CHARATERIZATION OF A DISPERSIVE SYSTEM USING QUADRATIC TIME-FREQUENCY REPRESENTATIONS

J. BERNARD, S. MONTRESOR, J-H. THOMAS and C. DEPOLLIER

Laboratoire d'acoustique de l'Université du Maine (LAUM UMR-CNRS 6613)
Av. O. Messiaen, 72085 LE MANS, France (Europe)
phone: +33 (0) 2 43 83 35 53, email: jerome.bernard@univ-lemans.fr

ABSTRACT

Quadratic time-frequency representations (QTFR) to characterize dispersive media are interesting because of their capacity to concentrate the energy of the signal in a two-dimensional representation plane. In this work, some distributions of a special class of QTFR (P_k-Bertrand) are used to analyse non-stationary signals passing through dispersive systems like porous media. The entropy of Renyi is used to determine the well matched distribution and therefore to evaluate the group delay law of the analysed signal.

1. INTRODUCTION

The aim of this work is to study the propagation of an acoustic wave in porous media using QTFR, for example the Bertrand distribution. Because of the dispersive characteristic of such a system, the wave velocity in the porous structure is a complex function of the frequency, i. e., the energy of the signal carried by each frequency has its own velocity. So, a sample of porous medium is excited by an ultrasonic pulse, and the QTFR of the output signal is observed to evaluate the group delay law, function of the frequency, and then the parameters of the material.

The relevance of such signal processing tools is their capacity to concentrate the energy of a signal in a two-dimensional plane (time-frequency, time-scale,...) along its group delay law. In the case of propagation in porous media, the group delay law is compared to a power law in the time-frequency plane. Finally, the properties of the Power Class of QTFR are used. This class of representations allows to generate a large number of QTFR and each of them matches with an adapted power law in terms of lisibility and concentration of energy in the time-frequency plane. Hence, to choose the most relevant representation, concentration measurement tools as Renyi entropy are used.

Here, the situation where a signal passes through a dispersive system which causes a power law group delay to the signals is considered. So, a QTFR which localizes efficiently this kind of signals is needed as well as the Wigner-Ville QTFR localizes the linear chirp signals.

In the first section of this paper, porous media and sound propagation in them is studied to show the relevance of quadratic time-frequency representations of the response of a porous medium excited by an ultrasonic pulse.

Next, in the second section, signal processing tools adapted to non-stationnary signals are presented and more particularly the affine class of quadratic distributions and the affine Wigner representations. The entropy of Renyi, as a concentration measurement tool, is used in order to choose the best distribution which localizes the energy of the signal along its power group delay law and therefore to evaluate it.

Finally, results obtained using these time-frequency tools are discussed in the last section. For that, an ultrasonic pulse is propagated in a sample of porous medium and output signal representations using different distributions are compared. The energetic attenuation curve as a function of the frequency is given directly from input and output signal representations.

2. SOUND PROPAGATION IN POROUS MEDIA

2.1 Short description of the porous medium

In this problem, a homogeneous isotropic porous material is considered. The porosity of this material is noted ϕ, and the saturing fluid is characterized by its compressibility modulus K_a, its viscosity η and its density ρ_f. It is also assumed that the frame of the porous solid is not deformable when it is subjected to an acoustic wave; so this wave propagates only in the fluid and the porous medium can be seen as an equivalent fluid with a complex compressibility modulus and a complex density. This is shown by the Euler and mass conservation equations :

$$\rho_f \alpha(\omega) \frac{\partial v_i}{\partial t} = -\nabla_i p, \quad (1)$$

$$\frac{\beta(\omega)}{K_a} \frac{\partial p}{\partial t} = -\nabla . v. \quad (2)$$

with ω the angular frequency, $\alpha(\omega)$ the dynamic tortuosity, $\beta(\omega)$ the dynamic compressibility of the fluid defined in [1], and v, p, respectively the particule velocity and the acoustic pressure. The sound velocity is derived from the expressions of $\alpha(\omega)$ and $\beta(\omega)$ and yields the equation :

$$c(\omega) = \sqrt{\frac{K_a}{\rho_f \alpha(\omega) \beta(\omega)}}. \quad (3)$$

In this expression, the velocity is a complex function of the frequency ω and thus explains the relevance of the time-frequency representations to study wave propagation in porous media. But this relation is still not sufficient to characterize the properties of the medium, in particularly the effects of these properties on the propagation phenomena. So, the time expressions of (1) and (2) are needed to have access to the time evolution of the acoustic wave.

2.2 Sound propagation in the medium

To evaluate the dispersion and the attenuation due to the medium, the propagation equation is derived in eq. 4, with the dispersion modelled by a fractionnal derivative as in [2].

$$\frac{\partial^2 v}{\partial x^2} - A\frac{\partial^2 v}{\partial t^2} - B\int \frac{\partial^2 v/\partial t'^2}{\sqrt{t-t'}}\,dt' - C\frac{\partial v}{\partial t} = 0, \quad (4)$$

with A, B and C which depend on the physical properties of the material and represent respectively the velocity of the wave in the fluid included in the material, the dispersion and the attenuation. Hence the material can be seen as a system which propagates, filters and attenuates the input signal. In the last part of this paper, experimented results are shown with an ultrasonic pulse as an input signal.

3. SIGNAL PROCESSING TOOLS

3.1 Time-frequency representations

This section is a brief description of the time-frequency representations [3, 4] and more specially of the affine class of distributions, because of its relevance for analysing physical phenomena.

3.1.1 *The affine class*

The affine class is the group of time-frequency representations which are covariant by time shift and scales changes, as seen in eq. 5.

$$\begin{array}{ccc} x(t) & \longrightarrow & \frac{1}{\sqrt{a}}x(\frac{t-t_0}{a}) \\ \downarrow & & \downarrow \\ \Omega_x(t,f) & \longrightarrow & \Omega_x(\frac{t-t_0}{a}, af), \end{array} \quad (5)$$

with $\Omega(t,f)$ the quadratic time-frequency distribution of the affine class.

These time-frequency distributions are well adapted to the representation of physical phenomena, wideband radar or self-similar processes for example. But the sound propagation in porous media introduces wave dispersion and a special group delay law in the output signal. So, without *a priori* on the shape of this signal, more adapted distributions, as affine Wigner distributions are needed.

3.1.2 *Affine Wigner distributions*

Quadratic time-frequency representations of the Power class (PC_k) are quite suitable for the analysis of signals passing through dispersive systems (porous media for example) with a power function group delay $\tau_k(f) \propto f^{k-1}$ [5]. Hence for this study, time frequency distributions of Bertrand, which are members of both the affine class and the PC_k, are used. This class of quadratic time-frequency representations were defined by J. and P. Bertrand [6]. The general expression is :

$$P_x^{(k)}(t,f) = f\int \underbrace{X[\lambda_k(u)f]}_{dilatation}\underbrace{X^*[\lambda_k(-u)f]}_{compression}\underbrace{\mu_k(u)e^{i2\pi tf\zeta_k(u)}}_{pseudo-Fourier}du, \quad (6)$$

with $P_x^k(t,f)$ the time-frequency representation of Bertrand, $f > 0$ and $\mu_k(u)$ an arbitrary positive and continuous function and :

$$\lambda_k(u) = \left(k\frac{e^{-u}-1}{e^{-ku}-1}\right)^{1/(k-1)}, k \neq 0, 1. \quad (7)$$

$$\zeta_k(u) = \lambda_k(u) - \lambda_k(-u). \quad (8)$$

These distributions depending on the parameter k, are covariant by scale changes and by translations along a power law as seen in eq. 9.

$$\begin{array}{ccc} X(f) & \longrightarrow & e^{-j2\pi\Phi_k(f)}X(f) \\ \downarrow & & \downarrow \\ P_x^k(t,f) & \longrightarrow & P_x^k[t-\frac{d}{df}\Phi_k(f), f], \end{array} \quad (9)$$

with $X(f)$ the Fourier transform of the time signal $x(t)$.

The good localization of this kind of time-frequency representations along the group delay law f^{k-1} due to their covariance properties ensures few interference terms. Hence in this paper the aim is to find the well-matched representation to an *a priori* unknown signal and then its power law, introduced by the dispersive characteristics of porous media.

3.2 Entropy of Renyi

3.2.1 *Definition*

Thanks to the analogy between signal energy in the time-frequency plane and probability densities, the entropy can be used to evaluate the energy concentration and the information content of the time-frequency plane. Because of the non-positivity of some distributions, a particular expression of the entropy was developped by Renyi [7] :

$$H_\alpha(\Omega_x) = \frac{1}{1-\alpha}log_2\int\int \Omega_x^\alpha(t,f)\,dt\,df, \quad (10)$$

with the parameter order $\alpha > 0$.

In this work, the entropy of Renyi of third order is used on normalized versions of P_k-Bertrand to determine the more adapted among them in terms of localization along the unknown power group delay of signals propagated through porous media.

4. RESULTS

An ultrasonic pulse (bandwidth : 60-420 kHz)[9] is propagated in a sample of porous medium and time-frequency representations are used to evaluate some properties of the propagation in this medium. First, the input signal and two output signals (experimental and theoritical) and their Wigner-Ville distributions (computed using [8]) are represented in fig. 1, 2 and 3. And from these representations the energetic attenuation curve as a function of the frequency is obtained. The physical predictions of wave propagation (dispersion relationship with a complex wave number) are respected : high frequencies are more attenuated than low frequencies (see fig. 4). Then, the third order entropy of Renyi is used to evaluate the interval of the k of P_k-Bertrand which minimizes $H_\alpha(P_x^k)$ and then ensures the better localization along the group delay law of the signal and less interference terms. As example, fig. 6, fig. 7 and fig. 8 represent different Bertrand distributions (respectively k=-0.1, k=-2 and k=2) of the experimental output signal. These representations are more satisfactory than the fig. 2b from Wigner-Ville distribution in terms of energy concentration. The Bertrand distribution computed with a parameter k (see fig. 6) in an interval near the minimum of the third order Renyi entropy

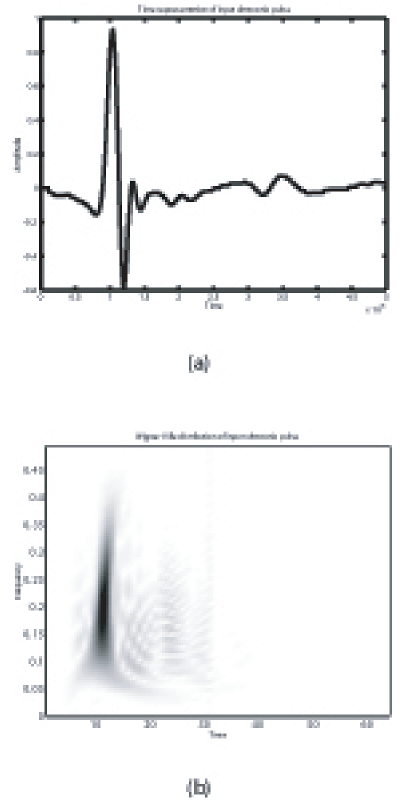

Figure 1: Input ultrasonic pulse : (a) time signal, (b) Wigner-Ville distribution

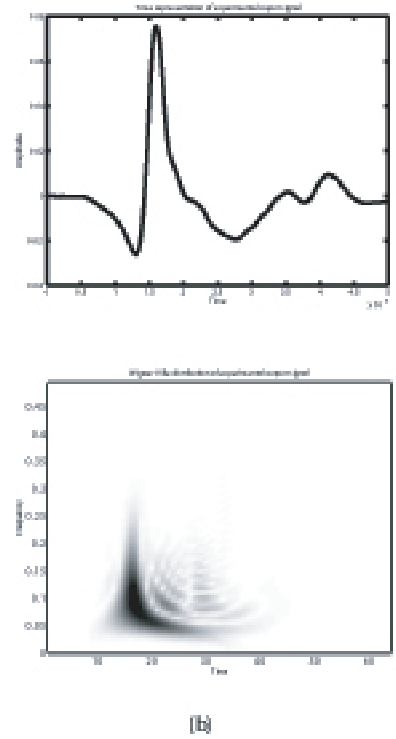

Figure 2: Experimental output signal : (a) time signal, (b) Wigner-Ville distribution

function shown on g. 5 seems more relevant than the representations on g. 7 and 8 : they are visually less interference terms.

5. CONCLUSION

The propagation of acoustic waves through porous media has properties like attenuation of high frequencies and dispersion, i. e., the energy carried by low frequencies propagates more slowly than the one carried by high frequencies. These properties can be characterized by Quadratic time-frequency representations, in particularly the -Bertrand distributions which localize the energy of the signal along a special power group delay law.

From these representations, the energetic attenuation curve was computed. The choice of an acceptable representation, in terms of localization and concentration of energy in the time-frequency plane, was done from the third order Renyi entropy function to evaluate the power dispersion law of unknown analyzed signals.

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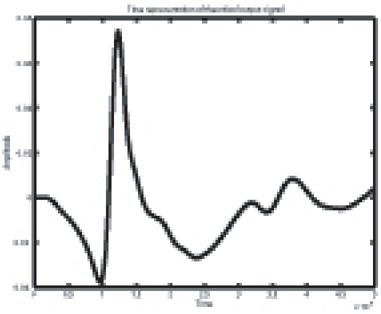
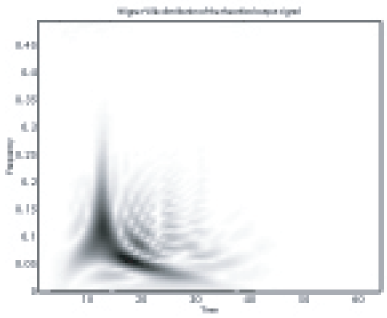


(a)



(b)

Figure 3: Theoritical output signal : (a) time signal, (b) Wigner-Ville distribution

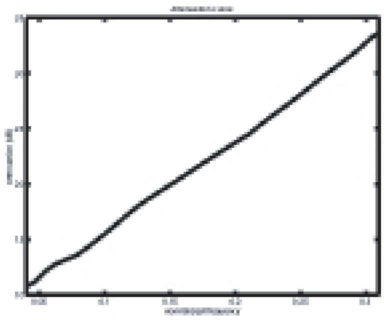


Figure 4: Energetic attenuation curve

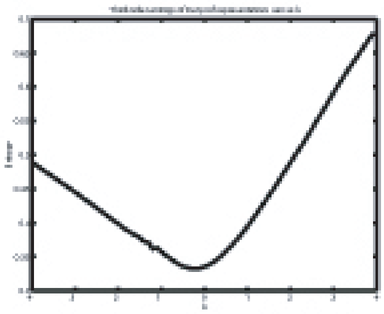


Figure 5: Third order entropy as function of

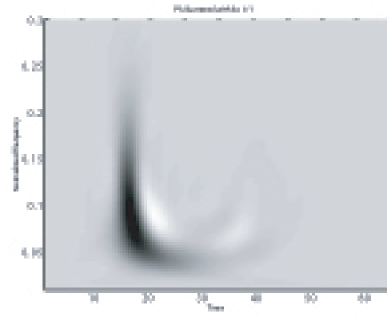


Figure 6: -Bertrand with k=-0.1

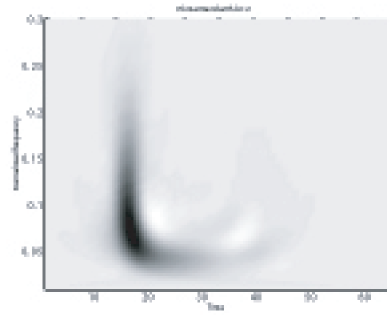


Figure 7: -Bertrand with k=-2

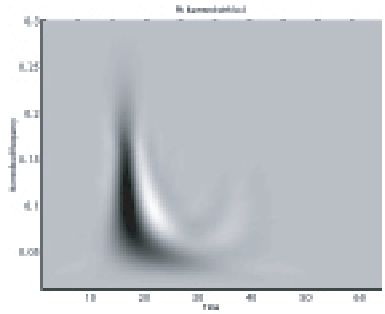


Figure 8: -Bertrand with k=2

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