-1](S_{\widehat{i}}(R_i) - S_{\widehat{i}}(R_{i+1})) + \Delta\right\} \\
\text{with } \Delta &= \left[S_{\widehat{i}}(R_i) - S_{\widehat{i}}(R_{i+1})\right] - \left[S_{\widehat{i}}(R_{i+1}) - S_{\widehat{i}}(R_{i+2})\right]
\end{aligned}$$

where the first term is unambiguously negative, whereas the sign of Δ is a priori indeterminate. Using the definition of $S_i(\epsilon)$ for $i > \widehat{i}$, we have:

$$\begin{aligned}
S_{\widehat{i}}(R_{i+1}) - S_{\widehat{i}}(R_{i+2}) &= \sum_{j=0}^{T-\widehat{i}-2} [\beta(1-\lambda)]^j \left(\max\{R_{i+1} - R_{\widehat{i}+j}; 0\} - \max\{R_{i+2} - R_{\widehat{i}+j}; 0\}\right) \\
S_{\widehat{i}}(R_i) - S_{\widehat{i}}(R_{i+1}) &= \sum_{j=0}^{T-\widehat{i}-1} [\beta(1-\lambda)]^j \left(\max\{R_i - R_{\widehat{i}+j}; 0\} - \max\{R_{i+1} - R_{\widehat{i}+j}; 0\}\right)
\end{aligned}$$

There exist some ages $\widehat{i}+p$, $\widehat{i}+n$ and $\widehat{i}+m$ after which the marginal jobs, respectively R_i, R_{i+1} and R_{i+2}, are closed after $\widehat{i}$. The associated productivity thresholds are such that $\max\{R_i - R_{\widehat{i}+p}; 0\} = 0$, $\max\{R_{i+1} - R_{\widehat{i}+n}; 0\} = 0$ and $\max\{R_{i+2} - R_{\widehat{i}+m}; 0\} = 0$. As $R_i > R_{i+1} > R_{i+2}$, we have $p > n > m$. From the previous expressions, we deduce that:

$$\begin{aligned}
S_{\widehat{i}}(R_{i+1}) - S_{\widehat{i}}(R_{i+2}) &= \left(\sum_{j=0}^{\widehat{i}+m-1} [\beta(1-\lambda)]^j\right)(R_{i+1} - R_{i+2}) \\
&\quad + [\beta(1-\lambda)]^m \sum_{j=0}^{n-m} [\beta(1-\lambda)]^j (R_{i+1} - R_{\widehat{i}+m+j})
\end{aligned}$$

$$\begin{aligned}
S_{\widehat{i}}(R_i) - S_{\widehat{i}}(R_{i+1}) &= \left(\sum_{j=0}^{\widehat{i}+n-1} [\beta(1-\lambda)]^j\right)(R_i - R_{i+1}) \\
&\quad + [\beta(1-\lambda)]^n \sum_{j=0}^{p-n} [\beta(1-\lambda)]^j (R_i - R_{\widehat{i}+n+j})
\end{aligned}$$

The surplus gap between two ages comes from the initial difference in the productivity thresholds, which persists after $\widehat{i}$, plus the additional advantage of having a longer job duration for the younger worker. We thus deduce that:

$$\begin{aligned}\Delta &= \left(\sum_{j=0}^{\widehat{\imath}+n-1}[\beta(1-\lambda)]^j\right)(R_i - R_{i+1}) - \left(\sum_{j=0}^{\widehat{\imath}+m-1}[\beta(1-\lambda)]^j\right)(R_{i+1}-R_{i+2}) \\ &\quad \underbrace{+[\beta(1-\lambda)]^m\left(\sum_{j=0}^{p-n}[\beta(1-\lambda)]^{j+n-m}(R_i - R_{\widehat{\imath}+n+j}) - \sum_{j=0}^{n-m}[\beta(1-\lambda)]^j(R_{i+1}-R_{\widehat{\imath}+m+j})\right)}_{\Omega}\end{aligned}$$

Ω gives the relative value associated with the additional durations of the different jobs ($p-n$ versus $n-m$). If $p-n \equiv \tau \leq n-m$, we have:[34]

$$\Omega = \widetilde{\Omega} - [\beta(1-\lambda)]^{m+\tau+1}\sum_{j=0}^{n-m-\tau-1}[\beta(1-\lambda)]^j(R_{i+1}-R_{\widehat{\imath}+m+\tau+1+j})$$

where the gap between Ω and $\widetilde{\Omega}$ is unambiguously negative, given that $\widetilde{\Omega}$ is defined as follows:

$$\begin{aligned}\widetilde{\Omega} &= [\beta(1-\lambda)]^m\left\{\left(\sum_{j=0}^{\tau}[\beta(1-\lambda)]^j\right)(R_i - R_{i+1}) - \sum_{j=0}^{\tau}[\beta(1-\lambda)]^j(R_{\widehat{\imath}+n+j} - R_{\widehat{\imath}+m+j})\right. \\ &\quad \left. +[1-[\beta(1-\lambda)]^\tau]\left[-R_i\left(\sum_{j=0}^{\tau}[\beta(1-\lambda)]^j\right) + \sum_{j=0}^{\tau}[\beta(1-\lambda)]^j R_{\widehat{\imath}+n+j}\right]\right\}\end{aligned}$$

We note that the two last terms in the brackets are negative. Finally, we obtain the following expression:

$$\mathcal{V}_{i+1}(\widehat{\imath}, R_i) - \mathcal{V}_{i+2}(\widehat{\imath}, R_{i+1}) = -\Psi_{1,i+1} + \Psi_{2,i+1}(R_i - R_{i+1}) - \Psi_{3,i+1}(R_{i+1} - R_{i+2}) \quad (52)$$

where $\Psi_{j,i+1} > 0$ for $j = 1, 2, 3$. The term $-\Psi_{1,i+1}$ captures all the negative components isolated at each step of the solution calculation of $\mathcal{V}_{i+1}(\widehat{\imath}, R_i) - \mathcal{V}_{i+2}(\widehat{\imath}, R_{i+1})$. Using equation (52), we then obtain:

$$\begin{aligned}R_{i+1} - R_i &= \frac{-\Upsilon(e_{i+1}) + \Upsilon(e_i)}{\left(\gamma p(\theta)[1+\beta(1-\lambda)P_{i+1}(\widehat{\imath})]\right)(1+\Phi_{2,i}\Psi_{2,i+1})} \\ &\quad + \frac{-\Phi_{2,i}\Psi_{1,i+1}}{1+\Phi_{2,i}\Psi_{2,i+1}} + \frac{\Phi_{1,i}+\Phi_{2,i}\Psi_{3,i+1}}{1+\Phi_{2,i}\Psi_{2,i+1}}(R_{i+2}-R_{i+1})\end{aligned}$$

where $\Phi_{x,y} > 0$, $\forall x, y$. A sufficient condition for $R_{i+1} - R_i < 0$ is that $-\Upsilon(e_{i+1}) + \Upsilon(e_i) < 0$. If e_i is age-decreasing and $\Upsilon'(x) < 0$, $\forall x \in [e_{\widehat{\imath}-1}, e_1]$, i.e. $\lambda < \gamma p(\theta)e_i, \forall i \leq \widehat{\imath} - 1$, then $-\Upsilon(e_{i+1}) + \Upsilon(e_i) < 0$ and $R_{i+1} - R_i < 0, \forall i < \widehat{\imath} - 1$, provided that the terminal condition $R_{\widehat{\imath}-1} > R_{\widehat{\imath}}$ holds.

[34]The condition $p - n \leq n - m$ implies that $R_{\widehat{\imath}+j+2} - R_{\widehat{\imath}+j+1} \geq R_{\widehat{\imath}+j+1} - R_{\widehat{\imath}+j}$, $\forall j \geq 0$. Given that $e_i > e_{i+1}$, this implies that $\Upsilon''(e_i) \leq 0$, which holds for a large class of the function ϕ.

At age $\widehat{i}$, the age-dynamics of reservation productivity change. For $i = \widehat{i}-1$, we have $\gamma p(\theta) R_{\widehat{i}-1} = \gamma p(\theta) b - \Upsilon(e_{\widehat{i}-1}) - \gamma p(\theta)(1-\lambda)\beta S_{\widehat{i}}(R_{\widehat{i}-1})$, and for $i \geq \widehat{i}$, we have $\gamma p(\theta) R_i = \gamma p(\theta) b - \Upsilon(e_i)$. We then deduce that:

$$\begin{aligned} R_{\widehat{i}} - R_{\widehat{i}-1} &= \frac{\Upsilon(e_{\widehat{i}-1}) - \Upsilon(e_{\widehat{i}}) + \gamma p(\theta)(1-\lambda)\beta S_{\widehat{i}}(R_{\widehat{i}-1})}{\gamma p(\theta)} \\ R_{\widehat{i}+1} - R_{\widehat{i}} &= \frac{\Upsilon(e_{\widehat{i}}) - \Upsilon(e_{\widehat{i}+1})}{\gamma p(\theta)} \end{aligned}$$

The first equation shows that the gap between $R_{\widehat{i}}$ and $R_{\widehat{i}-1}$ depends, through $S_{\widehat{i}}(R_{\widehat{i}-1})$, on the sequence of future reservation productivities. Nevertheless, note that if $R_{\widehat{i}+1} > R_{\widehat{i}-1}$ the solution for $S_{\widehat{i}}(R_{\widehat{i}-1})$ is simply $R_{\widehat{i}-1} - R_{\widehat{i}}$.

We consider a first-order approximation around $e_i = e_{\widehat{i}}$, leading to $\Upsilon(e_{\widehat{i}}) - \Upsilon(e_{\widehat{i}+1}) \approx \gamma p(\theta)(e_{\widehat{i}+1} - e_{\widehat{i}})^2 \phi''(e_{\widehat{i}})$ and $\Upsilon(e_{\widehat{i}-1}) - \Upsilon(e_{\widehat{i}}) \approx -\gamma p(\theta)(e_{\widehat{i}-1} - e_{\widehat{i}})^2 \phi''(e_{\widehat{i}})$. This implies that:

$$\begin{aligned} R_{\widehat{i}} - R_{\widehat{i}-1} &\approx \frac{-\gamma(e_{\widehat{i}-1} - e_{\widehat{i}})^2 p(\theta)\phi''(e_{\widehat{i}}) + \gamma p(\theta)(1-\lambda)\beta S_{\widehat{i}}(R_{\widehat{i}-1})}{\gamma p(\theta)} \\ R_{\widehat{i}+1} - R_{\widehat{i}} &\approx \frac{\gamma(e_{\widehat{i}+1} - e_{\widehat{i}})^2 p(\theta)\phi''(e_{\widehat{i}})}{\gamma p(\theta)} \end{aligned}$$

Then, for $e_{\widehat{i}-1} = e_{\widehat{i}} + \delta$ and $e_{\widehat{i}+1} = e_{\widehat{i}} - \delta$, we obtain $R_{\widehat{i}+1} - R_{\widehat{i}-1} \approx \frac{\gamma p(\theta)(1-\lambda)\beta S_{\widehat{i}}(R_{\widehat{i}-1})}{\gamma p(\theta)} > 0$ and then $S_{\widehat{i}}(R_{\widehat{i}-1}) = R_{\widehat{i}-1} - R_{\widehat{i}}$. We then deduce that:

$$R_{\widehat{i}} - R_{\widehat{i}-1} \approx \frac{-\gamma(e_{\widehat{i}-1} - e_{\widehat{i}})^2 p(\theta)\phi''(e_{\widehat{i}})}{\gamma p(\theta)[1 + \gamma p(\theta)\beta(1-\lambda)]} < 0$$

The terminal condition $R_{\widehat{i}-1} > R_{\widehat{i}}$ holds.

E The first-best allocation

In order to simplify the exposition, we present in turn the cases with age-decreasing and age-increasing profiles for reservation productivity.

E.1 The age-increasing profile case ($R_{i,t+i} < R_{i+1,t+i+1}$)

When $R_{i,t+i} < R_{i+1,t+i+1}$, the dynamics of unemployment and production depend on the proportion of jobs that become obsolete through aging:

$$\begin{aligned}
u_{i+1,t+i+1} &= [1 - e_{i,t+i}p(\theta_{t+i})(1 - G(R_{i+1,t+i+1}))]u_{i,t+i} \\
&\quad +(1-\lambda)n_{i,t+i}(R_{i+1,t+i+1}) + \lambda G(R_{i+1,t+i+1})(1 - u_{i,t+i}) \\
y_{i+1,t+i+1} &= (1-\lambda)[y_{i,t+i} - y_{i,t+i}(R_{i+1,t+i+1})] \\
&\quad +[\lambda(1-u_{i,t+i}) + u_{i,t+i}e_{i,t+i}p(\theta_{t+i})]\int_{R_{i+1,t+i+1}}^{1} xdG(x) \\
y_{i,t+i}(R_{i+1,t+i+1}) &= (1-\lambda)[y_{i-1,t+i-1}(R_{i+1,t+i+1}) - y_{i-1,t+i-1}(R_{i,t+i})] \\
&\quad + [\lambda(1-u_{i;t+i}) + u_{i,t+i}e_{i,t+i}p(\theta_{t+i})]\int_{R_{i,t+i}}^{R_{i+1,t+i+1}} xdG(x) \\
n_{i,t+i}(R_{i+1,t+i+1}) &= (1-\lambda)[n_{i-1,t+i-1}(R_{i+1,t+i+1}) - n_{i-1,t+i-1}(R_{i,t+i})] \\
&\quad +[\lambda(1-u_{i,t+i}) + u_{i,t+i}e_{i,t+i}p(\theta_{t+i})][G(R_{i+1,t+i+1}) - G(R_{i,t+i})]
\end{aligned}$$

By successive iterations, it is possible to express $n_{i,t+i}(R_{i+1,t+i+1})$ and $y_{i,t+i}(R_{i+1,t+i+1})$ as functions of the sum of all past job creations in the productivity range $[R_{i,t+i}; R_{i+1,t+i+1}]$

$$\begin{aligned}
u_{i+1,t+i+1} &= [1 - e_{i,t+i}p(\theta_{t+i})(1 - G(R_{i+1,t+i+1}))]u_{i,t+i} + \lambda G(R_{i+1,t+i+1})(1-u_{i,t+i}) \\
&\quad +[G(R_{i+1,t+i+1}) - G(R_{i,t+i})]\sum_{j=1}^{i-1}(1-\lambda)^j[\lambda(1-u_{i-j,t+i-j}) + u_{i-j,t+i-j}e_{i-j,t+i-j}p(\theta_{t+i-j})]
\end{aligned} \tag{53}$$

$$\begin{aligned}
y_{i+1,t+i+1} &= (1-\lambda)y_{i,t+i} + [\lambda(1-u_{i,t+i}) + u_{i,t+i}e_{i,t+i}p(\theta_{t+i})]\int_{R_{i+1,t+i+1}}^{1} xdG(x) \\
&\quad - \left[\int_{R_{i,t+i}}^{R_{i+1,t+i+1}} xdG(x)\right]\sum_{j=1}^{i-1}(1-\lambda)^j[\lambda(1-u_{i-j,t+i-j}) + u_{i-j,t+i-j}e_{i-j,t+i-j}p(\theta_{t+i-j})]
\end{aligned} \tag{54}$$

The planner then maximizes the objective (23) subject to the constraints (53) and (54) respectively, associated with the Lagrangean multipliers $\nu_{i,t+i}$ et $\mu_{i,t+i}$.

The first-order condition with respect to $R_{i,t+i}$ is as follows:

$$\begin{aligned}
0 = {} & \nu_{i-1,t+i-1}\left[\begin{array}{l} (e_{i-1,t+i-1}p(\theta_{t+i-1}) - \lambda)G'(R_{i,t+i})u_{i-1,t+i-1} + \lambda G'(R_{i,t+i}) \\ +G'(R_{i,t+i})\sum_{j=1}^{i-2}(1-\lambda)^j\left[\begin{array}{l}\lambda(1-u_{i-1-j,t+i-1-j}) \\ +u_{i-1-j,t+i-1-j}e_{i-1-j,t+i-1}p(\theta_{t+i-1-j})\end{array}\right]\end{array}\right] \\
& +\mu_{i-1,t+i-1}\left[\begin{array}{l} -[\lambda(1-u_{i-1,t+i-1}) + u_{i-1,t+i-1}e_{i-1,t+i-1}p(\theta_{t+i-1})]R_{i,t+i}G'(R_{i,t+i}) \\ -R_{i,t+i}G'(R_{i,t+i})\sum_{j=1}^{i-2}(1-\lambda)^j\left[\begin{array}{l}\lambda(1-u_{i-1-j,t+i-1-j}) \\ +u_{i-1-j,t+i-1-j}e_{i-1-j,t+i-1-j}p(\theta_{t+i-1-j})\end{array}\right]\end{array}\right] \\
& +\beta\nu_{i,t+i}\left[-G'(R_{i,t+i})\sum_{j=1}^{i-1}(1-\lambda)^j\left[\begin{array}{l}\lambda(1-u_{i-j,t+i-j}) \\ +u_{i-j,t+i-j}e_{i-j,t+i-j}p(\theta_{t+i-j})\end{array}\right]\right] \\
& +\beta\mu_{i,t+i}\left[+R_{i,t+i}G'(R_{i,t+i})\sum_{j=1}^{i-1}(1-\lambda)^j\left[\begin{array}{l}\lambda(1-u_{i-j,t+i-j}) \\ +u_{i-j,t+i-j}e_{i-j,t+i-j}p(\theta_{t+i-j})\end{array}\right]\right]
\end{aligned}$$

After some simplification, the first-order condition with respect to $R_{i,t+i}$ can be rewritten as follows:

$$\begin{aligned} 0 &= -\nu_{i-1,t+i-1} + \mu_{i-1,t+i-1} R_{i,t+i} \\ &+ \sum_{j=1}^{T-1-i} \beta^j (1-\lambda)^j \mu_{i-1+j,t+i-1+j} (R_{i+j,t+i+j} - R_{i-1+j,t+i-1+j}) \end{aligned}$$

Second, the first-order condition with respect to production $y_{i,t+i}$ is

$$\mu_{i-1,t+i-1} = \beta[1 + \mu_{i,t+i}(1-\lambda)]$$

This can be rewritten as:

$$\mu_{i,t+i} = \beta \sum_{j=1}^{T-1-i} \beta^{j-1}(1-\lambda)^{j-1}$$

Third, the first-order condition with respect to unemployment $u_{i,t+i}$ is:

$$\begin{aligned} 0 &= -\nu_{i-1,t+i-1} + \beta \left\{ \begin{array}{l} [(b - \phi(e_{i,t+i}) - c e_{i,t+i}\theta_{t+i})] \\ +\nu_{i,t+i} \left[\begin{array}{c} 1 - e_{i,t+i} p(\theta_{t+i}) \\ +(e_{i,t+i} p(\theta_{t+i}) - \lambda) G(R_{i+1,t+i+1}) \end{array} \right] \\ +\mu_{i,t+i} [e_{i,t+i} p(\theta_{t+i}) - \lambda] \int_{R_{i+1,t+i+1}}^1 x dG(x) \end{array} \right\} \\ &+ (e_{i,t+i} p(\theta_{t+i}) - \lambda) \sum_{j=1}^{T-2-i} \beta^j (1-\lambda)^j \left[\begin{array}{l} \nu_{i+j,t+i+j} \left[\begin{array}{c} G(R_{i+1+j,t+i+1+j}) \\ -G(R_{i+j,t+i+j}) \end{array} \right] \\ -\mu_{i+j,t+i+j} \int_{R_{i+j,t+i+j}}^{R_{i+j+1,t+i+j+1}} x dG(x) \end{array} \right] \end{aligned}$$

Fourth, the first-order condition with respect to search intensity $e_{i,t+i}$ is:

$$\begin{aligned} 0 &= -\phi'(e_{i,t+i}) - c\theta_{t+i} + p(\theta_{t+i}) \left\{ \begin{array}{l} -\nu_{i,t+i}(1 - G(R_{i+1,t+i+1})) \\ +\mu_{i,t+i} \int_{R_{i+1,t+i+1}}^1 x dG(x) \end{array} \right\} \\ &+ p(\theta_{t+i}) \sum_{j=1}^{T-2-i} \beta^j (1-\lambda)^j \left\{ \begin{array}{l} \nu_{i+j,t+i+j} \left[\begin{array}{c} G(R_{i+1+j,t+i+1+j}) \\ -G(R_{i+j,t+i+j}) \end{array} \right] \\ -\mu_{i+j,t+i+j} \int_{R_{i+j,t+i+j}}^{R_{i+1+j,t+i+1+j}} x dG(x) \end{array} \right\} \end{aligned}$$

Last, the first-order condition with respect to θ_{t+i} is different from the other conditions due to non-directed search: T generations with different ages overlap in the period $t+i$. The first-order condition is then:

$$\begin{aligned} \sum_{\tau=1}^{T-1} c u_{\tau,t+i} e_{\tau,t+i} &= \sum_{\tau=1}^{T-2} \Bigg[u_{\tau,t+i} e_{\tau,t+i} p'(\theta_{t+i}) \left\{ \begin{array}{l} -\nu_{\tau,t+i}(1 - G(R_{\tau+1,t+i+1})) \\ +\mu_{\tau,t+i} \int_{R_{\tau+1,t+i+1}}^1 x dG(x) \end{array} \right\} \\ &+ u_{\tau,t+i} e_{\tau,t+i} p'(\theta_{t+i}) \sum_{j=1}^{T-2-\tau} \beta^j (1-\lambda)^j \left\{ \begin{array}{l} \nu_{\tau+j,t+i+j} \left[\begin{array}{c} G(R_{\tau+1+j,t+i+1+j}) \\ -G(R_{\tau+j,t+i+j}) \end{array} \right] \\ -\mu_{\tau+j,t+i+j} \int_{R_{\tau+j,t+i+j}}^{R_{\tau+1+j,t+i+1+j}} x dG(x) \end{array} \right\} \Bigg] \end{aligned}$$

Combining these first-order conditions in order to eliminate the multipliers, and using repeatedly that $\int_z^1 xdG(x) = z[1-G(z)] + \int_z^1 (1-G(x))dx$, we obtain after some tedious manipulation:

$$\begin{aligned}
R_{i,t+i} &= b - \phi(e_{i,t+i}) - ce_{i,t+i}\theta_{t+i} \\
&\quad -(\lambda - e_{i,t+i}p(\theta_{t+i}))\beta \sum_{j=1}^{T-1-i} \beta^{j-1}(1-\lambda)^{j-1} \int_{R_{i+j,t+i+j}}^1 (1-G(x))dx \\
\frac{c}{q(\theta_{t+i})} &= [1-\eta(\theta_{t+i})]\beta \sum_{\tau=1}^{T-1} \frac{e_{\tau,t+i}u_{\tau,t+i}}{\widetilde{u}_{t+i}} \left(\sum_{j=1}^{T-1-\tau} \beta^{j-1}(1-\lambda)^{j-1} \int_{R_{\tau+j,t+i+j}}^1 (1-G(x))dx \right) \\
\phi'(e_{i,t+i}) &= -c\theta_{t+i} + p(\theta_{t+i})\beta \sum_{j=1}^{T-1-i} \beta^{j-1}(1-\lambda)^{j-1} \int_{R_{i+j,t+i+j}}^1 (1-G(x))dx
\end{aligned}$$

Considering these equations at the steady state, and given that the job surplus for all $\epsilon > R_i$ is

$$\begin{aligned}
S_i(\epsilon) &= \epsilon - R_i + (1-\lambda)\beta S_{i+1}(\epsilon) \\
&\Rightarrow \int_{R_{i+1}}^1 S_{i+1}(x)dG(x) = \sum_{j=1}^{T-1-i} \beta^{j-1}(1-\lambda)^{j-1} \int_{R_{i+j}}^1 (1-G(x))dx
\end{aligned}$$

we obtain equations (27), (28), (30), (31) and (32).

E.2 The age-decreasing profile case ($R_{i,t+i} > R_{i+1,t+i+1}$)

When $R_{i,t+i} > R_{i+1,t+i+1}$, equations (24) and (25) become:

$$u_{i+1,t+i+1} = [1 - e_{i,t+i}p(\theta_{t+i}) + (e_{i,t+i}p(\theta_{t+i}) - \lambda)G(R_{i+1,t+i+1})]u_{i,t+i} + \lambda G(R_{i+1,t+i+1}) \quad (55)$$

$$y_{i+1,t+i+1} = (1-\lambda)y_{i,t+i} + [\lambda(1-u_{i,t+i}) + u_{i,t+i}e_{i,t+i}p(\theta_{t+i})] \int_{R_{i+1,t+i+1}}^1 xdG(x) \quad (56)$$

These two constraints are special cases of those when $R_{i,t+i} < R_{i+1,t+i+1}$, as there are now no jobs destroyed by the aging process. The planner then maximizes the objective (23) subject to constraints (55) and (56). The Lagrangean multipliers associated with these two constraints are respectively $\nu_{i,t+i}$ and $\mu_{i,t+i}$. The first-order conditions with respect to $R_{i,t+i}$, $y_{i,t+i}$, $u_{i,t+i}$, $e_{i,t+i}$ and θ_{t+i} are then a simpler version of those obtained when $R_{i,t+i} < R_{i+1,t+i+1}$:

$$\begin{aligned}
\nu_{i-1,t+i-1} &= \mu_{i-1,t+i-1}R_{i,t+i} \\
\mu_{i-1,t+i-1} &= \beta[1 + (1-\lambda)\mu_{i,t+i}] \\
\nu_{i-1,t+i-1} &= \beta \left\{ \begin{array}{l} [(b - \phi(e_{i,t+i}) - ce_{i,t+i}\theta_{t+i})] + \nu_{i,t+i}(1-\lambda) \\ +(e_{i,t+i}p(\theta_{t+i}) - \lambda)\left[-\nu_{i,t+i}(1-G(R_{i+1,t+i+1})) + \mu_{i,t+i}\int_{R_{i+1,t+i+1}}^1 xdG(x)\right] \end{array} \right\} \\
\phi'(e_{i,t+i}) &= -c\theta_{t+i} + p(\theta_{t+i})\left[-\nu_{i,t+i}(1-G(R_{i+1,t+i+1})) + \mu_{i,t+i}\int_{R_{i+1,t+i+1}}^1 xdG(x)\right] \\
c\sum_{\tau=1}^{T-1} e_{\tau,t+i}u_{\tau,t+i} &= p'(\theta_{t+i})\sum_{\tau=1}^{T-2} e_{\tau,t+i}u_{\tau,t+i}\left[-\nu_{\tau,t+i}(1-G(R_{\tau+1,t+i+1})) + \mu_{\tau,t+i}\int_{R_{\tau+1,t+i+1}}^1 xdG(x)\right]
\end{aligned}$$

Combining these first-order conditions in order to substitute for the multipliers, we obtain after some tedious manipulation:

$$\begin{aligned}
R_{i,t+i} &= b-\phi(e_{i,t+i})-ce_{i,t+i}\theta_{t+i}-[\lambda-e_{i,t+i}p(\theta_{t+i})]\beta P_{i+1}(T)\int_{R_{i+1,t+i+1}}^{1}(1-G(x))dx \\
&\quad +\beta(1-\lambda)P_{i+1}(T)(R_{i+1,t+i+1}-R_{i,t+i}) \\
\phi'(e_{i,t+i}) &= -c\theta_{t+i}+p(\theta_{t+i})\beta P_{i+1}(T)\int_{R_{i+1,t+i+1}}^{1}(1-G(x))dx \\
\frac{c}{q(\theta_{t+i})} &= [1-\eta(\theta_{t+i})]\beta\sum_{j=1}^{T-1}\left(\frac{e_{j,t+i}u_{j,t+i}}{\widetilde{u}_{t+i}}P_{i+1}(T)\int_{R_{j+1,t+i+1}}^{1}(1-G(x))dx\right)
\end{aligned}$$

with $\mu_{i,t+i}=P_{i+1}(T)$, $\forall t$, $p'(\theta_{t+i})=[1-\eta(\theta_{t+i})]q(\theta_{t+i})$ and $\widetilde{u}_{t+i}=\sum_{i=1}^{T-1}e_{i,t+i}u_{i,t+i}$.

Considering the steady state of this economy, the job surplus for all $\epsilon>R_i$ is given by

$$\begin{aligned}
S_i(\epsilon) &= \epsilon-R_i+(1-\lambda)\beta[S_{i+1}(\epsilon)-S_{i+1}(R_i)] \\
&\Rightarrow \left\{\begin{array}{l}\int_{R_{i+1}}^{1}S_{i+1}(x)dG(x)=P_{i+1}(T)\int_{R_{i+1}}^{1}(1-G(x))dx \\ S_{i+1}(R_i)=P_{i+1}(T)(R_i-R_{i+1})\end{array}\right.
\end{aligned}$$

It is then possible to derive equations (27), (28), (30) and (31), whereas equation (32) yields $n_i(R_{i+1})=0$.

F Heterogeneity in an infinite-life environment

We assume that there is productivity heterogeneity across workers ($y(j),\ j=1,...,N$) in an infinite-life environment along the lines of Davis [2001]:

$$\begin{array}{rclrcl}
 & & \text{Equilibrium} & & & \text{First best} \\
S_j &=& \frac{y(j)-b-\frac{1}{1-\gamma}c\theta\tau_j+c\theta\tau_j}{1-\beta(1-s)} & S_j^\star &=& \frac{y(j)-b-\frac{1}{1-\eta(\theta^\star)}c\theta^\star\tau_j^\star+c\theta^\star}{1-\beta(1-s)} \\
\frac{c}{q(\theta)} &=& \beta(1-\gamma)\sum_{j=1}^{N}\left(\frac{u_j}{\widetilde{u}}S_j\right) & \frac{c}{q(\theta^\star)} &=& \beta[1-\eta(\theta^\star)]\sum_{j=1}^{N}\left(\frac{u_j^\star}{\widetilde{u}^\star}S_j^\star\right) \\
u_j &=& \frac{s}{s+p(\theta)} & u_j^\star &=& \frac{s}{s+p(\theta^\star)} \\
\tau_j &=& \frac{S_j}{\sum_{j=1}^{N}\left(\frac{u_j}{\widetilde{u}}S_j\right)} & \tau_j^\star &=& \frac{S_j^\star}{\sum_{j=1}^{N}\left(\frac{u_j^\star}{\widetilde{u}^\star}S_j^\star\right)}
\end{array}$$

In this case, all jobs have the same expected duration. The condition for the equivalence of the equilibrium and first-best allocations here is: $\sum_{j=1}^{N}\frac{u_j}{\widetilde{u}}\tau_j=1$, which obviously holds, given the definition of τ_j.