Etude et contrôle du vent acoustique dans les générateurs d'ondes thermoacoustiques annulaires.

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Abstract

This paper deals with the role of acoustic streaming in thermoacoustic engines. Attention is focused on the use of membranes in annular thermoacoustic prime movers to control the effect of acoustic streaming generated, which is known to play an important role in the thermoacoustic amplification process. The experimental results show that the position of membrane in the closed loop waveguide strongly influences the onset of thermoacoustic instability. Also, the results obtained exhibit additional nonlinear effects due to the reverse influence of streaming induced variations of temperature distribution on the thermoacoustic amplification process, but also due to the mechanical characteristics of the membrane itself, which cause under some circumstances the significant generation of third and fifth harmonics in acoustic pressure oscillations. An analytical model is presented to investigate the influence of membrane position along the waveguide on the onset of thermoacoustic instability. The generation of odd harmonics is also explained qualitatively.

Introduction

Since the early 80's, there has been a renewal of interest in thermoacoustic devices, both prime movers (i.e. thermal to acoustic energy converters) and heat pumps so that a great variety of devices has been built. The linear analytical theory is now well developped, but not sufficient to understand the complexity of such heat engines. Thermoacoustic engines involve numerous nonlinear processes which need to be finely characterized, because their control should lead to a significant increase in efficiency. Among these mechanisms, the excitation of acoustic streaming, i.e. the acoustically induced generation of a mean (nonoscillating) flow, plays a crucial role in the saturation of acoustic wave amplitude in thermoacoustic prime movers, by inducing via forced convection variations of the temperature field in the device, with subsequent variations of the thermoacoustic amplification process. An important research effort has been devoted recently to the description of acoustic streaming standing wave thermoacoustic devices. However, recent experimental development in thermoacoustics show that the most efficient devices have complicated geometries, which notably involve the use of a closed-loop resonator to allow the development of travelling acoustic waves [1].

The thermoacoustic device which is studied here is one of those « travelling wave type » thermoacoustic heat engine : this is an annular thermoacoustic prime mover (presented in Fig. 1(a)), basically composed of a closed-loop waveguide and a stack of solid plates submitted to a strong temperature gradient. When the temperature gradient exceeds some critical value, the thermoacoustic amplification process (which occurs in the stack, inside boudary layers) results in the self excitation of resonant acoustic waves (at frequency f≈153 Hz). It should be mentioned here that in such a device, due to the existence of a closed-loop path, the most important share of streaming induced heat transfer is that of a mean nonzero mass flow through the duct (directed clockwise in Fig. 1(a)), sometimes called « Gedeon streaming » [2].

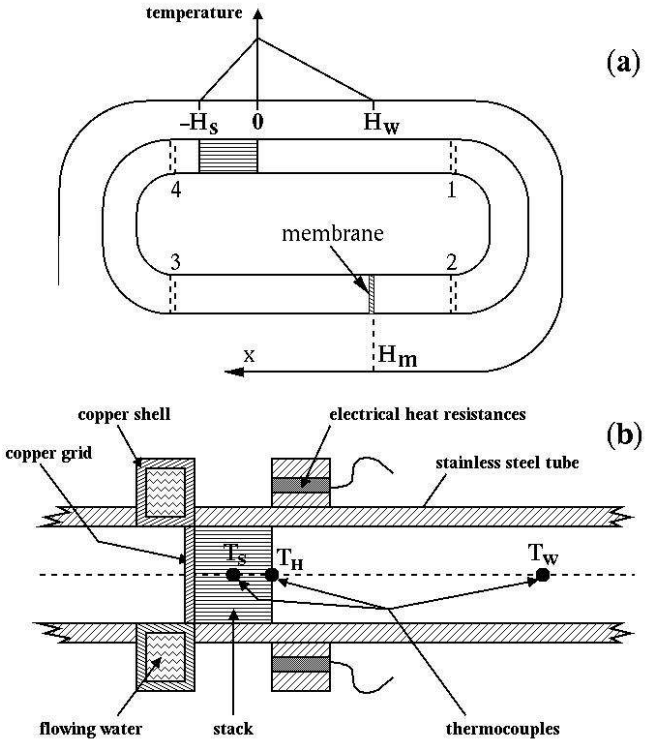

Figure 1 : (a) schematic diagram of the experimental apparatus. **(b)** detailed representation of the thermoacoustic core

While it seems to be commonly acknowledged that forced convection due to acoustic streaming has a harmful influence because it tends to reduce the externally imposed temperature gradient along the stack, it is not so clear what effect can have acoustic streaming on the efficiency of thermoacoustic engines [3]. However, solutions have been proposed which provide significant increase of engine efficiency by the use of passive elements in the resonator to control acoustic streaming. In their development of a thermoacoustic Stirling engine capable to reach 41% of the Carnot efficiency, Backhaus et al. [1] make use notably of a « jet pump ». More recently, Luo et al. [4] presented a thermoacoustically driven refrigerator with double thermoacoustic Stirling cycles where elastic membranes were placed at strategic positions in both loops to « suppress Gedeon streaming losses ». In this paper, we study the transient and steady regime behavior of an annular thermoacoustic prime mover when the nonzero mass flow

component of acoustic streaming is suppressed by the introduction of an elastic membrane in the waveguide. The influence of the position of the membrane along the device on the onset conditions and on the spectral content of the acoustic wave is investigated. An analytical model is presented which aims at predicting the onset of thermoacoustic instability as a function of membrane position.

Experiments

The thermoacoustic device in study is schematically presented in Fig. 1. The torus-shaped stainless steel tube of length L=2.24 m is filled with air at atmospheric pressure, and the stack is a honeycombed ceramic material with square channels of cross-section 0.9×0.9 mm². The complete description of the experimental apparatus is given in ref. [5]. In the following, experimental results are presented when an elastic membrane is introduced in the resonator : this cellophane® membrane was placed at various positions along the tube, referred as points 1 to 4 in Fig. 1 (a).

For each of the measurements presented in the following, the heating power supply is gradually increased, and both temperatures along the stack and acoutic pressure levels are measured. In particular, the influence of membrane position on the value of the hot stack end temperature T_H when the onset occurs is investigated. Table 1 presents the results obtained for T_H at onset, for various positions of membrane.

Membrane position	1	2	3	4
T_H (°C)	175	162	177	194

Table 1: Hot stack end temperature T_H at onset in function of membrane position (see Fig. 1(a)). In case when no membrane is installed, T_H=152 °C.

The results show that the position of membrane significantly impacts the onset temperature gradient, since a difference of more than 30 °C in T_H is observed when membrane is placed at point 4 compared to point 2. Moreover, it is noteworthy that, whatever membrane position is in these experiments, the onset temperature gradient is higher than its value when no membrane is installed (for which T_H= 152 °C).

The influence of membrane on the dynamics of wave amplitude growth was also investigated. It appears that the introduction of a membrane leads to complicated transient regimes, which also strongly depend on membrane position. For instance, figure 2 presents the onset of thermoacoustic instability when membrane is installed at position 1. It appears from Fig. 2(a) that the initial exponential acoustic pressure amplitude growth (occuring at time t ≈ 90 s) is followed by intermittent slow variations of pressure amplitude. Moreover, it appears from Fig. 2 (b) which present the associated acoustic energy distribution in a time-frequency domain, that the acoustic wave is not purely sinusoidal. In particular, the introduction of membrane is responsible for the significant generation of odd harmonics : in steady regime, the ratio p_2/p_1 of second harmonic amplitude (oscillating at f≈300 Hz) to fundamental amplitude is of about 1 %, while the ratio p_3/p_1 of third harmonic (f≈450 Hz) to fundamental is of about 10 %.

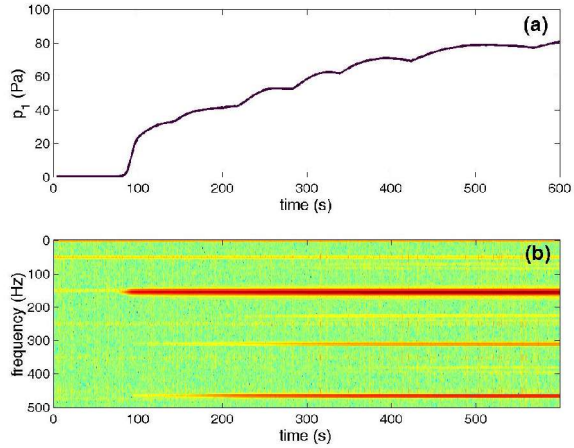

Figure 2: (a) Gradual evolution of the root mean square amplitude p_1 of acoustic pressure (oscillating at frequency f ≈ 150 Hz) when membrane is placed at position 1 in Fig. 1 (a). **(b)** Spectrogram.

The effect of slow, intermittent variations with time of acoustic pressure amplitude has been observed to be very important under some circumstances, and significant variations in temperature distribution along the thermoacoustic core were associated with acoustic pressure variations. For instance, when membrane was placed at position 1, abrupt increases/decreases in acoustic pressure (approximately from 100 to 300 Pa) were accompanied by abrupt decreases/increases of T_H (with temperature drops of about 15 °C) simultaneously with abrupt increases/decreases of T_W (see Fig. 1(a)), with temperature drops of about 10 °C. Importantly, the observation of this effect depends on membrane position, since when membrane was placed at positions 2 or 4, the amplitude of acoustic pressure (together with temperature distribution) was always stable in the steady regime, and the spectral content of acoustic wave did not exhibit strong third harmonic generation. It is also noticeable that, after gradually increasing the heating power supply Q, there was a critical value of Q for which the amplitude of acoustic pressure finally became stable, whatever membrane position is. Finally, it must also be mentionned that for a given value of heating power supply, whatever membrane position, the acoustic pressure level was always at least twice lower than its value without membrane.

Discussion

On the role of membrane position

In a recent publication [6], the analytical description of sound amplification in annular thermoacoustic prime movers has been proposed. The amplification/attenuation of the acoustic wave is obtained by calculating the thermoacoustic amplification coefficient α (and the corresponding onset frequency f) which depends on the temperature distribution in the entire thermoacoustic core (i.e. the region $[-H_S,H_W]$ in Fig. 1 (a)). Here, assuming a linear temperature distribution along the thermoacoustic core (as plotted in Fig. 1(a)), it is possible to predict the hot temperature T_H which corresponds to the onset of thermoacoustic instability. From this model, it is quite straightforward to predict the onset temperature T_H when a membrane is introduced at position $x=H_m$. This is

done by calculating α from the scattering matrix of region $[-H_S,H_m]$ (instead of that of region $[-H_S,H_W]$ in the initial model).

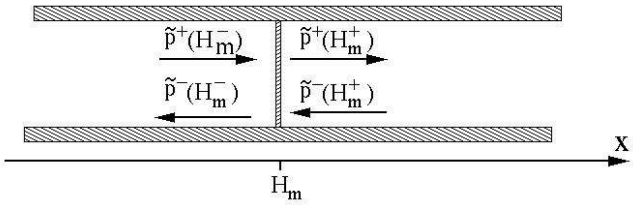

Figure 3 : Reflected and transmitted acoustic waves through the membrane.

According to the notation introduced in Fig. 3, the reflected and transmitted waves through the membrane are described with its scattering matrix as follows :

$$\begin{pmatrix} \tilde{p}^+(H_m^+) \\ \tilde{p}^-(H_m^-) \end{pmatrix} = \begin{pmatrix} T & R \\ R & T \end{pmatrix} \begin{pmatrix} \tilde{p}^+(H_m^-) \\ \tilde{p}^-(H_m^+) \end{pmatrix}, \quad (1)$$

where the complex amplitude of acoustic pressure $\tilde{p}(x,\omega) \equiv \tilde{p}(x)$ is separated into its two counterpropagating components $\tilde{p}^+$ and $\tilde{p}^-$ which propagate respectively in the +x and –x directions. T and R=1-T are the transmission and the reflexion coefficients of the membrane. Introducing angular frequency ω, velocity of sound in air c, that of waves in membrane c_m, T (and R) can be obtained from ref. [7] :

$$T \approx \frac{J_2(k_m r_w)}{i\frac{k\ell}{2} J_0(k_m r_w) + J_2(k_m r_w)}, \quad (2)$$

where $k=\omega/c$, $k_m=\omega/c_m$, $\ell=\sigma/\rho$ is the ratio of mass per unit area of membrane to density of air, and r_w is the radius of membrane. Then, assuming linear propagation of a plane wave in the cold part of the resonator, we get the transfer matrix of the membrane

$$\begin{pmatrix} \tilde{p}(H_m^+) \\ \tilde{v}(H_m^+) \end{pmatrix} = \begin{pmatrix} 1 & -iZ_W k_W \ell \frac{J_0(k_m r_w)}{J_2(k_m r_w)} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{p}(H_m^-) \\ \tilde{v}(H_m^-) \end{pmatrix}, \quad (3)$$

where $\tilde{v}$ denotes acoustic velocity, and where

$$k_w = \frac{\omega}{c}\sqrt{\frac{1+(\gamma-1)f_\kappa}{1-f_v}}, \quad (4)$$

$$Z_w = \frac{\rho\omega}{1-f_v}\frac{1}{k_w}, \quad (5)$$

are the wave number and the acoustic impedance which account for thermal and viscous losses in the waveguide (see ref. [6], Eqs. (3),(4) and (22)). Also, it is easy to express the transfer matrix from $x=H_W$ to $x=H_m^-$ while the analytical method to obtain the transfer matrix of the thermoacoustic core ($-H_S \le x \le H_W$) in function of temperature distribution is given in ref [6]. Finally, the transfer matrix from $x=-H_S$ to $x=H_m^+$ is obtained, and simple calculations allow to get the corresponding scattering matrix. Then, invoking § 2.2 in ref. [6], the thermoacoustic amplification α is obtained.

Thus, to calculate the onset temperature T_H in function of membrane position, we just need to know the mechanical characteritics of elastic membrane, i.e. σ and c_m. The former is measured using a microbalance. The latter is obtained with a very simple experiment : the membrane vibrations are induced by a small impact on its center, while the frequency ν_{01} of the fundamental mode is measured with a microphone. Invoking the theoretical relation $\nu_{01}=\beta_{01}\times c_m/(2r_w)$, with $\beta_{01}=0.7655$ [8] then gives c_m.

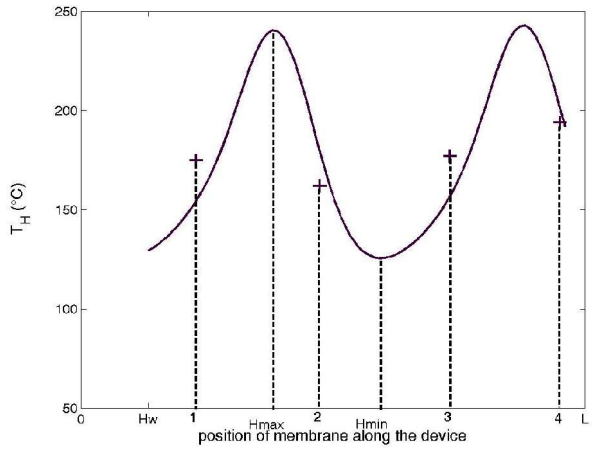

Figure 4 : Calculated variations of the onset temperature T_H at hot stack end with membrane position along the device (solid line) compared to experimental results (+). The calculated onset temperature T_H without membrane is 154 °C.

Figure 4 presents the calculated hot temperature T_H at onset in function of membrane position. It would be fallacious to conclude that there is good agreement between experiments and calculation, due to the lack of measurement points, but it seems however that the model match reasonably the only 4 available experimental data. The results exhibit significant variations of the onset conditions with position of membrane, since T_H varies from 130 °C (if membrane is placed at $H_m=H_{min}\approx1.30$ m from hot stack end) to 240 °C (if $H_m=H_{max}\approx0.85$ m). Moreover, it is remarkable that there is a region ($1.15\ m \le H_m \le 1.6\ m$) where the onset temperature is lower than its value $T_H\approx154$ °C when no membrane is installed. It was however not possible in practice to install a membrane in this region to validate this result. Figure 5 presents the distribution of the acoustic field along the device for the cases of no membrane (solid line), membrane at $H_m=H_{min}$ (dashed line), and membrane at $H_m=H_{max}$ (dotted line). Note that the amplitudes of acoustic pressure |p| and acoustic flow |u| are arbitrary (see ref. [6, §2.3]). From the results presented in Fig. 5, it appears that the introduction of a membrane influences the spatial distribution of acoustic variables which control the thermoacoustic amplification process in the stack. Thus, the introduction of a membrane leads, via mixing of counterpropagating acoustic waves, to variations of the heat engine's cycle in the stack where sound is amplified. There is consequently an optimum position of membrane corresponding to a minimum in onset temperature, and probably to a maximum in the engine's efficiency in the steady regime.

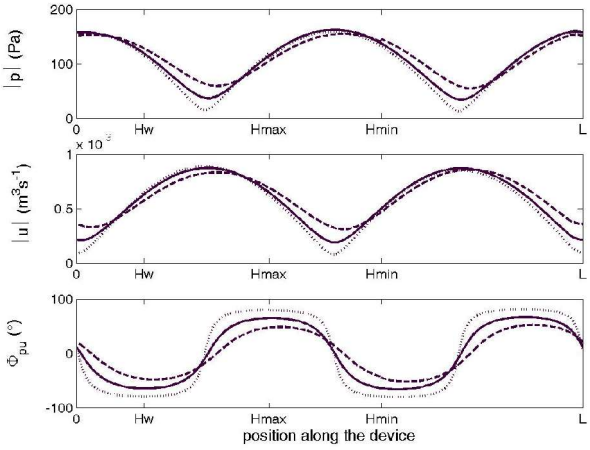

Figure 5 : Spatial distribution of acoustic pressure p, acoustic flow u, and phase shift between pressure and velocity oscillations ϕ_{pu} along the device. Solid line : without membrane ; dashed line : with membrane at $x=H_{min}$; dotted line : with membrane at $x=H_{max}$.

On the slow, intermittent variations of acoustic pressure amplitude

In a previous paper [3], the observation in an annular thermoacoustic prime mover of a periodic switch on/off of thermoacoustic instability has been reported and explained. In the present case, the processes observed are different but similar : there are slow, almost periodic variations of acoustic pressure amplitude (instead of switch on/off). In both cases, temperature variations are strongly coupled to acoustic pressure variations, so that the processes observed here may be explained similarly. On the one hand, the gradual evolution of acoustic pressure amplitude is controlled by the temperature distribution in the entire thermoacoustic core (in particular, thermoacoustic amplification strongly depends on the shape of the temperature field in the passive part of the thermoacoustic core $x\in[0,H_W]$). On the other hand, the temperature field is varying due to stack heating and nonlinear effects : in particular, while « Gedeon streaming » is suppressed by introduction of a membrane, there are still upward and downward streaming currents across the duct, which, as observed in experiments, impact the temperature field in the entire thermoacoustic core. Importantly, while the effect of temperature distribution on thermoacoustic amplification is instantaneous, there is a time delay in the acoustically induced variation of temperature field, so that complicated dynamics of wave amplitude variation is observed.

On the odd harmonics generation

Interaction of an acoustic fluid with nonlinear vibration of a clamped circular membrane is a complex problem which requires to be thoroughly considered. However, from the existing theory of nonlinear vibrations [9], some qualitative considerations can give us indications on the reason why the introduction of a membrane gives rise here to the development of odd harmonics. Assuming low frequency excitation ($f<\nu_{01}$) and neglecting viscosity, the deflexion w (r,t) of a circular plate submitted to an external excitation f (r,t) (i.e. the acoustic pressure drop across membrane) is [9, p. 509] :

$$\rho_m h\partial_{tt}w + D\nabla^4 w = \frac{1}{r}\partial_r(\partial_r F\partial_r w) + f\,, \qquad (6)$$

where ρ_m and h are density and thickness of plate, D is flexural rigidity, and F is a stress function which satisfies

$$\frac{1}{r}\partial_r F - \partial^2_{rr}F - r\partial^3_{rrr}F = Eh\frac{1}{2}(\partial_r w)^2\,, \qquad (7)$$

where E is Young modulus. Note that $D\nabla^4 w$ can be neglected in Eq. (6) because D is proportional to h^3 and $h<<1$ in case of membrane. Anyway, it is clear from the right hand side of Eq. (7), which is quadratic in $\partial_r w$, that the term $\partial_r(\partial_r F\partial_r w)$ in Eq. (6) exhibits cubic nonlinearity. Consequently, acoustic excitation of membrane at angular frequency ω gives rise to high amplitude vibrations of membrane, so that cubic nonlinearity of membrane induces 3ω (rather than 2ω) and higher odd harmonics generation. Moreover, arguing that the deflexion of membrane w(x,t) matches acoustic displacement $\xi(x,t)$ of fluid in contact with membrane (with $\tilde{\xi} = \tilde{v}/i\omega$), it is clear that cubic nonlinearity will be more effective if membrane is placed close to a velocity maximum in the waveguide. This is in agreement with experimental observations.

Conclusion

In this paper, experiments have shown that the introduction of a membrane in annular thermoacoustic prime movers induces additionnal complexity in the analysis of nonlinear processes controlling the thermoacoustic amplification process: the nonlinear vibrations of membrane give rise under some circumstances to odd harmonics generation, while the influence of acoutic streaming remains significant. Complementary work is now in progress in order to characterize precisely the development of acoustic streaming: a plexiglass closed-loop acoustic resonator has been built, in which the acoustic field is generated and controlled by two loudspeakers with appropriate phasing, and Laser Doppler Velocimetry measurements of acoustic streaming velocity should help to reach better understanding.

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