Etude et contrôle du vent acoustique dans les générateurs d’ondes thermoacoustiques annulaires.

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Abstract
This paper deals with the role of acoustic streaming in thermoacoustic engines. Attention is focused on the use of membranes in annular thermoacoustic prime movers to control the effect of acoustic streaming generated, which is known to play an important role in the thermoacoustic amplification process. The experimental results show that the position of membrane in the closed loop waveguide strongly influences the onset of thermoacoustic instability. Also, the results obtained exhibit additional nonlinear effects due to the reverse influence of streaming induced variations of temperature distribution on the thermoacoustic amplification process, but also due to the mechanical characteristics of the membrane itself, which cause under some circumstances the significant generation of third and fifth harmonics in acoustic pressure oscillations. An analytical model is presented to investigate the influence of membrane position along the waveguide on the onset of thermoacoustic instability. The generation of odd harmonics is also explained qualitatively.

Introduction
Since the early 80’s, there has been a renewal of interest in thermoacoustic devices, both prime movers (i.e. thermal to acoustic energy converters) and heat pumps so that a great variety of devices has been built. The linear analytical theory is now well developed, but not sufficient to understand the complexity of such heat engines. Thermoacoustic engines involve numerous nonlinear processes which need to be finely characterized, because their control should lead to a significant increase in efficiency. Among these mechanisms, the excitation of acoustic streaming, i.e. the acoustically induced generation of a mean (nonoscillating) flow, plays a crucial role in the saturation of acoustic wave amplitude in thermoacoustic prime movers, by inducing via forced convection variations of the temperature field in the device, with subsequent variations of the thermoacoustic amplification process. An important research effort has been devoted recently to the description of acoustic streaming standing wave thermoacoustic devices. However, recent experimental development in thermoacoustics show that the most efficient devices have complicated geometries, which notably involve the use of a closed-loop resonator to allow the development of travelling acoustic waves [1].

The thermoacoustic device which is studied here is one of those « travelling wave type » thermoacoustic heat engine: this is an annular thermoacoustic prime mover (presented in Fig. 1(a)), basically composed of a closed-loop waveguide and a stack of solid plates submitted to a strong temperature gradient. When the temperature gradient exceeds some critical value, the thermoacoustic amplification process (which occurs in the stack, inside boundary layers) results in the self excitation of resonant acoustic waves (at frequency f≈153 Hz). It should be mentioned here that in such a device, due to the existence of a closed-loop path, the most important share of streaming induced heat transfer is that of a mean nonzero mass flow through the duct (directed clockwise in Fig. 1(a)), sometimes called « Gedeon streaming » [2].

Figure 1 : (a) schematic diagram of the experimental apparatus. (b) detailed representation of the thermoacoustic core

While it seems to be commonly acknowledged that forced convection due to acoustic streaming has a harmful influence because it tends to reduce the externally imposed temperature gradient along the stack, it is not so clear what effect can have acoustic streaming on the efficiency of thermoacoustic engines [3]. However, solutions have been proposed which provide significant increase of engine efficiency by the use of passive elements in the resonator to control acoustic streaming. In their development of a thermoacoustic Stirling engine capable to reach 41% of the Carnot efficiency, Backhaus et al. [1] make use notably of a « jet pump ». More recently, Luo et al. [4] presented a thermoacoustically driven refrigerator with double thermoacoustic Stirling cycles where elastic membranes were placed at strategic positions in both loops to « suppress Gedeon streaming losses ». In this paper, we study the transient and steady regime behavior of an annular thermoacoustic prime mover when the nonzero mass flow
component of acoustic streaming is suppressed by the introduction of an elastic membrane in the waveguide. The influence of the position of the membrane along the device on the onset conditions and on the spectral content of the acoustic wave is investigated. An analytical model is presented which aims at predicting the onset of thermoacoustic instability as a function of membrane position.

Experiments
The thermoacoustic device in study is schematically presented in Fig. 1. The torus-shaped stainless steel tube of length \(L=2.24\,\text{m}\) is filled with air at atmospheric pressure, and the stack is a honeycombed ceramic material with square channels of cross-section \(0.9\times0.9\,\text{mm}^2\). The complete description of the experimental apparatus is given in ref. [5]. In the following, experimental results are presented when an elastic membrane is introduced in the resonator: this cellophane\textregistered membrane was placed at various positions along the tube, referred as points 1 to 4 in Fig. 1 (a).

For each of the measurements presented in the following, the heating power supply is gradually increased, and both temperatures along the stack and acoustic pressure levels are measured. In particular, the influence of membrane position on the value of the hot stack end temperature \(T_H\) when the onset occurs is investigated. Table 1 presents the results obtained for \(T_H\) at onset, for various positions of membrane.

<table>
<thead>
<tr>
<th>Membrane position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_H(°C))</td>
<td>175</td>
<td>162</td>
<td>177</td>
<td>194</td>
</tr>
</tbody>
</table>

Table 1: Hot stack end temperature \(T_H\) at onset in function of membrane position (see Fig. 1(a)). In case when no membrane is installed, \(T_H=152\,°C\).

The results show that the position of membrane significantly impacts the onset temperature gradient, since a difference of more than \(30\,°C\) in \(T_H\) is observed when membrane is placed at point 4 compared to point 2. Moreover, it is noteworthy that, whatever membrane position is in these experiments, the onset temperature gradient is higher than its value when no membrane is installed (for which \(T_H=152\,°C\)).

The influence of membrane on the dynamics of wave amplitude growth was also investigated. It appears that the introduction of a membrane leads to complicated transient regimes, which also strongly depend on membrane position. For instance, figure 2 presents the onset of thermoacoustic instability when membrane is installed at position 1. It appears from Fig. 2(a) that the initial exponential acoustic pressure amplitude growth (occurring at time \(t=90\,\text{s}\) is followed by intermittent slow variations of pressure amplitude. Moreover, it appears from Fig. 2 (b) which present the associated acoustic energy distribution in a time-frequency domain, that the acoustic wave is not purely sinusoidal. In particular, the introduction of membrane is responsible for the significant generation of odd harmonics: in steady regime, the ratio \(p_2/p_1\) of second harmonic amplitude (oscillating at \(f=300\,\text{Hz}\)) to fundamental amplitude is of about 1%, while the ratio \(p_3/p_1\) of third harmonic (\(f=450\,\text{Hz}\)) to fundamental is of about 10%.

Discussion
On the role of membrane position
In a recent publication [6], the analytical description of sound amplification in annular thermoacoustic prime movers has been proposed. The amplification/attenuation of the acoustic wave is obtained by calculating the thermoacoustic amplification coefficient \(\alpha\) and the corresponding onset frequency \(f\) which depends on the temperature distribution in the entire thermoacoustic core (i.e. the region \([-H_w,H_w]\) in Fig. 1 (a)). Here, assuming a linear temperature distribution along the thermoacoustic core (as plotted in Fig. 1(a)), it is possible to predict the hot temperature \(T_H\) which corresponds to the onset of thermoacoustic instability. From this model, it is quite straightforward to predict the onset temperature \(T_H\) when a membrane is introduced at position \(x=H_m\). This is
done by calculating $\alpha$ from the scattering matrix of region $[-H, H]$ (instead of that of region $[-H_s, H_s]$ in the initial model).

$$\begin{pmatrix} \tilde{p}^-(H_m^-) \\ \tilde{v}^-(H_m^-) \end{pmatrix} = \begin{pmatrix} T & R \\ R & T \end{pmatrix} \begin{pmatrix} \tilde{p}^+(H_m^+) \\ \tilde{v}^+(H_m^+) \end{pmatrix},$$  \hspace{1cm} (1)

where the complex amplitude of acoustic pressure $\tilde{p}(x, \omega) = \tilde{p}(x)$ is separated into its two counterpropagating components $\tilde{p}^+$ and $\tilde{p}^-$ which propagate respectively in the $+x$ and $-x$ directions. $T$ and $R=1-T$ are the transmission and the reflection coefficients of the membrane. Introducing angular frequency $\omega$, velocity of sound in air $c$, that of waves in membrane $c_m$, $T$ (and $R$) can be obtained from ref. [7]:

$$T = \frac{J_1(k_r r_m)}{\frac{i}{2}k J_0(k_r r_m) + J_1(k_r r_m)},$$  \hspace{1cm} (2)

where $k = \omega c$, $k_m = \omega c_m$, $\ell = \sigma / \rho$ is the ratio of mass per unit area of membrane to density of air, and $r_m$ is the radius of membrane. Then, assuming linear propagation of a plane wave in the cold part of the resonator, we get the transfer matrix of the membrane

$$\begin{pmatrix} \tilde{p}(H_m^-) \\ \tilde{v}(H_m^-) \end{pmatrix} = \begin{pmatrix} 1 - i Z_w k_m \ell J_0(k_r r_m) & J_1(k_r r_m) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{p}(H_m^+) \\ \tilde{v}(H_m^+) \end{pmatrix},$$  \hspace{1cm} (3)

where $\tilde{v}$ denotes acoustic velocity, and where

$$k_w = \frac{\omega}{c} \sqrt{\frac{j + (\gamma - j) r_m}{l - f_v}},$$ \hspace{1cm} (4)

$$Z_w = \frac{\rho c}{l - f_v} \frac{l}{k_w},$$ \hspace{1cm} (5)

are the wave number and the acoustic impedance which account for thermal and viscous losses in the waveguide (see ref. [6], Eqs. (3),(4) and (22)). Also, it is easy to express the transfer matrix from $x=H_m$ to $x=H_m^-$ while the analytical method to obtain the transfer matrix of the thermoacoustic core ($-H_s \leq x \leq H_s$) in function of temperature distribution is given in ref [6]. Finally, the transfer matrix from $x=-H_s$ to $x=H_m^-$ is obtained, and simple calculations allow to get the corresponding scattering matrix. Then, invoking § 2.2 in ref. [6], the thermoacoustic amplification $\alpha$ is obtained.

Thus, to calculate the onset temperature $T_{on}$ in function of membrane position, we just need to know the mechanical characteristics of elastic membrane, i.e. $\sigma$ and $c_m$. The former is measured using a microbalance. The latter is obtained with a very simple experiment: the membrane vibrations are induced by a small impact on its center, while the frequency $f_{01}$ of the fundamental mode is measured with a microphone. Invoking the theoretical relation $f_{01} = \beta_0 \lambda c_m / \left(2 \pi r_m\right)$, with $\beta_0=0.7655$ [8] then gives $c_m$.

Figure 3: Reflected and transmitted acoustic waves through the membrane.

According to the notation introduced in Fig. 3, the reflected and transmitted waves through the membrane are described with its scattering matrix as follows:

$$\begin{pmatrix} \tilde{p}^-(H_m^-) \\ \tilde{v}^-(H_m^-) \end{pmatrix} = \begin{pmatrix} T & R \\ R & T \end{pmatrix} \begin{pmatrix} \tilde{p}^+(H_m^+) \\ \tilde{v}^+(H_m^+) \end{pmatrix},$$  \hspace{1cm} (1)

where $\tilde{p}(x, \omega) = \tilde{p}(x)$ is separated into its two counterpropagating components $\tilde{p}^+$ and $\tilde{p}^-$ which propagate respectively in the $+x$ and $-x$ directions. $T$ and $R=1-T$ are the transmission and the reflection coefficients of the membrane. Introducing angular frequency $\omega$, velocity of sound in air $c$, that of waves in membrane $c_m$, $T$ (and $R$) can be obtained from ref. [7]:

$$T = \frac{J_1(k_r r_m)}{\frac{i}{2}k J_0(k_r r_m) + J_1(k_r r_m)},$$  \hspace{1cm} (2)

where $k = \omega c$, $k_m = \omega c_m$, $\ell = \sigma / \rho$ is the ratio of mass per unit area of membrane to density of air, and $r_m$ is the radius of membrane. Then, assuming linear propagation of a plane wave in the cold part of the resonator, we get the transfer matrix of the membrane

$$\begin{pmatrix} \tilde{p}(H_m^-) \\ \tilde{v}(H_m^-) \end{pmatrix} = \begin{pmatrix} 1 - i Z_w k_m \ell J_0(k_r r_m) & J_1(k_r r_m) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{p}(H_m^+) \\ \tilde{v}(H_m^+) \end{pmatrix},$$  \hspace{1cm} (3)

where $\tilde{v}$ denotes acoustic velocity, and where

$$k_w = \frac{\omega}{c} \sqrt{\frac{j + (\gamma - j) r_m}{l - f_v}},$$ \hspace{1cm} (4)

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Figure 4: Calculated variations of the onset temperature $T_{on}$ at hot stack end with membrane position along the device (solid line) compared to experimental results (+). The calculated onset temperature $T_{on}$ without membrane is $154 \degree C$.

Figure 4 presents the calculated hot temperature $T_{on}$ at onset in function of membrane position. It would be fallacious to conclude that there is good agreement between experiments and calculation, due to the lack of measurement points, but it seems however that the model match reasonably the only 4 available experimental data. The results exhibit significant variations of the onset conditions with position of membrane, since $T_{on}$ varies from $130 \degree C$ (if membrane is placed at $H_m=H_{non}=1.30 \ m$ from hot stack end) to $240 \degree C$ (if $H_m=H_{non}=0.85 \ m$). Moreover, it is remarkable that there is a region ($1.15 \ m \leq H_m \leq 1.6 \ m$) where the onset temperature is lower than its value $T_{on}=154 \degree C$ when no membrane is installed. It was however not possible in practice to install a membrane in this region to validate this result. Figure 5 presents the distribution of the acoustic field along the device for the cases of no membrane (solid line), membrane at $H_m=H_{non}$ (dashed line), and membrane at $H_m=H_{non}$ (dotted line). Note that the amplitudes of acoustic pressure $|p|$ and acoustic flow $u$ are arbitrary (see ref. [6, §2.3]). From the results presented in Fig. 5, it appears that the introduction of a membrane influences the spatial distribution of acoustic variables which control the thermoacoustic amplification process in the stack. Thus, the introduction of a membrane leads, via mixing of counterpropagating acoustic waves, to variations of the heat engine’s cycle in the stack where sound is amplified. There is consequently an optimum position of membrane corresponding to a minimum in onset temperature, and probably to a maximum in the engine’s efficiency in the steady regime.
On the odd harmonics generation

Interaction of an acoustic fluid with nonlinear vibration of a clamped circular membrane is a complex problem which requires to be thoroughly considered. However, from the existing theory of nonlinear vibrations [9], some qualitative considerations can give us indications on the reason why the introduction of a membrane gives rise here to the development of odd harmonics. Assuming low frequency excitation $f \ll f_0$ and neglecting viscosity, the deflection $w(r,t)$ of a circular plate submitted to an external excitation $f(r,t)$ (i.e. the acoustic pressure drop across membrane) is [9, p. 509]:

$$
\rho_m h \partial_{nt} w + \nabla^4 w = \frac{I}{r} \partial_r (\partial_r F \partial_r w) + f,
$$

(6)

where $\rho_m$ and $h$ are density and thickness of plate, $D$ is flexural rigidity, and $F$ is a stress function which satisfies

$$
\frac{I}{r} \partial_r F - \partial_r^2 F - \tau \partial_{nt} F = Eh \frac{I}{2} (\partial_r w)^2,
$$

(7)

where $E$ is Young modulus. Note that $\nabla^4 w$ can be neglected in Eq. (6) because $D$ is proportional to $h^3$ and $h^{\simeq 1}$ in case of membrane. Anyway, it is clear from the right hand side of Eq. (7), which is quadratic in $\partial w$, that the term $\partial_r (\partial_r F \partial_r w)$ in Eq. (6) exhibits cubic non-linearity. Consequently, acoustic excitation of membrane at angular frequency $\omega$ gives rise to high amplitude vibrations of membrane, so that cubic non-linearity of membrane induces $3\omega$ (rather than $2\omega$) and higher odd harmonics generation. Moreover, arguing that the deflexion of membrane $w(x,t)$ matches acoustic displacement $\xi(x,t)$ of fluid in contact with membrane (with $\xi = \vec{v}/i\omega$), it is clear that cubic non-linearity will be more effective if membrane is placed close to a velocity maximum in the waveguide. This is in agreement with experimental observations.

**Conclusion**

In this paper, experiments have shown that the introduction of a membrane in annular thermoacoustic prime movers induces additional complexity in the analysis of nonlinear processes controlling the thermoacoustic amplification process: the nonlinear vibrations of membrane give rise under some circumstances to odd harmonics generation, while the influence of acoustic streaming remains significant. Complementary work is now in progress in order to characterize precisely the development of acoustic streaming: a plexiglass closed-loop acoustic resonator has been built, in which the acoustic field is generated and controlled by two loudspeakers with appropriate phasing, and Laser Doppler Velocimetry measurements of acoustic streaming velocity should help to reach better understanding.

**References**