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On the use of acoustic streaming for heat transfer applications

Guillaume Penelet, Pierrick Lotton, Vitalyi Gusev[1], Michel Bruneau

Laboratoire d'Acoustique de l'Universite du Maine, UMR CNRS 6613;
avenue Olivier Messiaen, 72085 Le Mans cedex 9, France, guillaume.penelet@univ-lemans.fr
[1] Laboratoire de Physique de l'Etat Condense, UMR CNRS 6087, avenue Olivier Messiaen, 72085 Le Mans cedex 9, France

Abstract: This study aims at investigating a possibility to use surface acoustic waves (SAWs) in order to drive acoustic streaming and other nonlinear effects leading to heat transport through a microchannel. As a beginning in this exploratory study, a solid-fluid interface submitted to a temperature gradient is considered, and three distinct mechanisms of heat transfer are estimated analytically. The type of SAW that could be used for an efficient transfer of heat is also discussed.

Key words: Acoustic streaming, nonlinear heat transport, surface acoustic waves.

A. Introduction

The recent development of thermoacoustic engines has brought to light the interest in studying acoustic streaming, i.e. the time averaged motion of fluid induced by high amplitude acoustic field. At the same time, rapid advances in micromechanical systems have produced a variety of devices, among which some experimental demonstration of fluid motion induced in a microchannel by the propagation of guided acoustic waves [1]. Viscosity effects are indeed very important in microchannels, so that nonlinear effects driven by viscosity such as acoustic streaming may be successfully used in microdevices as a micropump or a micromixer [2].

Attention is focused in this paper on the possibility to use acoustic streaming in microdevices in order to extract heat from electronic components. In this recently initiated study, a very simple geometry is first considered for conveniency, i.e. a solid-fluid interface submitted to a longitudinal temperature gradient (see Fig. 1(a)), where the oscillations (at order 1) of fluid/solid parcels give rise (at order 2) to a non-oscillating heat flux along the interface. Different mechanisms which could drive this effect are considered : steady streaming due to spatial variations of viscosity, thermoacoustic heat transport along the interface, and steady streaming due to transverse vibrations of the solid wall (in z-direction). Simplified analytical models are provided and the type of SAW which may be used for an efficient micro-heat-exchanger is also discussed.

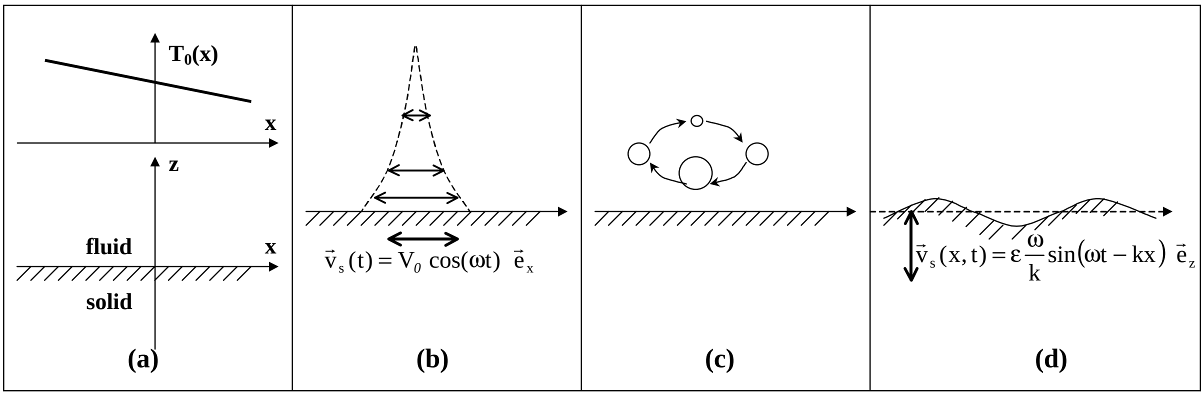

Fig.1. (a) Schematic representation of the problem under consideration : a solid-fluid interface, where the mean temperature field $T_0(x)$ may vary linearly with coordinate x. **(b)** Vibration of the solid along x-axis will drive in the fluid a second order mean flow (with subsequent heat flow) due to dependence of viscosity on temperature. **(c)** The propagation of a plane acoustic wave in the fluid gives rise to thermoacoustic heat transport in the vicinity of the solid wall. **(d)** Transverse oscillations propagating along x-axis on the interface lead to a second order steady streaming in the fluid.

B. Different mechanisms which may induce heat transport along the interface

B.1. Steady streaming due to dependence of viscosity on temperature

A first mechanism of heat transfer is illustrated in Fig. 1(b), where the excitation of vibrations along x-axis in the solid induce oscillation of fluid in the viscous boundary layer. Here, variations of viscosity along x-axis (due for instance to mean temperature variations) lead to a second order mean flow in the fluid, which can be easily estimated in case of small variations of viscosity.

Assuming constant fluid density ρ_0, the two-dimensional mass and momentum conservations equations are

$$\partial_x u + \partial_z w = 0 \qquad (1.a)$$

$$\rho_0\left(\partial_t u + \partial_x u^2 + \partial_z uw\right) = -\partial_x P + \eta\Delta u + 2\partial_x \eta \partial_{xt} u \qquad (1.b)$$

$$\rho_0\left(\partial_t w + \partial_x uw + \partial_z w^2\right) = -\partial_z P + \eta\Delta w + \partial_x \eta\left(\partial_z u + \partial_x w\right) \qquad (1.c)$$

where u and w are x and z components of velocity, respectively, P is the pressure in the fluid, η is the shear viscosity of fluid, and Δ denotes Laplacian operator. The boundary layer approximation (u>>w, ∂_z>>∂_x) can be applied to (1), leading in the absence of temperature gradient to the classical result

$$u(z,t) = V_0 e^{-kz}\cos(\omega t - kz), \quad (2)$$

where $k=(\omega/2\nu)^{½}$, and $\nu=\eta/\rho_0$ is cinematic viscosity.

However, assuming weak dependence of viscosity with x-coordinate in the form $\eta(x)=\eta_0+\varepsilon\eta_1(\varepsilon x)$ with ε<<1, the longitudinal velocity u oscillating at angular frequency ω can be decomposed into its order ε^0 component

$$u_0(z,t) = V_0 e^{-k_0 z}\cos(\omega t - k_0 z), \quad (3)$$

with $k_0=(\omega/2\nu_0)^{½}$, and its order ε^1 component (due to variations of viscosity)

$$u_1(\varepsilon x, z, t) = k_1 V_0 e^{-k_0 z}\left\{\sin(\omega t - k_0 z) - \cos(\omega t - k_0 z)\right\} \quad (4)$$

with $k_1=-k_0\nu_1(\varepsilon x)/2\nu_0$. Each unknown f can be expanded as $f=f_0+\varepsilon f_1+\varepsilon^2 f_2+\ldots$, so that at order ε^1, we get $w_1=0$ and $P_1=0$. At order ε^2, the mass conservation law equation is $\partial_x u_1+\partial_x u_2+\partial_x w_2=0$ where w_2 is the tranverse velocity oscillating at angular frequency ω, so that the time-average of this equation leads to $\partial_x\langle u_2\rangle$, where $\langle ..\rangle$ denotes time averaging and $\langle u_2\rangle$ is the longitudinal component of steady streaming velocity. Moreover, the time average of the momentum conservation equations leads to

$$\eta_0 \partial^2_{zz}\left\langle u_2\right\rangle = \rho_0 \partial_z\left\langle u_0 w_2\right\rangle + \partial_x\left\langle P_2\right\rangle, \quad (5)$$

and $\partial_z\langle P_2\rangle=0$, so that $\langle P_2\rangle$ does not depend on z, and is consequently equal to zero because the oscillating perturbation has no influence for z>>1. Finally, integrating twice equation (5), and accounting for boundary conditions $\partial_z\langle u_2\rangle|_{z\to\infty}=0$ and $\langle u_2\rangle|_{z=0}=0$ the second order in ε longitudinal velocity of steady streaming is obtained :

$$\varepsilon^2\left\langle u_2\right\rangle = \varepsilon d_x \eta_1 \frac{V_0^2}{4\eta_0\omega} e^{-2k_0 z}\left(e^{k_0 z}\cos(k_0 z) - k_0 z - 1\right). \quad (6)$$

This streaming velocity reaches maximum at approximate distance of $2\delta_{\nu 0}$ where $\delta_{\nu 0}=k_0^{-1}$ denotes the viscous boundary layer thickness, and rapidly decreases far from the solid wall. It is quadratic in V_0 and proportional to the viscosity gradient $d_x\eta_1$ (note that in case of gases, η grows with temperature T_0, while in case of liquids η generally decreases with increasing temperature).

B.2. Thermoacoustic heat pumping in the vicinity of the solid wall

A second mechanism of heat transfer is the classical thermoacoustic heat pumping effect which employs the interaction between acoustic and thermal waves in the vicinity of the solid-fluid interface. This mechanism is illustrated in Fig. 1(c), where the propagation of a traveling plane acoustic wave in the fluid in the presence of a temperature field $T_0(x)$ is considered. The acoustic pressure is written $p_1(x,t) = \mathrm{Re}(\tilde{p}_1(x)e^{j\omega t})$ where $\tilde{p}_1(x) = \tilde{P}_1 e^{-jkx}$, $k=\omega/c_0$ is the acoustic wave number (c_0 is the sound speed in the fluid), and where subscript 1 refers to order 1 quantities, i.e. acoustic quantities oscillating at angular frequency ω. Far from the solid, as the fluid oscillates along the solid at acoustic frequency, it experiences temperature fluctuations, which are due both to adiabatic compression and expansion accompanying acoustic pressure fluctuation and to the local temperature $T_0(x)$ itself . However, at a distance of about one thermal boundary layer thickness $\delta_\kappa=(2\kappa/\omega)^{½}$ (where κ denotes thermal diffusivity of fluid), the thermal contact between the oscillating fluid and the solid results in a time delay between pressure and temperature fluctuations, which leads to a nonzero, second order time-averaged heat flux. This well-known effect can be described through the linear thermoacoustic theory [3] : using both the momentum conservation and heat transfer equations with appropriate boundary conditions, the amplitude $\tilde{u}_1(x,z)$ of the acoustic wave velocity and the amplitude $\tilde{T}_1(x,z)$ of temperature oscillations in the fluid are obtained, and substituted in the expression of the second order time averaged thermoacoustic heat flux along x per unit area driven by the thermoacoustic effect

$$\left\langle q_2\right\rangle = \frac{1}{2}\rho_0 T_0 \mathrm{Re}\left(\tilde{T}_1\tilde{u}_1^*\right) - \frac{1}{2}T_0\beta\mathrm{Re}\left(\tilde{p}_1\tilde{u}_1^*\right) \qquad (7)$$

where * denotes complex conjugate and β is the thermal expansion coefficient of fluid. After calculations, the resulting lineic heat flux $\left\langle Q_2\right\rangle = \int_0^\infty \left\langle q_2\right\rangle .dz$ is

$$\left\langle Q_2\right\rangle = -\frac{\delta_\kappa}{4}\frac{1}{1+\sigma}\frac{\left|\tilde{P}_1\right|^2}{\rho_0 c_0}\times \left\{\left(1-\sqrt{\sigma}\right)T_0\beta + \frac{C_p}{\omega c_0}\frac{1+\sqrt{\sigma}+\sigma}{1+\sqrt{\sigma}} d_x T_0\right\}, \qquad (8)$$

where σ and C_p are the Prandtl number and isobaric heat capacity per unit mass of fluid, respectively. From (8) it appears that if the temperature gradient is positive, the thermoacoustic heat flux is negative (i.e. from hot to cold). However, if $d_x T_0$ is negative (or equivalently if temperature growth is opposite to the direction of wave propagation), there is a critical temperature gradient

$$d_x T_0\big|_{cr} = -\frac{1-\sigma}{1+\sqrt{\sigma}+\sigma}\frac{T_0\beta\omega c_0}{C_p} \qquad (9)$$

below which the direction of heat flux is reversed.

B.3. Steady streaming due to oscillating boundaries

The third mechanism of heat transfer which is considered here is illustrated in Fig. 1(d) where the propagation of a traveling surface wave in the solid leads via nonlinear effects to a mean fluid flow, with subsequent heat transport in the presence of a mean temperature gradient. A simplified analytical description of the resulting steady streaming velocity is proposed here in case of small vibration amplitudes, no temperature gradient, incompressible fluid, and purely transverse oscillations of boundaries. The fluid flow $\vec{v} = (u, w)$ is described by Eqs. (1) (where spatial variations of viscosity are neglected) taking into account the boundary conditions due to vibrating wall :

$$\vec{v}(x, z = \varepsilon k^{-1}\cos(\omega t - kx), t) = (0, -\varepsilon\omega k^{-1}\sin(\omega t - kx)) \quad (10)$$

Scaling for conveniency lengths by k^{-1}, time by ω^{-1}, velocities by $\varepsilon\omega k^{-1}$, pressure by $\varepsilon\eta\omega$, and defining the non-dimensional parameter α by $\alpha^2=\rho_0\omega\eta^{-1}k^{-2}$, Eqs (1) and (10) are rewritten in dimensionless form as

$$\nabla.\vec{v} = 0, \qquad \alpha^2\left(\partial_t\vec{v} + \varepsilon\vec{v}.\vec{\nabla}\vec{v}\right) = -\vec{\nabla}P + \Delta\vec{v}, \qquad (11)$$

$$\text{and } \vec{v}(x, z = \varepsilon\cos(t - x), t) = (0, -\sin(t - x)). \quad (12)$$

Considering small displacements ($\varepsilon<<1$), each unknown is expanded in the form $f=f_0+\varepsilon f_1+\ldots$while the velocity at the moving wall is also expanded around position z=0 :

$$\vec{v}(x, z = \varepsilon\cos(t - x), t) = \vec{v}(x, 0, t) + \varepsilon\cos(t - x)\partial_z\vec{v}(x, 0, t) + \ldots \quad (13)$$

At order ε^0, introducing the stream function φ_0 so that $u_0=-\partial_z\varphi_0$ and $w_0=\partial_x\varphi_0$ which satisfies $\Delta\Delta\varphi_0-\alpha^2\partial_t\Delta\varphi_0=0$, we seek a solution in the form

$$\varphi_0(x, z, t) = \mathrm{Re}(\tilde{\varphi}_0(x, z, t)) = \mathrm{Re}\left(\tilde{\psi}(z)e^{j(t-x)}\right). \quad (14)$$

Accounting for continuity of velocities at z=0 and requiring that velocities should be finite when $z\to\infty$, the following solution for $\tilde{\psi}$ is obtained :

$$\tilde{\psi}(z) = \frac{-me^{-y} + e^{-my}}{m - 1}, \quad (15)$$

with $m=1+j\alpha^2$. At order ε^1, the time averaged mass and momentum conservations equations are

$$\nabla.\vec{v}_1 = 0, \qquad \alpha^2\left\langle\vec{v}_0.\vec{\nabla}\vec{v}_0\right\rangle = -\vec{\nabla}\langle P_1\rangle + \Delta\langle u_1\rangle, \qquad (16)$$

with boundary conditions

$$\langle\vec{v}_1(x, 0, t)\rangle = -\langle\cos(t - x)\partial_z\vec{v}_0(x, 0, t)\rangle \quad (17)$$

The geometry of the problem allows us to seek a solution $\langle\vec{v}_1(z)\rangle$ independent of x. Requiring that $\left\langle\partial_z\vec{v}_1(z)\right\rangle\Big|_{z\to\infty} = 0$, Eqs (16) are solved, leading to $\langle w_1\rangle=0$ and

$$\langle u_1\rangle = \frac{\alpha^2}{2}\int_0^z\left(\int_0^{z'}(\psi_r d_z\psi_i - \psi_i d_z\psi_r)dz''\right)dz' + C_1 z + C_2 \quad (18)$$

where $\psi_r = \mathrm{Re}(\tilde{\psi})$, $\psi_i = \mathrm{Im}(\tilde{\psi})$,

$$C_1 = -\frac{\alpha^2}{2}\int_0^\infty(\psi_r d_z\psi_i - \psi_i d_z\psi_r)dz, \quad (19)$$

$$\text{and } C_2 = \langle u_1\rangle\big|_{z=0} = \frac{1}{2}d_{zz}^2\psi_r\big|_{z=0}. \quad (20)$$

Note that, due to the choice in the scaling of variables, the longitudinal steady streaming is actually of order ε^2 in magnitude. Note also that this solution (18) is not realistic far from the wall ($\langle u_2\rangle|_{z\to\infty}=C_2\neq 0$), due to the absence of boundaries in the half-space z>0. In the expression (18) of the dimensionless steady streaming velocity, there are two sources which drive the mean flow. The first one, present in constant C_2, is the peristaltic pumping induced by nonlinearity of boundaries, which drive the fluid in the direction of wave propagation ; the second source, due to classical Reynolds stresses in the viscous boundary layers do not depend on the direction of wave propagation.

C. Discussion.

When exciting SAW in a microchannel subject to a longitudinal temperature gradient, each of the above mentioned processes of heat transport may be involved. As a preliminary estimate, the heat fluxes associated to each mechanism should be evaluated. However, ameliorations should be included in the model before reasonable estimates of the global heat transport may be provided. First of all, a fluid-solid interface could be replaced in calculations by a fluid film bounded by two solid half space ; the fluid film thickness would be of about a few boundary layer thicknesses, for which at least the first and second heat transport mechanisms are the most efficient. Also, the analytical models presented above should be ameliorated. In particular, compressibility of fluid should be taken into account in the first and third mechanisms, and the possibility of a nonzero longitudinal component in the oscillating interface for the third mechanism should be considered. In the presence of a SAW, the actual motion of a solid particle at the fluid-solid interface is indeed elliptical more than purely transverse, so that the associated steady streaming may be substantially different from that estimated here.

Another important question arises when trying to transport heat via SAW, i.e. what type of SAW may be the more interesting for the given purpose ? Consider an interface (z=0) of an isotropic solid and an inviscid fluid. Introducing the velocity potential functions φ_s, ψ_s, and φ_f so that $u_s=\partial_x\varphi_s-\partial_z\psi_s$, $w_s=\partial_z\varphi_s+\partial_x\psi_s$, $u_f=\partial_x\varphi_f$, and $w_f=\partial_z\varphi_f$ where subscripts s and f refer to solid and fluid, respectively, we seek the harmonic travelling wave solution as

$$\varphi_s = Ae^{qz}e^{j(\omega t - kx)}, \quad (21)$$

$$\psi_s = Be^{sz}e^{j(\omega t - kx)}, \quad (22)$$

$$\varphi_f = Ce^{-k_z z}e^{j(\omega t - kx)}, \quad (23)$$

where k is the wave number of the traveling wave, $q^2=k^2-(\omega/c_L)^2$, $s^2=k^2-(\omega/c_t)^2$, $k_z^2=k^2-(\omega/c_0)^2$, c_L and c_t are the speeds of longitudinal and transverse waves in the solid, respectively. The wave number is determined by ensuring boundary conditions at z=0, i.e. continuity of transverse velocities, and continuity of the tranverse and longitudinal components of the stress tensor. This leads to the following equation (the Scholte determinant) :

$$\left(k^2+s^2\right)^2-4k^2qs+\frac{q}{k_z}\frac{\rho_f}{\rho_s}\left(\frac{\omega}{c_t}\right)^4=0\,, \quad (24)$$

which always has a real root k_S and, in most cases, also has a complex root k_{LRW}. The real root k_S corresponds to the Scholte wave, i.e. an acoustic wave which is trapped near the solid-fluid interface and which propagates along x. The complex root corresponds to the leaky Rayleigh wave, which is similar to the Rayleigh wave for a solid-vacuum interface, except that it is leaky due to continuous sound radiation in the fluid.

Table 1 presents different characteristics relative to Leaky Rayleigh Wave and Scholte wave propagation at 1 MHz, in case of a silicon-water interface (ρ_s=2329 kg.m^{-3}, c_L=8432 m.s^{-1},c_t=4673 m.s^{-1}, ρ_f=1000 kg.m^{-3},c_0=1481 m.s^{-1}), for a total energy density of 1 J.m^{-3} (for which particle displacements at the interface are of about a few nanometers). The two types of SAW present fairly different characteristics. In case of LRW propagation, approximately 25% of the acoustic energy is located into the fluid, and the wave is attenuated after a few centimeters (Im(k_{LRW})≈2.9 cm). However, the Scholte wave is unattenuated (if viscosity is neglected) and all its the energy is located into the fluid : the Scholte wave is similar to a bulk wave traveling along the surface at a velocity very close to sound speed in the fluid (c_{sw}=1480.2), but localized in depth (k_z^{-1}≈7mm).

Table 1. Various characteristics of Leaky Rayleigh Wave (LRW) and Scholte wave (SW) propagation in case of a silicon-water interface, at 1MHz, and for a total energy density E_{tot}=1J.m^{-3}. Note that viscosity is not taken into account here.

	LRW	SW
k	(1455.3+33.8×j) m^{-1}	4244.9 m^{-1}
E_f/E_{tot}	25.6 %	99.7 %
$\lvert w_{s,f}\rvert_{z=0}$	43 10^{-2} m.s^{-1}	2.5 10^{-2} m.s^{-1}
$\lvert u_s\rvert_{z=0}$	28.5 10^{-2} m.s^{-1}	8.3 10^{-3} m.s^{-1}
$\lvert u_f\rvert_{z=0}$	15.7 10^{-2} m.s^{-1}	75.4 10^{-2} m.s^{-1}

Both the LRW and the SW can be easily generated on a fluid-solid interface using interdigitated transducers. Each type of SAW seem to have advantages and disadvantages for the given purpose of heat transport (LRW may be however more interesting due to larger amplitudes of wall vibration) but before drawing any conclusion, viscosity should actually be considered in the calculations [4] because continuity of longitudinal velocities is not ensured here (in case of SW, the longitudinal velocity in the fluid at the interface is approximately 10^2 times bigger than in the solid).

Works are now in progress in order account for the weaknesses of the model that may impact significantly the estimate of the global heat transport associated to SAW propagation in a microchannel.

D. Conclusion

In this paper, three mechanisms of nonlinear heat transport in the vicinity of an oscillating fluid-solid interface submitted to a longitudinal temperature gradient have been considered and estimated with simplified analytical models. Each of these nonlinear effects may be involved in the heat transfer associated to LRW or SW propagation in a microchannel. The analytical models presented here must now be improved (account of compressibility and of the actual wall motion) in order to provide indications on the feasibility and the heat flux that can reach a SAW-driven heat exchanger.

E. Aknowledgements

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F. Litterature

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