Auto-oscillateurs thermoacoustiques : effets non linéaires et comportement dynamique

Guillaume Penelet

Laboratoire d'Acoustique de l'Université du Maine, UMR CNRS 6613
avenue Olivier Messiaen, 72085 Le Mans cedex 9, France
guillaume.penelet@univ-lemans.fr

Plan

Préambule : de quelle thermoacoustique parle-t-on ?

Théorie linéaire de la thermoacoustique

Exemple : le tube de Sondhauss

Effets de saturation non linéaires

Dynamique des auto-oscillateurs thermoacoustiques

Forçage des auto-oscillateurs thermoacoustique

Conclusion

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Thermoacoustique = interaction thermique/acoustique . . .

$\Rightarrow$ de quelle thermoacoustique parle-t-on ici ?

- Tomographie thermoacoustique,
- Haut-parleurs thermoacoustiques,
- Effet piston,
- Instabilités de combustion,
- Machines thermoacoustiques (pompes à chaleur et moteurs thermoacoustiques).

→ Tomographie thermoacoustique

Méthode d'imagerie médicale basée sur la réponse de certains tissus à une impulsion EM

chauffage => détente thermoélastique => onde acoustique.

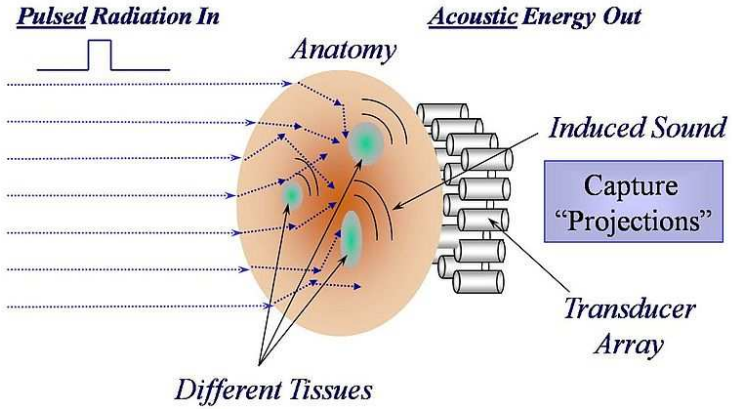


Schéma de principe de la tomographie TA

→ Haut-parleurs thermoacoustiques.

→ Effet piston.

→ Instabilités de combustion.

→ Machines thermoacoustique.

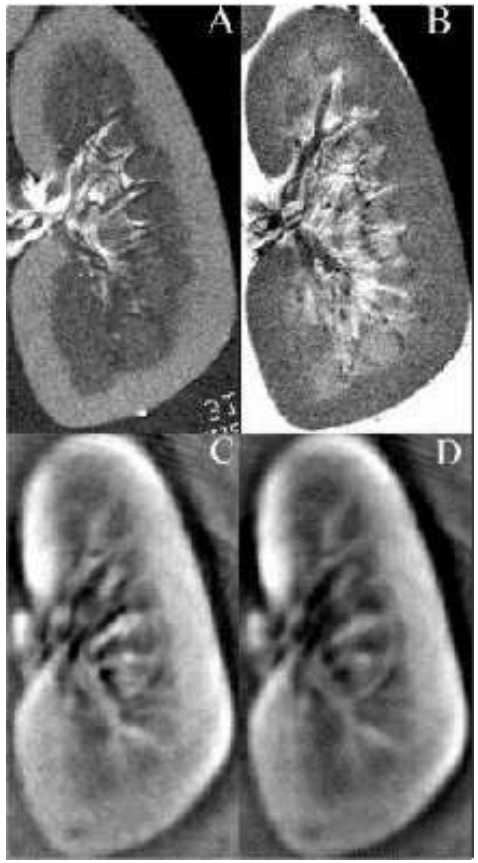
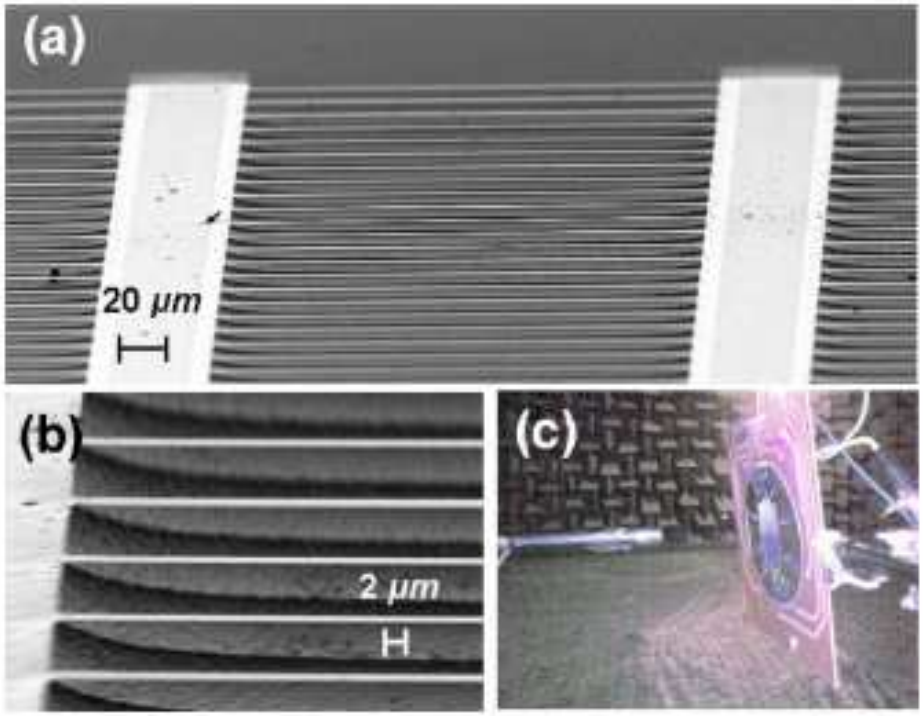


Image 3D d'un rein d'agneau obtenu par tomographie TA (C,D), comparée à celle obtenue par IRM. D'après [1]

[1] Kruger R.A. et al., Radiology 216 :279-283, 2000.

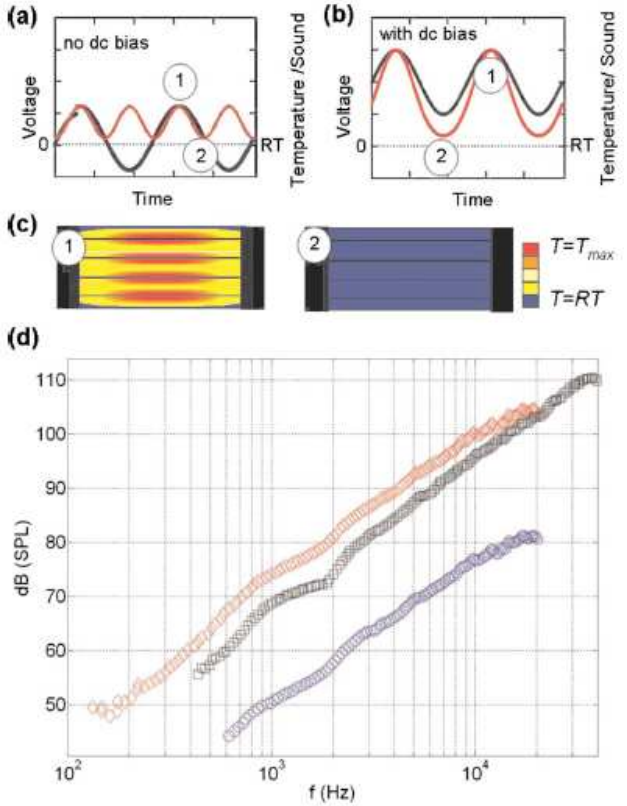
→ Tomographie thermoacoustique

→ **Haut-parleurs thermoacoustiques.**
chauffage fluctuant d'une surface solide
=> rayonnement acoustique



Haut-parleur thermoacoustique. D'après [1]

→ Effet piston.
→ Instabilités de combustion.
→ Machines thermoacoustiques



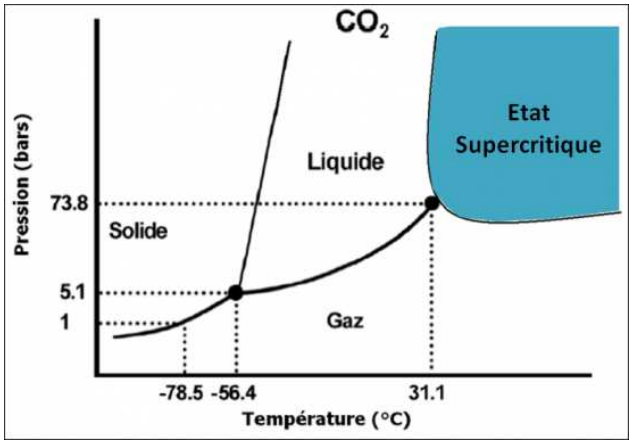
Réponse en fréquence d'un HP TA. D'après [1]

Ecouteurs thermoacoustiques. D'après [2]

[1] Nyskacen et al. Appl. Phys. Let., 95 :163102, 2009. [2] US Patent n°8625822, Tsinghua University (Beijing)

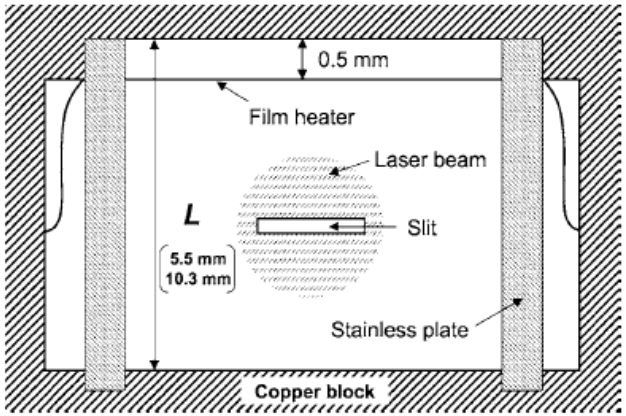
→ Tomographie thermoacoustique
→ Haut-parleurs thermoacoustiques.

→ **Effet piston**
Un 4^{e} mode de transport de la chaleur !

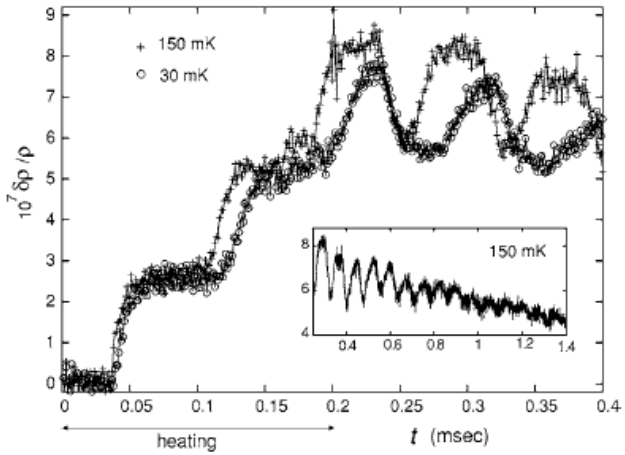


Autour du point critique, $\chi \longrightarrow \infty$ et $\kappa \longrightarrow 0$
Incrément de chauffage local
$\Rightarrow$ expansion abrupte de la C.L. (effet piston)
$\Rightarrow$ compression adiabatique de la cellule & ondes TA
$\Rightarrow$ T $\uparrow$ avec un temps caractéristique $\tau_{PE} \ll \tau_{\kappa}$

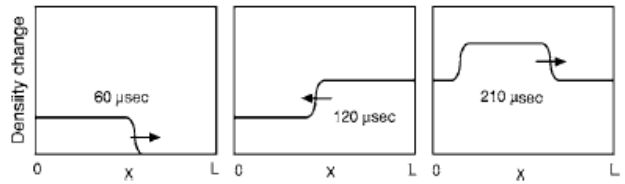
→ Instabilités de combustion.
→ Machines thermoacoustiques



Mesure des ondes TA dans du CO2 au point critique[1]



Réponse $\Delta\rho/\rho$ à un pulse de chauffage[1].



$\Delta\rho(x, t)$ au cours du chauffage[1].

[1] Y. Miura et al., Phys. Rev. E, 74 :010101, 2006

→ Tomographie thermoacoustique
→ Haut-parleurs thermoacoustiques.
→ Effet piston

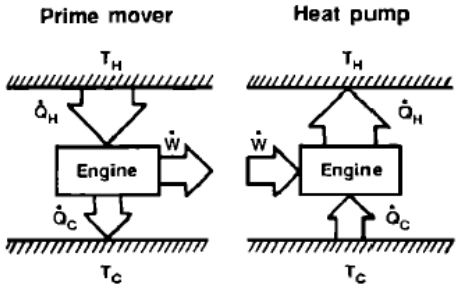
→ **Instabilités de combustion**
Génération spontanée d'oscillations acoustiques résultant d'une interaction complexe d'une flamme et d'un écoulement dans une chambre de combustion.

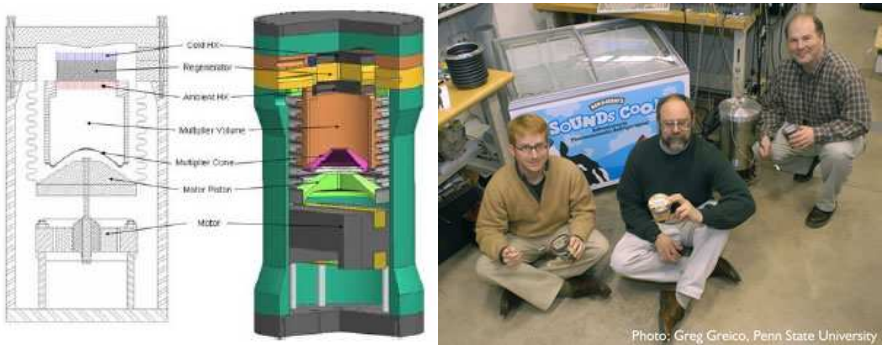
→ Machines thermoacoustiques

- Tomographie thermoacoustique
- Haut-parleurs thermoacoustiques.
- Effet piston
- Instabilités de combustion

- **Machines thermoacoustique**

Machine thermique cyclique faisant usage d'un travail mécanique de nature **acoustique**.





Congélateur thermoacoustique réalisé à l'université de Penn State pour le glacier ben & Jerry's[1]

Machine TA tritherme (moteur+frigo) réalisé au Los Alamos National Laboratory[2].

[1] http://www.acs.psu.edu/thermoacoustics/refrigeration/benandjerrys.htm, [2] G. W. Swift, J. J. Wollan, GasTIPS, Volume 8, Number 4, pages 21-26 (Fall 2002).

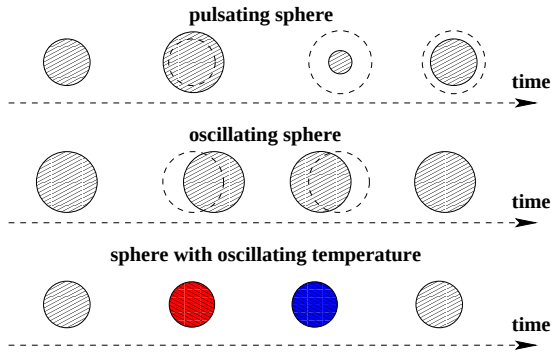
Elementary sources of sound emission, unsteady heat release

- The *linearized* equations of motion for an *inviscid* and *non-heat-conducting* ideal gas

$$\partial_t \rho' + \rho_0 \mathrm{div}\mathbf{v} = Q(\mathbf{r}, t), \qquad Q \text{ :mass addition}$$

$$\rho_0 \partial_t \mathbf{v} + \mathbf{grad} p = \mathbf{f}(\mathbf{r}, t), \qquad \mathbf{f} \text{ : force}$$

$$\rho_0 C_p \partial_t \tau - \partial_t p = \dot{q}(\mathbf{r}, t), \qquad \dot{q} \text{ : Heat release}$$



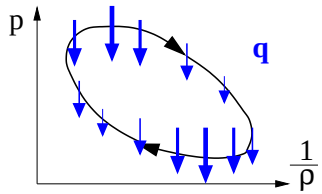
- The resulting inhomogeneous wave equation (using $p = RT_0\rho' + R\rho_0\tau$) is

$$\Delta p - \frac{1}{c_0^2}\partial_{tt}^2 p = -\frac{\partial_t \dot{q}}{T_0 C_p} - \rho_0 \partial_t Q + \mathrm{div}\mathbf{f}$$

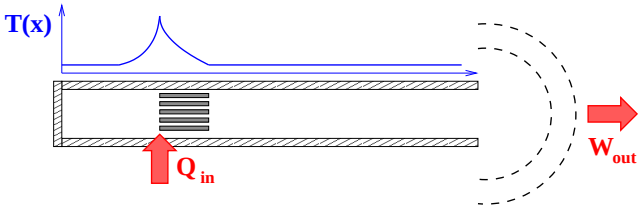
- Contrarily to Q and f, $\dot{q}$ is difficult to derive in realistic cases because
  - ♭ The heat conductivity of the gas should be considered
  - ♭ Generally, $\dot{q} = \dot{q}(p, \mathbf{v})$, which may lead to self-sustained oscillations (e.g. Rijke tube etc . . .)

The Rayleigh criterium ($\int p\dot{q}.dt > 0$)

" . . .at the phase of greatest condensation heat is received by the air, and at the phase of greatest rarefaction heat is given up from it, and thus there is a tendency to maintain the vibrations " J.W. Strutt (Lord Rayleigh), « The theory of sound », Dover, New York, 2nd edition, § 322i, p. 231, 1945)



TA Laser



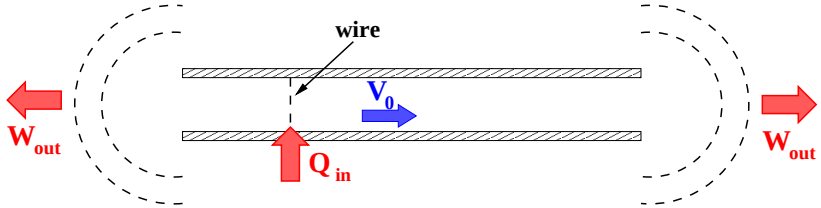
$$\dot{q} = f(d_x T, p, v_x, stack \ldots)$$

⋆ not acoustically compact

⋆ amplification of the first mode ?

$$\int p\dot{q}.dt > 0 \Rightarrow d_x T < 0$$

Rijke tube



$$\dot{q} = f(T_{wire}, v_x, V_0 \ldots)$$

⋆ acoustically compact
$(\dot{q} \approx \dot{q}(t)\delta(x - x_{wire}))$

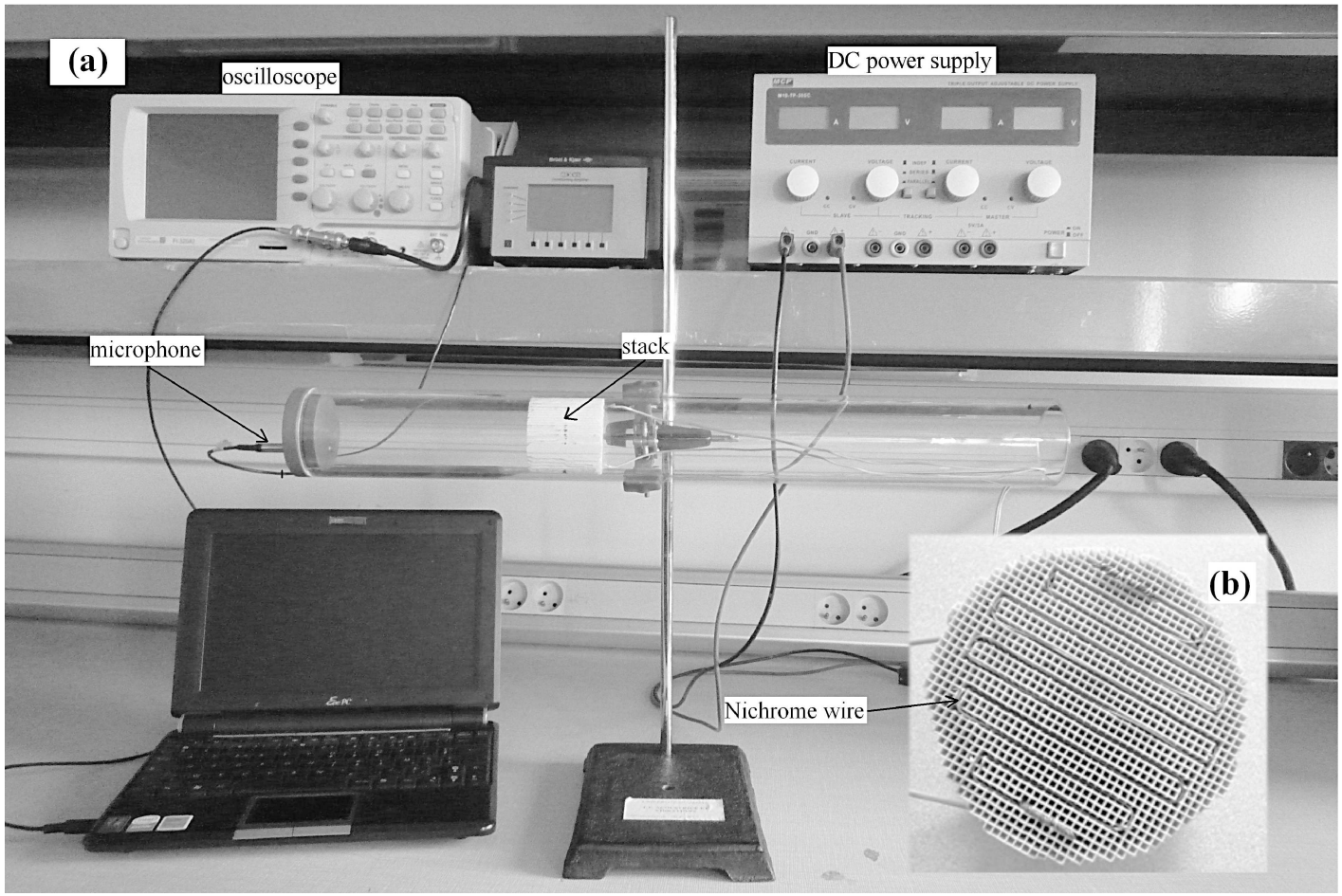
⋆ amplification of the first mode ?

$$\int p\dot{q}.dt > 0 \Rightarrow x_w < \frac{L}{2}$$

Common features

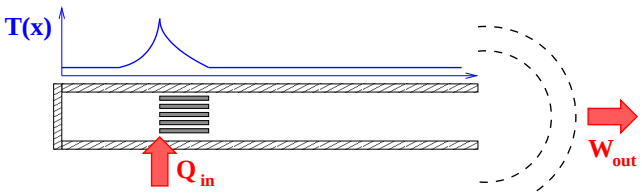
⋆ Autonomous oscillators driven by heat (which both fill the "Rayleigh criterium")
⋆ Simple in terms of geometry, but complicated operation above threshold . . .

The TA Laser : a prototypical example

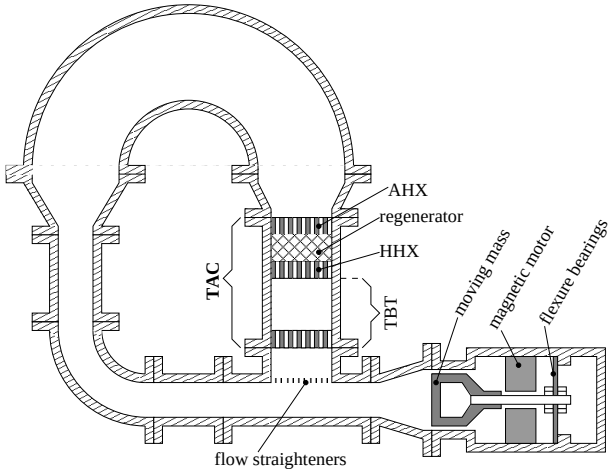


- Heat input => sound output (self-sustained acoustic oscillations at the frequency of the most unstable mode(s))
- Keywords : heat engine (prime-movers and heat pumps), autonomous oscillator
- Paradigmatic of the challenges that need to be taken up to understand more deeply the processes controlling the operation of TA engines

Moteur thermoacoustique "quart d'onde"



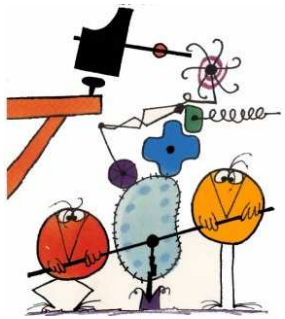
Générateur thermo-acousto-électrique



⋆ Application d'un ∇T le long d'un matériau poreux placé dans un réseau de guide d'onde ⇒ auto-oscillations à la fréquence du mode le plus instable

⋆ Géométrie ± simple, mais fonctionnement complexe au delà du seuil (pompage θ_{ac}, "vent acoustique", perte de charge singulière, propa NL, ...)

⇒ comprendre le plus simple ... ...pour mieux appréhender le "compliqué".



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Exemple : le tube de Sondhauss

Effets de saturation non linéaires

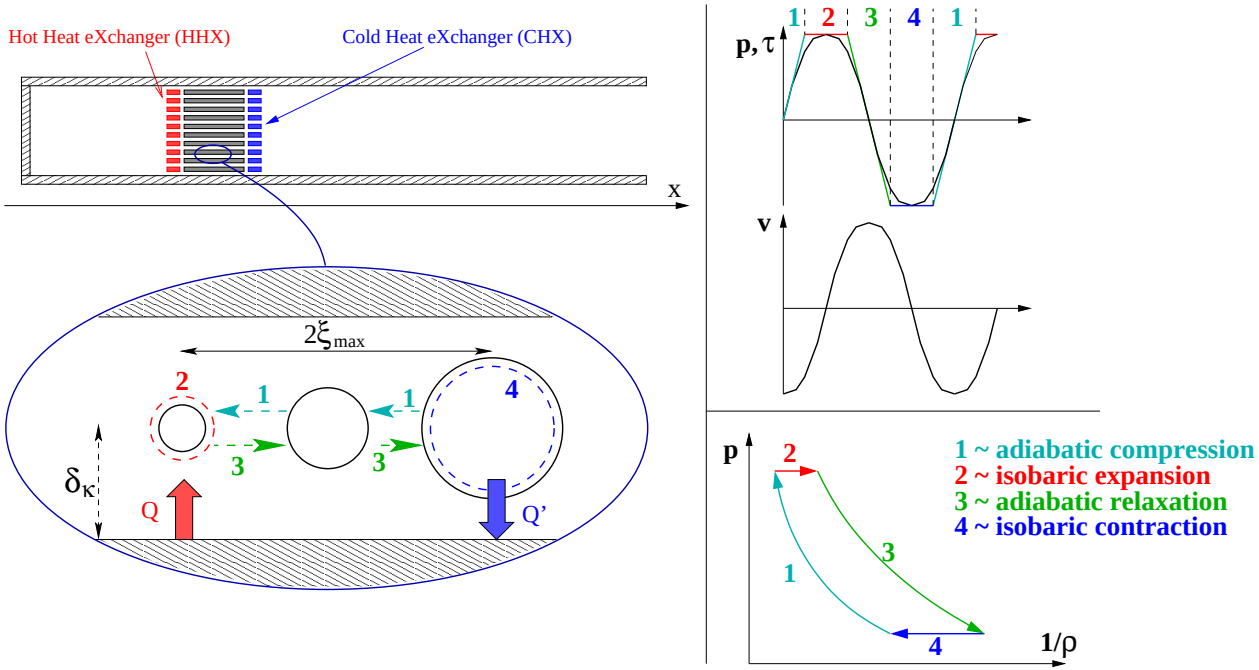
Dynamique des auto-oscillateurs thermoacoustiques

Forçage des auto-oscillateurs thermoacoustique

Conclusion

The TA Laser : a conceptual, very simplified approach . . .

⋆ Simplistic description => Lagrangian approach, follow a gas parcel submitted to a standing wave through a channel with an axial temperature gradient $d_x T_0$



=> Work production if $|d_x T|\xi_{max} > \tau_{max}$ and $d_x T < 0$

⋆ But actual processes are actually more complex (heat exchange occurs continuously)

⋆ A key point : need of an imperfect thermal contact

=>"efficient" gas parcels should be at about δ_κ from the wall, where $\delta_\kappa = \sqrt{2\kappa/\omega}$ is the (frequency-dependent) thermal boundary layer thickness

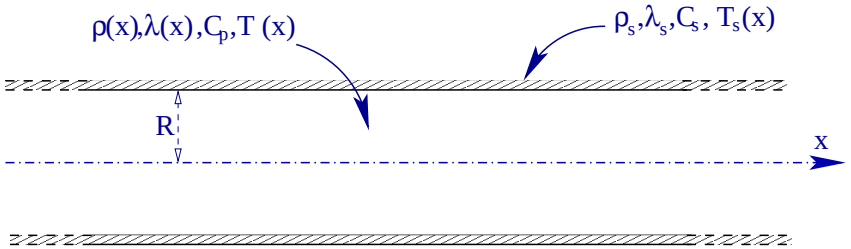
⋆ Threshold of thermoacoustic instability (for a given mode) ?

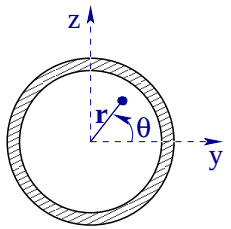
=> Sound amplification must be larger than losses

⋆ Plane wave propagation through a viscous and heat-conducting gas along a duct submitted to a temperature gradient.

⋆ Both the production of acoustic work and the "bucket brigade" heat transport by sound along the duct are then described by second-order quantities.

the "bucket brigade"





Main assumptions		
⋆ **Low amplitudes**	⋆ **Typical wavelength » R**	⋆ **No mean flow**
$P(x,r,t) = P_0 + p(x,r,t)$	=> planes waves	⋆ **ideal gas**
$\rho(x,r,t) = \rho_0(x) + \rho'(x,r,t)$	$p(x,r,t) = p(x,t)$	⋆ $\rho_s C_s \gg \rho_0 C_p$
$T(x,r,t) = T_0(x) + \tau(x,r,t)$	=>"boundary layer approximation"	⋆ $\lambda_s \gg \lambda$
$\mathbf{v}(x,r,t) \ll c_0$	$\lvert\partial_r \zeta\rvert \gg \lvert\partial_x \zeta\rvert$	
$S(x,r,t) = S_0(x) + s(x,r,t)$	$(\zeta = p, \rho', \tau, \mathbf{v}, s)$	

[1] N. Rott, Adv. Appl. Mech., 1980 ; [2] G.W. Swift, J. Acoust. Soc. Am., 1988.

⋆ Equations fondamentales linéarisées

$$\begin{aligned}
\rho d_t \mathbf{V} = -\nabla P + \mu\nabla^2\mathbf{V} + (\eta+\mu/3)\nabla(\nabla.\mathbf{V}), \quad &\Rightarrow \quad \rho_0 i\omega\tilde{v}_x = -d_x\tilde{p} + \mu(1/r)\partial_r(r\partial_r\tilde{v}_x)\\
\partial_t\rho + \nabla.(\rho\mathbf{V}) = 0 \quad &\Rightarrow \quad i\omega\tilde{\rho}' + \rho_0\partial_x\tilde{v}_x + \tilde{v}_x d_x\rho_0 = 0\\
\rho T d_t S = \lambda\Delta T + O_2(\mathbf{v}) \quad &\Rightarrow \quad \rho_0 T_0 (i\omega\tilde{s} + \tilde{v}_x\partial_x S_0) = \lambda(1/r)\partial_r(r\partial_r\tilde{\tau})\\
dS = (C_p/T)dT - (\alpha/\rho)dP \quad &\Rightarrow \quad T_0\tilde{s} = C_p\tilde{\tau} - \tilde{p}/\rho_0\\
d\rho = -\rho\alpha dT + \rho\chi_t dP \quad &\Rightarrow \quad \tilde{\rho}' = -(\rho_0/T_0)\tilde{\tau} + (\gamma/c_0^2)\tilde{p}
\end{aligned}$$

⋆ Conditions aux limites (tuyau d'axe x et de rayon R) $\Rightarrow \tilde{\mathbf{v}}(x, r=R) = \mathbf{0}$, $\tilde{\tau}(x, r=R) = 0$.

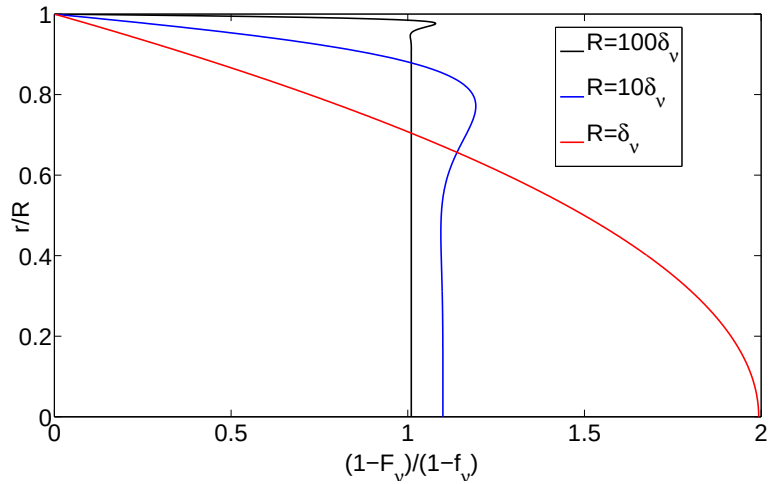
$$\begin{aligned}
\tilde{v}_x(x,r) &= i/(\omega\rho_0)d_x\tilde{p}\,[1 - F_\nu(r)],\\
\tilde{\tau}(x,r) &= \tilde{p}/(\rho_0 C_p)\,[1 - F_\kappa(r)] - 1/(\rho_0\omega^2)d_x\tilde{p}d_x T_0\,[1 - (\mathrm{Pr}F_\nu(r) - F_\kappa(r))/(\mathrm{Pr}-1)],\\
\tilde{\rho}' &= 1/\omega^2\,[1 - (\mathrm{Pr}F_\nu - F_\kappa)/(\mathrm{Pr}-1)+]\,d_x T_0/T_0 d_x\tilde{p} + 1/c_0^2\,[1 + (\gamma-1)F_\kappa]\,\tilde{p}\\
\tilde{s} &= -\tilde{p}/(\rho_0 T_0)F_\kappa - C_p/(\rho_0\omega^2)d_x\tilde{p}d_x T_0/T_0\,[1 - (\mathrm{Pr}F_\nu - F_\kappa)/(\mathrm{Pr}-1)]
\end{aligned}$$

$$F_{\nu,\kappa}(r) = \frac{J_0(k_{\nu,\kappa}r)}{J_0(k_{\nu,\kappa}R)}$$

$$k_{\nu,\kappa} = \frac{1-i}{\delta_{\nu,\kappa}}$$

$$\delta_\nu = \sqrt{\frac{2\nu}{\omega}},\ \delta_\kappa = \sqrt{\frac{2\kappa}{\omega}}$$

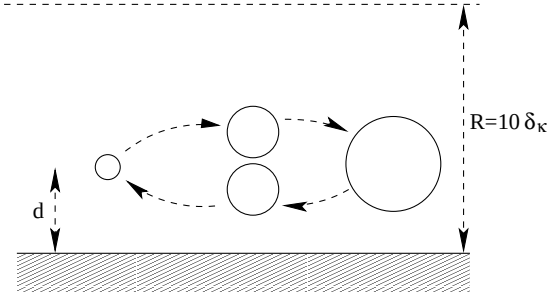
$$f_{\nu,\kappa} = \langle F_{\nu,\kappa}\rangle = \frac{2}{k_{\nu,\kappa}R}\frac{J_1(k_{\nu,\kappa}R)}{J_0(k_{\nu,\kappa}R)}$$



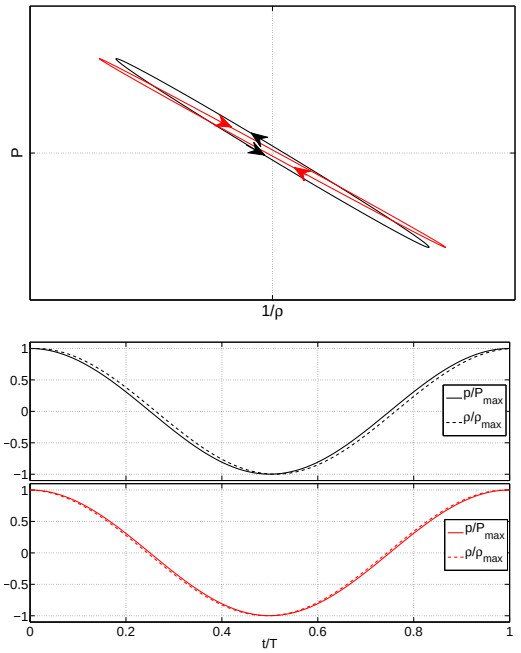
⋆ Onde plane stationnaire dans un tuyau "large"

$$\tilde{p}(x) = P_{max} \cos(k_0 x)$$

$$\tilde{\rho'} = \frac{1}{\omega^2}\left[1 - \frac{\mathrm{Pr} F_\nu - F_\kappa}{\mathrm{Pr} - 1} + \right] \frac{d_x T_0}{T_0} d_x \tilde{p} + \frac{1}{c_0^2}\left[1 + (\gamma - 1) F_\kappa\right] \tilde{p}$$



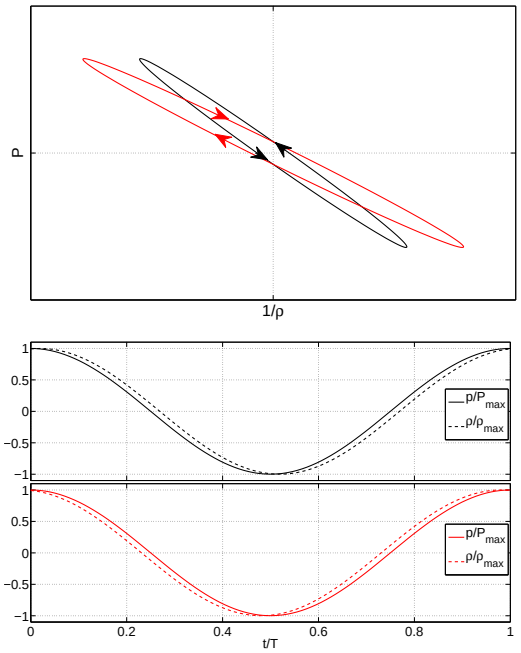
d $= \delta_\kappa / 3$



$d_x T_0 = 0$

$d_x T_0 = -220 K/m$

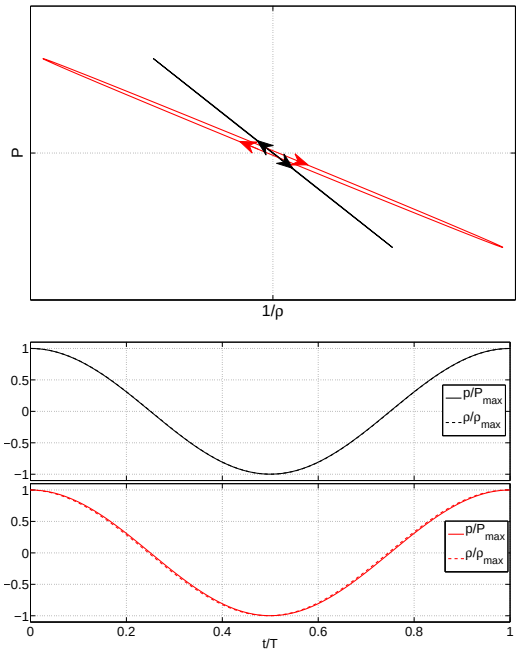
d $= \delta_\kappa$



$d_x T_0 = 0$

$d_x T_0 = -220 K/m$

d $= 3\delta_\kappa$



$d_x T_0 = 0$

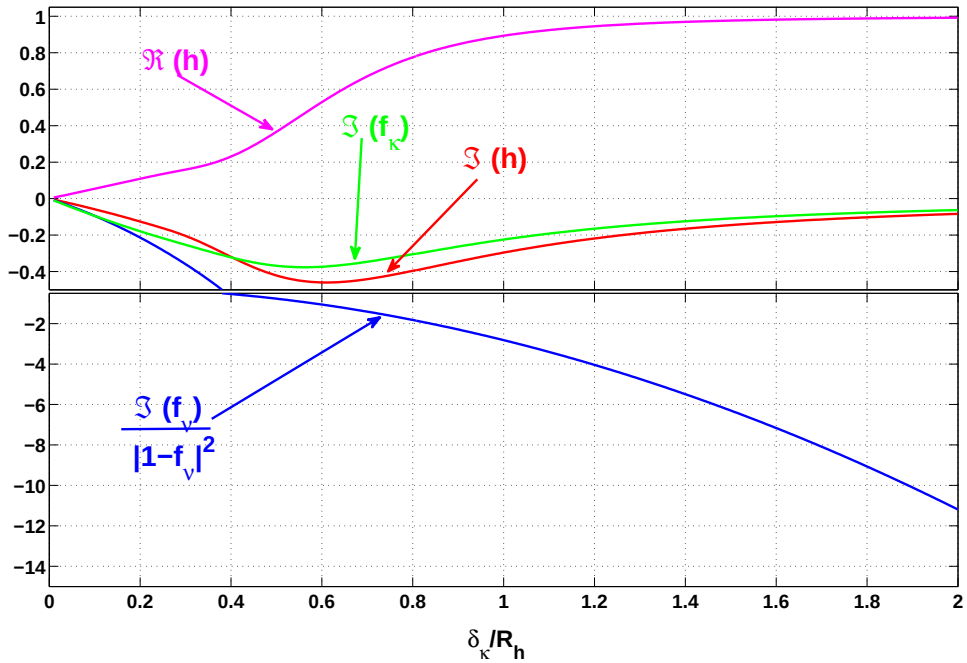
$d_x T_0 = -220 K/m$

⋆ From the knowledge of the acoustic variables, it is possible to calculate the time-average second order acoustic power produced (or absorbed) per unit volume $\mathfrak{w}_2$ ($[\mathfrak{w}_2] = \mathrm{W.m^{-3}}$) :

$$\mathfrak{w}_2 = \partial_x(\overline{p < v_x >}).$$

⋆ After some calculations[3],

$$\begin{aligned}
\mathfrak{w}_2 &= \mathfrak{w}_\kappa + \mathfrak{w}_\nu + \mathfrak{w}_{sw} + \mathfrak{w}_{tw} \\
\mathfrak{w}_\kappa &= \frac{1}{2}\frac{(\gamma-1)\omega}{\rho_0 c_0^2}\Im\left(f_\kappa\right)|\tilde{p}|^2 \\
\mathfrak{w}_\nu &= \frac{1}{2}\omega\rho_0\frac{\Im(f_\nu)}{|1-f_\nu|^2}|\langle\tilde{v}_x\rangle|^2 \\
\mathfrak{w}_{sw} &= -\Im(h)\frac{d_x T_0}{T_0}\frac{1}{2}\Im\left(\tilde{p}\langle\tilde{v}_x^*\rangle\right) \\
\mathfrak{w}_{tw} &= \Re(h)\frac{d_x T_0}{T_0}\frac{1}{2}\Re\left(\tilde{p}\langle\tilde{v}_x^*\rangle\right) \\
h &= \frac{f_\kappa - f_\nu}{(1-\mathrm{Pr})(1-f_\nu)}
\end{aligned}$$



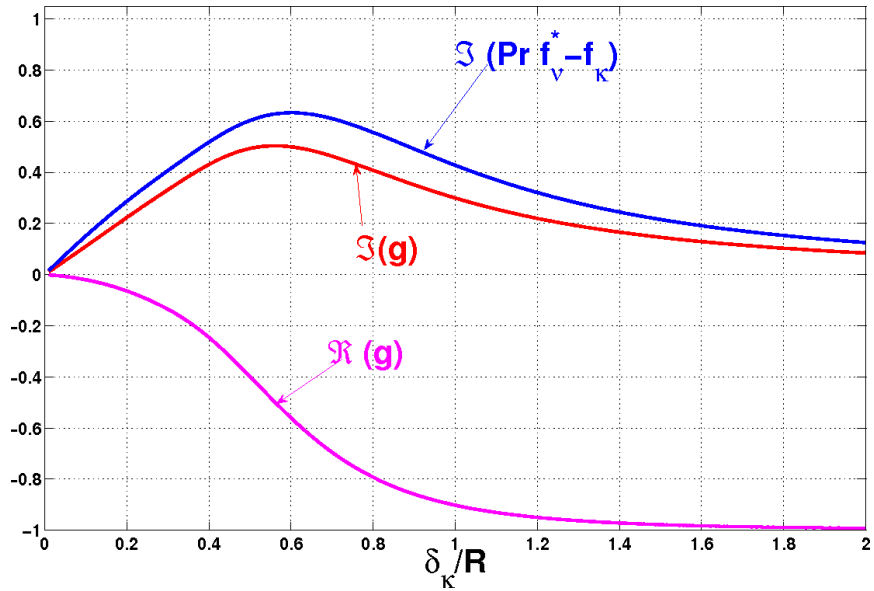
[3] A. Tominaga, Cryogenics, 1995.

⋆ From the knowledge of the acoustic variables, it is also possible to calculate the thermoacoustic heat flux q_2 ($[q_2] = \mathrm{W.m^{-2}}$) :

$$q_2 = \rho_0 T_0 (\overline{\langle s v_x \rangle}), \quad s = \frac{C_p}{T_0}\tau - \frac{1}{\rho_0 T_0} p$$

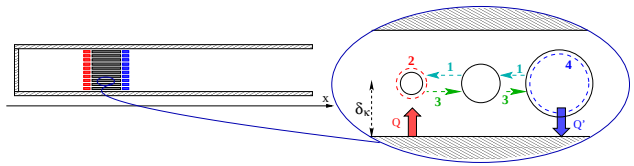
⋆ After some calculations,

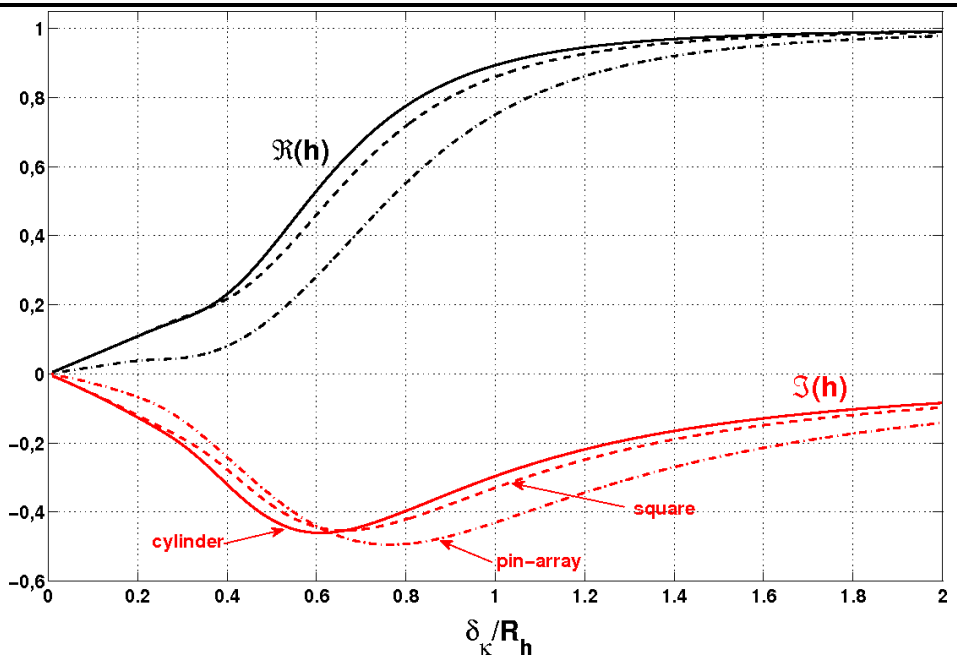
$$\begin{aligned} q_2 &= q_{sw} + q_{tw} + \lambda_{ac} d_x T_0 \\ q_{sw} &= -\Im(g)\frac{1}{2}\Im\left(\tilde{p}\langle \tilde{v}_x^* \rangle\right) \\ q_{tw} &= \Re(g)\frac{1}{2}\Re\left(\tilde{p}\langle \tilde{v}_x^* \rangle\right) \\ \lambda_{ac} &= \frac{\rho_0 C_p}{2\omega(1-\mathrm{Pr}^2)} \frac{\Im\left(\mathrm{Pr} f_\nu^* - f_\kappa\right)}{|1-f_\nu|^2} |\langle \tilde{v}_x \rangle|^2 \\ g &= \frac{f_\nu^* - f_\kappa}{(1+\mathrm{Pr})(1-f_\nu^*)} \end{aligned}$$

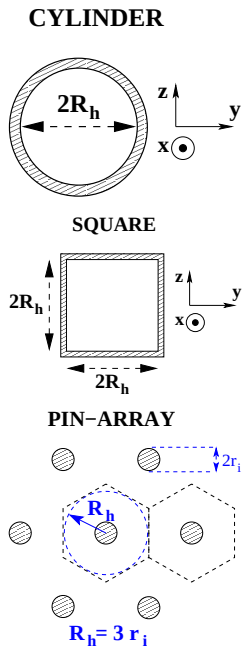


$$\mathfrak{w}_2 = \underbrace{\mathfrak{w}_\kappa + \mathfrak{w}_\nu}_{\text{losses}} + \underbrace{\overbrace{\mathfrak{w}_{sw}}^{\propto -\Im(h)} + \overbrace{\mathfrak{w}_{tw}}^{\propto \Re(h)}}_{\text{production}}$$

$$h = (f_\kappa - f_\nu)/[(1-\mathrm{Pr})(1-f_\nu)]$$



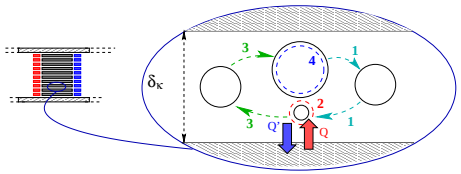


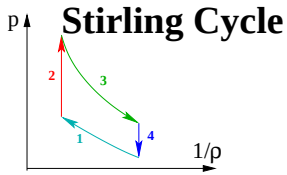


⋆ In case of a standing wave, one need $\delta_\kappa \leq R_h$ (stack) and always have $|\Im(h)| < 1$
=> intrinsic irreversibility[4] due to the need of an imperfect thermal contact
⋆ But in case of a travelling wave phasing , we have $|\Im(h)| \approx 1$ if $\delta_\kappa \geq R_h$
=> a travelling wave phasing combined with a regenerator ($\delta_\kappa \geq R_h$) should be better !

[4] J. C. Wheatley et al, J. Acoust. Soc. Am., 1983.

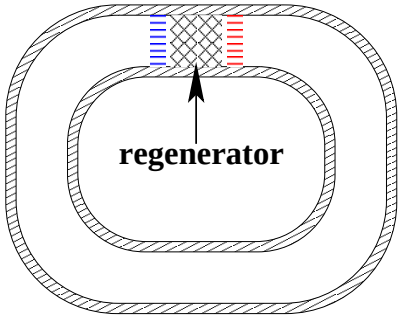
$$\mathfrak{w}_2 = \underbrace{\mathfrak{w}_\kappa + \mathfrak{w}_\nu}_{\text{losses}} + \underbrace{\overbrace{\mathfrak{w}_{sw}}^{\propto -\Im(h)} + \overbrace{\mathfrak{w}_{tw}}^{\propto \Re(h)}}_{\text{production}}$$





=> use a **regenerator**($\delta_\kappa \geq R_h$) and a **travelling wave** phasing between p and v_x

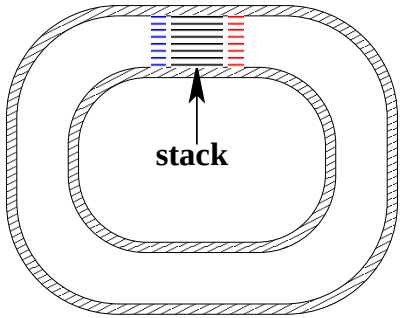
⋆ [Ceperley,JASA,1979]



=> closed-loop resonator
- $\tilde{p}, \tilde{v}_x \propto e^{-i(kx-\omega t)}$,
- $Z = \frac{\tilde{p}}{\tilde{v}_x} = \rho_0 c_0$
- $f \approx c_0/L$

=> does not sing...
because $\mathfrak{w}_{tw} < \mathfrak{w}_\nu$

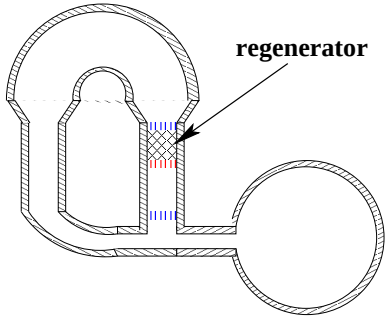
⋆ [Yazaki et al., PRL, 1998]



=> closed-loop resonator
- $\tilde{p}, \tilde{v}_x \propto e^{-i(kx-\omega t)}$,
- $Z = \frac{\tilde{p}}{\tilde{v}_x} = \rho_0 c_0$
- $f \approx c_0/L$

=> does sing...
. . .but employs a stack

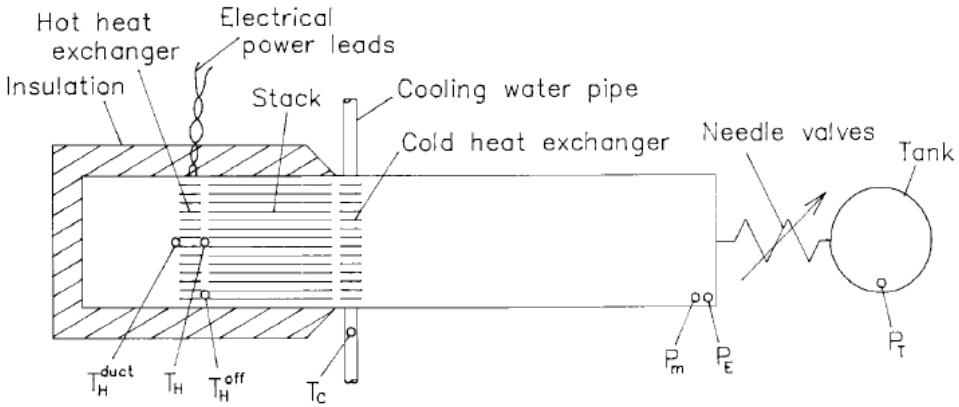
⋆ [Backhaus et al., Nature,1999]



=> acoustic feedback loop
- **local** TW phasing,
- $Z >> \rho_0 c_0$ **locally**
- $f << c_0/L$

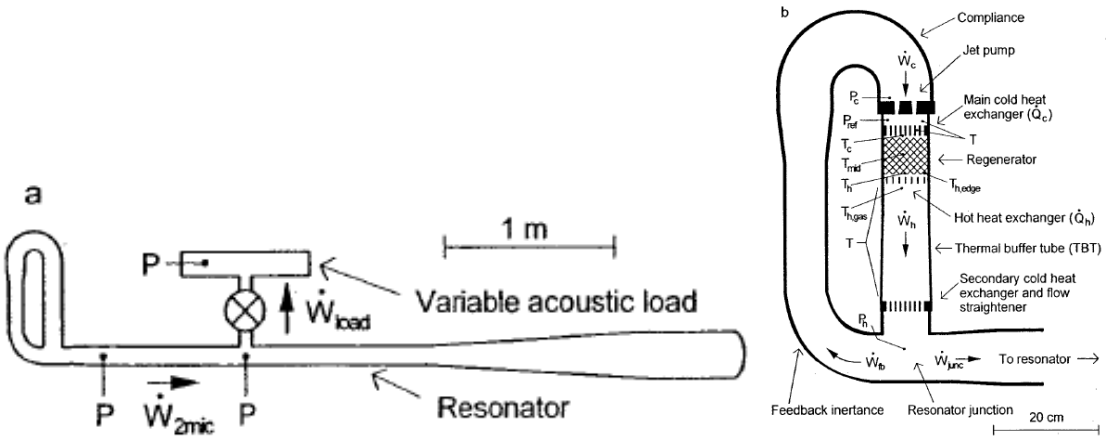
=> does sing...
. . .quite well

A Standing wave engine [Swift, JASA, 1992]



- fluid† : 13.8 bar Helium
- frequency : 120 Hz
- drive ratio p/P_0 : 6%
- T_H : 1000 K
- W_{load} : 630 W
- Q_H : 7 kW
- $\eta = 0.09$ $(0.13\eta_c)$

A thermoacoustic-Stirling engine [Backhaus & Swift, Nature, 1999]



- fluid† : 30 bar Helium
- frequency : 80 Hz
- drive ratio p/P_0 : 6-10%
- T_H : 1000 K
- W_{load} : 710 W
- Q_H : 2.4 kW
- $\eta = 0.3$ $(0.42\eta_c)$

† Need a large γ, low Pr, and high P_0 (in 1-bar air at 120 $\mathrm{dB_{SPL}}$, $W_{ac} \approx 10^{-2}\mathrm{W.m^{-2}}$ ...)

Advantages of θ_{ac} engines (and heat pumps)

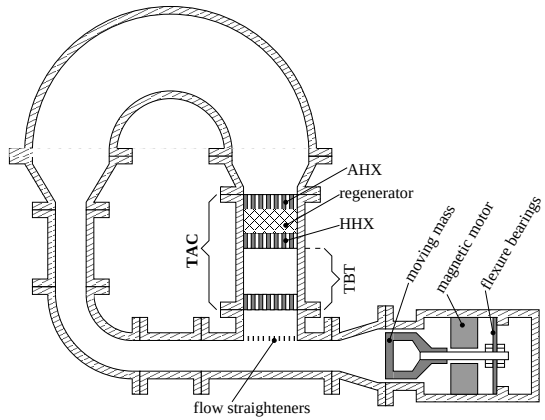
⋆ Simple, easy to build => potential reliability, low cost
⋆ Working gas = pressurized inert gas => harmless, environmental friendly
⋆ No or not much moving parts => resists wear
⋆ Decent efficiencies(currently up to 32% in thermo-acoustic conversion for engines [5]), with potential improvements.

Limitations of θ_{ac} engines (and heat pumps)

⋆ Moderate output powers => up to a few kilowatts
⋆ decent efficiencies => to be improved.
⋆ A development which remains limited to a few research laboratories and start-up companies

Potential applications of θ_{ac} engines (and heat pumps)

⋆ Waste heat recovery, micro-cogeneration
⋆ Thermoacoustically driven pulse-tube refrigeration (cryogeny, liquefaction of natural gas in methane tankers . . .)
⋆ Domestic refrigeration ? electronics cooling ? . . .



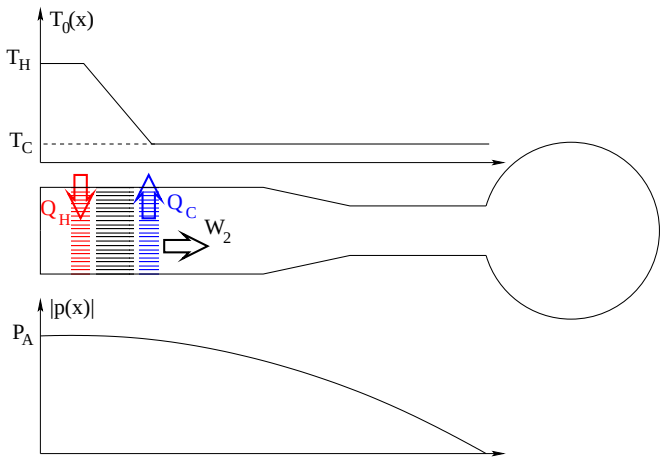
Sketch of a thermo-acousto-electric generator. Such kind of engine has been demonstrated to achieve a global efficiency of 18 % [6]

[5] Tijani et al., J. Appl. Phys., 2011 ; [6] Backhaus et al., Appl. Phys. Lett., 2004.

⋆ The basic principles of design tools may be summarized as

- A **two-port** modeling of acoustic wave propagation under an assigned $T_0(x)$ through the device, leading to a **characteristic equation**
- An **energy balance** under an assigned heat input Q_H, which accounts for the **thermoacoustic heat transport by sound** Q_2.

⋆ A simplistic illustration would be, for instance, as follows



1. Assign Q_H and set $P_A = 0$ (and thus $Q_2 = 0$)
2. Solve heat transfer => get $T_0(x)$
3. Solve acoustics => get spatial distribution $\tilde{p}(x)$ and working frequency f
4. Fix arbitrary amplitude P_A
5. Calculate thermoacoustic heat transport Q_2
6. Solve heat transfer => get $T_0(x)$ (accounting for Q_2)
7. Repeat steps 3-6 until equilibrium, i.e. $Q_H = Q_C + W_2$, with $Q_C = Q_{C_0} + Q_2$.

Limitations of the existing design tools

⋆ Up to now, the design of TA engines is always realized from the linear TA theory.

⋆ A famous, free-downloadable software developed at Los Alamos : DELTA-EC[9]

⋆ Efficient tool for design purposes, which works quite-well for the prediction of steady-state operation of Low-Amplitude TA engines

But . . .

⋆ These tools are restricted to the linear (or weakly nonlinear) regime

⋆ These tools are restricted to a 1-D description of the phenomena

⋆ They predict performance in steady-state

⋆ The user need to be experienced in the field*

[9] W.C. Ward et al., J. Acoust. Soc. Am., 1994

* « Intuition is required from the start and successful solutions yield intuition », W.C. Ward, Acoustics'08, Paris, 2008

Plan

Préambule : de quelle thermoacoustique parle-t-on ?

Théorie linéaire de la thermoacoustique

Exemple : le tube de Sondhauss

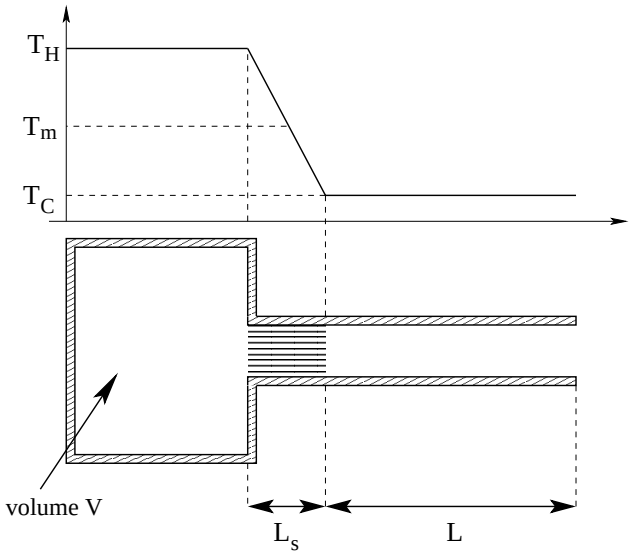
Effets de saturation non linéaires

Dynamique des auto-oscillateurs thermoacoustiques

Forçage des auto-oscillateurs thermoacoustique

Conclusion

⋆ Système considéré ∼ tube de Sondhauss[1]



⋆ Objectif = le décrire comme 1 oscillateur 1 d.d.l. (oscillateur à amortissement < 0 ?)

→ hypothèse B.F. + analogies électriques

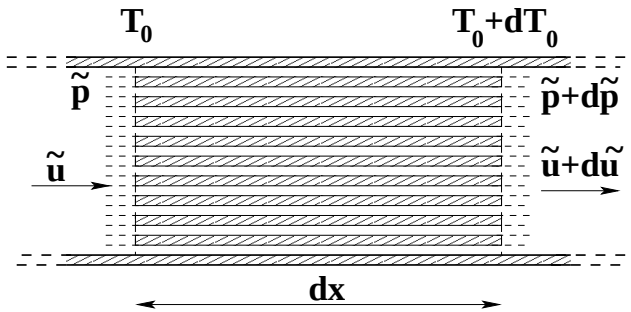
→ Approximation (fausse) de son fonctionnement par une D.D.E.

→ Résolution analytique par la méthode des échelle multiples

→ Modélisation de la saturation NL par pompage thermoacoustique.

C. Sondhauss, Ann. Phys.(Leipzig) 79 :1,1850

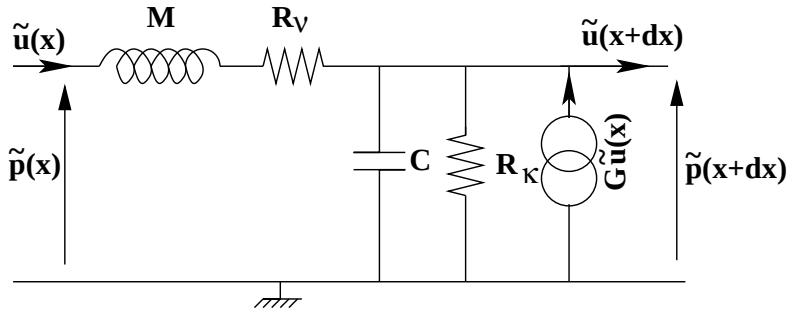
⋆ portion de stack de longueur $dx \ll \lambda$



$$d\tilde{p} = -\frac{i\omega\rho_0 dx}{\phi S}\frac{1}{1-f_\nu}\tilde{u}$$

$$d\tilde{u} = -\frac{i\omega\phi S dx}{\gamma P_0}\left[1+(\gamma-1)f_\kappa\right]\tilde{p} + \frac{(f_\kappa - f_\nu)}{(1-f_\nu)(1-\sigma)}\frac{dT_0}{T_0}\tilde{u}$$

⋆ Analogie électro-acoustique



$$d\tilde{p} = -(i\omega M + R_\nu)\tilde{u}$$

$$d\tilde{u} = -\left(i\omega C + 1/R_\kappa\right)\tilde{p} + G\tilde{u}$$

$$M = \frac{\rho_0 dx}{\phi S}\frac{1-\Re(f_\nu)}{|1-f_\nu|^2}$$

$$C = \frac{\phi S dx}{\gamma P_0}\left(1+(\gamma-1)\Re(f_\kappa)\right)$$

$$R_\nu = \frac{\omega\rho_0 dx}{\phi S}\frac{\Im(-f_\nu)}{|1-f_\nu|^2}$$

$$\frac{1}{R_\kappa} = \frac{\gamma-1}{\gamma}\frac{\omega\phi S dx\,\Im(-f_\kappa)}{P_0}$$

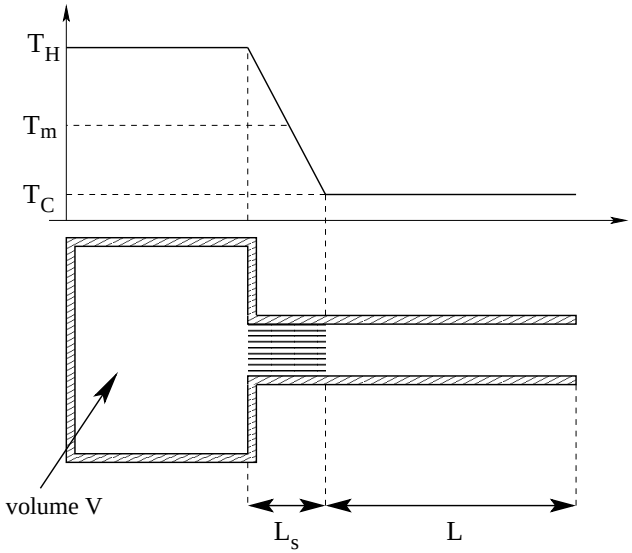
$$G = \frac{f_\kappa - f_\nu}{(1-f_\nu)(1-\sigma)}\frac{dT_0}{T_0}$$

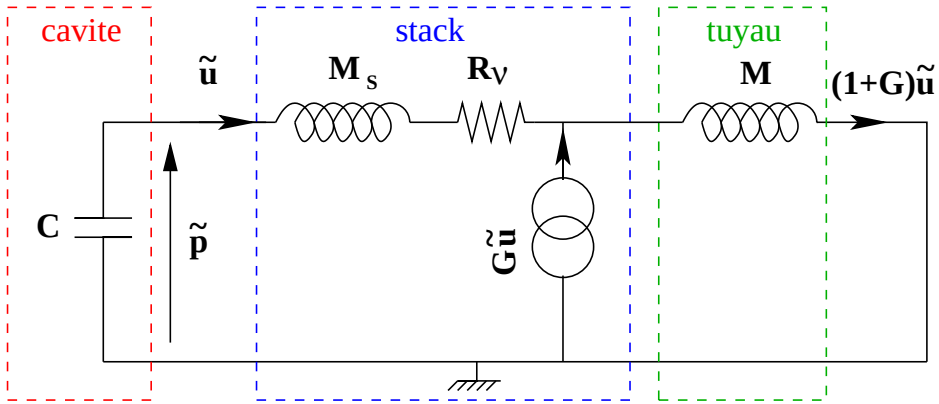
⋆ Approximation "quasi adiabatique"

$f_{\nu,\kappa} \sim (1-i)\epsilon_{\nu,\kappa}$, avec $\epsilon_{\nu,\kappa} = \delta_{\nu,\kappa}/R_s \ll 1$ (et $\epsilon_{\nu,\kappa} \propto \omega^{-1/2}$)

$M \propto (1-\epsilon_\nu)$, $C \propto 1+(\gamma-1)\epsilon_\kappa$, $R_\nu \propto \omega\epsilon_\nu$, $\frac{1}{R_\kappa} \propto \omega\epsilon_\kappa$, $G \propto (1-i)\epsilon_\kappa\frac{dT_0}{T_0} \in \mathbb{C}$

⋆ Schéma électrique équivalent





⋆ Equation caractéristique

$$(i\omega)^2\,\tilde{u} - (i\omega)^{3/2}\,\tilde{f}(T_H)\tilde{u} + \omega_0^2\tilde{u} = 0$$

$$\frac{1}{\omega_0^2(T_H)} = \left(M + \frac{\rho_0 L_s}{\Phi S}\frac{T_C}{T_m}\right) C$$

$$|\tilde{f}| = \frac{2\sqrt{\kappa}}{R_s}\left(\frac{T_m}{T_c}\right)^{\beta/2}\frac{\sqrt{\mathrm{Pr}}}{1+\sqrt{\mathrm{Pr}}}\frac{\Delta T}{T_m}\frac{1}{1+\frac{L_s}{\phi L}\frac{T_C}{T_m}}\sqrt{1+\left(\sqrt{\mathrm{Pr}}\left(1+\sqrt{\mathrm{Pr}}\right)\frac{L_s}{\phi L}\frac{T_C}{T_m}\right)^2}$$

$$Arg(\tilde{f}) = \arctan\left(\sqrt{\mathrm{Pr}}\left(1+\sqrt{\mathrm{Pr}}\right)\frac{T_C}{\Delta T}\frac{L_s}{\phi L}\right)$$

$$(i\omega)^2\,\tilde{u} - (i\omega)^{3/2}\,\tilde{f}(T_H)\tilde{u} + \omega_0^2\tilde{u} = 0$$

⋆ On *présume* que la solution est un signal *harmonique* de pulsation $\omega \approx \omega_0$ et d'amplitude *lentement* variable. En posant $\omega = \omega_0 + \Delta\omega$ avec $\Delta\omega = (\omega - \omega_0) \ll \omega_0$ il vient :

$$-(i\omega)^{3/2}\,\tilde{f}(T_H) \approx \tfrac{1}{2}\omega_0\sqrt{\omega_0}|\tilde{f}|e^{-i\omega_0 t_1} - i\omega \times \tfrac{3}{2}\sqrt{\omega_0}|\tilde{f}|e^{-i\omega_0 t_2}$$

avec $\omega_0 t_1 = \frac{5\pi}{4} - Arg(\tilde{f})$ et $\omega_0 t_2 = \frac{7\pi}{4} - Arg(\tilde{f})$

⋆ On prend ensuite la transformée de Fourier inverse, en remarquant que

$$\tilde{u}e^{-i\omega_0 t_{1,2}} \approx \tilde{u}e^{-i\omega t_{1,2}} + i\tfrac{\Delta\omega}{\omega}\tilde{u}e^{-i\omega t_{1,2}}$$

de sorte que (sous réserve d'une grosse[†] approximation)

$$\mathfrak{F}^{-1}\left[-(i\omega)^{3/2}\,\tilde{f}\tilde{u}(\omega)\right] \approx \frac{\omega_0^{3/2}|\tilde{f}|}{2}u(t-t_1) - \frac{3\omega_0^{1/2}|\tilde{f}|}{2}d_t u(t-t_2).$$

$\Rightarrow$ obtention d'une équation différentielle à retard

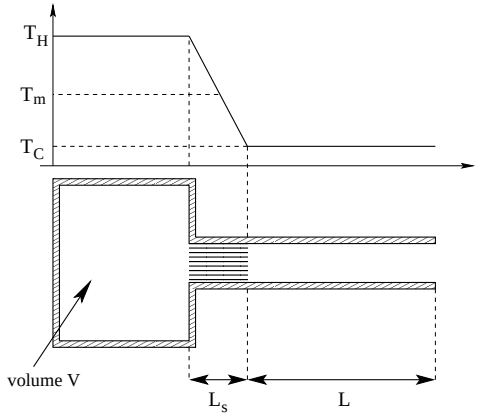
$$\boxed{d_{tt}^2 u(t) - \tfrac{3}{2}\sqrt{\omega_0}|\tilde{f}|d_t u(t-t_2) + \tfrac{1}{2}\omega_0\sqrt{\omega_0}|\tilde{f}|u(t-t_1) + \omega_0^2 u(t) \approx 0}$$

dont les retards $t_{1,2}$ traduisent un *effet mémoire* qui est l'essence des phénomènes mis en jeu dans les couches limites viscothermiques.

[†] **Transformation d'une dérivée fractionnaire en une fonction à retard !**

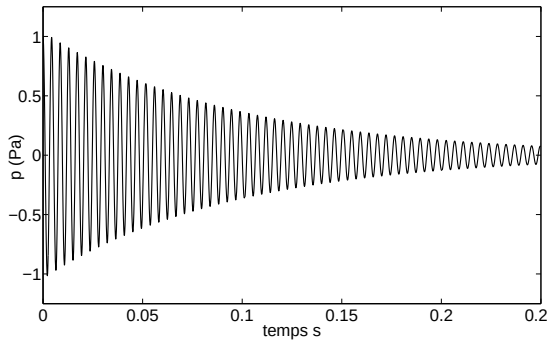
$$\ddot{p} - \frac{3}{2}\sqrt{\omega_0}|\tilde{f}|\dot{p}(t-t_2) + \frac{1}{2}\omega_0\sqrt{\omega_0}|\tilde{f}|p(t-t_1) + \omega_0^2 p \approx 0$$

$\Rightarrow$ Utilisation d'un solveur de DDE et calcul de $p(t \geq 0)$ pour différentes valeurs de T_H, et pour $p(t \leq 0) = 1$ Pa

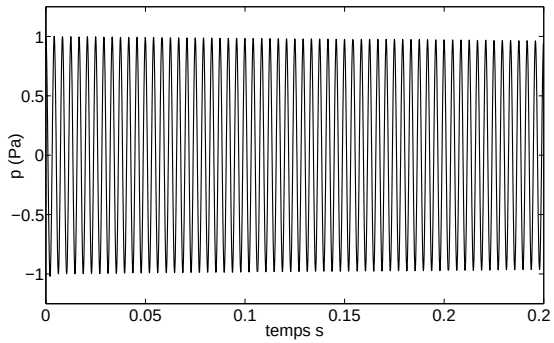


Valeurs des paramètres de calcul

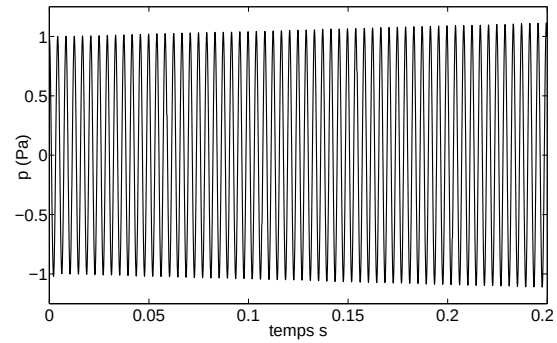
fluide (à $T_C = 300K$)		tuyau	
viscosité	$\mu = 1.84\,10^{-5}$Pa.s	longueur	$L = 12$cm
conductivité	$\lambda = 2.26\,10^{-2}$W.m^{-1}.K^{-1}	section	$S = \pi \times 3^2 cm^2$
masse volumique	$\rho_0 = 1.2$kg.m^{-3}		
capacité calorifique	$C_p = 1003$J.kg^{-1}.K^{-1}	stack	
coef. polytropique	$\gamma = 1.4$	longueur	$L_s = 4$cm
cavité		rayon pore	$R_s = 3$mm
		porosité	$\phi = 0.95$
volume	$V = 1$l		



$T_H = 300K$



$T_H = 460K$



$T_H = 470K$

$\star$ Changement de variable $\theta \equiv \omega_0 t \Rightarrow$

$$d^2_{\theta\theta} p - 3\zeta d_\theta p(\theta - \theta_2) + \zeta p(\theta - \theta_1) + p = 0$$

avec $\theta_{1,2} = \omega_0 \theta_{1,2}$ et $\zeta = \frac{1}{2} \frac{|\tilde{f}|}{\sqrt{\omega_0}} \ll 1$ $(\zeta \propto (\Delta T / T_m)(\sqrt{\frac{2\nu_m}{\omega_0}} / R_s) \ll 1)$

$\star$ On cherche une solution sous la forme :

$$p(\theta) = p(\tau, s) = p_0(\tau, s) + \zeta p_1(\tau, s) + \ldots, \text{ avec } s \equiv \zeta\theta \text{ et } \tau \equiv \theta$$

$$\begin{aligned}
d_\theta p &\approx \partial_\tau p + \zeta \partial_s p \\
d^2_{\theta\theta} p &\approx \partial^2_{\tau\tau} p + 2\zeta \partial^2_{\tau s} p \\
p(\theta - \theta_1) &\approx p(\tau - \theta_1, s) - \zeta\theta_1 \partial_s p(\tau - \theta_1, s) + \ldots \\
d_\theta p(\theta - \theta_2) &\approx \partial_\tau p(\tau - \theta_2, s) - \zeta\theta_2 \partial^2_{\tau s} p(\tau - \theta_2, s) + \zeta \partial_s p(\tau - \theta_2, s)
\end{aligned}$$

$\star$ Ordre ζ^0

$$\partial^2_{\tau\tau} p_0 + p_0 = 0 \Rightarrow p_0(\tau, s) = A(s) e^{it} + c.c., \text{ avec } A(s) = R(s) e^{i\Phi(s)} \in \mathbb{C}$$

$\star$ Ordre ζ^1 :

$$\partial^2_{\tau\tau} p_1 + p_1 = \underbrace{\left(-2id_s A + 3iAe^{-i\theta_2} - Ae^{-i\theta_1}\right)}_{=0 \text{ condition de solvabilité}} e^{i\tau} + c.c.$$

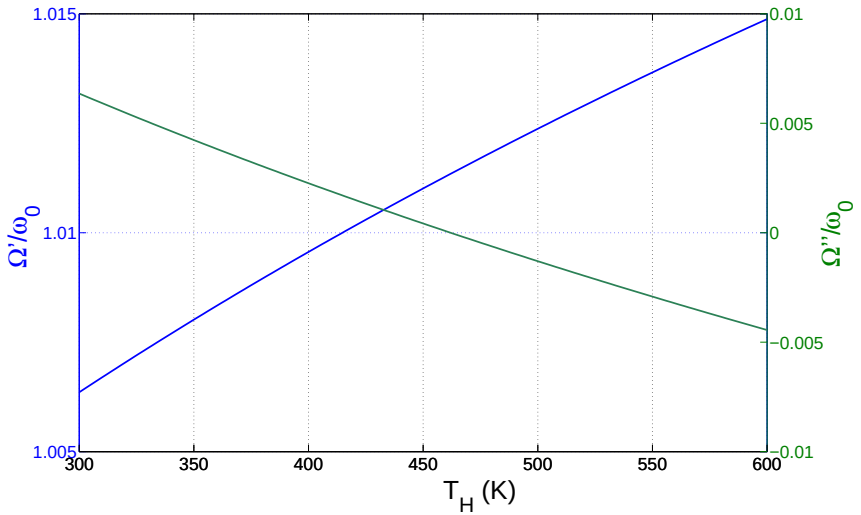
Résolution analytique :méthode des temps multiples (2)

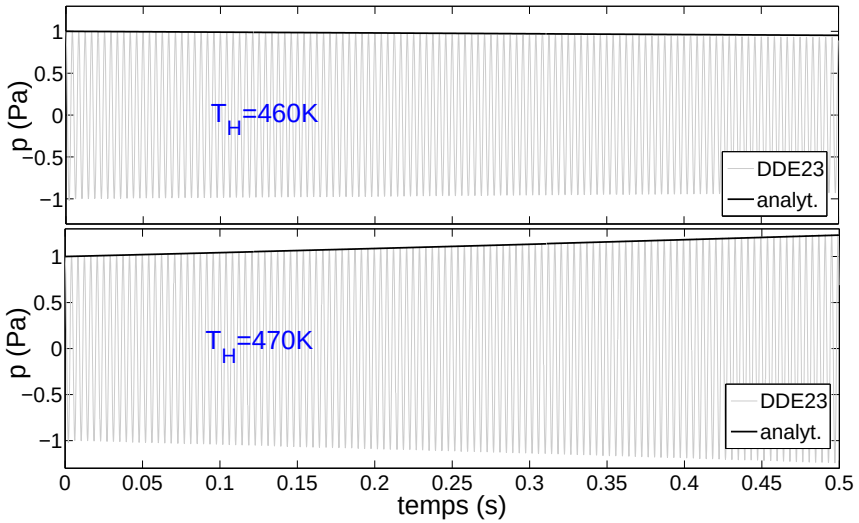
$$\begin{cases} d_s A = \frac{1}{2} A \left(3e^{-i\theta_2} + ie^{-i\theta_1} \right) \\ A(s) = R(s) e^{i\Phi(s)} \end{cases} \Rightarrow \begin{cases} d_s R = \left(\frac{3}{2} \cos\theta_2 + \frac{1}{2} \sin\theta_1 \right) R \\ d_s \Phi = \frac{1}{2} \left[\cos\theta_1 - 3\sin\theta_2 \right] \end{cases}$$

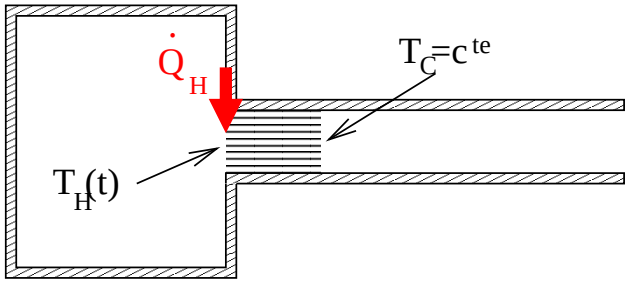
$$\boxed{p(t) \approx 2R(0) e^{-\Omega'' t} \cos\left(\Omega' t + \Phi(0) \right)}$$

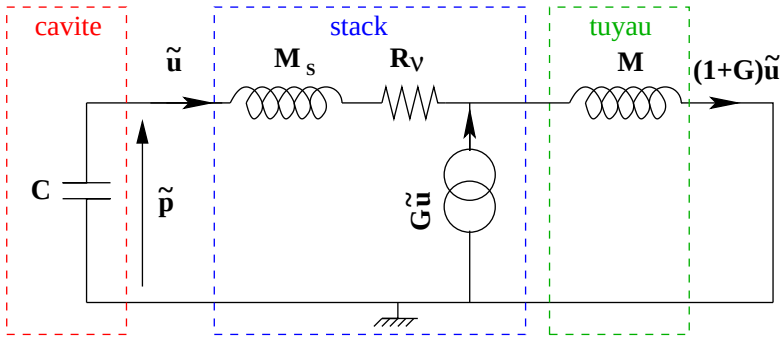
,avec

$$\begin{cases} \Omega'' = -\frac{1}{2} \left(3\cos\theta_2 + \sin\theta_1 \right) \zeta\omega_0 \\ \Omega' = \omega_0 \left[1 + \left(\frac{1}{2} \cos\theta_1 - \frac{3}{2} \sin\theta_2 \right) \zeta \right] \end{cases} \text{ et } \begin{cases} \zeta(T_H) = \frac{1}{2} \frac{|\tilde{f}|}{\sqrt{\omega_0}} \propto (\Delta T / T_m)(\delta_\kappa / R_s) \ll 1 \\ \theta_1(T_H) = \frac{5\pi}{4} - Arg(\tilde{f}) = \theta_2(T_H) - \frac{\pi}{2} \end{cases}$$









$$\dot{Q}_H \Rightarrow \Delta T \nearrow \Rightarrow \text{auto-oscillations} \nearrow \Rightarrow \dot{Q}_2 \propto p^2 \Rightarrow \Delta T \searrow$$

$\Rightarrow$ un effet de saturation NL décrit par la théorie linéaire ...

⋆ **Amplification TA**

$$p(t, \zeta t) \approx P(\zeta t) \cos\left(\Omega'(\zeta t) t\right)$$

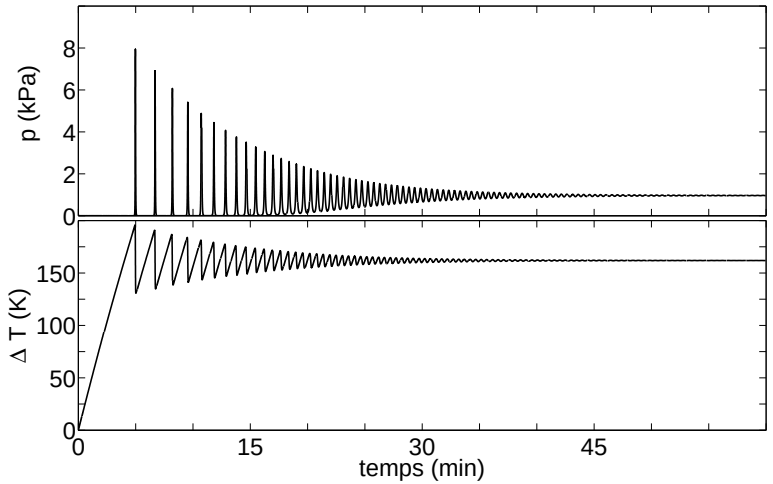
⋆ **Equation de la thermocinétique :**

$$\rho_s C_s V_s d_t T_H = \dot{Q}_H + \dot{Q}_c + \dot{Q}_2$$

$$\begin{aligned} \dot{Q}_c &= -\lambda_s S \Delta T / L_s \\ \dot{Q}_2 &= \rho_0 T_c S \langle \overline{s v_x} \rangle \approx -\lambda_{ac} S \Delta T / L_s \\ &\text{avec } \lambda_{ac}(\zeta t) \propto P^2(\zeta t) \end{aligned}$$

⋆ Bilan[‡]

$$\begin{cases} d_t P = \sigma(T_H) P \\ \rho_s C_s d_t T_H = \dot{Q}_H / V_s - (\lambda_s + \Gamma_\lambda P^2) \Delta T / L_s^2 \end{cases}$$



Transitoire de déclenchement et de saturation pour un chauffage $\dot{Q}_H$ de 15 W, en prenant $\rho_s C_s = 1.8\,10^5 J.m^{-3}.K^{-1}$ et $\lambda_s = 0.17 W.m^{-1}.K^{-1}$

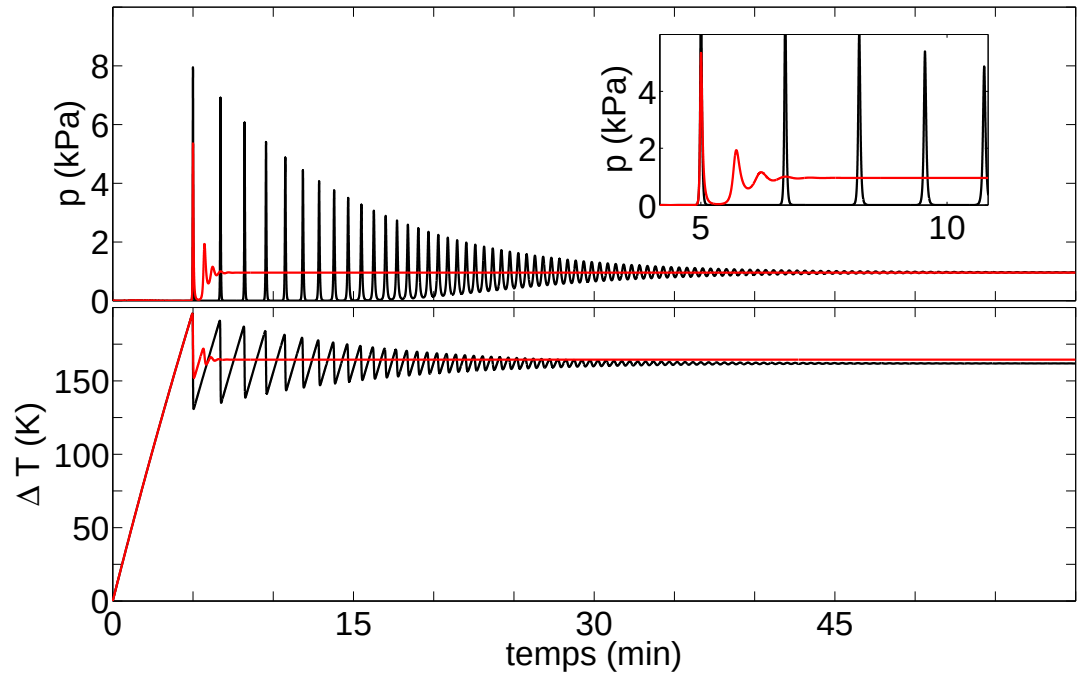
[‡] **sous réserve que $d_t T_H \sim \zeta T_H$ ou $\ll \zeta T_H$**

$$\left\{ \begin{array}{c} d_t P = \sigma(T_H) P \\ \rho_s C_s d_t T_H = \dot{Q}_H / V_s - (\lambda_s + \Gamma_\lambda P^2) \Delta T / L_s^2 \end{array} \right. \Rightarrow \left\{ \begin{array}{c} \Delta T|_\infty = \Delta T|_{seuil} \\ P_\infty = \sqrt{\frac{1}{\Gamma_\lambda} \left(\frac{\dot{Q}_H L_s^2}{V_s \Delta T|_\infty} - \lambda_s \right)} \end{array} \right.$$

Ajoutons arbitrairement un terme de saturation quadratique :

$$\left\{ \begin{array}{c} d_t P' = \sigma(T'_H) P' - \alpha P'^2 \\ \rho_s C_s d_t T'_H = \dot{Q}_H / V_s - (\lambda_s + \Gamma_\lambda P'^2) \Delta T' / L_s^2 \end{array} \right.$$

et choisissons $\alpha = 10^{-4}$ de sorte que $P'_\infty \approx 0.99 P_\infty$



Transitoire de déclenchement et saturation. En noir : seul le pompage thermoacoustique est pris en compte. En rouge : ajout arbitraire d'une saturation quadratique avec $\alpha = 10^{-4}$

Plan

Préambule : de quelle thermoacoustique parle-t-on ?

Théorie linéaire de la thermoacoustique

Exemple : le tube de Sondhauss

Effets de saturation non linéaires

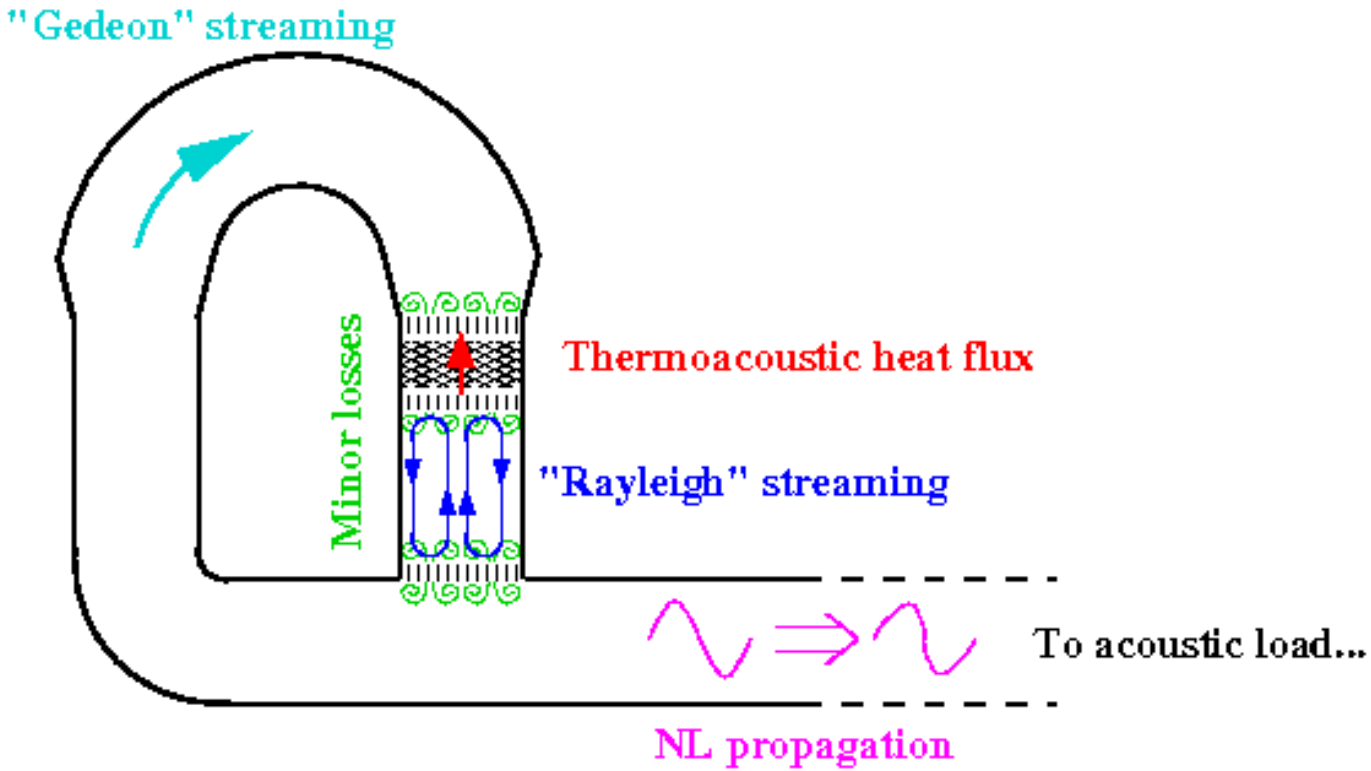
Dynamique des auto-oscillateurs thermoacoustiques

Forçage des auto-oscillateurs thermoacoustique

Conclusion

Autonomous oscillators => nonlinear saturation processes . . .

⋆ Thermoacoustic engines = **self-sustained oscillators**
⋆ Most of the **nonlinear** processes controlling wave saturation are **well identified . . .**



⋆ Some of them are well-predicted by theory (at least for moderate pressure levels) . . .
⋆ . . .but most of them are poorly described

A second-order effect which is described by the linear thermoacoustic theory . . .

$$q_2 = \frac{1}{2}\Re\left[\frac{f_\nu^* - f_\kappa}{(1+\mathrm{Pr})(1-f_\nu^*)}\tilde{p}\langle\tilde{v}_x^*\rangle\right] + \frac{\rho_0 C_p}{2\omega(1-\mathrm{Pr}^2)}\frac{\Im\left(\mathrm{Pr}f_\nu^* - f_\kappa\right)}{|1-f_\nu|^2}|\langle\tilde{v}_x\rangle|^2 d_x T_0$$

. . .but actual stack/regenerators employ materials of complicated geometry. . .

mesh grids

NiCr foam

RVC foam

. . .which are also anisotropic.

$\Rightarrow$ How to know $T_0(x)$ from Q_{in} ? Is T_0 uniform through a section ?

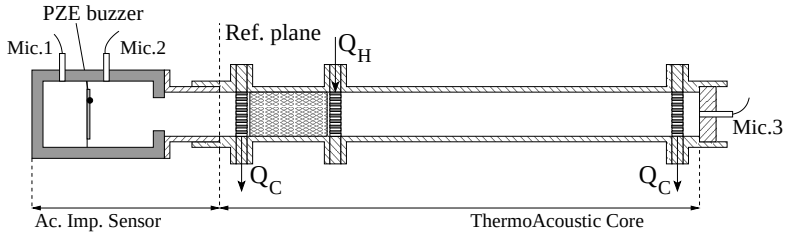
. . .and finally the problem is the same for the evaluation of $\mathfrak{w}_2$!

$\Rightarrow$ **Measure the sound scattered by the TAC**

$\rightharpoonup$ to evaluate both $f_{\nu,\kappa}$[1−4] (with $Q_H = 0$)

$\rightharpoonup$ to predict the onset threshold of a TA engine[5,6] (as a function of Q_H)

$\rightharpoonup$ and/or to evaluate the thermophysical properties of the stack[7]



Sketch of the experimental set-up used by Bannwart et al.[6] to measure the T-matrix of the TAC as a function of Q_H

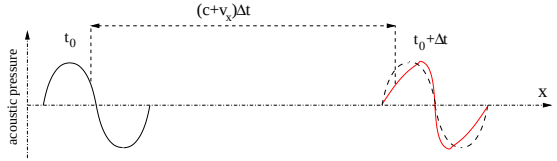
[1] Hayden et al., JASA, 1997 ; [2] Wilen, JASA, 2001 ; [3] Petculescu et al., JASA, 2001 ; [4] Y. Ueda et al., JASA, 2009 ; [5] M. Guedra et al., JASA, 2011 ; [6] F.C. Bannwart et al., JASA, 2013 ; [7] M. Guedra et al., submitted to Appl. Therm. Eng., 2014.

⋆ **High amplitude oscillations ⇒ wave distorsion . . .**

→ e.g. a simple wave traveling along $x \uparrow$ in an inviscid, non heat conducting gas[1] :

$$\partial_t p + (c + v_x)\,\partial_x p = 0,$$

→ An extensively studied topic[2,3]



⋆ **Impact on the operation of TA engines ?**

Outside the stack . . .

→ Higher harmonics generation, and even shock waves[4] !

→ Increase of losses, since $\langle \mathfrak{w}_{\nu,\kappa} \rangle \propto \sqrt{\omega}$

Within the stack . . .

→ Decrease of TA amplification ? (since δ_κ depends on ω)

⋆ **Some works tried to account for it . . .**

Outside the stack . . .

→ Solve NL propagation[5]

→ Determine optimal shape of the resonator[5]

Within the stack . . .

→ Numerical computation[7,8] (no linearization)

. . .but a lack of quantitative comparison

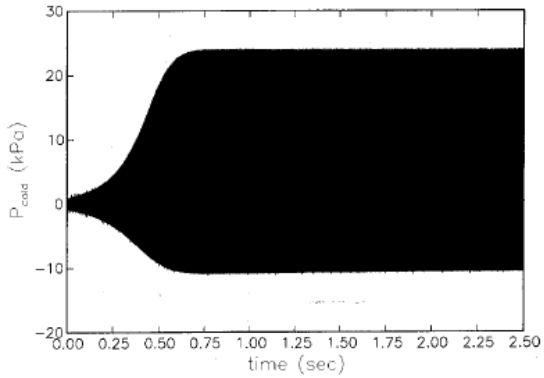
→ Most calculations made with an assigned ΔT . . .

⇒ **Do high amplitude effects play a key role ?**

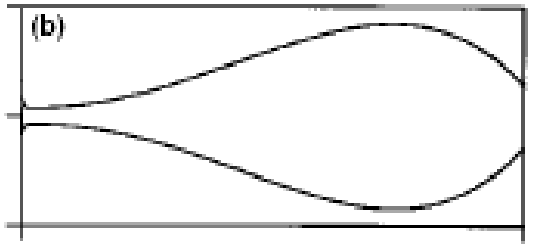
→ Most TA resonator's are inharmonic

→ p_2, v_{x_2} lead to $\mathfrak{w}_4$, $\mathfrak{q}_4$ . . .

⇒ Other saturating processes are probably worth considering



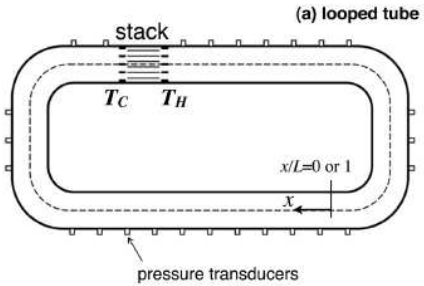
Transient regime in a SW TA engine under an assigned ΔT, from [7]



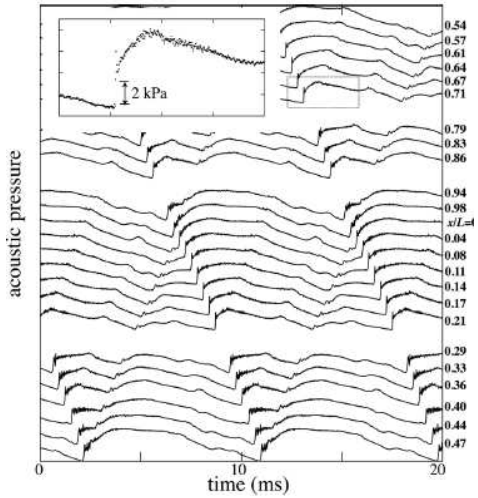
An optimum resonator's shape that maximize the Q of a resonator, from [6]

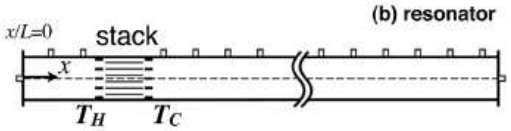
[1] A.D. Pierce, *Acoustics*, Acoustical Society of America, NY, 1991, [2] Rudenko & Soluyan, consultant bureau, NY, 1977 ; [3] Hamilton & Blackstock, Ac. Soc. Am., 1998 ; [4] Biwa et al., JASA, 2011, [5] Gusev et al., Acust. Acta Acust., 2000 ; [6] Ilinskii et al., JASA, 2001 ; [7] Karpov et al., JASA, 2002 ; [8] hamilton et al., JASA, 2002

⋆ Still, the study of NL propagation in TA engines is an interesting topic . . .
⋆ Recently, shock waves reported in a closed-loop, stack based TA engine [Biwa et al., JASA, 2011], and acoustic intensity measurements were performed.

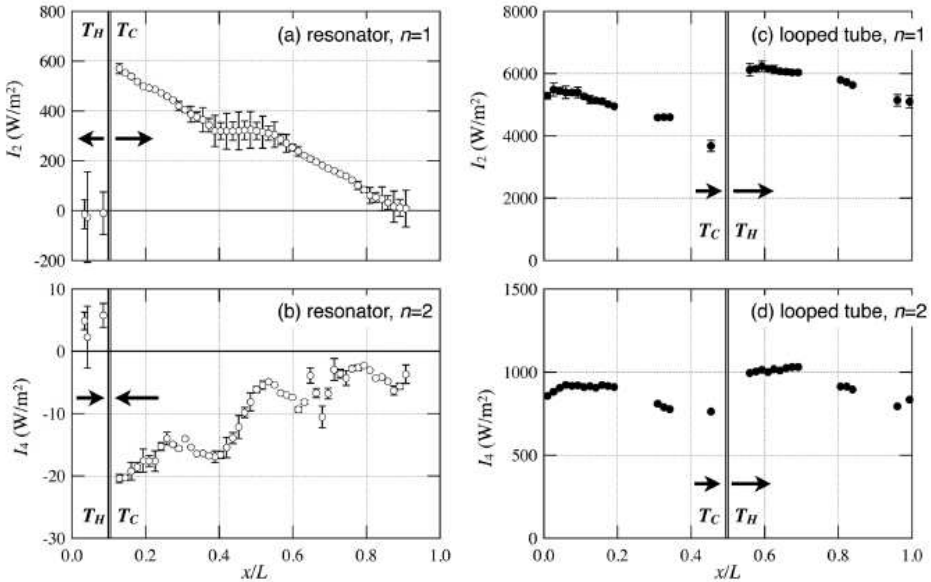


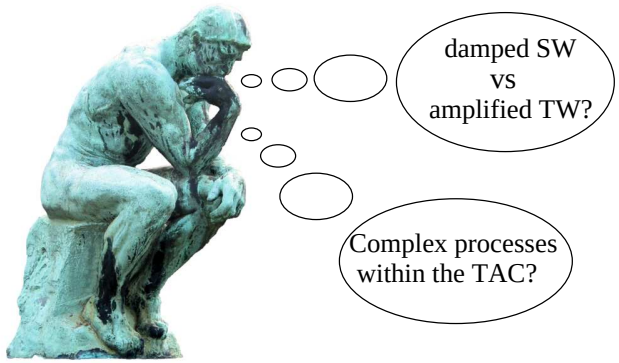
TW engine
=> shock wave along $x \uparrow$



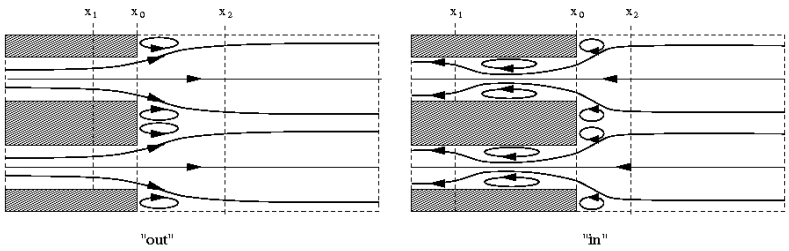


SW engine
=> no shock waves





⋆ Geometrical singularities => Vorticity ...



→ "Minor" losses[1−4]

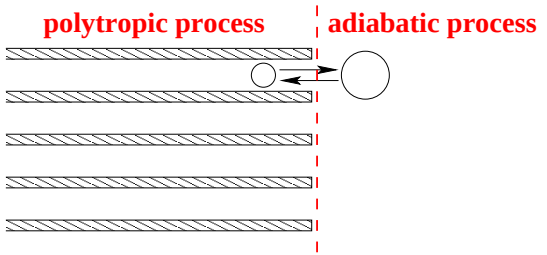
→ Basic modeling

$$\omega t \leq \pi,\ \Delta p_{out}(t) = -\frac{1}{2} K_{out} \rho_0 u^2(t)$$

$$\omega t \geq \pi,\ \Delta p_{in}(t) = \frac{1}{2} K_{in} \rho_0 u^2(t)$$

$$W_{minor} \propto \int_0^{\frac{\pi}{\omega}} \Delta p.u.dt + \int_{\frac{\pi}{\omega}}^{\frac{2\pi}{\omega}} \Delta p.u.dt \propto U^3$$

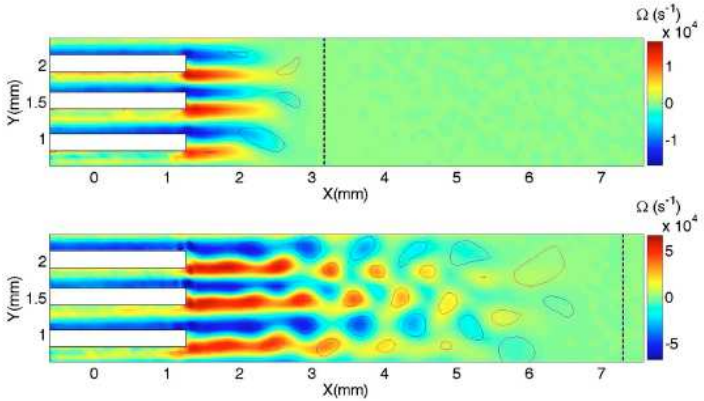
...but thermal effects as well



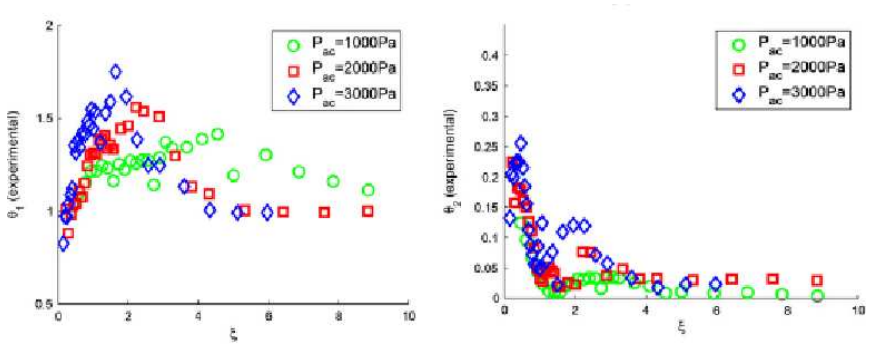
→ Abrupt transition from polytropic to adiabatic

⇒ higher harmonics in τ[5−6]

→ Heat transport accompanying vortex shedding ?



Vorticity field at the edge of a stack obtained with PIV (from [4])



Amplitudes of τ_1 and τ_2 as a function of the distance to the stack obtained with CWA (from [6])

⇒ **Are established results for steady flows applicable to oscillating flows ?**

[1] G.W., Acoustical Society of America, NY, 2001 ;[2] Blanc-Benon et al., C.R. Meca. 2003 ; [3] Marx et al., JASA, 2003. [4] Berson et al.,JASA, 2008 ;[5] Gusev et al., JASA, 2001, [6] Berson et al., Int. Journ. Heat Mass Transf., 2011.

⋆ Start with the Navier-Stokes equation, with $\partial_y >> \partial_x$, then make successive approximations ($\zeta = \zeta_0 + \zeta_1 + \zeta_2 \ldots$) up to second order, and after time averaging, one gets :

$$\boxed{\nu_0 \partial^2_{yy} \overline{v_{x_2}} = 1/\rho_0 \partial_x \overline{p_2} + \partial_x \left(\overline{v^2_{x_1}}\right) + \partial_y \left(\overline{v_{x_1} v_{y_1}}\right) - \partial_y \left(\overline{\nu_1 \partial_y v_{x_1}}\right)}$$

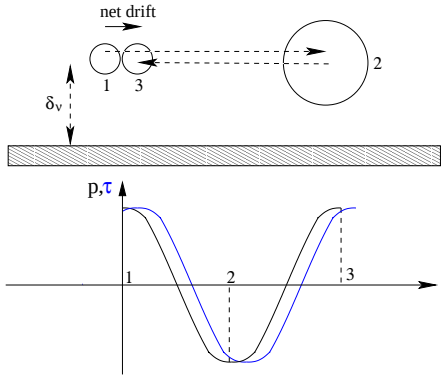
⋆ A steady velocity $\overline{v_{x_2}}$ generated by acoustic oscillations :

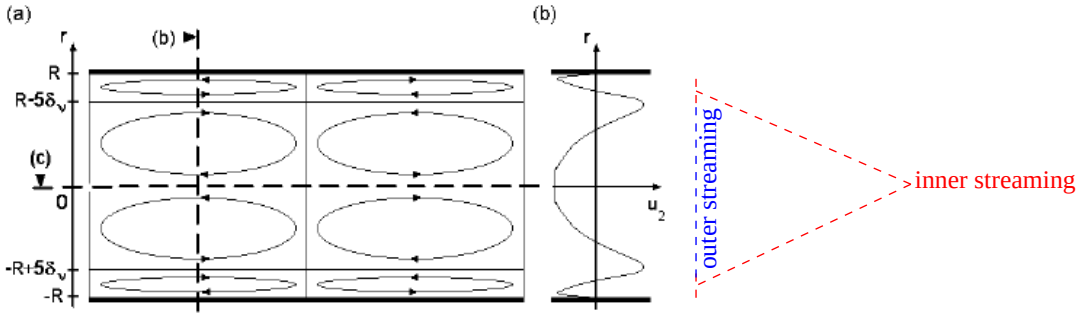
- because of the **convective derivative** :
$$\partial_x \left(\overline{v^2_{x_1}}\right) + \partial_y \left(\overline{v_{x_1} v_{y_1}}\right)$$

- because ν depends on temperature ($\nu = \nu_0(T_0) + \nu_1(\tau)$)
$$\partial_y \left(\overline{\nu_1 \partial_y v_{x_1}}\right)$$

- . . .and actually **many other sources** of streaming

$$\nu \propto T^{1+\beta}$$
$$\int_1^2 \nu.dt > \int_2^3 \nu.dt$$



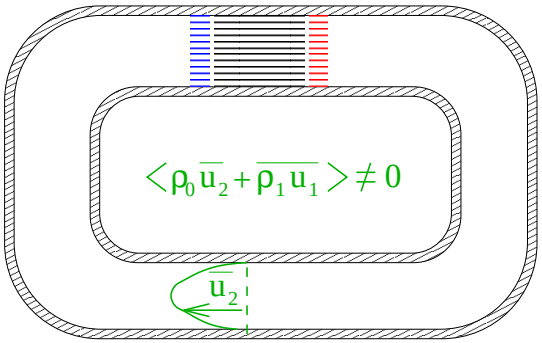


Since the pioneering theoretical works of Rayleigh (1883) and Schlichting (1932),

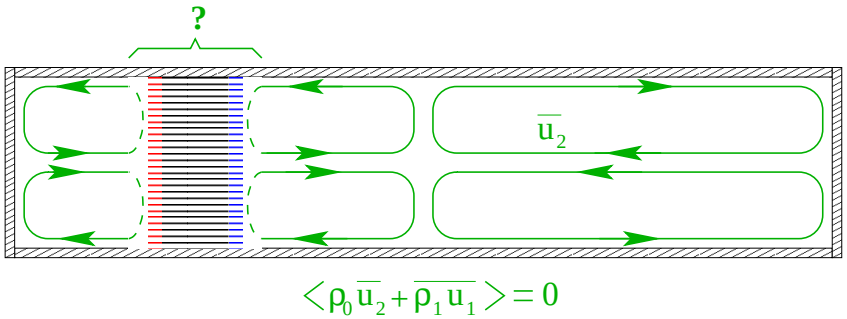
→ **many analytical/numerical/experimental studies including**

- ⋆ thermal effects[1−4] ($d_x T_0$, $\mu = \mu(T)$, $\lambda = \lambda(T)$),
- ⋆ acoustic streaming in closed-loop devices[5−7] ("Gedeon streaming"),
- ⋆ high amplitude effects[8−11] (e.g. inertia effects as $Re_{nl} = \sqrt{\mathrm{M/Sh}} \approx 1$)
- ⋆ the coupling between $\overline{u}_2$ and $d_x T_0$ (reciprocal impact[10−11])

closed loop, traveling wave engine



standing wave engine



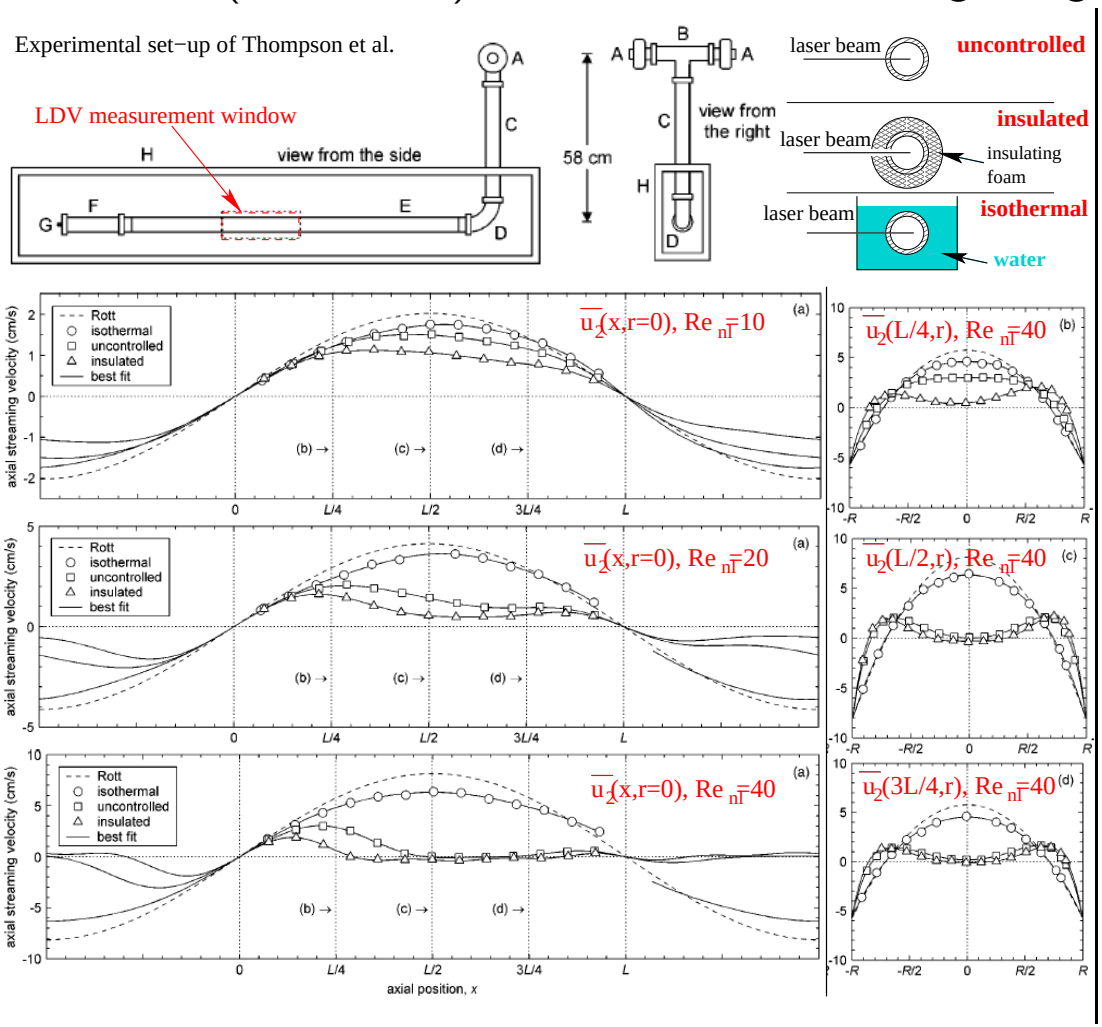
→ **but still many open question . . .**

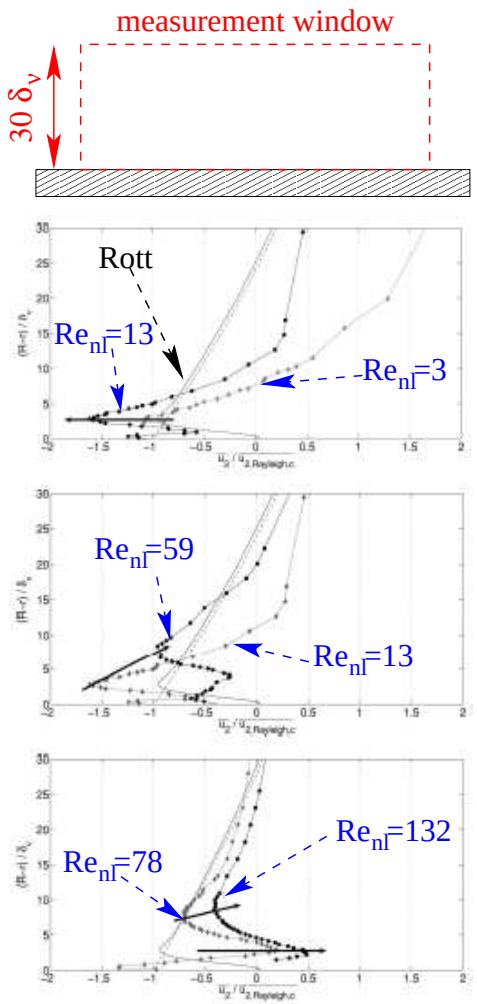
- ⋆ Acoustic streaming in "non-empty" resonators (additional cells ?)
- ⋆ Amount of heat-transport by $\langle \rho_0 \overline{U}_2 + \rho_1 u_1 \rangle$?
- ⋆ What about the dynamics of streaming establishment ?

[1] N. Rott, Z.A.M.P., 1974 ; [2] Bailliet et al., JASA, 2001 ; [3] Hamilton et al., JASA, 2003 ; [4] Thompson et al., JASA, 2005, [5] D.C. Gedeon, Cryocoolers, 1997 ; [6] V. Gusev et al., JASA, 2000 ; [7] desjouy et al., JASA, 2009 ; [8] Menguy et al., JASA,2000 ; [9] Moreau et al., JASA, 2008 ; [10] Daru et al., Wave Motion, 2013 ; [11] Reyt et al., JASA, 2013

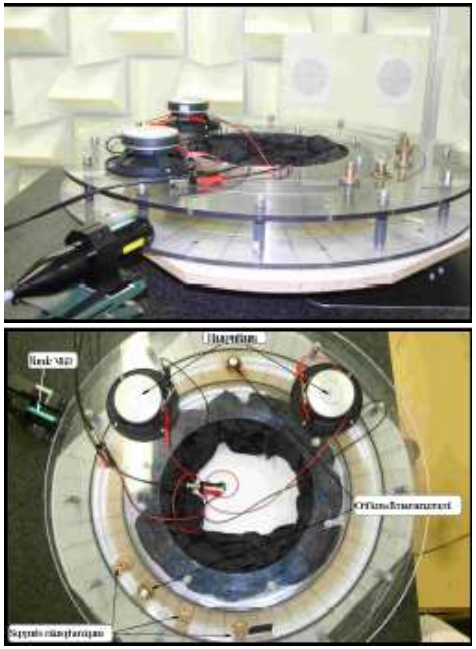
⋆ Thompson et al. (JASA, 2004) : outer streaming at high $\mathrm{Re_{nl}}$ ($\mathrm{Re_{nl}} = \sqrt{\mathrm{M/Sh}}$, impact of temperature distribution and fluid inertia

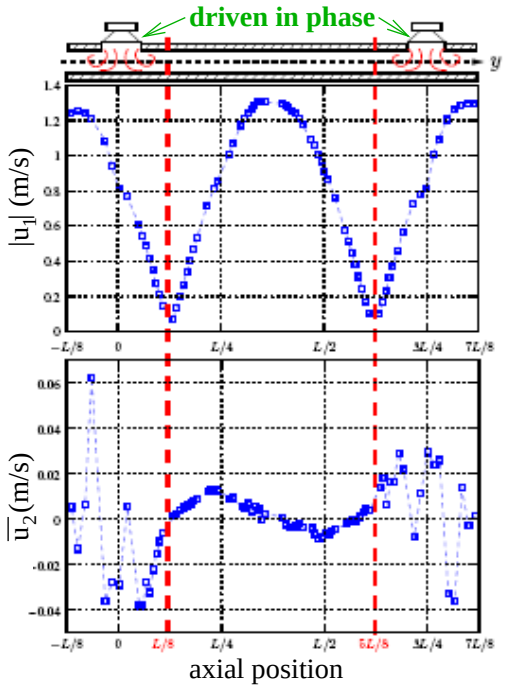
⋆ Moreau et al. (JASA, 2007) : inner and outer streaming at high $\mathrm{Re_{nl}}$.

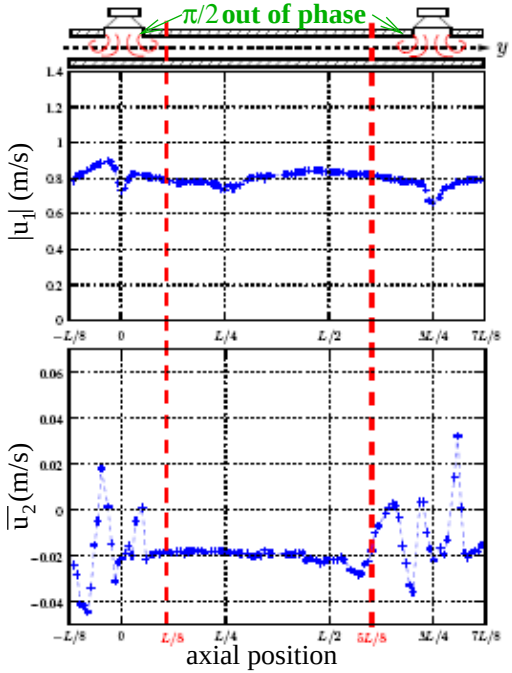




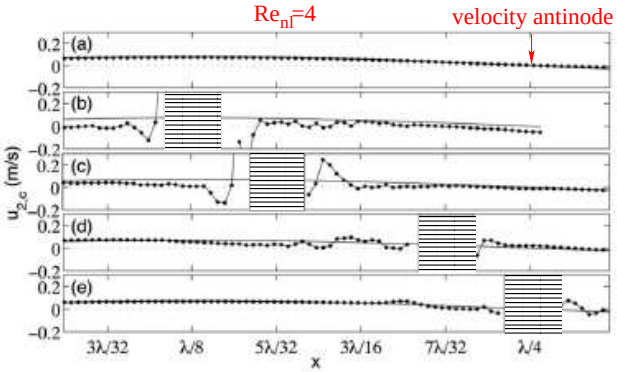
Acoustic streaming in a closed-loop resonator (Desjouy et al.,JASA,2009)



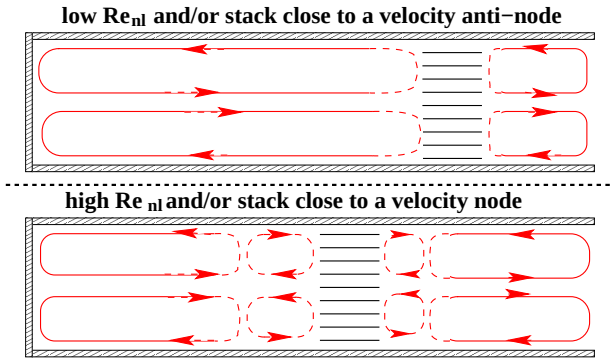




Acoustic streaming in SW resonators with a stack (Moreau et al., JASA, 2009)



?



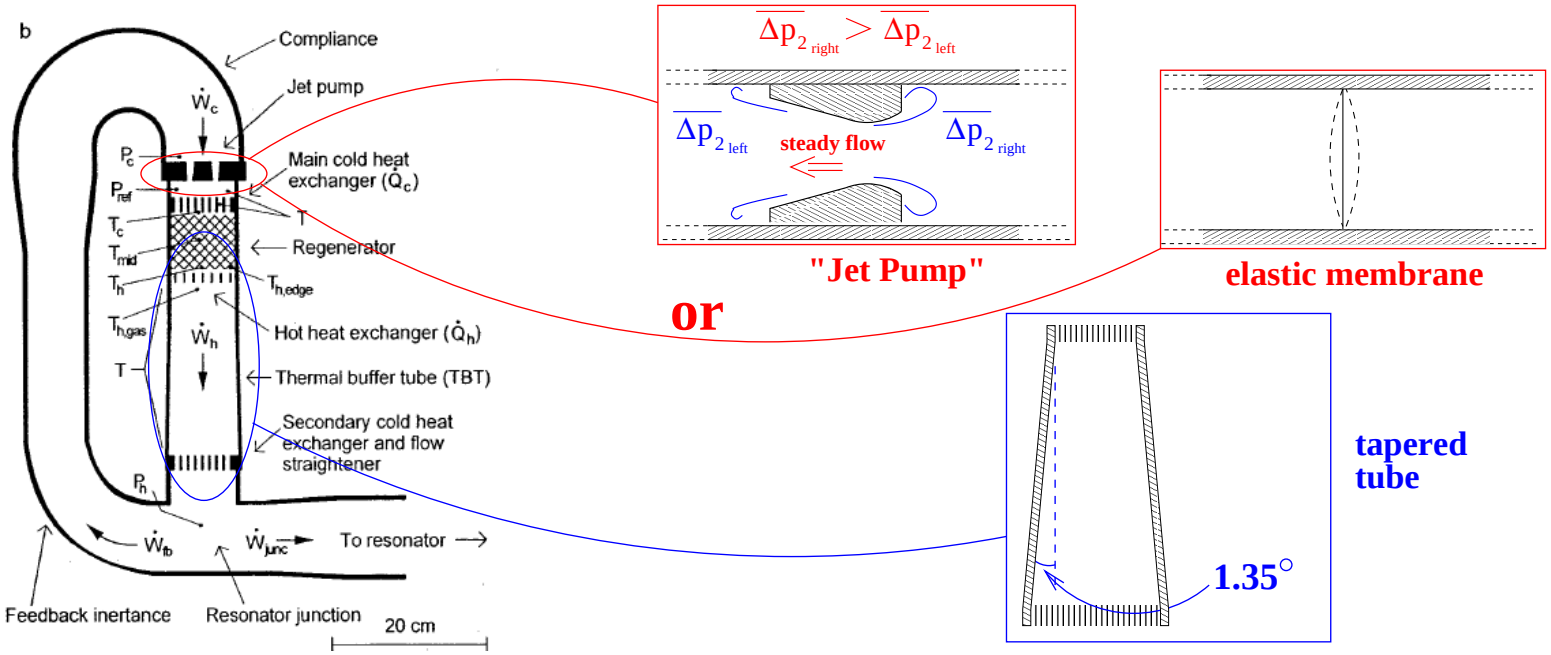
Acoustic streaming from the engineering standpoint

⋆ Acoustic streaming well-described only in simple devices at low amplitudes .

=> Empirical solutions to remove acoustic streaming

=>"jet pumps"[1] (makes $\rho_0 \overline{u_2} + \overline{\rho_1 u_1} = 0$) or membranes[2] (makes $\overline{u_2} = 0$) to suppress Gedeon streaming

=>"tapered" tube[1,3] to diminish Rayleigh streaming in the TBT

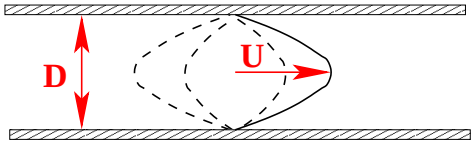


⋆ Open questions : Heat transport by $\rho_0 \overline{v_2} + \overline{\rho_1 v_1}$? Is acoustic streaming always undesirable for TA engines ?

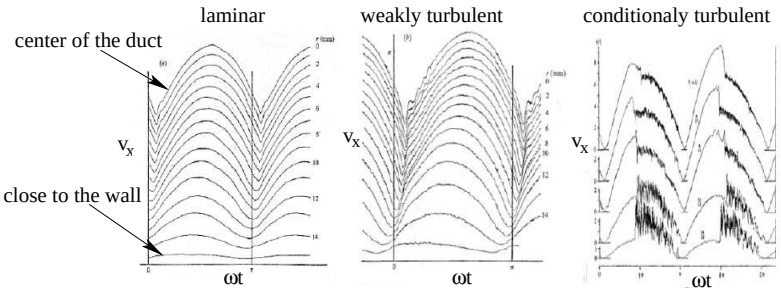
[1] Backhaus et al., JASA, 99 ; [2] Tijani et al, JASA, 2011 ; [3] Olson et al., Cryogenics, 1997

Transition to turbulence extensively studied . . .but in steady flows

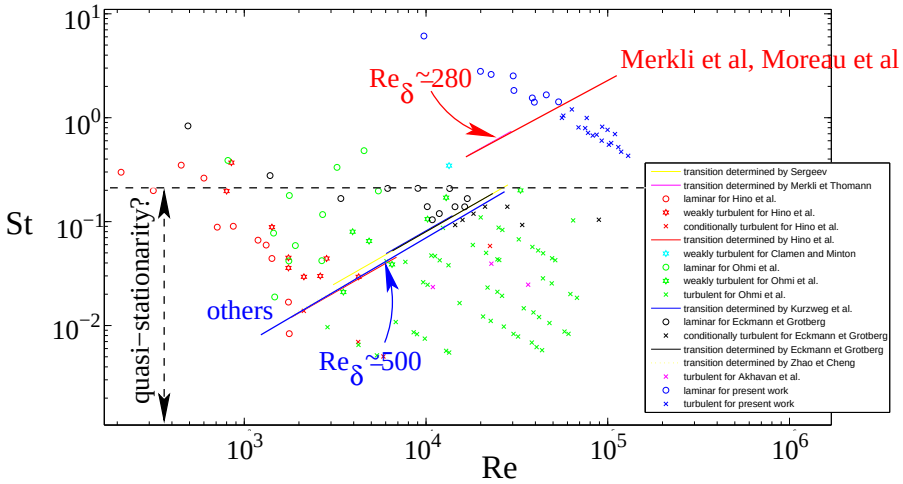
⇒ Only a few works in oscillating flows[1−8]
- → Hot Wire and LDV measurements
- → only in straight ducts
- → without considering wall's roughness
- → should depend on **2** dimensionless numbers

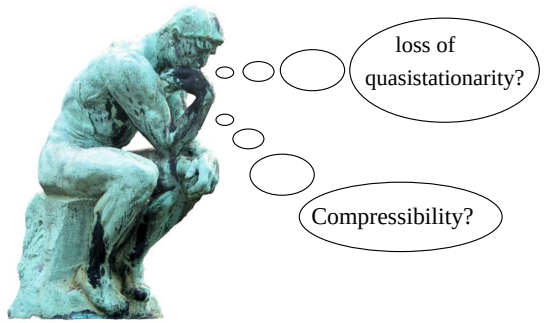


$$\mathrm{St} = \frac{\omega D}{U} \;\&\; \mathrm{Re} = \frac{UD}{\nu}$$



HW measurements of v_x as a function of the oscillating amplitude and distance to the wall (from [3]).





And still many open questions . . .

- → Evaluation of subsequent losses ?
- → Turbulence near T-junctions, in coiled ducts, etc . . .

[1] Merkli & Thomann, JFM, 1975 ; [2] Sergeev, Fluid Dyn., 1966 ; [3] Hino et al., JFM, 1976 ; [4] Ohmi et al.,JSME, 1982 ; [5] Kurzweg et al., Phys. Fluid A, 1989 ; [6] Eckman et al.,JFM, 1991 ; [7] Zhao et al. Int. J. Heat Fluid Flow, 1996 ; [8] S. Moreau, Phd, Poitiers, 2006.

Evaluation of losses due to turbulence (DELTA-EC)

$\Rightarrow$ modification of $\langle \mathfrak{w}_\nu \rangle$ from the time-averaging of well-known results of fully developped steady flows :

- **steady flows**

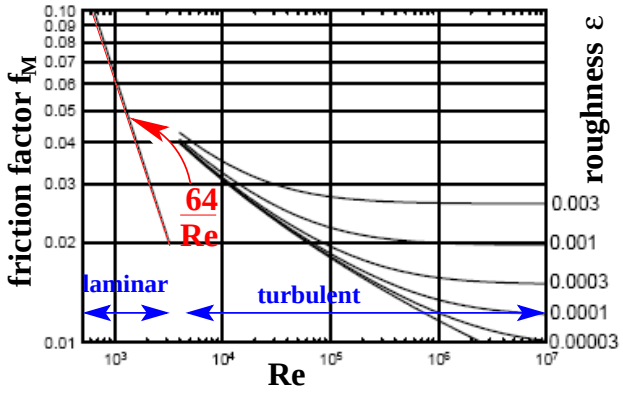
$$\Delta p = f_M \frac{L}{D} \frac{1}{2} \rho_0 \langle v_x \rangle^2$$

$$\langle \mathfrak{w}_{\nu,turb} \rangle = \frac{\Delta p \langle v_x \rangle}{L} = \frac{\rho_0 f_M \langle v_x \rangle^3}{2D}$$

- **oscillating flows** $(\mathrm{Re(t)} = \frac{\langle v_x(t) \rangle D}{\nu})$

$$\langle \mathfrak{w}_{\nu,turb} \rangle = \frac{\rho_0 \overline{f_M |\langle v_x \rangle|^3}}{2D},$$

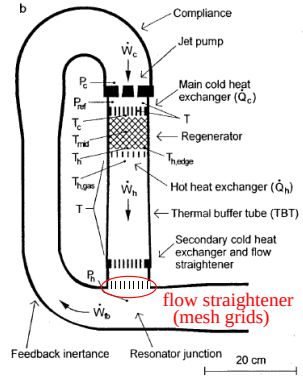
$$f_M(\mathrm{Re(t)}) \approx f_M + \frac{df_M}{d\mathrm{Re}} (|\mathrm{Re(t)}| - \mathrm{Re_{max}})$$



where both f_M and $d_{\mathrm{Re}} f_M$ are evaluated at $\mathrm{Re_{max}}$ from Moody chart, knowing ϵ and Re (approach considered unsatisfactory by the authors themselves . . .)

Avoiding turbulence from the engineering standpoint

- polish internal surfaces of ducts
- use of "flow straighteners"



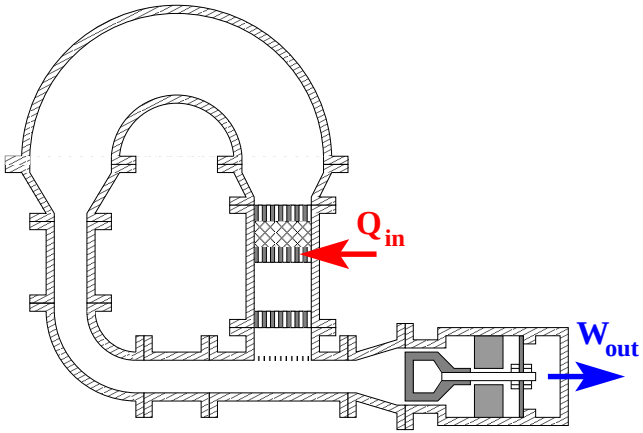
⋆ Linear theory

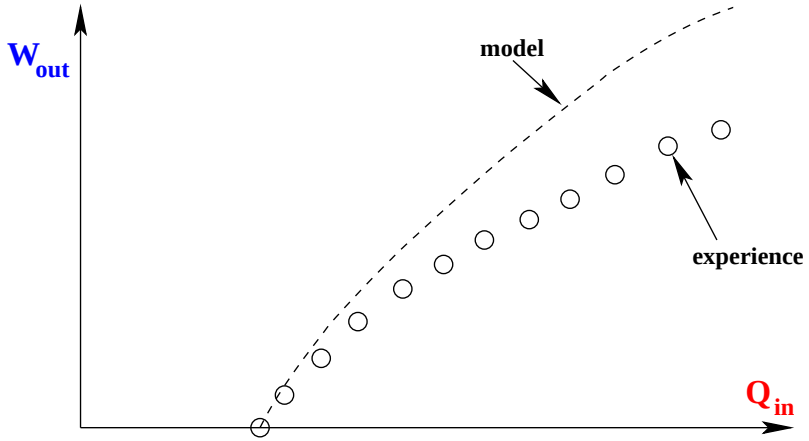
Design tools are available, but

- they are based on the linearization to 1st order of the governing equations,
- they are based on a 1-D description both acoustics and heat transport,
- they predict steady-state operation.

⋆ Nonlinear processes

NL process	Academic understanding	Appropriate modeling	Impact on TA Engines
TA Heat pumping	☺	☺	significant
NL acoustics	☺		not much
Streaming	☺	☹	significant
Edge effects	☺	☺	significant
Turbulence	☹	☹	?





⋆ How to fit experiments and theory ? => adjust any poorly known parameter ?

⋆ How to dissociate the role of each NL process ? => study the transient regime

Plan

Préambule : de quelle thermoacoustique parle-t-on ?

Théorie linéaire de la thermoacoustique

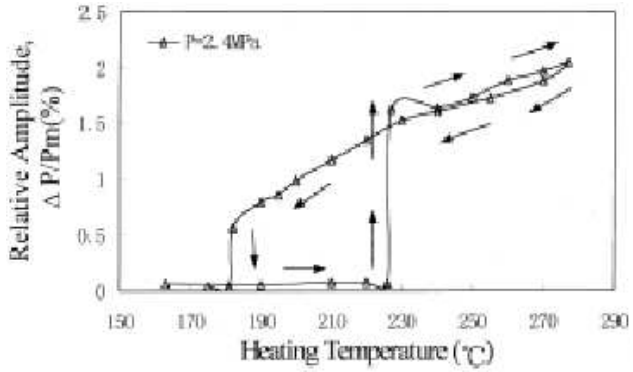
Exemple : le tube de Sondhauss

Effets de saturation non linéaires

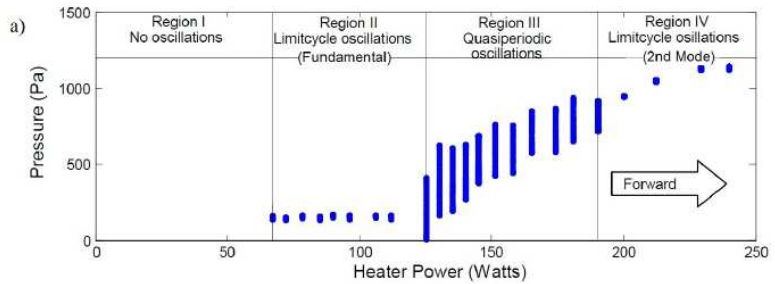
Dynamique des auto-oscillateurs thermoacoustiques

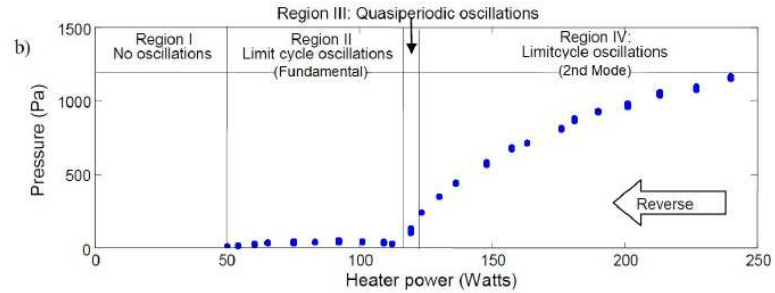
Forçage des auto-oscillateurs thermoacoustique

Conclusion

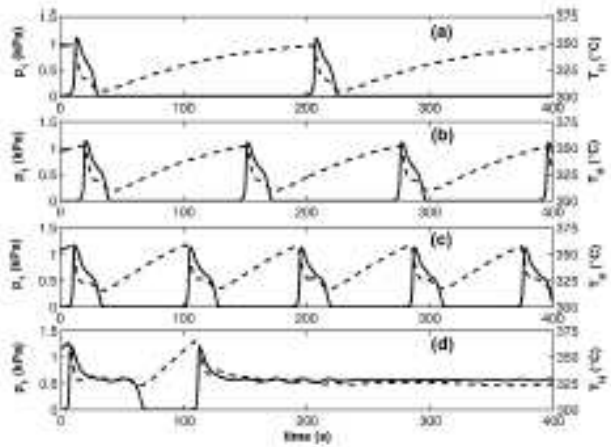


Hysteresis[6]

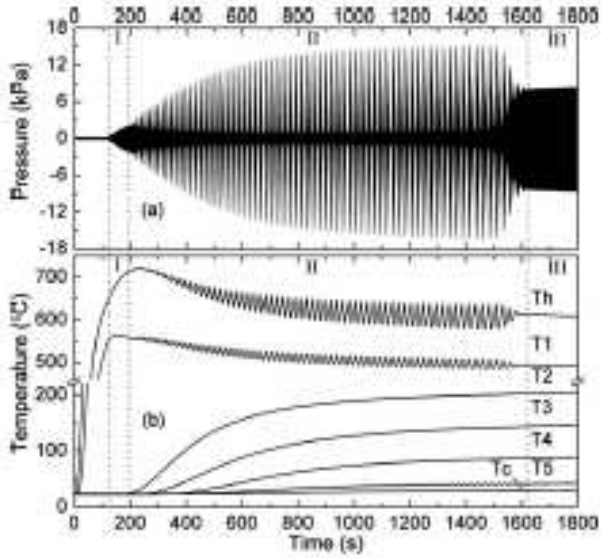




compétition de modes, régimes QP[8]



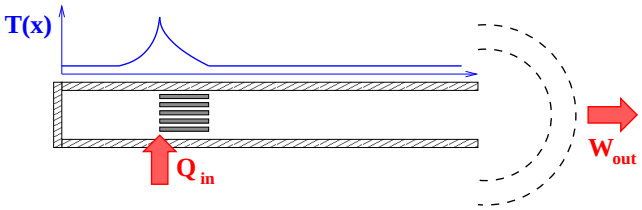
Switch on/off[2,4]



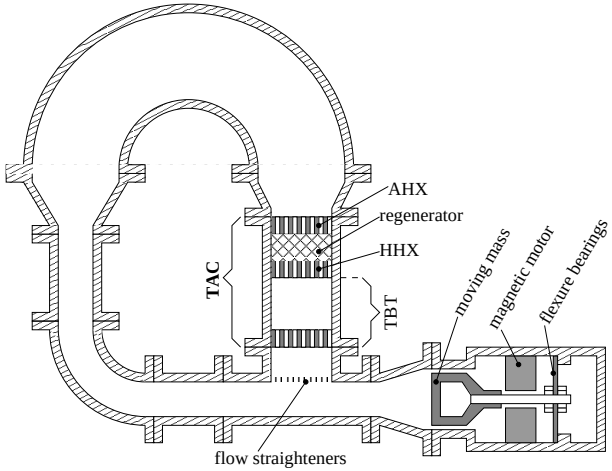
"Fishbone-like instability"[7]

[1] Penelet et al., Cryogenics, 2002 ; [2] Swift, JASA, 1992, [3] Zhou et al., Cryogenics, 1998 ; [4] Penelet et al., Phys. Let. A, 2006 ; [5] Penelet et al., Int. J. Heat Mass Trans., 2012 ; [6] Chen et al., Cryogenics, 1999 ; [7] Yu et al. JASA, 2010. ; [8] Unni et al., N3L Workshop, Munich, 2013

Moteur thermoacoustique "quart d'onde"

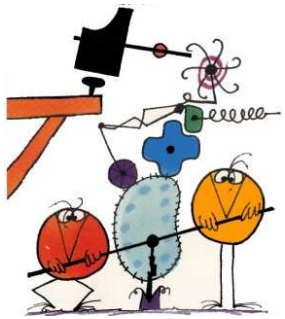


Générateur thermo-acousto-électrique

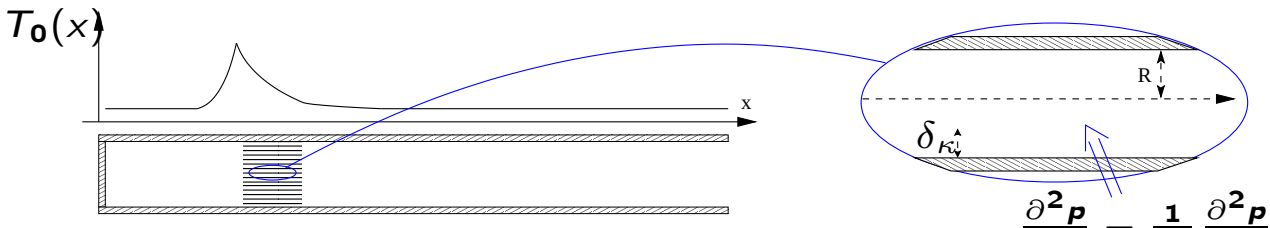


⇒ comprendre le plus simple . . .

. . .pour mieux appréhender le "compliqué".



⋆ Find of a Green function and make use of the kirchhoff-Helmholtz integral theorem ?



$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = q(x,t)?$$

⋆ But even in the limit of a quasi-adiabatic interaction, the inhomogeneous wave equation in the time domain is not simple[1]

$$\frac{\partial^2 p}{\partial t^2} - \frac{\partial}{\partial x}\left(c_0^2 \frac{\partial p}{\partial x}\right) \approx -\frac{2c_0^2\sqrt{\nu}}{R}\left[\underbrace{C\frac{\partial^{-1/2}}{\partial t^{-1/2}}\left(\frac{\partial^2 p}{\partial x^2}\right)}_{\text{thermoviscous effects}} + \underbrace{(C+C_T)\frac{1}{T_0}\frac{dT_0}{dx}\frac{\partial^{-1/2}}{\partial t^{-1/2}}\left(\frac{\partial p}{\partial x}\right)}_{\text{thermoacoustic amplification}}\right]$$

where the fractional derivatives

$$\frac{\partial^{-1/2} f(x,t)}{\partial t^{-1/2}} = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{t}\frac{f(x,\tau)}{\sqrt{t-\tau}}d\tau$$

have the meaning of *memory effects which are the essence of the thermoacoustic phenomena*[1] .
⋆ Moreover, $T_0(x)$ is controlled by p (and vice versa ...)

⇒ let's try a simpler approach, based on the quasi-steady state assumption
(seek for a slowly varying harmonic solution ...)

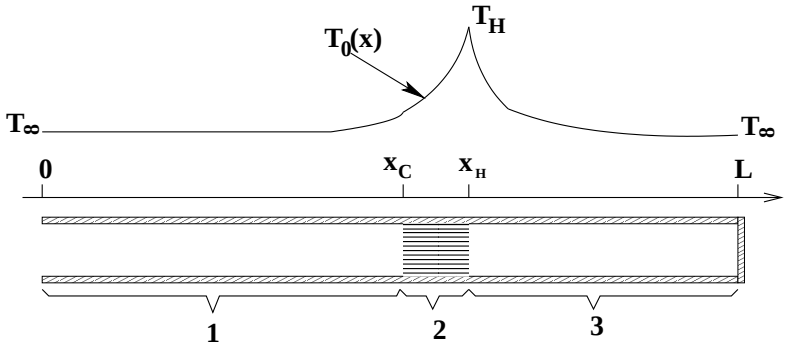
[1] N. Sugimoto, J. Fluid Mech. 658 :89-116, 2010 :

⋆ **Description of wave amplitude growth/attenuation.**

♭ For a given temperature distribution $T_0(x)$ through the device, each element is described by its T-matrix[1,2],

$$\begin{pmatrix} \tilde{p}(L) \\ \tilde{u}(L) \end{pmatrix} = M_3 \times M_2 \times M_1 \times \begin{pmatrix} \tilde{p}(0) \\ \tilde{u}(0) \end{pmatrix}$$

$$= \begin{bmatrix} \mathcal{M}_{pp} & \mathcal{M}_{pu} \\ \mathcal{M}_{up} & \mathcal{M}_{uu} \end{bmatrix} \times \begin{pmatrix} \tilde{p}(0) \\ \tilde{u}(0) \end{pmatrix}$$



which, combined with $\tilde{p}(0) = 0$ and $\tilde{u}(L) = 0$ leads to a characteristic equation[2]

$$=> \mathcal{M}_{uu} = 0$$

♭ Allow the angular frequency ω to be complex[2], and find $\omega = \Omega + i\epsilon$ so that

$$\mathcal{M}_{uu}\left(\omega, T_0(x), \dots\right) = 0.$$

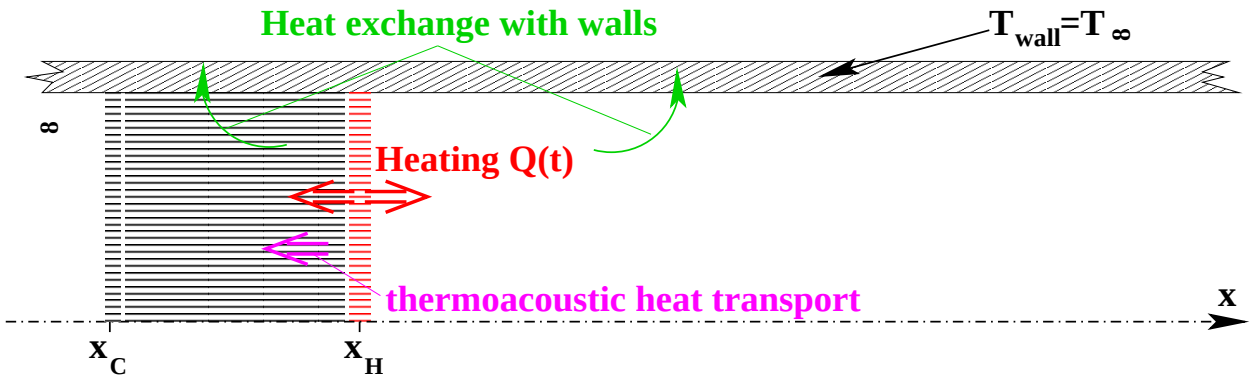
♭ Quasi-steady state assumption (i.e. $\epsilon \ll \Omega$) :

$$\frac{dP}{dt} \approx \epsilon\left[T_0(x)\right] P$$

where $P = |\tilde{p}(L)|$, and $\epsilon = \Im(\omega)$ is the amplification rate ($\epsilon > 0$: amplification, $\epsilon < 0$: attenuation)

[1] Penelet et al., Acust. Acta Acust., 2005 ; [2] Guedra et al., Acust. Acta Acust. 2012.

⋆ Unsteady heat transfer[1] (simplified and summarized).



$$x \in [0, x_c] \cup [x_h, L], \qquad \frac{\partial T_0}{\partial t} = \frac{1}{\rho_f C_f} \frac{\partial}{\partial x} \left(\lambda_f \frac{\partial T_0}{\partial x} \right) - \frac{T_0 - T_\infty}{\tau_f}$$

$$x \in [x_c, x_h] \qquad \frac{\partial T_0}{\partial t} = \frac{1}{\rho_s C_s} \frac{\partial}{\partial x} \left(\lambda_s \frac{\partial T_0}{\partial x} \right) - \frac{T_0 - T_\infty}{\tau_s} - \frac{1}{\rho_s C_s} \frac{\partial \varphi_{ac}}{\partial x},$$

$$x = x_c \qquad \lambda_f \left. \frac{\partial T_0}{\partial x} \right|_{x_c^-} = \lambda_s \left. \frac{\partial T_0}{\partial x} \right|_{x_c^+} - \varphi_{ac}(x_c^+),$$

$$x = x_h \qquad \lambda_s \left. \frac{\partial T_0}{\partial x} \right|_{x_h^-} = \lambda_f \left. \frac{\partial T_0}{\partial x} \right|_{x_h^+} + \varphi_{ac}(x_h^-) + \frac{Q(t)}{\pi R^2},$$

$$x \in [x_c, x_h] \qquad \varphi_{ac} = \frac{1}{2} \frac{\rho_f T_0}{\pi R^2} \Re \left(\langle \tilde{s} \tilde{v}_x^* \rangle \right).$$

Nomenclature	
$\rho_f C_f$ (resp. $\rho_s C_s$)	volumetric heat capacity of fluid (resp. of stack)
λ_f (resp. λ_s)	thermal conductivity of fluid (resp. of stack)

[1] M. Guedra et al., J. Appl. Phys., 2014 ;

⋆ **Modeling of the transient regime : summary.**

Equations describing unsteady heat transfer (including thermoacoustic heat pumping) ...

$$
\begin{aligned}
x \in [0, x_c] \cup [x_h, L], \qquad & \frac{\partial T_0}{\partial t} = \frac{1}{\rho_f C_f} \frac{\partial}{\partial x} \left(\lambda_f \frac{\partial T_0}{\partial x} \right) - \frac{T_0 - T_\infty}{\tau_f} \\
x \in [x_c, x_h] \qquad & \frac{\partial T_0}{\partial t} = \frac{1}{\rho_s C_s} \frac{\partial}{\partial x} \left(\lambda_s \frac{\partial T_0}{\partial x} \right) - \frac{T_0 - T_\infty}{\tau_s} - \frac{1}{\rho_s C_s} \frac{\partial \varphi_{ac}}{\partial x}, \\
x = x_c \qquad & \lambda_f \left. \frac{\partial T_0}{\partial x} \right|_{x_c^-} = \lambda_s \left. \frac{\partial T_0}{\partial x} \right|_{x_c^+} - \varphi_{ac}(x_c^+), \\
x = x_h \qquad & \lambda_s \left. \frac{\partial T_0}{\partial x} \right|_{x_h^-} = \lambda_f \left. \frac{\partial T_0}{\partial x} \right|_{x_h^+} + \varphi_{ac}(x_h^-) + \frac{Q(t)}{\pi R^2}, \\
x \in [x_c, x_h] \qquad & \varphi_{ac} = \frac{1}{2} \frac{\rho_f T_0}{\pi R^2} \Re \left(\langle \tilde{s} \tilde{v}_x^* \rangle \right) = f(P^2, x).
\end{aligned}
$$

...coupled to that describing wave amplification :

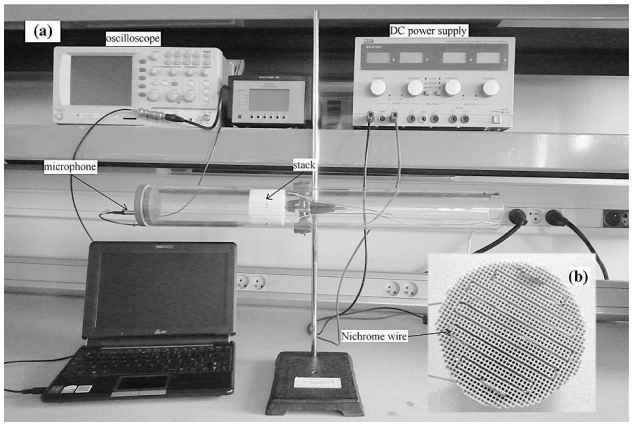
$$
\frac{dP}{dt} = \epsilon \left[T_0(x) \right] P
$$

=> Nonlinear set of differential and partial differential equations, solved using a finite difference scheme

⋆ **Main assumptions.**

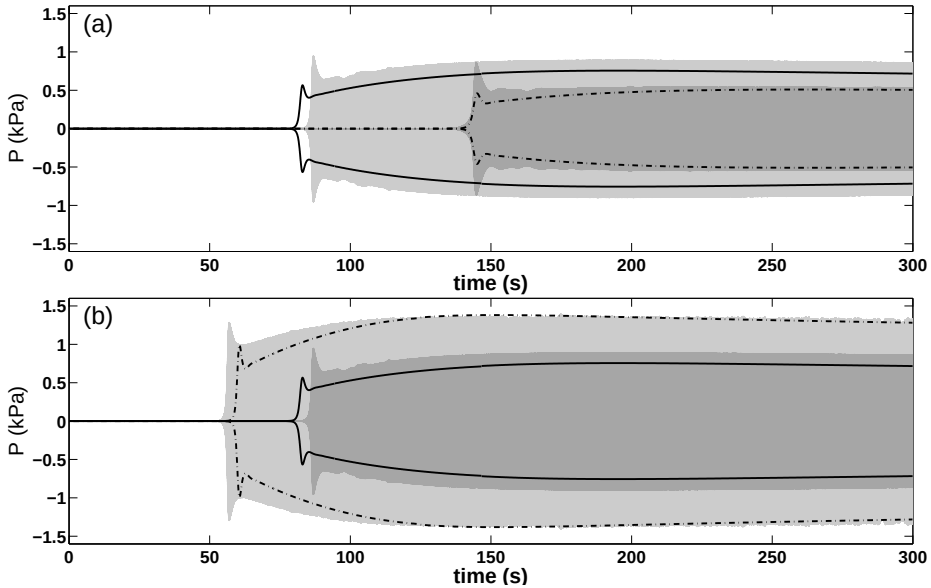
- The variations of $T_0(x)$ are negligible at the time-scale of a few acoustic periods
- $\epsilon \ll \Omega$
- T_0 is uniform through a cross-sectional area

The device



$L = 0.5m$: length of the duct
$L_s = 0.05m$: length of the stack
$r_s = 0.9mm$: the diameter of a pore
d : the distance stack-rigid plug (variable)
Q_{in} : heating power
$t = 0$: time at which heating is switched on

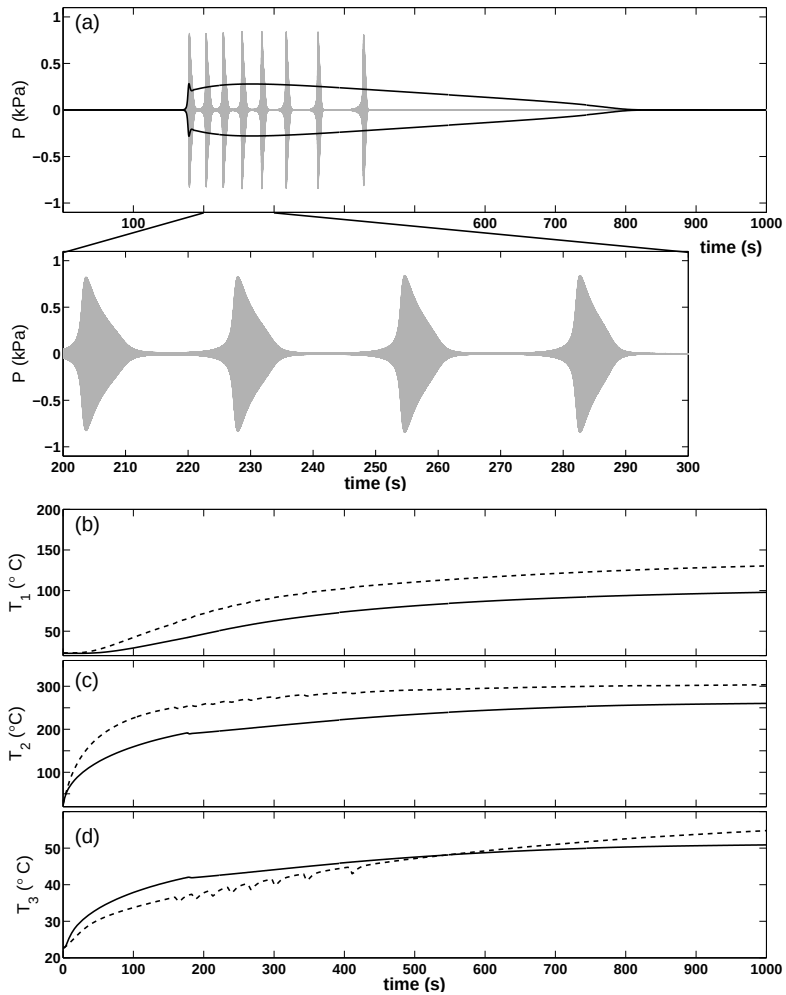
Comparison theory/experiments



(a) :(d=20cm, Q_{in}=23.3 W)/(d=20cm, Q_{in}=18.6 W)
(b) :(d=14cm, Q_{in}=23.3 W)/(d=20cm, Q_{in}=23.3W)

M. Guedra et al., J. Appl. Phys., 2014 ;

Comparison with experiments (2)



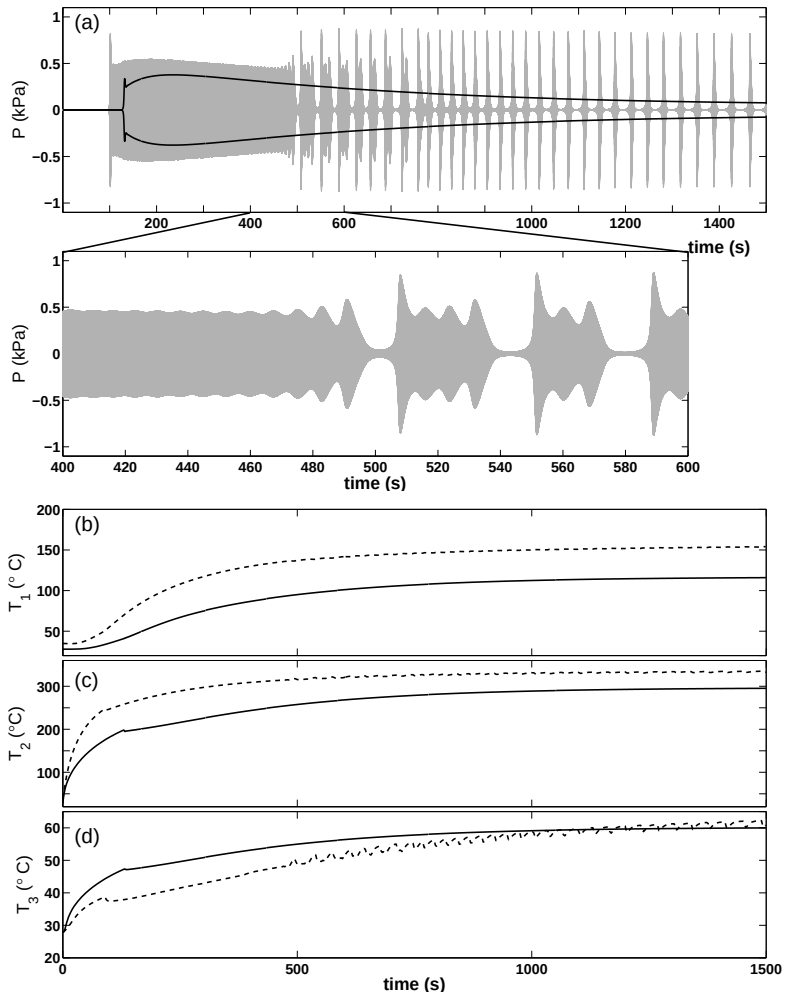
d=26 cm, Q_{in} = 23.3 W

Solid line=model ; dashed line=experiments

T_1 : temperature in the middle of the stack.

T_2 : temperature at hot side of the stack.

T_3 : temperature 2.5 cm far from the stack.



d=26 cm, Q_{in} = 26.2 W

Solid line=model ; dashed line=experiments

T_1 : temperature in the middle of the stack.

T_2 : temperature at hot side of the stack.

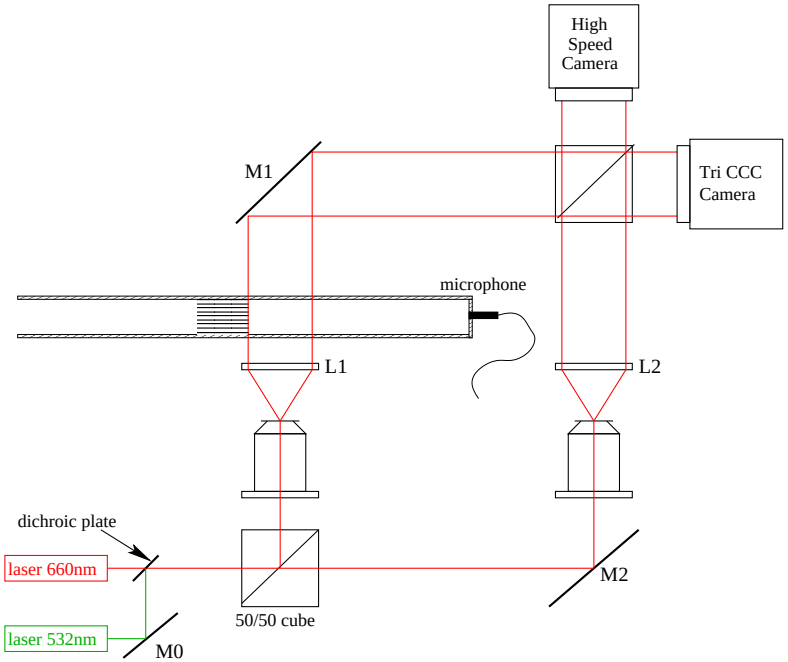
T_3 : temperature 2.5 cm far from the stack.

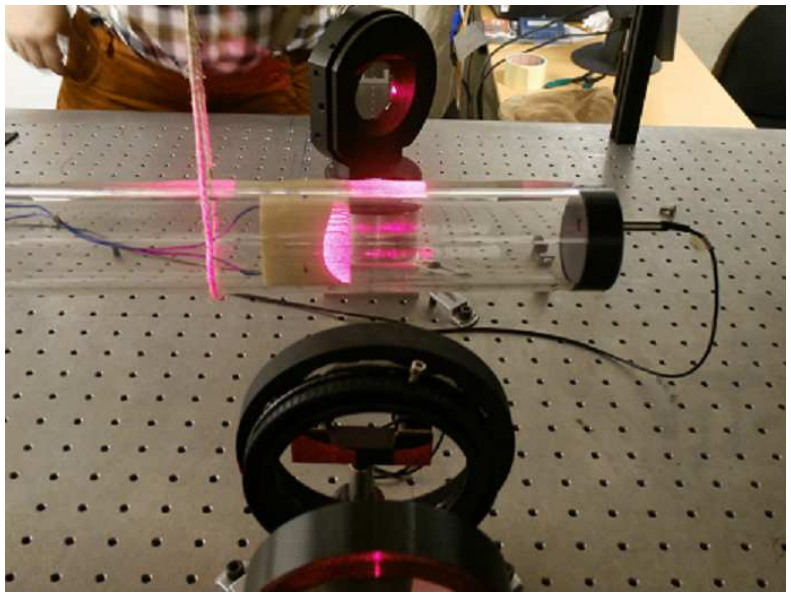
⋆ **Phénomène responsable du déclenchement-arrêt ?**

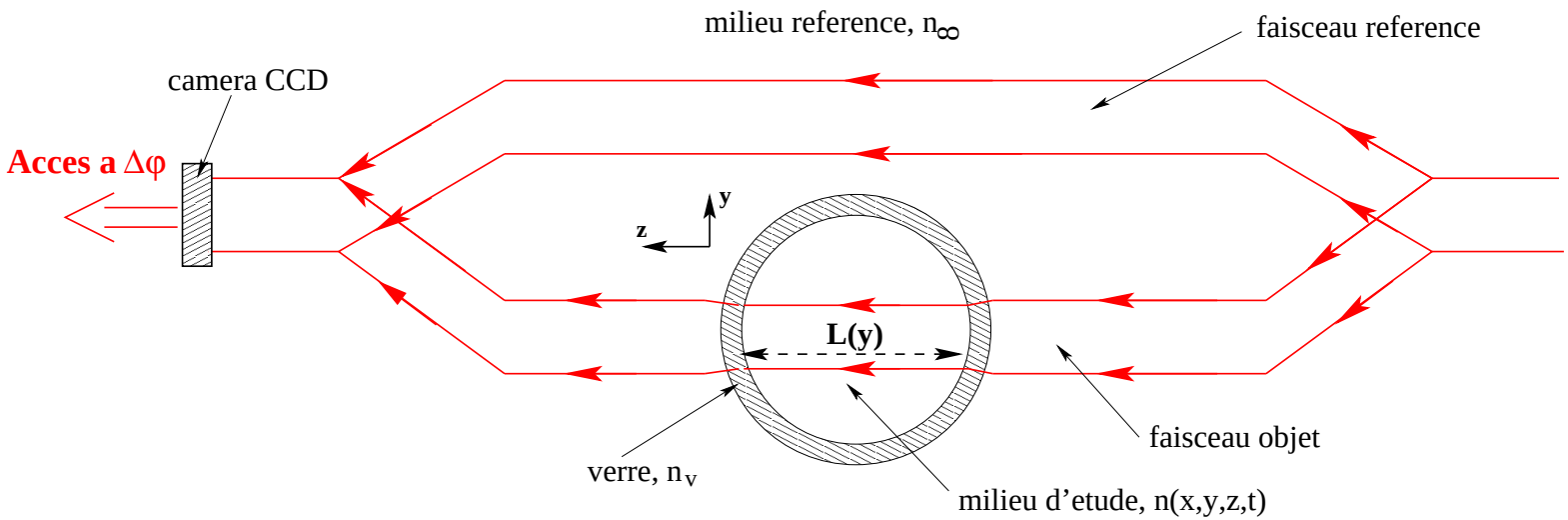
⇒ transport de chaleur acoustiquement induit, avec temps caractéristique $\tau \neq \tau_{ampl}$.

⇒ vent acoustique ? pompage thermoacoustique ? effets de bords ?

⋆ **Moyen expérimentaux envisageables : PIV, LDV, fil chaud/froid, interférométrie optique . . .**







$$\frac{\lambda\Delta\varphi_{ref}(x,y)}{2\pi} = \int_L \left[n^{ref}(x,y,z) - n_\infty\right].dz + 2\int_{L_v}\left[n_v(x,y,z) - n_\infty\right].dz$$

$$\frac{\lambda\Delta\varphi(x,y,t)}{2\pi} = \int_L \left[n(x,y,z,t) - n_\infty\right].dz + 2\int_{L_v}\left[n_v(x,y,z) - n_\infty\right].dz$$

$$\Longrightarrow \quad \frac{\lambda\left[\Delta\varphi(x,y,t) - \Delta\varphi_{ref}(x,y)\right]}{2\pi} = \int_L \left[n(x,y,z,t) - n^{ref}(x,y,z)\right].dz$$

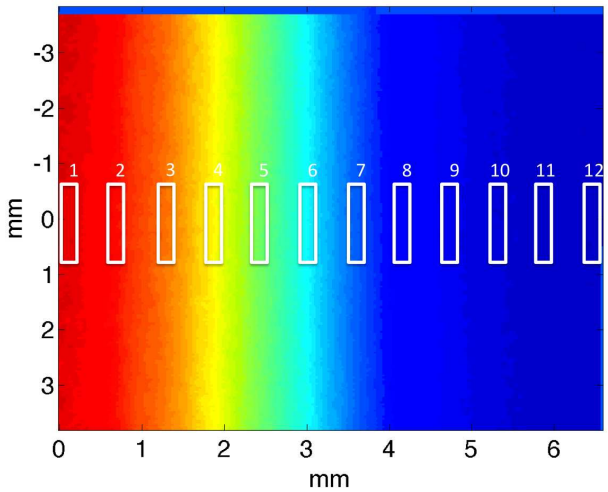
$\Rightarrow$ accès à $\langle n\rangle = (1/L)\int n.dz$, puis $\langle\rho\rangle = (1/L)\int \rho.dz$ via la relation de Gladstone-Dale[§] :

$\hat{r} = \frac{2}{3}\frac{1}{\rho(x,y,t)}(n(x,y,t)-1)$ $\quad$ $\hat{r} = 0.1516 m^3 kg^{-1} m^{-1}$ pour de l'air[¶] à 288 K et $\lambda = 607$ nm

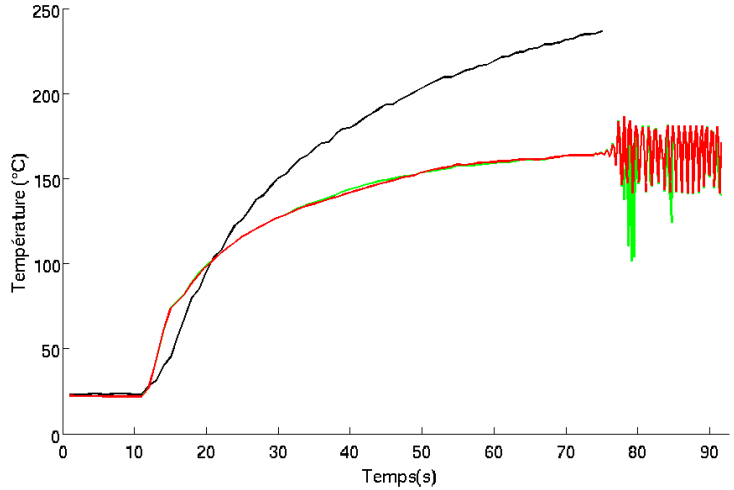
[§] NB : si $P = P_0 + p_{ac}$ connue, alors accès à $T = T_0 + \tau_{ac}$ ($P_0 = \rho_0 R T_0$ & propa. linéaire)

[¶] NB2 : $\hat{r} = \hat{r}(\lambda, T) \Rightarrow$ utilisation de deux longueurs d'onde ?

- Avant les auto-oscillations, $P_0 = P_{atm}$. => $T_0(t)$ directement déduit de $\rho_0(t)$



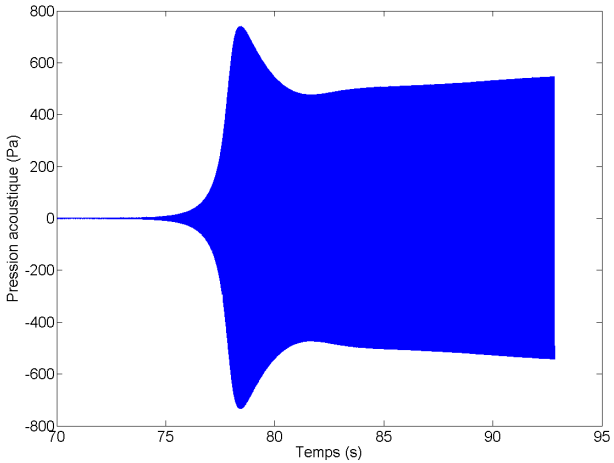
Exemple d'interférogramme, avec définition de zones d'intérêt



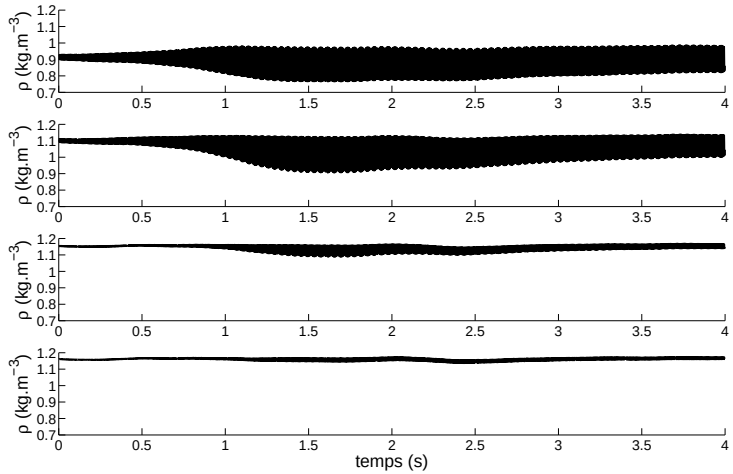
Température en zone 1 lors du chauffage (initié à $t = 0$), pour $x_s = 24cm$ et $Q_{in} = 25.1W$

- pas d'effet de chromatisme ...
- mesure de $T = T_0(t) + \tau_{ac}$ assurément fausse après le déclenchement ...
- ...mais peut-on supposer que $P = P_0 + p_{mic}\cos(k_0 x_s)\cos(\omega t)$ (i.e. pas d'effets de bords sur P) pour calculer T_0 et τ_{ac} au delà du seuil ? Dans le doute, évitons de le faire ...

- $x_s = 24$ cm, $Q_{in} = 19.4W$, début acqui. caméra rapide à $t \approx 77$ s ($f_e = 1kHz$), durée=4 s.



pression acoustique instantanée lors du déclenchement



masse volumique instantanée ($\langle\rho\rangle = \langle\rho_0\rangle + \langle\rho_{ac}\rangle$) dans les zones 3 ($x_s - x \approx 1.3mm$), 6 ($\approx 3mm$), 9 ($\approx 4.7mm$), et 12 ($\approx 6.4mm$), de haut en bas

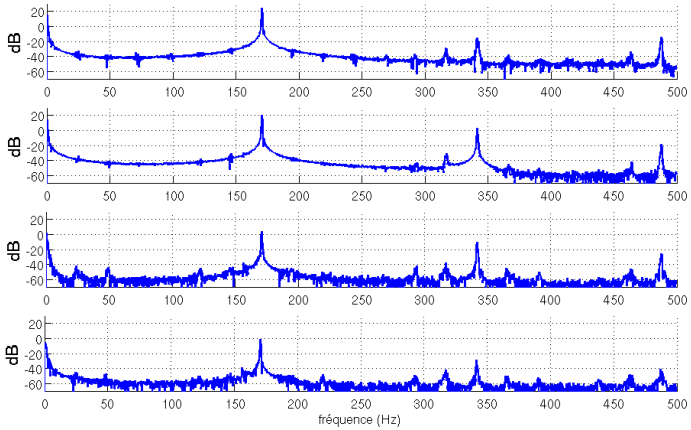
- Quelques estimations (au plus fort des auto-oscillations ...)

zone	1	2	3	4	5	6	7	8	9	10	11	12
$d/(2d_{ac})$ [‖] (%)	8	28	48	68	88	108	128	148	168	188	208	228
ρ_{max}/ρ_0 (%)	7.5	9.6	12.1	**12.9**	10.8	10.3	8.0	5.5	3.1	1.7	1.2	0.9

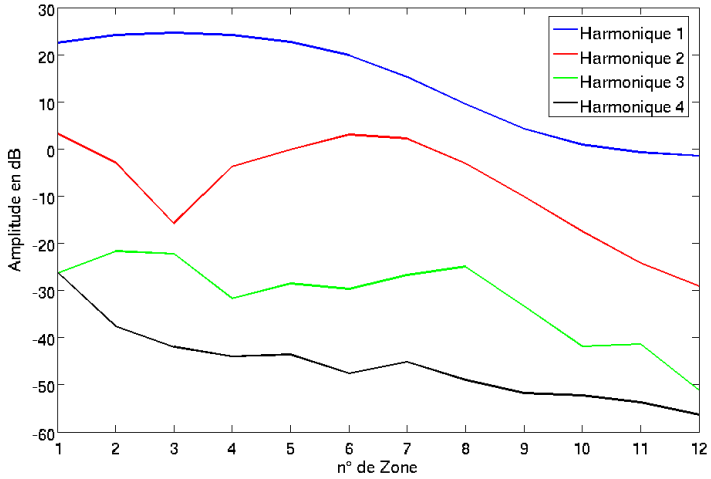
alors même que $P_{max}\cos(k_0 x_s)/P_0 \approx 0.7\%$ ...

‖ $d_{ac} \approx \frac{P_{max}}{\omega c_0}\sin(k_0 x_s) \approx 1.4mm$

⋆ Analyse spectrale des signaux de masse volumique instantanée

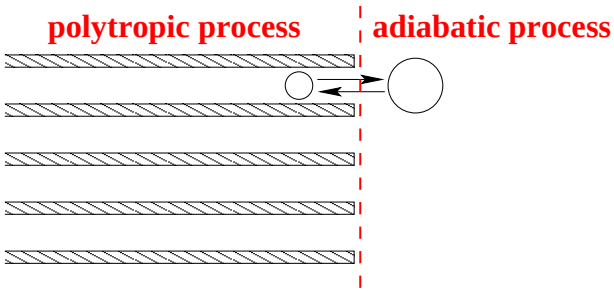


Spectres de ρ dans les zones 3, 6, 9, et 12 (de haut en bas)

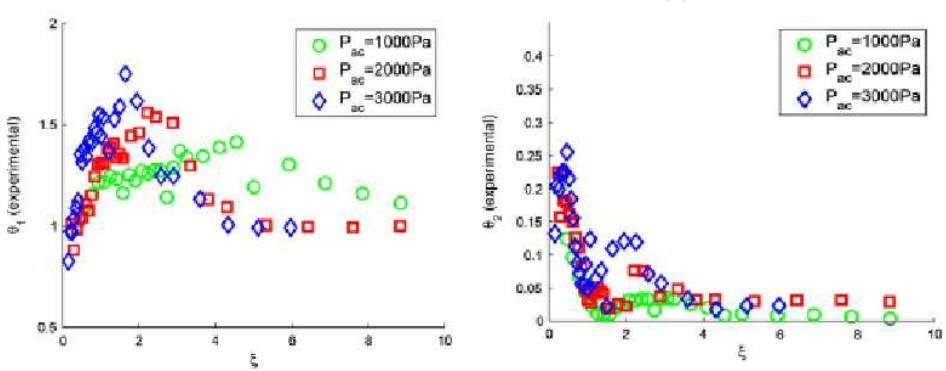


Amplitude des harmoniques en fonction de la zone d'observation

⇒ Effets de bords "thermiques"** ?



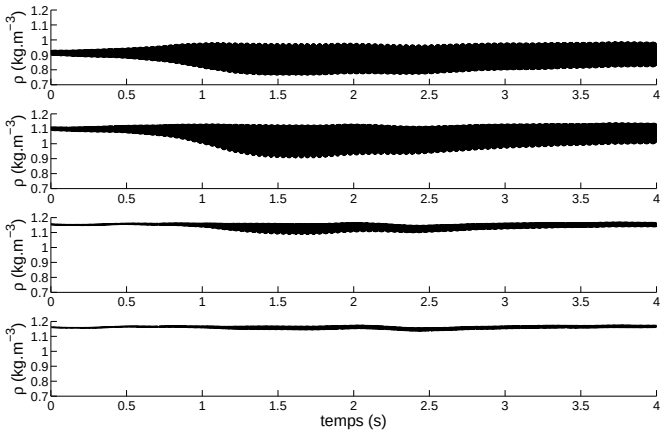
Passage abrupte d'une zone polytropique vers une zone adiabatique



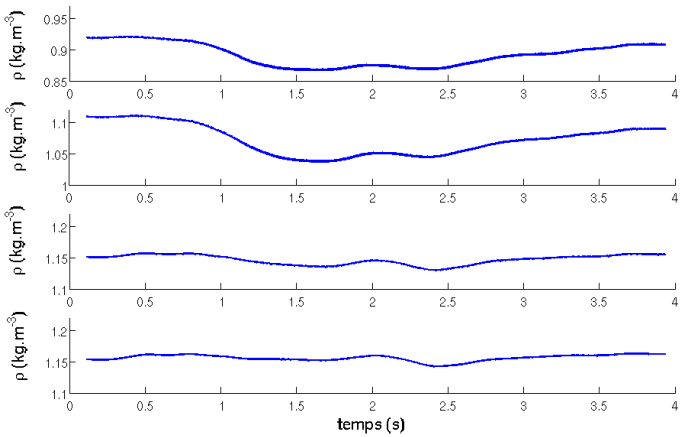
Mesures par fil froid des fluctuation de températures aux extrémités d'un stack, d'après [3]

** **[1] Gusev et al., JASA,109,2001. [2]Marx et al. JASA, 118, 2005, [3] Berson et al., I.J.H.M.T., 54, 2011**

⋆ moyenne glissante $\overline{\rho} = (1/T) \int \rho.dt$ ($T \approx 20$ période acoustiques)



ρ **dans les zones 3, 6, 9 et 12**

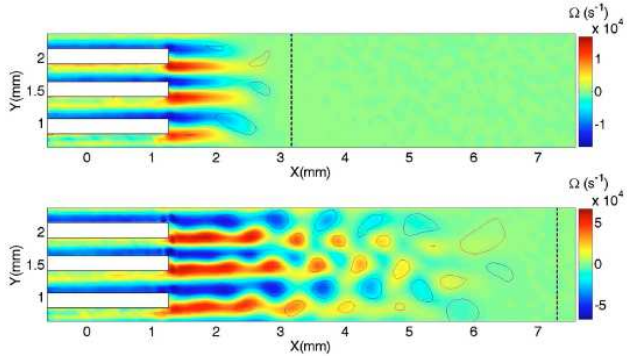


$\overline{\rho}$ **dans les zones 3, 6, 9 et 12**

$\Rightarrow$ variations significatives de $\overline{\rho}$ au cours du transitoire (jusqu'à $\approx 0.07\rho_0$ en zone 4)

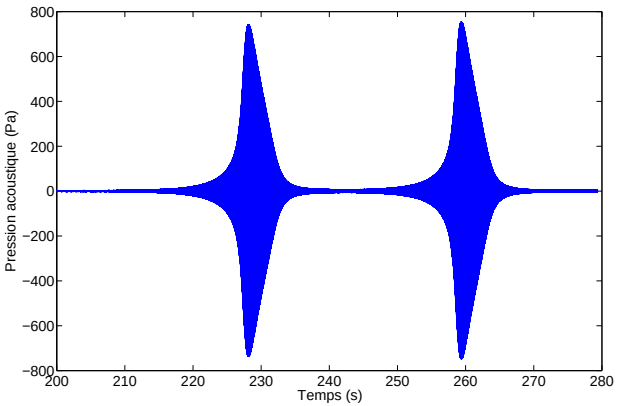
⋆ Explication possibles ?

- Réchauffement localisé (dissipation visqueuse, vent acoustique ?)
- Pertes de charges singulières $\overline{\rho} = f(\overline{T}, \overline{\mathbf{P}})$

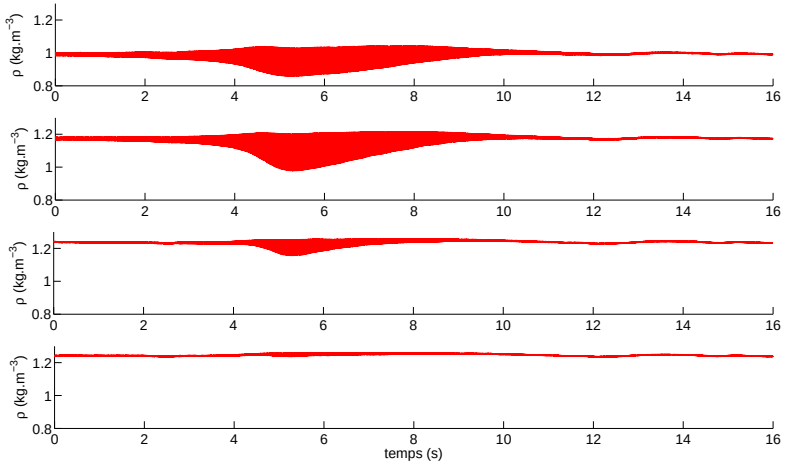


Images PIV (vorticité) en bout de stack, d'après [berson et al., JASA EL122, 2007]

- $x_s = 24$ cm, $Q_{in} = 19.4W$, début acqui. caméra rapide à $t \approx 218.3$ s ($f_e = 1kHz$), durée=16 s.



pression acoustique instantanée

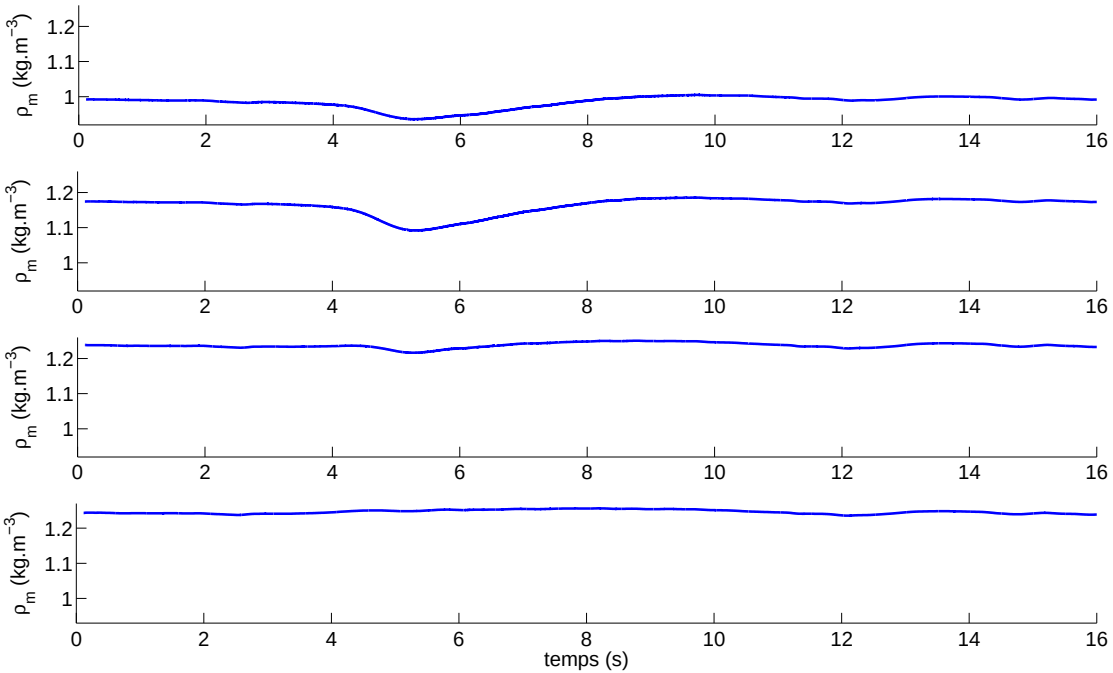


masse volumique instantanée ($\langle\rho\rangle = \langle\rho_0\rangle + \langle\rho_{ac}\rangle$) dans les zones 3 ($x_s - x \approx 1.3mm$), 6 ($\approx 3mm$), 9 ($\approx 4.7mm$), et 12 ($\approx 6.4mm$), de haut en bas

- Quelques estimations (au plus fort des auto-oscillations ...)

zone	1	2	3	4	5	6	7	8	9	10	11	12
$d/(2d_{ac})$ (%)	8	28	48	68	88	108	128	148	168	188	208	228
ρ_{max}/ρ_0 (%)	4.5	6.6	7.9	9.6	9.1	8.7	7	4.6	2.9	1.6	0.8	0.6

alors même que $P_{max}\cos(k_0 x_s)/P_0 \approx 0.7\%$ ...



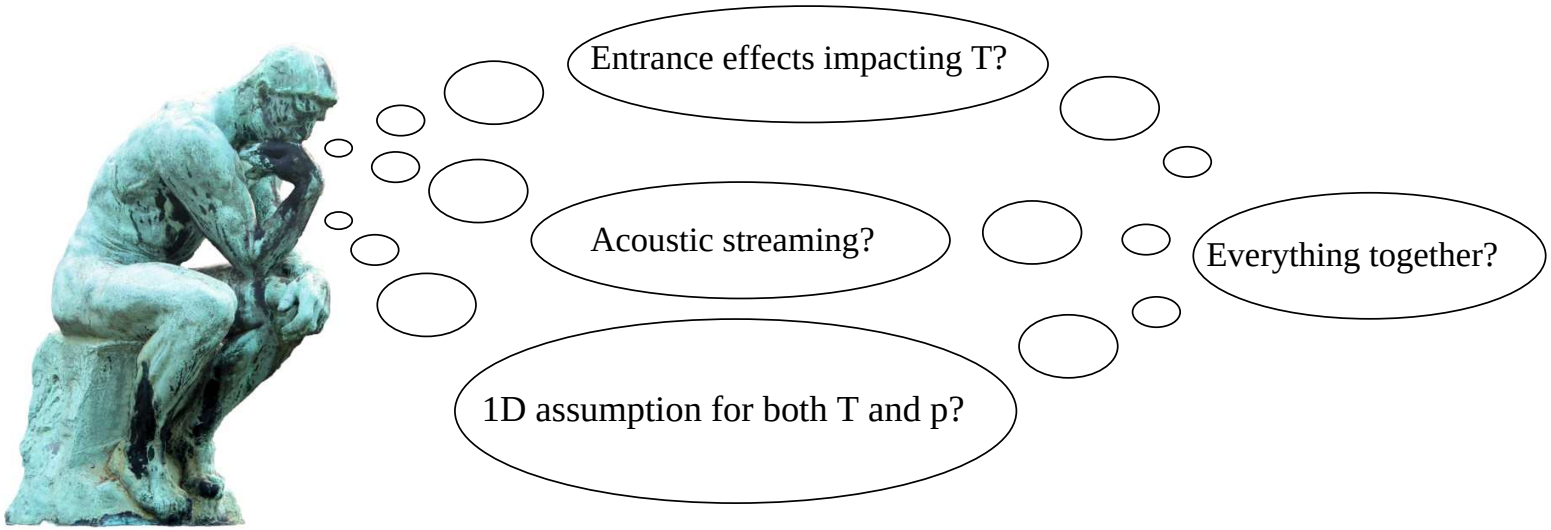
moyenne glissante ρ_m dans les zones 3,6,9,et 12.

- Quelques estimations grossières ($\Delta\rho_m/\rho_m = 2. * (\rho_{m,max} - \rho_{m,min})/(\rho_{m,max} + \rho_{m,min})$)

zone	1	2	3	4	5	6	7	8	9	10	11	12
$\Delta\rho_m/\rho_m$ (%)	2.2	5.4	7.2	7.8	9.2	8.7	6.8	4.1	2.7	1.7	1.8	1.6

The transient regime of TA Oscillators : the big deal . . .

What is the mechanism responsible for the switch on/off process ? . . .



. . .and how can we claim understanding TA engines if we cannot reproduce such an effect ?

Plan

Préambule : de quelle thermoacoustique parle-t-on ?

Théorie linéaire de la thermoacoustique

Exemple : le tube de Sondhauss

Effets de saturation non linéaires

Dynamique des auto-oscillateurs thermoacoustiques

Forçage des auto-oscillateurs thermoacoustique

Conclusion

The ThermoAcoustic Oscillator as an interesting dynamical system

⋆ An autonomous oscillator driven by heat

- which is almost out of control above threshold
- and which exhibits complicated dynamical behaviors (overshoot, "integrate and fire regime", ...)

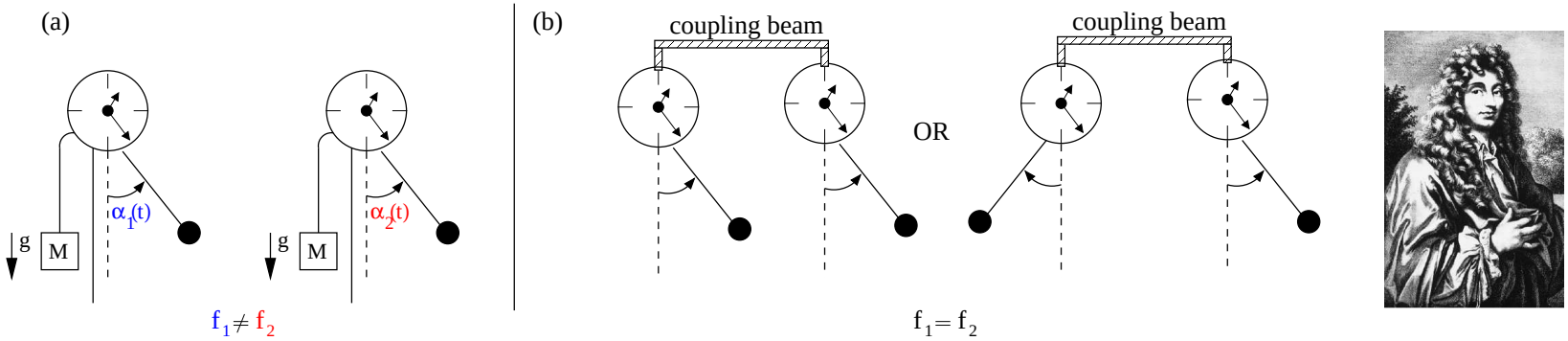
⋆ Investigate the nonlinear coupling between a TAO and an external sound source, with two objectives

- revisit universal aspects of synchronization[1] phenomena in the frame of thermoacoustics (notably for teaching purpose).
- investigate the active control of TA engine to ↗ their efficiency

[1] A. Pikovsky, M. Rosenblum, J. Kurths, « Synchronization : A Universal Concept in Nonlinear Science », Cambridge University Press, NY, 2001.

Synchronization phenomena

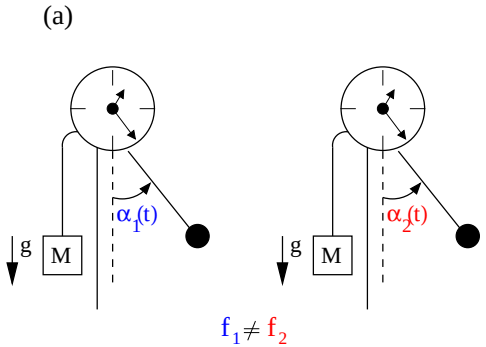
⋆ First observations : Huygens (1665, « sympathy of two pendulum clocks »)

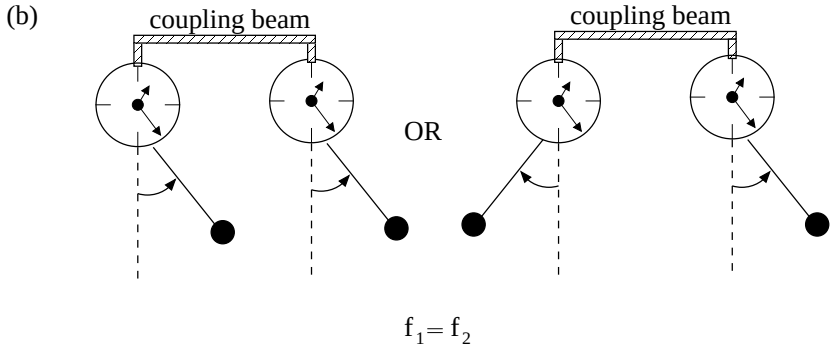


A movie made at Lancaster University (UK), which can be easily found on youtube

Synchronization phenomena

⋆ First observations : Huygens (1665, « sympathy of two pendulum clocks »)

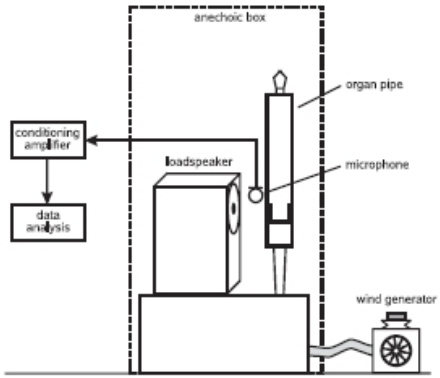




⋆ Use or observation of synchronization phenomena are nowadays abundant
- ♭ biology, medicine(singing crickets, circadian rythm, cardiac pacemaker . . .),
- ♭ electronics engineering(triode generators for radio communications . . .),
- ♭ mechanics(clocks, organ pipes . . .), physics/chemistry(lasers, B–Z reaction, . . .),
- ♭ social life(clapping audience).

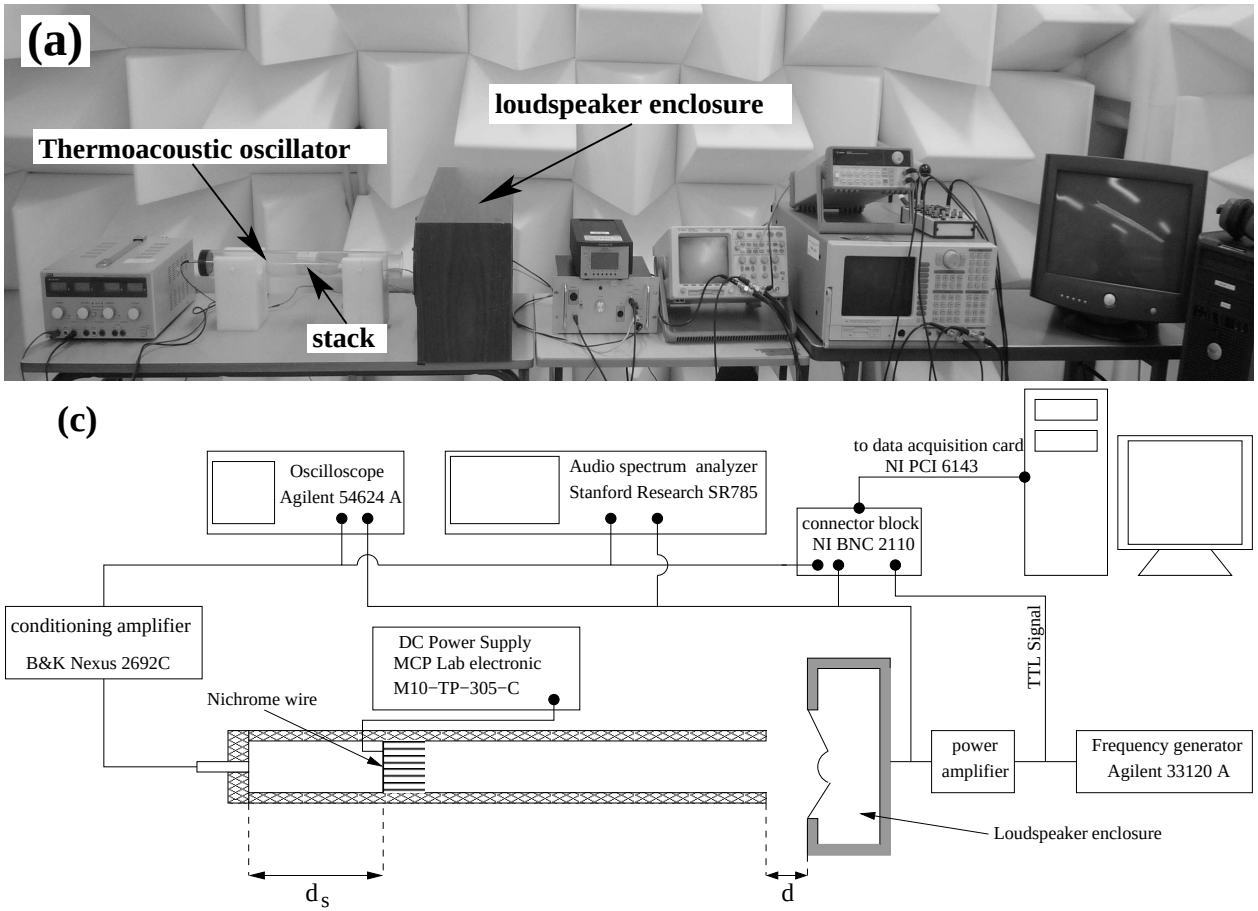
⋆ An example in the field of acoustics : synchronization of organ pipes

- ♭ Former study by Lord Rayleigh : mutual synchronization of two organ pipes and the quenching effect (oscillation death)
- ♭ . . .but even recent studies[1,2] :



Sketch of the exp. by Abel et al.[2]

[1] Abel et al, J. Acoust. Soc. Am. 119 :2467, 2006 ; [2] Abel et al. Phys. Rev. Let., 103 :114301, 2009.



(a) Photograph of the complete experimental set-up. (c) Sketch of the experimental set-up.

[Penelet & Biwa, Am. Journ. Phys., 2013]

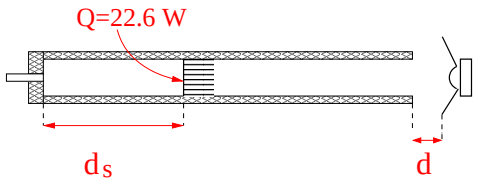
Synchronization by a sound source : experimental protocol

⋆ Experimental protocol

♭ Fix heat power Q=22.6 W ($Q > Q_{onset}$)

♭ Fix both d and d_s

♭ Proceed to measurements[††] by varying the driving frequency f and the loudspeaker voltage U_{rms}

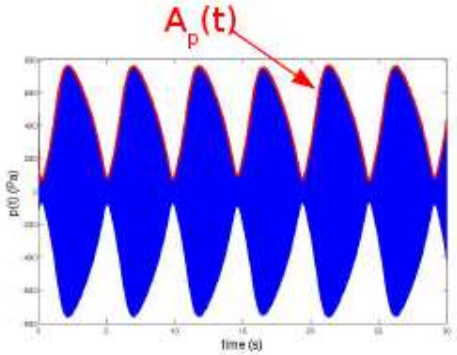


⋆ Signal processing

♭ Make both FFT and Hilbert transforms of $p(t)$ and $U(t)$

♭ Quantities of interest for data analysis :

- Frequency spectra $p(f)$ and $U(f)$
- Amplitude modulation $A_p(t)$
- Instantaneous phase difference $\Psi(t) = \Phi_p(t) - \Phi_U(t)$

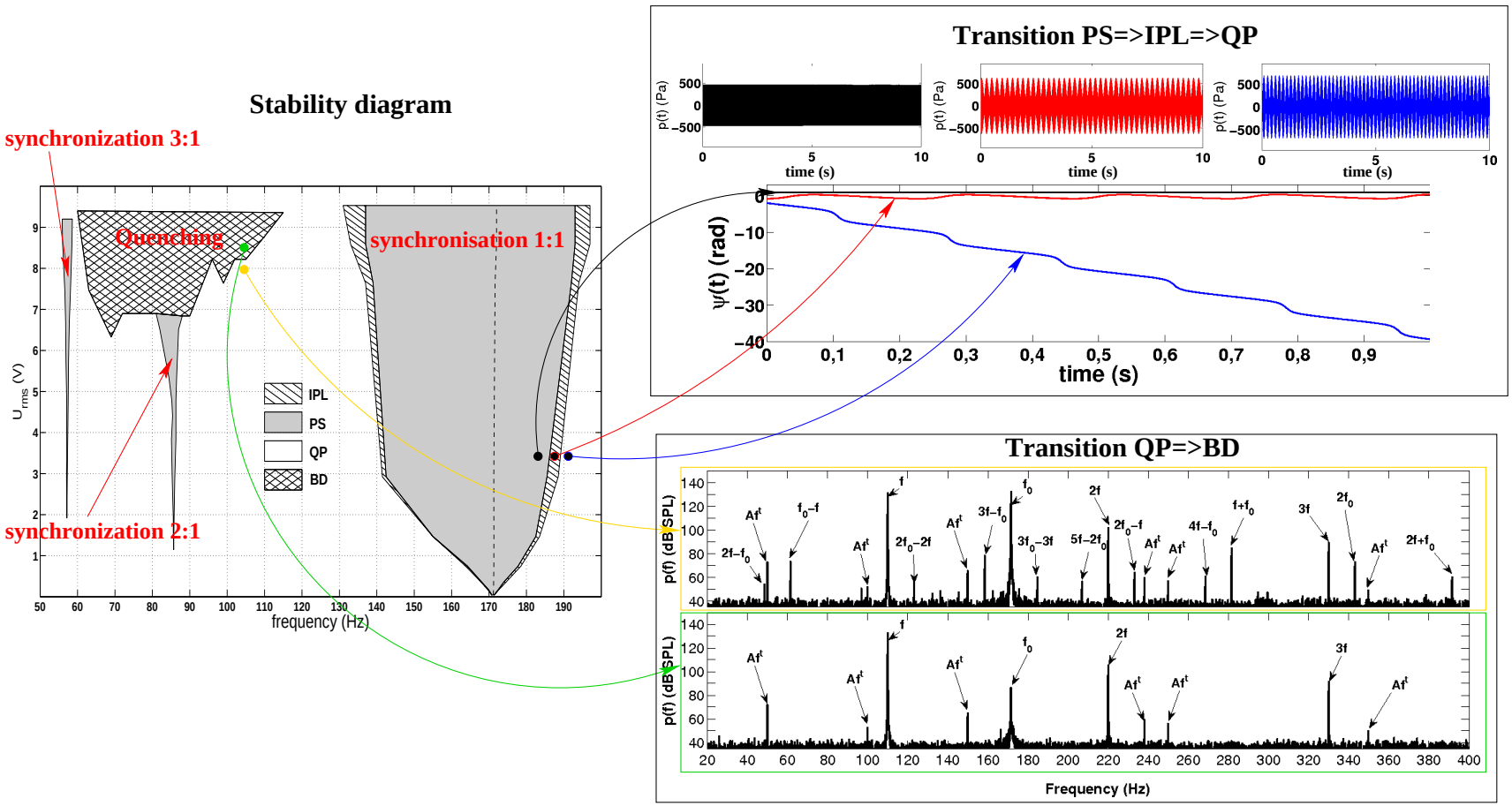


⋆ Different possible states

(PS) Perfect Synchronization	(IPL) Imperfect Phase Locking	(QP) QuasiPeriodicity	(BD) Beating Death
$f_{TAO} = f$ (or $f_{TAO} = \frac{f}{n}$) $A_p(t) = c^{te}$ $\Psi(t) = c^{te}$	$f_{TAO} \neq f$ (or $f_{TAO} \neq \frac{f}{n}$) $A_p(t) \neq c^{te}$ $\Psi(t) \neq c^{te}$ but bounded	$f_{TAO} \neq f$ (or $f_{TAO} \neq \frac{f}{n}$) $A_p(t) \neq c^{te}$ $\Psi(t) \neq c^{te}$ (unbounded)	$A_p(t) = c^{te}$

[††] Each set of measurements ∼ 12-24 hours of total duration !!

⋆ Stability diagram as a function of $U_{loudspeaker}$ and f, for d=1 mm, d_s=8 cm and Q=22.6 W

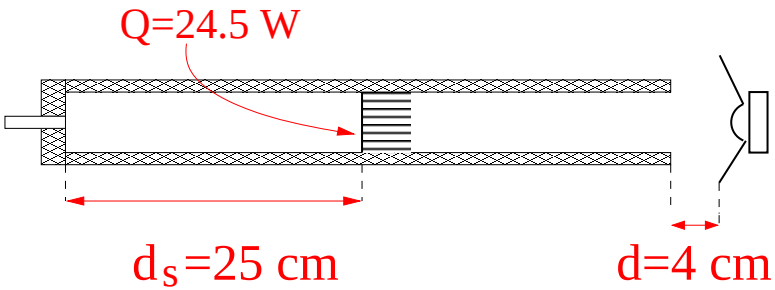


♭ Some universal aspects of synchronization are retrieved by this experiment‡‡
♭ Some aspects are intrinsic to the TAO itself :
=> why are the Arnold tongues asymetric ?
=> why does quenching occurs around $f \approx f_{TAO}/2$?

‡‡ **Only the 2nd experimental evidence of the phase trapping phenomenon . . .**

Other phenomena related to the external forcing of a TAO

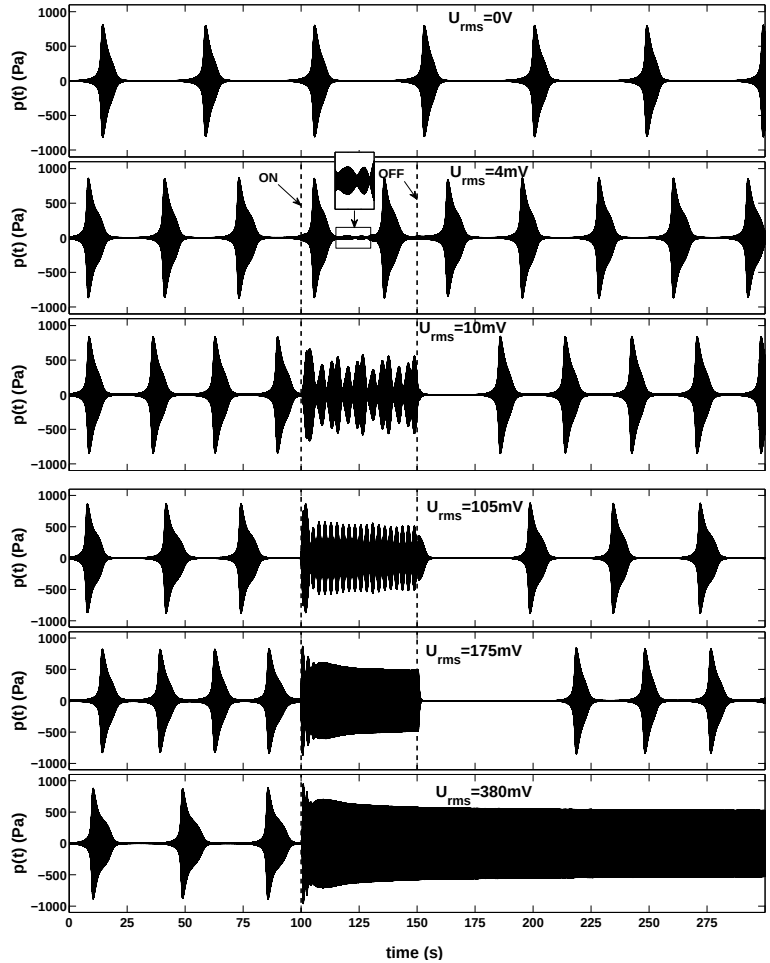
⋆ **What if the TAO is operated as a relaxational oscillator ?**



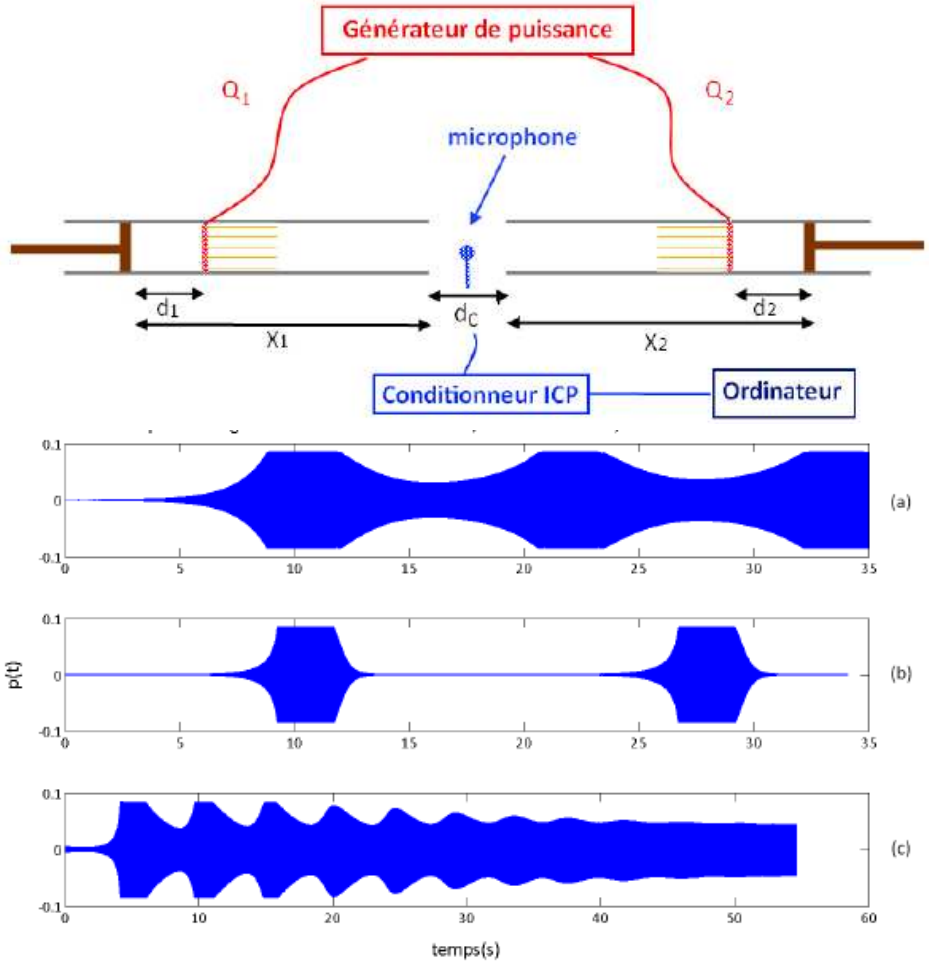
Loudpseaker off => "integrate and fire regime", with $f_{TAO} \approx 177.1$ Hz

⋆ **Experimental procedure**

1.- Settle f to 176.9 Hz ($\neq f_{TAO}$)
2.- Switch the loudspeaker on at t=100 s
3.- Switch the loudspeaker off at t=150 s
4.- Repeat steps 1-3 with another driving voltage



interpretation ? => Simply quenching or more complicated processes ?
=> does external forcing lead to the transformation of a bistable regime (with periodic switch between two stable states) into a stable one ?


Générateur de puissance
Q1
Q2
microphone
d1
dC
d2
X1
X2
Conditionneur ICP
Ordinateur
p(t)
(a)
(b)
(c)
temps(s)

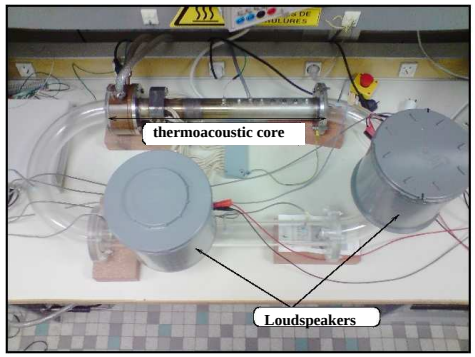
And what about forcing a TAO to increase its efficiency ?

Basic ideas

- Above threshold (and at fixed heat input Q_0), TA engines are "out of control"
- Is there a possibility to ↗ the efficiency of a TA engine with auxiliary acoustic sources ?

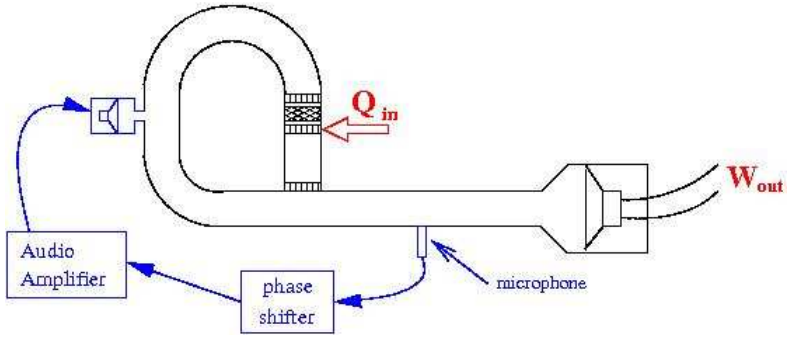
A proof of concept study performed succesfully[1]

- Closed-loop , stack based, TA engine
- Active control by 2 sources controlled in both amplitude and phase



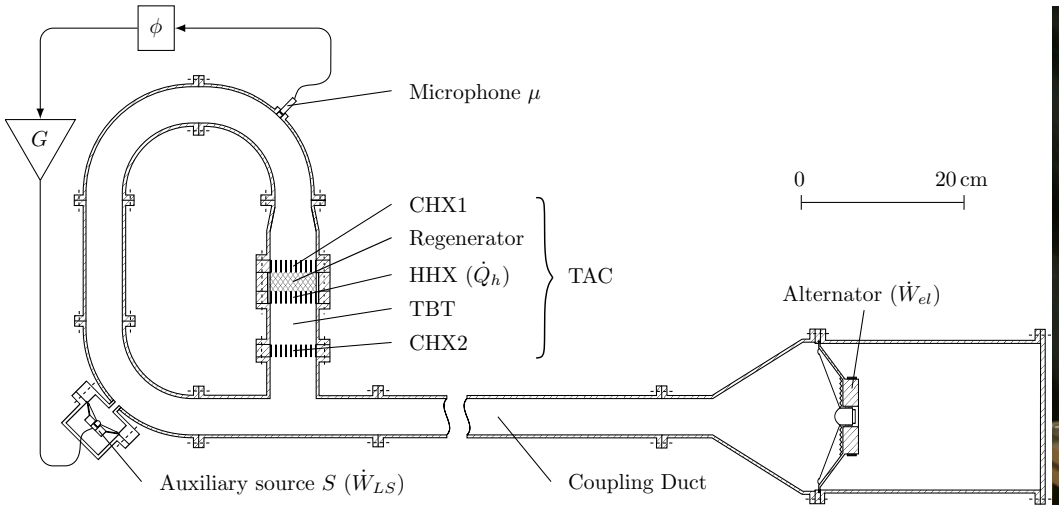
A new proof of concept study on a larger TA engine[2]...

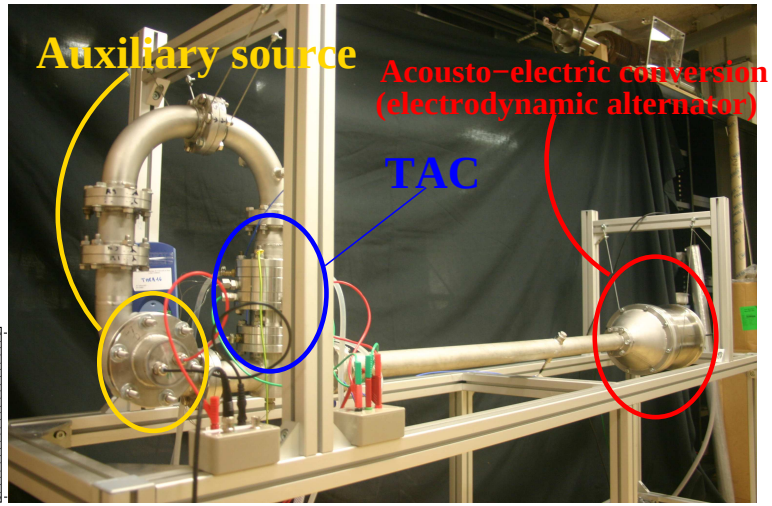
- ThermoAcoustic Stirling electricity generator
- Active control by 1 source and a feedback loop



[1] C. Desjouy et al., J. Appl. Phys., 2010. [2] C. Olivier et al., accepted for publication in J. Appl. Phys., 2014.

⋆ The device (a "Stirling type" Thermo-acousto-electric engine, working with 5 bars air)





⋆ The control parameters

- The heat input $\dot{Q}_h$
- The phase shift ϕ,
- The power supplied to the auxiliary source $\dot{W}_{LS}$ (controlled via the voltage gain G)

⋆ The output

Feedback loop switched off

- The output power $\dot{W}_{el_0}$
- The efficiency $\eta_0 = \dot{W}_{el_0}/\dot{Q}_h$
- The temperature difference ΔT_0

as a function of $\dot{Q}_h$

Feedback loop switched on

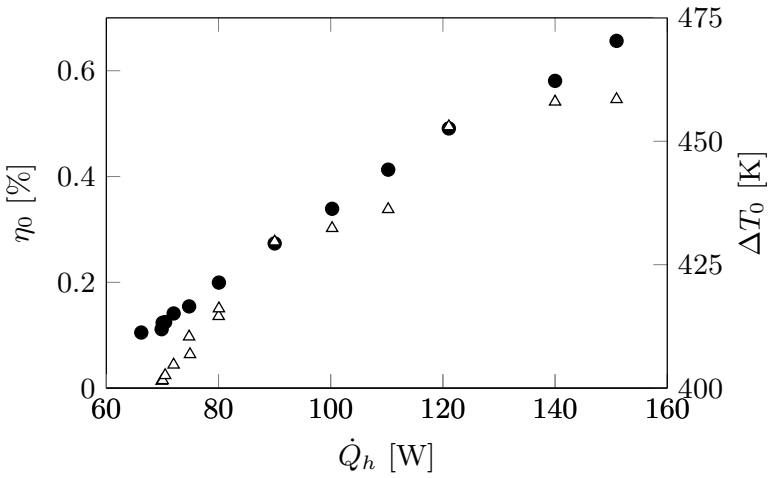
- $\dot{W}_{el}$ and $\Delta\dot{W}_{el} = \dot{W}_{el} - \dot{W}_{el_0}$
- $\eta = \dot{W}_{el}/\left(\dot{Q}_h + \dot{W}_{LS}\right)$ and η/η_0
- ΔT and $\Delta T - \Delta T_0$

as a function of $\dot{Q}_h$, ϕ and G

Active Control of a TAO : the results(1)

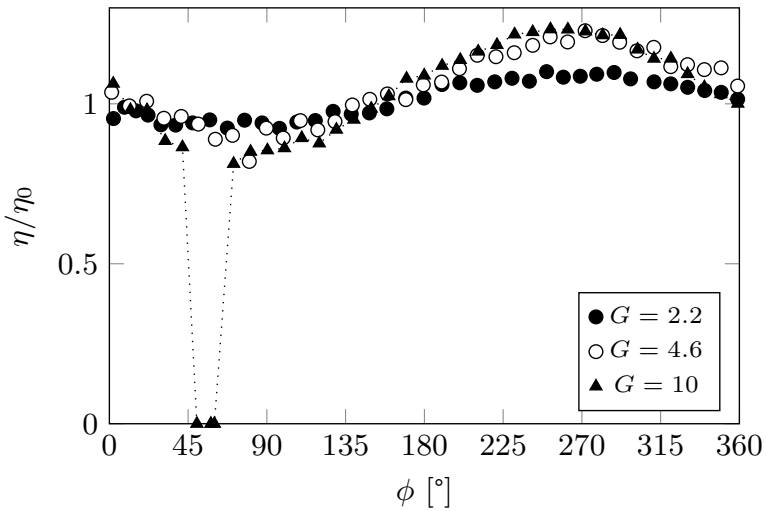
Feedback loop switched off (G=0)

$\eta_0(\dot{Q}_h)$ (•), $\Delta T_0(\dot{Q}_h)$ (△)



Feedback loop switched on (G ≠ 0)

$\dot{Q}_h$=75 W, $\eta/\eta_0(G,\phi)$



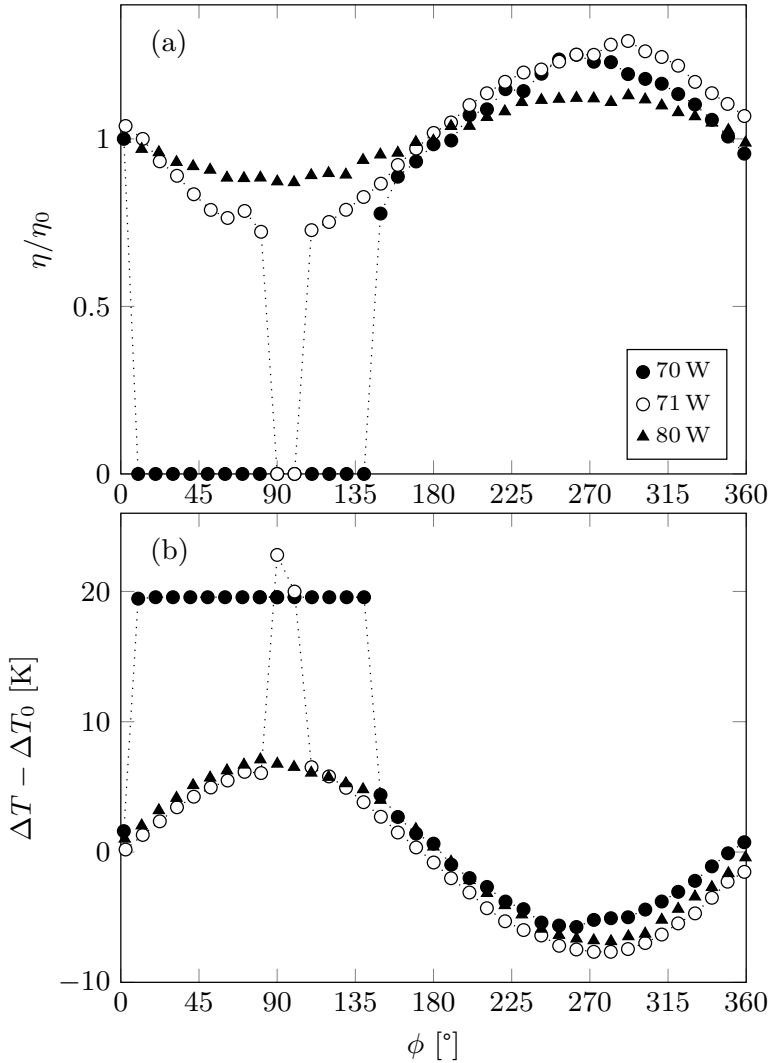
⇒ **The electroacoustic feedback loop impacts the operation of the TA engine**

→ Possibility to ↗ η (i.e. to have $\eta > \eta_0$)

→ Here, $\phi_{opt} \approx 270°$

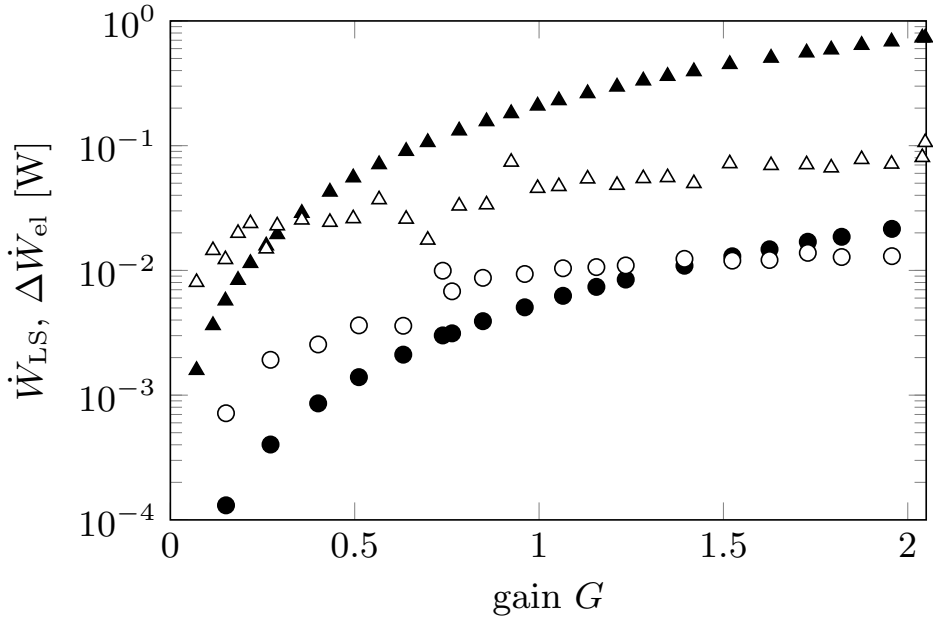
→ Possibility to destroy TA oscillations (here for $\phi \equiv 50 - 60°$ and $G \geq 10$)

The variation of $\eta(\Phi, \dot{Q}_h)$, $\Delta T(\Phi, \dot{Q}_h)$ with a fixed voltage gain G (here $G = 3.2$)



$\Rightarrow \eta > \eta_0$ while $\Delta T > \Delta T_0$!

The net output power gain $\Delta\dot{W}_{\mathrm{el}}$ compared to the power provided to the auxiliary source $\dot{W}_{LS}$, with an assigned phase shift (here $\phi = 270°$)



$\Delta\dot{W}_{\mathrm{el}}$ (○) and $\dot{W}_{LS}$ (●) as a function of G for $\dot{Q}_h = 70$W
$\Delta\dot{W}_{\mathrm{el}}$ (△) and $\dot{W}_{LS}$ (▲) as a function of G for $\dot{Q}_h = 150$W

$\Rightarrow$ Possibility to have $\Delta\dot{W}_{el} > \dot{W}_{LS}$ (indisputable proof of concept ...)

Plan

Préambule : de quelle thermoacoustique parle-t-on ?

Théorie linéaire de la thermoacoustique

Exemple : le tube de Sondhauss

Effets de saturation non linéaires

Dynamique des auto-oscillateurs thermoacoustiques

Forçage des auto-oscillateurs thermoacoustique

Conclusion

Conclusion

⋆ Conclusion of part 1-2 (linear theory, design tools, Sondhauss tube)

Design tools are available, but

- they are based on the linearization to 1^{st} order of the governing equations,
- they are based on a 1-D description both acoustics and heat transport,

⋆ Conclusion of part 3 (nonlinear processes)

NL process	Academic understanding	Appropriate modeling	Impact on TA Engines
TA Heat pumping	☺	☺	significant
NL acoustics	☺		not much
Streaming	☺	☹	significant
Edge effects	☺	☺	significant
Turbulence	☹	☹	?

⋆ Conclusion of part 4 (dynamics of TA Oscillators)

♭ Even the simplest TAO exhibit complicated behaviors which are not very well understood

♭ The study of the transient regime may provide a deeper physical insight on the mechanisms responsible for sound saturation ...

♭ and may therefore provide new opportunities to increase the engine's efficiencies

⋆ Conclusion of part 5 (External forcing of TA oscillators)

♭ The forcing of a TAO allows studying synchronization phenomena ...

♭ But it may also be used to increase the efficiency of TA engines

Remerciements

L'OR thermoacoustique du LAUM

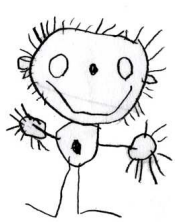
Pierrick LOTTON
(DR CNRS)

Guillaume PENELET
(MCF)

Gaëlle POIGNAND
(IR)

Côme OLIVIER
(Doc.)

Les anciens doctorants


Cyril DESJOUY

Matthieu GUEDRA

Flavio BANNWART

Autres collègues du LAUM ...

M. BRUNEAU, J.P. DALMONT, J. GILBERT, V. GUSEV, M. LECLERCQ, P. PICART

ou d'ailleurs ...

T. BIWA (Tohoku Univ., Japan)

merci de m'avoir écouté ...