Methods for transfer matrix evaluation applied to thermoacoustics

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ACOUSTICS 2012, Nantes, 26 April 2012
Session « Thermoacoustics »

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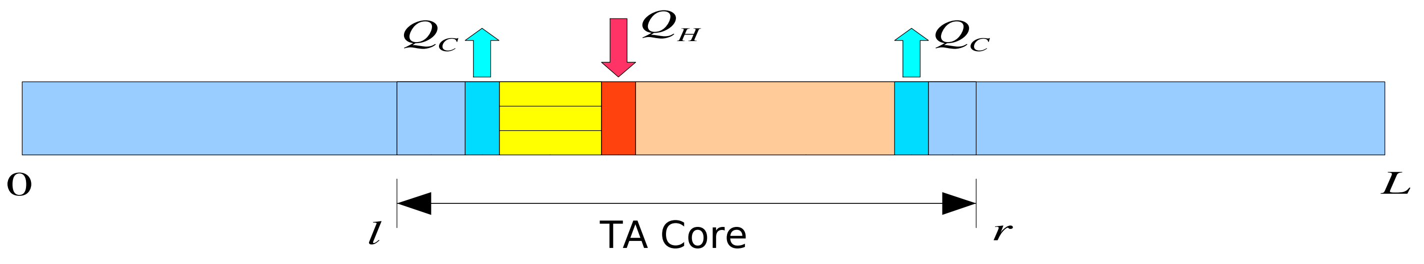
1) Thermoacoustic (TA) context

- **Onset conditions** required to be known for a **prime-mover design**:
  - Frequency of resonance ($\mathbf{f_{onset}}$),
  - Minimum needed heat power supply ($\mathbf{Q_{onset}}$).
- They can be calculated from the **Tranfer Matrix** (**T**) coefficients of the **TA Core.**
- **T** must be obtained at **different heating** conditions.
- **TA Core** definition: interval with **non-homogeneous temperature** comprising:
  - Stack or Regenerator,
  - Q_H and Q_C Heat Exchangers.



1) Thermoacoustic context

1.1) Stack and Regenerator definitions

- **STACK**
  - Pore diameter a few times **bigger** than thermal boundary layer thickness
  - Essentially **irreversible** heat tranfer
  - **Standing-wave** TA system

- **REGENERATOR**
  - Pore diameter much **smaller** than thermal boundary layer thickness
  - Quest for **reversible** heat transfer
  - **Traveling-wave** TA system

1) Thermoacoustic context

1.2) Definition of the Transfer Matrix (T)

Harmonic wave: $p(x,t) = \Re[\tilde{p}(x)e^{-j\varpi t}]$

Definition of **T** : $\begin{Bmatrix} \tilde{p}(x_r) \\ \tilde{U}(x_r) \end{Bmatrix} = \mathbf{T} \begin{Bmatrix} \tilde{p}(x_l) \\ \tilde{U}(x_l) \end{Bmatrix} = \begin{pmatrix} T_{pp} & T_{pu} \\ T_{up} & T_{uu} \end{pmatrix} \begin{Bmatrix} \tilde{p}(x_l) \\ \tilde{U}(x_l) \end{Bmatrix}$

- $\tilde{p}(x)$ and $\tilde{U}(x)$ are the **complex amplitudes** of the acoustic **pressure** and volume **velocity** .
- $p(x,t)$ is **measured** (or **simulated**) in an arbitrary frequency range, regarding the applied method specific limitations.
- The system is **reciprocal** if **det(T) = 1** (hypothesis verified without heating input)

2) Methods for the T evaluation

Analytical alternative:

- Requires accurate geometrical and thermophysical considerations.
- Works mostly for the stack case.
- Hence, it has important limitations.

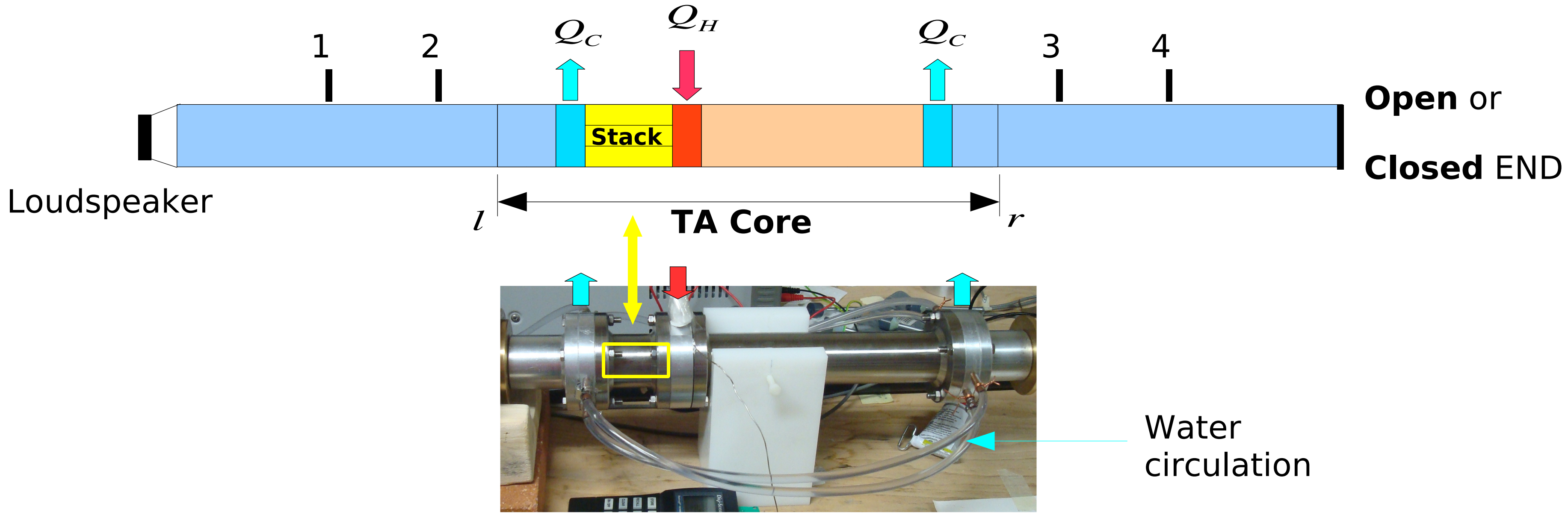
Experimental alternatives :

- **Two-Loads Method**
- **Impedance Method**

- They rely on acoustic pressure measurements.
- Advantages:
  - The **TA Core** is treated as a « **black box** » (measurements outside it).
  - No geometrical nor thermophysical considerations.
- Disadvantage:
  - Highly indirect.

2) Methods for the T evaluation

2.1) The Two-Loads Method



- Classical « two-loads » and 4 microphones method.
- $\tilde{p}(x_{l,r})$ and $\tilde{U}(x_{l,r})$ calculated from the measurements $\tilde{p}(x_i)$ [i=1,2,3,4].
- 2 set of $\tilde{p}(x_i)$ measurements with respective 2 different acoustic loads: **Open** or **Closed** End.
- Calculation of the **<u>T</u>** <u>coefficients</u> T_{pp}, T_{pu}, T_{up} and T_{uu} from its definition.

2) Methods for the T evaluation

2.1) The Two-Loads Method - Experimental Procedure

- Frequency range from **50Hz** to **200Hz**.
- **Different heating conditions** Q_H are imposed into the system, each one to whose corresponding **T** is measured.
- Q_H from 0W to 81W at a **constant increment** of 3W.
- A **time delay of 13 minutes** is required for each heat increment, aiming at the steady state regime accoplishment.
- The **cooling condition** Q_C is imposed by water circulation at room temperature.

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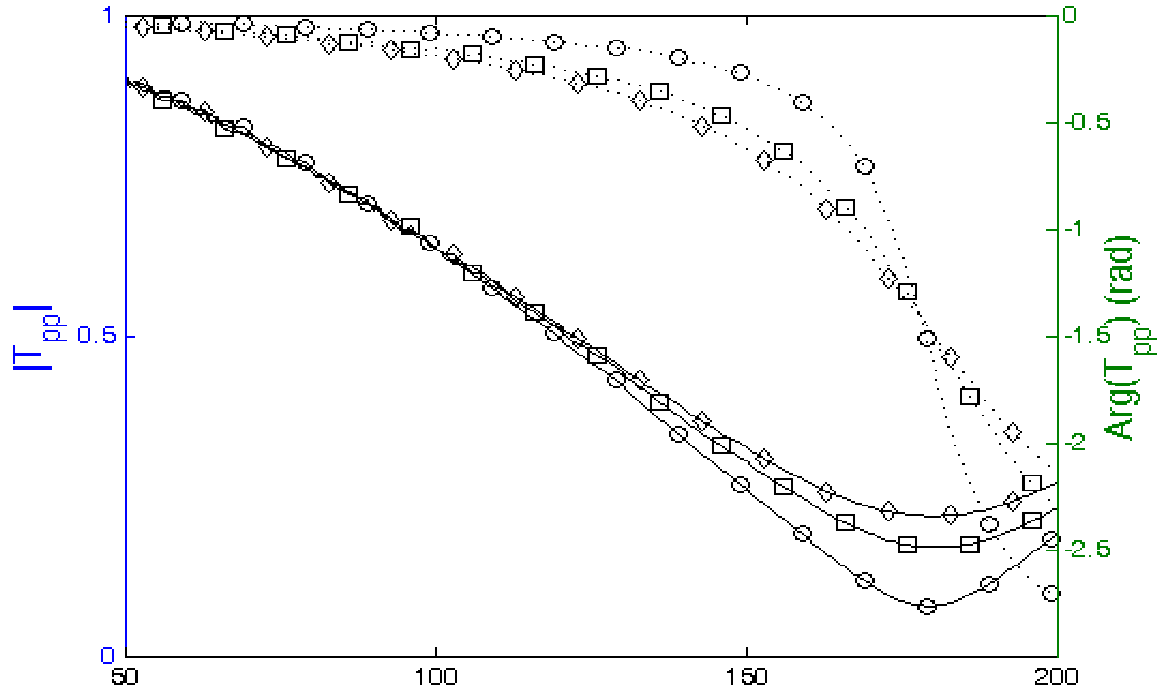
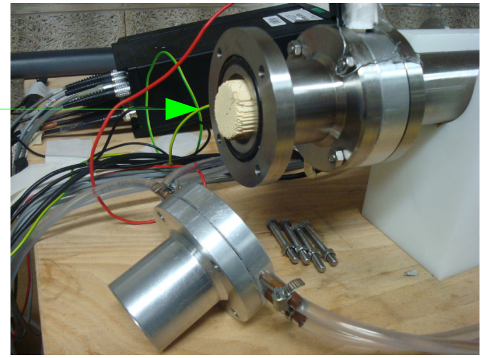
2) Methods for the T evaluation

2.1) The Two-Loads Method – Results - Ceramic Stack



Stack

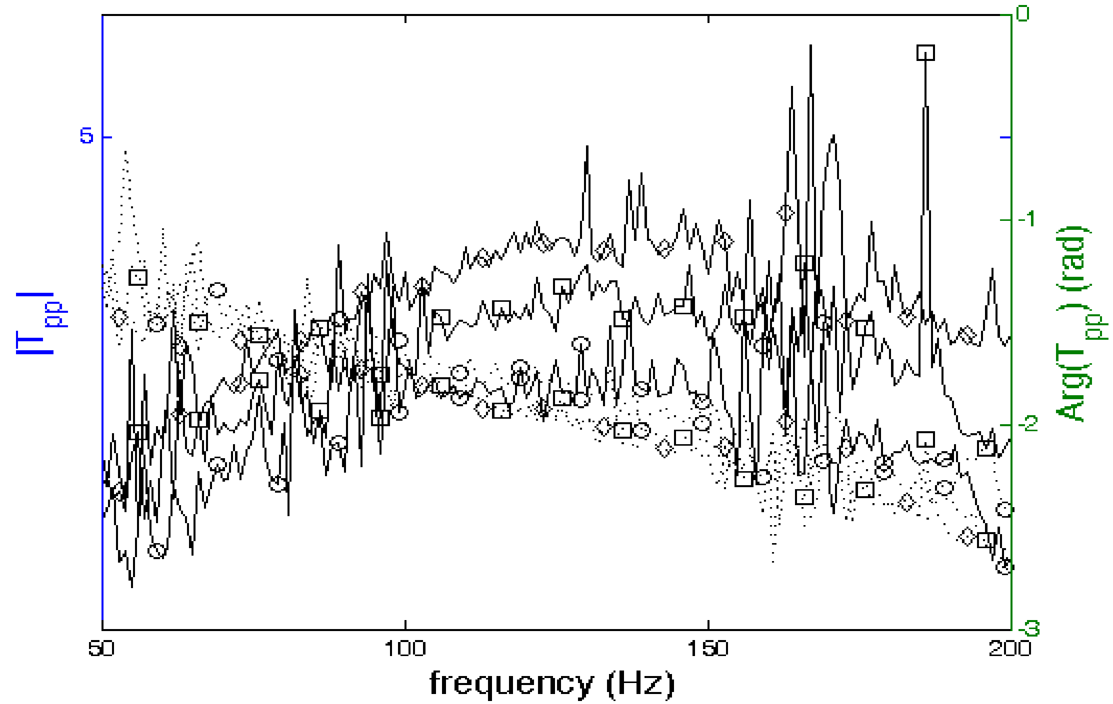


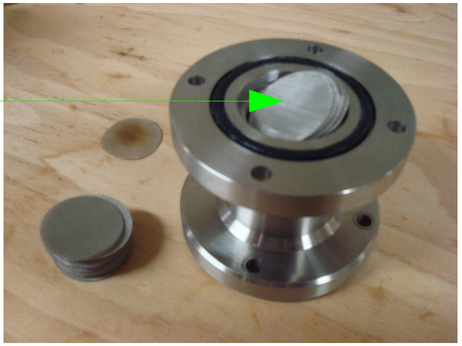
- T_{pp} Amplitude (solid lines)
  Phase (dotted lines)
  - (frequency domain in Hz).
- Three heating conditions are shown:
  - $Q_H = 0W$ (o),
  - $Q_H = 36W$ (square),
  - $Q_H = 72W$ (diamond).

- **Reciprocity** (det(**T**)) values for no heat input ($Q_H = 0W$):
  - Amp. Average: **1.010** (low bias level), with a std deviation of **3.3e-3** (low noise level); Theoretical value: **1**.
  - Phase Av. (rad): **1.9e-3** (low bias level), with a std deviation of **2.4e-3** (low noise level); Theoretical value: **0**.

2) Methods for the T evaluation

2.1) The Two-Loads Method – Results - Grid of Steel Pile





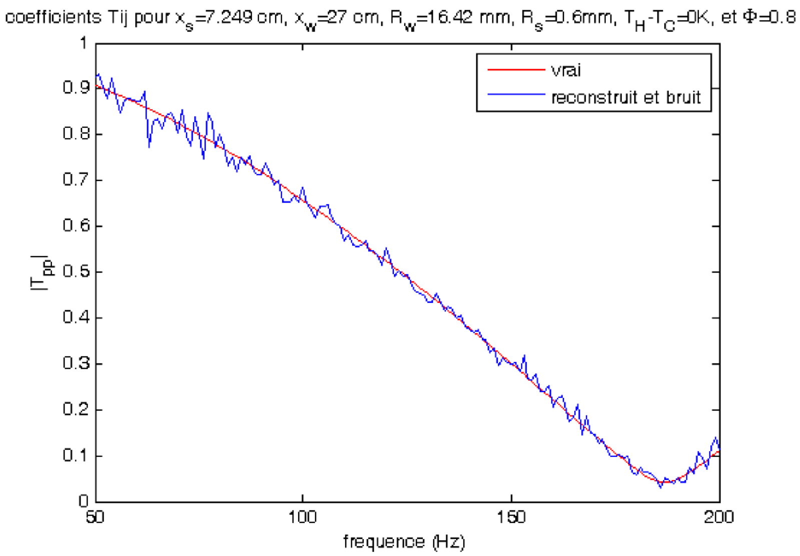
- T_{pp} Amplitude (solid lines)
  Phase (dotted lines)
  - (frequency domain in Hz).
- Three heating conditions are shown:
  - $Q_H = 0W$ (o),
  - $Q_H = 36W$ (square),
  - $Q_H = 72W$ (diamond).

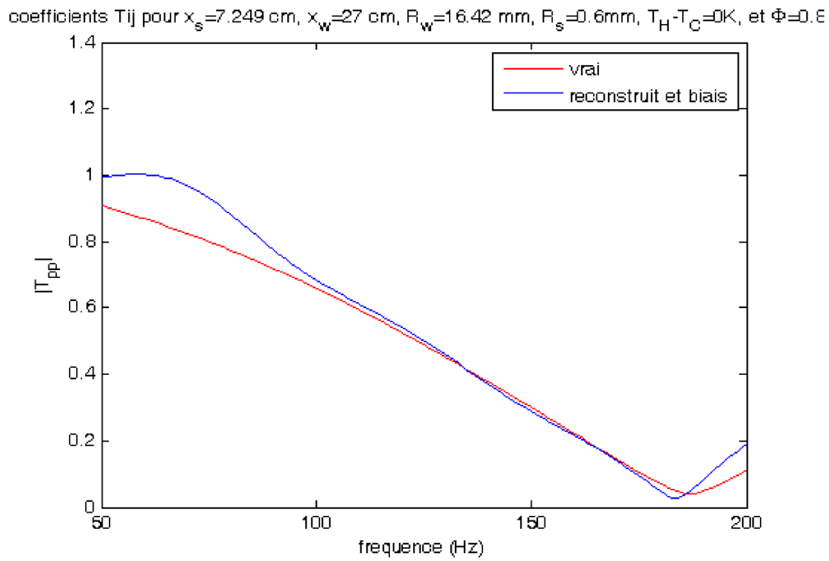
- **Reciprocity** (det(**T**)) values for no heat input ($Q_H = 0W$):
  - Amp. Average: **0.9958** (low bias level), with a std deviation of **0.1818 (high noise level)**; Theoretical value: **1**.
  - Phase Av. (rad): **1.9e-3** (low bias level), with a std deviation of **0.1265 (high noise level)**; Theoretical value: **0**.

2) Methods for the T evaluation

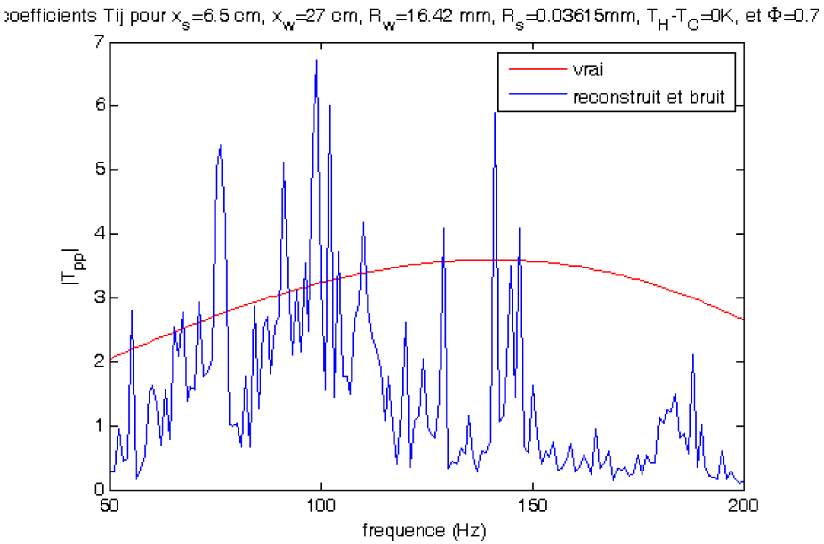
2.1) The Two-Loads Method – Sensitivity Analysis

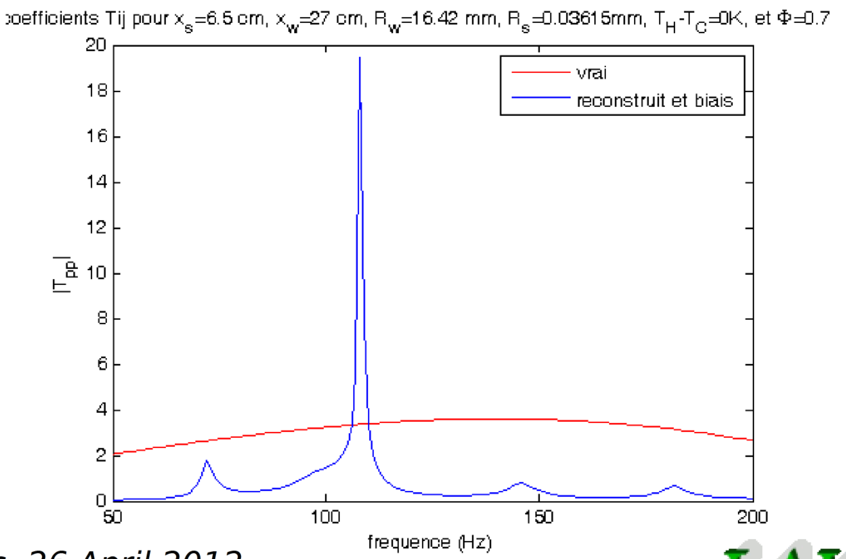
Ceramic Stack - Theory and Simulated **Noise** and **Bias**





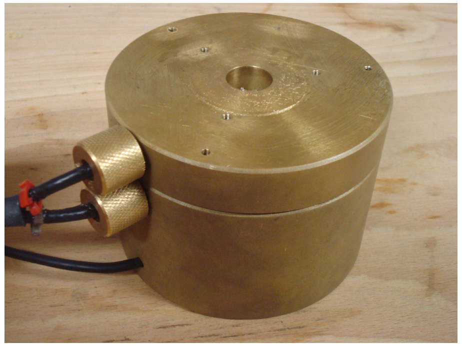
Grid of Steel Pile (regenerator) - Theory and Simulated **Noise** and **Bias**





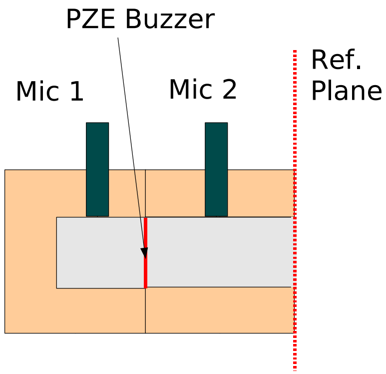
2) Methods for the T evaluation

2.2) The IMPEDANCE Method - Acoustic Impedance Sensor

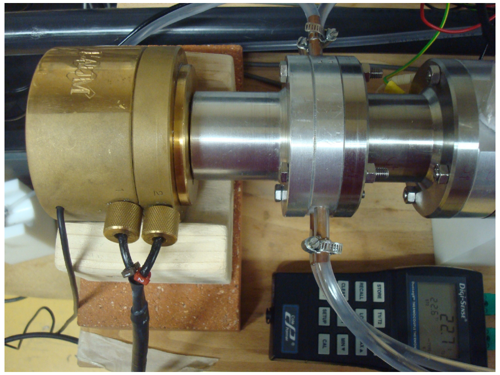


- The **<u>T coefficients</u>** are calculated from the **Impedance Matrix** (**Z**).
- The **<u>Z coefficients</u>** are measured by the **Acoustic Impedance Sensor**.
- Definition of **Z** :

$$\begin{Bmatrix} \tilde{p}_s \\ \tilde{p}_w \end{Bmatrix} = \mathbf{Z} \begin{Bmatrix} \tilde{U}_s \\ \tilde{U}_w \end{Bmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{Bmatrix} \tilde{U}_s \\ \tilde{U}_w \end{Bmatrix}$$



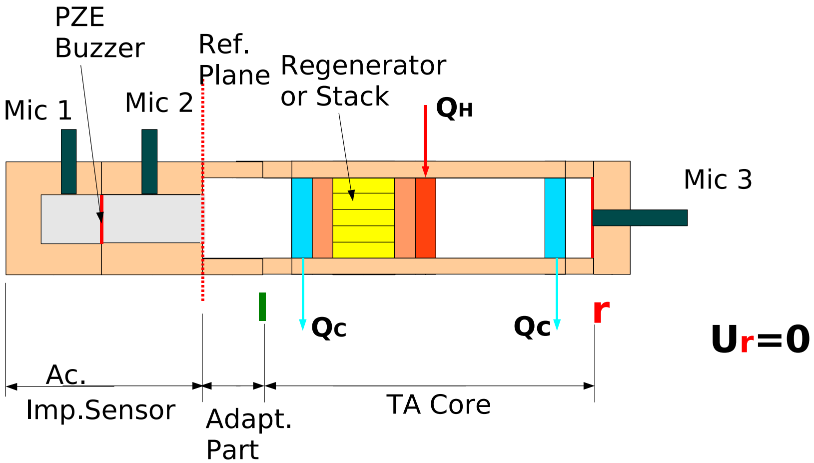
Attachement to the **TA Core**.

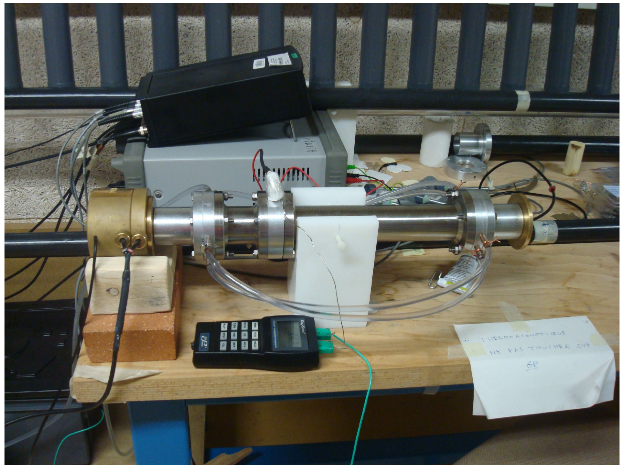


2) Methods for the T evaluation

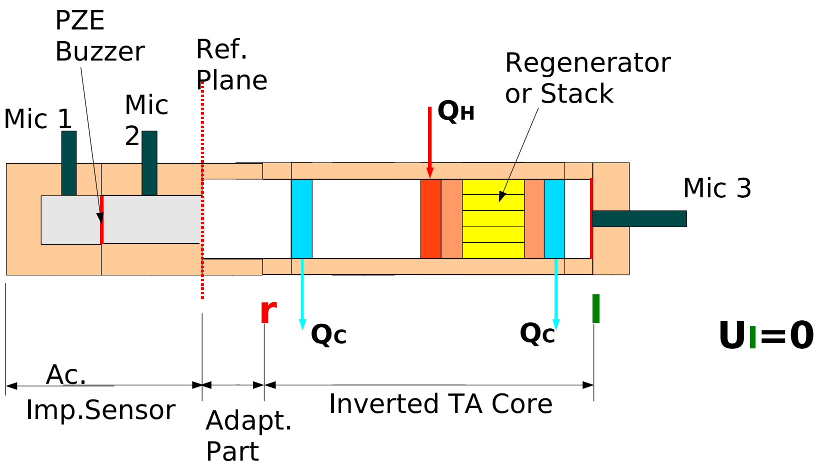
2.2) The IMPEDANCE Method – Configuration and Setup

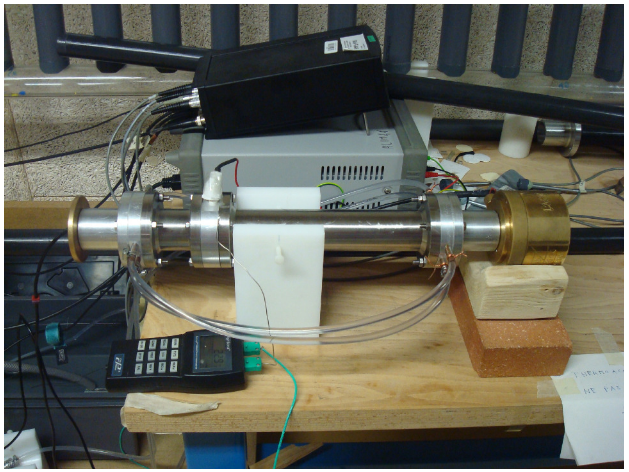
- **First** measurement: Z_{11} ans Z_{21} **DIRECT SENSE**





- **Second** mesurement: Z_{12} and Z_{22} **INVERSE SENSE**





2) Methods for the T evaluation

2.2) The IMPEDANCE Method – Calculation Procedure

- **First Measurement** :

$$Z_{11} = \frac{(H_{21}/K) - \beta}{1 - (\delta H_{21}/K)} \qquad Z_{21} = \frac{H_{31}(1 + \delta Z_{11})}{\delta K_T}$$

Where:

$$H_{21} = \frac{p_2}{p_1}\frac{s_2}{s_1} \qquad H_{31} = \frac{p_3}{p_1}\frac{s_3}{s_1}$$

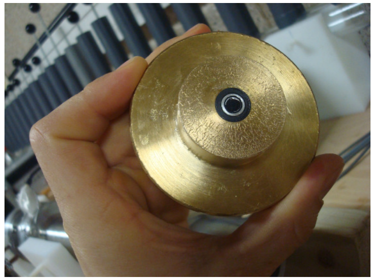
The **sensitivities** are implicit when proceeding with the calibration.

K and K_T are obtained from the calibration as well.

- **Second Measurement** :
  - Same equations
  - New H_{21} and H_{31}
  - Z_{12} and Z_{22} are accomplished

2) Methods for the T evaluation

2.2) The IMPEDANCE Method – Calibration

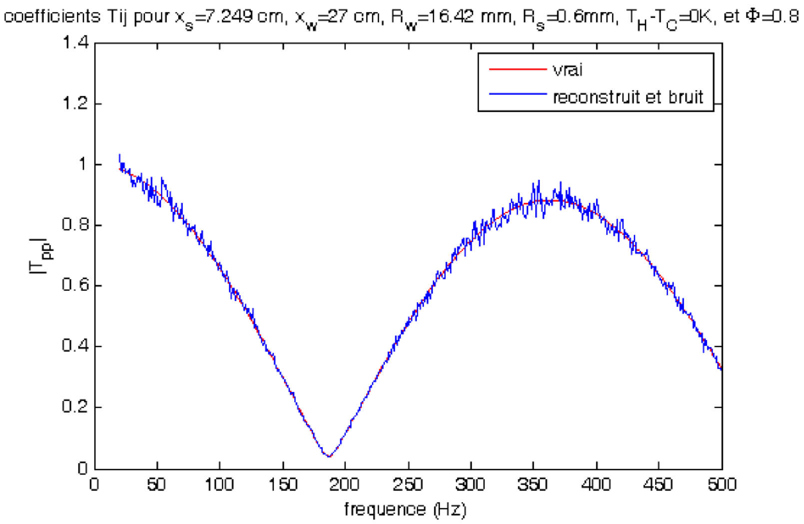


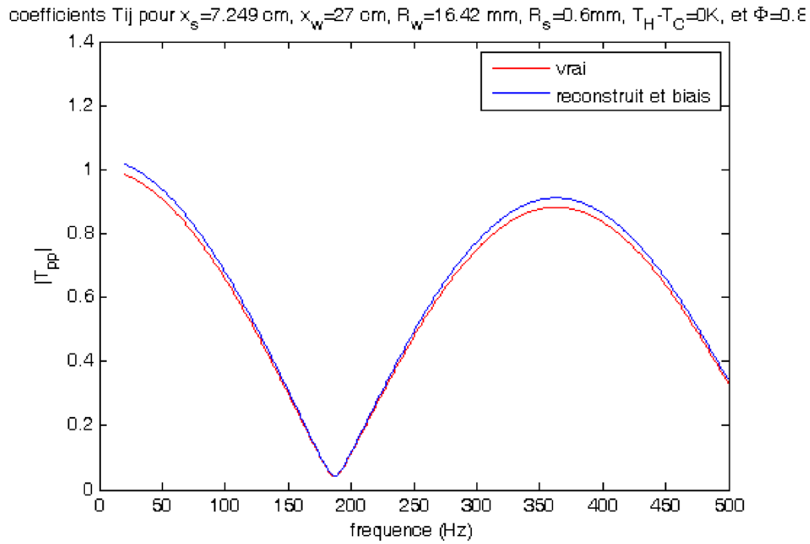
- **Ac. Imp. Sensor** is enclosed by a rigid plate (with **mic. 3**).
- H_{21} and H_{31} are measured.
- p_2 and p_3 are the same reference.
- The **calibration** is established (label « cal »).
- $K = \delta H_{21cal}$ $\quad\quad$ $K_T = H_{31cal}$
- Substitutions into **Z** equations leads to simple calculations.

2) Methods for the T evaluation

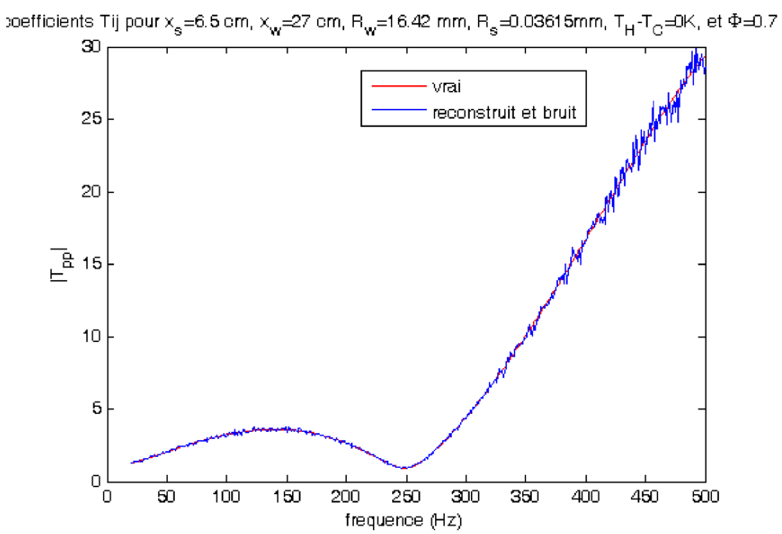
2.2) The IMPEDANCE Method – Sensitivity Analysis

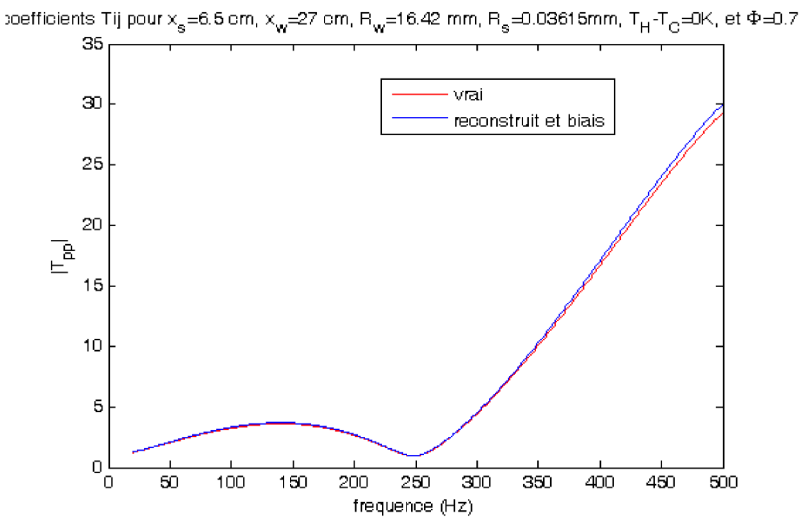
Ceramic Stack - Theory and Simulated **Noise** and **Bias**





Grid of Steel Pile (regenerator) - Theory and Simulated **Noise** and **Bias**





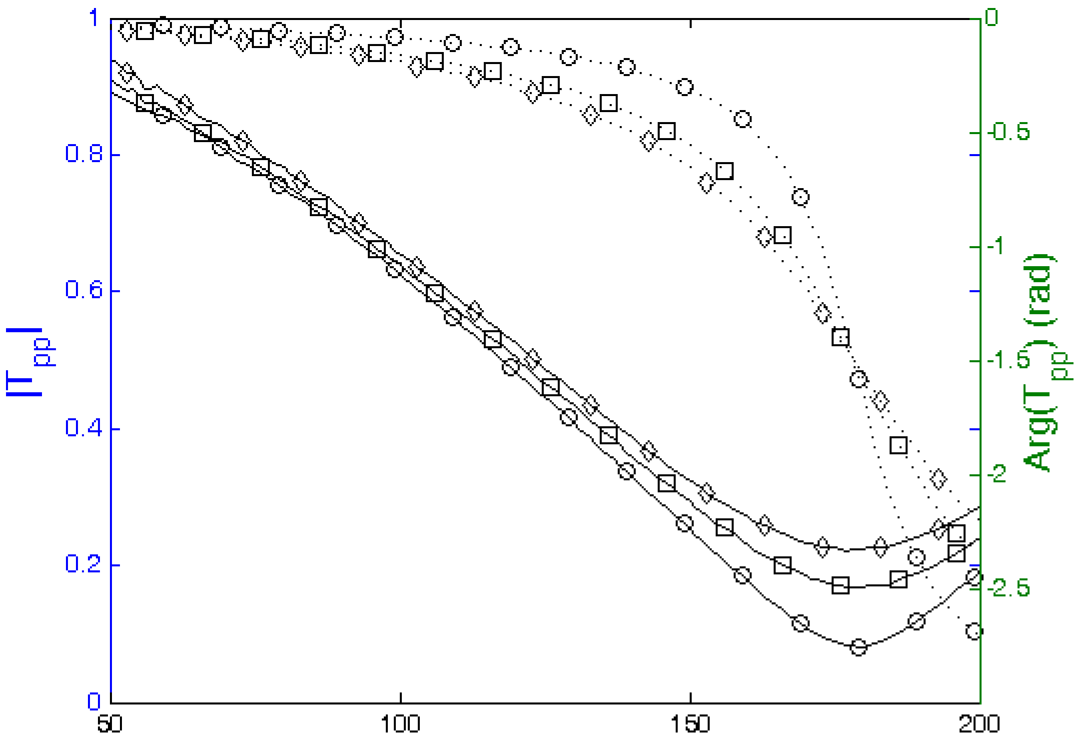
2) Methods for the T evaluation

2.2) The IMPEDANCE Method – Experimental Procedure

- Frequency range from **20Hz** to **500Hz**.
- **Different heating conditions** Q_H are imposed into the system, each one to whose corresponding **T** is measured.
- Q_H from 0W to 81W with a **constant increment** of **9W**.
- A **time delay of 13 minutes** is required for each heat increment, aiming at the steady state regime accoplishment.
- The **cooling condition** Q_C is imposed by water circulation at room temperature.

2) Methods for the T evaluation

2.2) The IMPEDANCE Method – Results - Ceramic Stack



- T_{pp} Amplitude (solid lines)
  Phase (dotted lines)
  - (frequency domain in Hz).

- Three heating conditions are shown:

  - $Q_H = 0W$ (o),
  - $Q_H = 36W$ (square),
  - $Q_H = 72W$ (diamond).

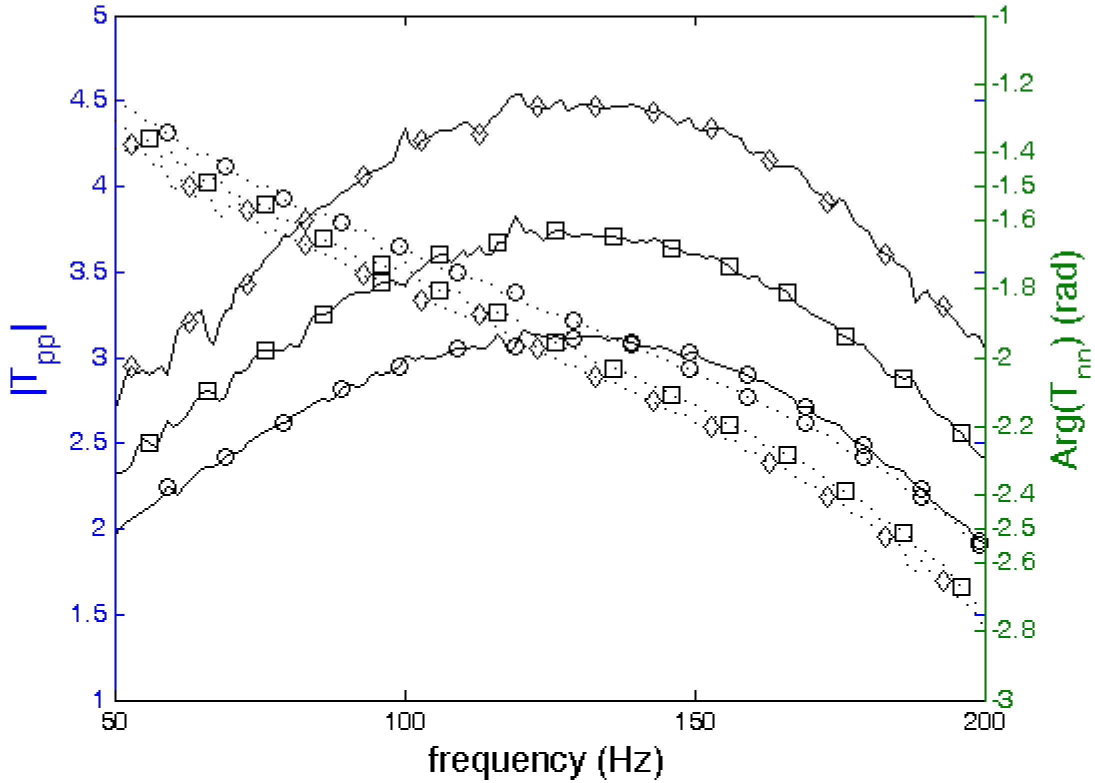
- **Reciprocity** (det(**T**)) values for no heat input ($Q_H = 0W$):

  - Amp. Average: **0.9815** (low bias level), with a std deviation of **4.5e-3** (low noise level); Ideal value: **1**.

  - Phase Av.(rad): **-1.4e-3** (low bias level), with a std deviation of **3.6e-3** (low noise level); Ideal value: **0**.

2) Methods for the T evaluation

2.2) The IMPEDANCE Method – Results - Grid of Steel Pile



- T_{pp} Amplitude (solid lines)
  Phase (dotted lines)
  - (frequency domain in Hz).

- Three heating conditions are shown:
  - $Q_H = 0W$ (o),
  - $Q_H = 36W$ (square),
  - $Q_H = 72W$ (diamond).

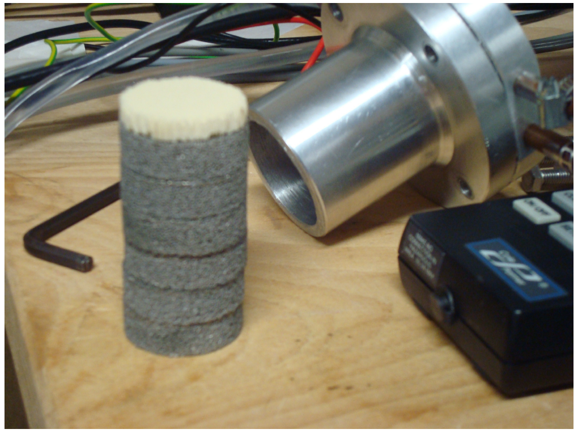
- **Reciprocity** (det(**T**)) values for no heat input ($Q_H = 0W$):
  - Amp. Average: **1.0037** (low bias level), with a std deviation of **0.0107** (low noise level); Ideal value: **1**.
  - Phase Av. (rad): **9.0e-3** (low bias level), with a std deviation of **7.2e-3** (low noise level); Ideal value: **0**.

2) Methods for the T evaluation

2.2) The IMPEDANCE Method – Ni-Cr alloy and RVC Foams

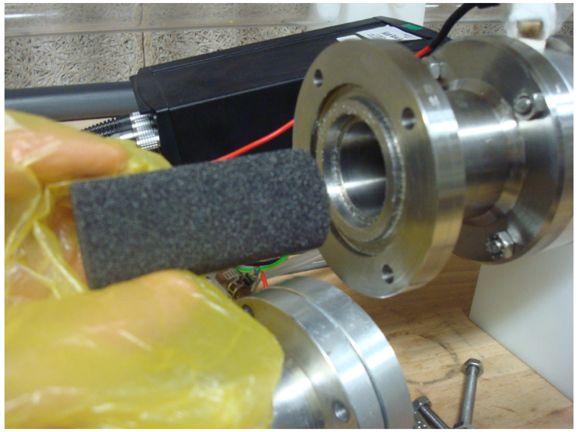
Ni-Cr alloy Foam

- **Reciprocity** (det(**T**)) values for no heat input ($Q_H = 0W$):
  - Amp. Average: **0.9288 (high bias level)**, with a std deviation of **0.0154** (med. noise level); Ideal value: **1**.
  - Phase Av. (rad): **-0.0102** (med. bias level), with a std deviation of **0.0101** (med. noise level); Ideal value: **0**.



RVC Foam

- **Reciprocity** (det(**T**)) values for no heat input ($Q_H = 0W$):
  - Amp. Average: **0.9893** (med. bias level), with a std deviation of **0.0317** (med. noise level); Ideal value: **1**.
  - Phase Av. (rad): **-0.0016** (low bias level), with a std deviation of **0.0233** (med. noise level); Ideal value: **0**.



3) Additional Investigation (Impedance Method)

– New measurements: more **accurate**

Ni-Cr alloy Foam

- **Reciprocity** (det(**T**)) values for no heat input ($Q_H = 0W$):
  - Amp. Average: **1.0014** **(low bias level)**, with a std deviation of **1.9e-3** **(low noise level)**; Ideal value: **1**.

RVC Foam

- **Reciprocity** (det(**T**)) values for no heat input ($Q_H = 0W$):
  - Amp. Average: **1.0002** (low bias level), with a std deviation of **2.3e-3** (low noise level); Ideal value: **1**.

Ceramic Stack

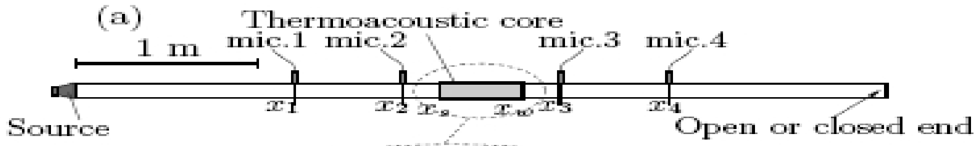
- **Reciprocity** (det(**T**)) values for no heat input ($Q_H = 0W$):
  - Amp. Average: **1.0017** (low bias level), with a std deviation of **2.1e-3** (low noise level); Ideal value: **1**.

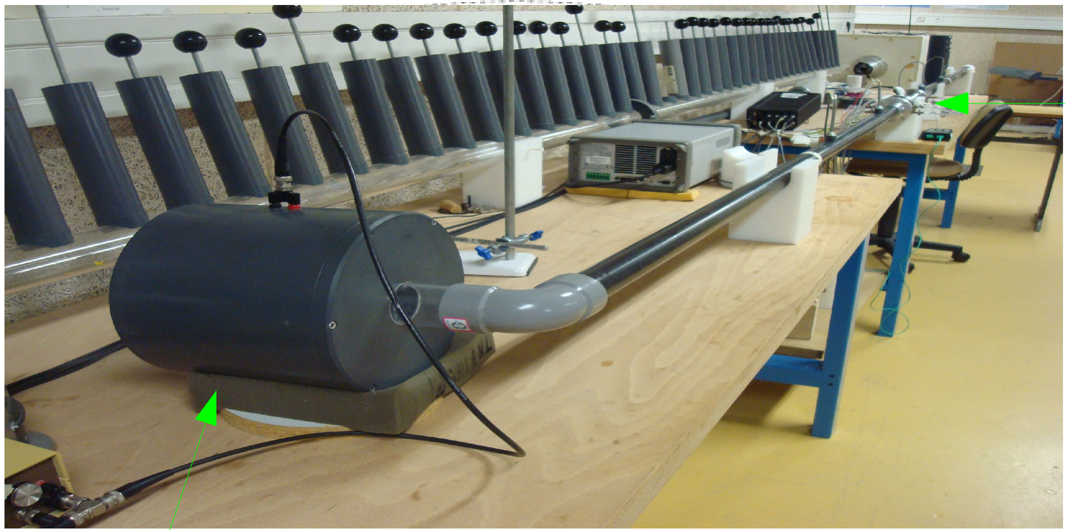
4) Conclusions and Perspectives

- A new method for the measurement of the **T** of the **TA Core** has been shown.
- The **Impedance Method**, contrarily to the **Two-Loads Method**, is accomplishing either for the <u>stack</u> or <u>regenerator</u>.
- The **Impedance Method** performance was confirmed also for the Ni-Cr alloy foam.
- Thermoacoustic investigations are under work to evaluated these materials performence.
- Futures works will be devoted to the design of TA machines.

2) Methods for the T matrix evaluation

2.1) The Two-Loads Method – Experimental Setup

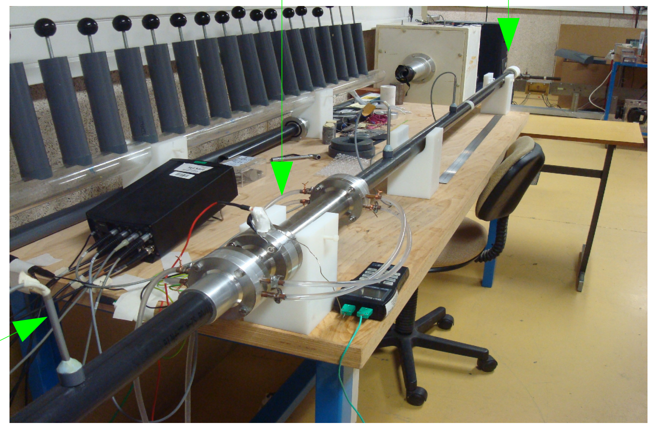




Closed end

TA Core

The Complete Assembling



Microphone

Loudspeaker

Right side detail


Digi-Sense
DualLogR THERMOCOUPLE THERMOMETER
190.1
OPEN
HOLD
MAX
MIN
CAL
T1/T2
LOG
PRINT
SETUP
STORE
RECALL
CLEAR
ON/OFF
Cole Parmer
VERN