

Source Separations and Identification by Structural Holography

Corentin Chesnais and Nicolas Totaro Laboratoire Vibration Acoustique

> Jean-Hugh Thomas LAUM/ ENSIM

Jean-Louis Guyader Laboratoire Vibration Acoustique

ABSTRACT

The source field reconstruction aims at identifying the excitation field measuring the response of the system. In Near-field Acoustic Holography, the response of the system (the radiated acoustic pressure) is measured on a hologram using a microphones array and the source field (the acoustic velocity field) is reconstructed with a back-propagation technique performed in the wave number domain. The objective of the present works is to use such a technique to reconstruct displacement field on the whole surface of a plate by measuring vibrations on a one-dimensional holograms. This task is much more difficult in the vibratory domain because of the complexity of the equation of motion of the structure.

The method presented here and called "Structural Holography" is particularly interesting when a direct measurement of the velocity field is not possible. Moreover, Structural Holography decreases the number of measurements required to reconstruct the displacement field of the entire plate. This method permits to separate the sources in the case of multi-sources excitations by considering them as direct or back waves. It's possible to compute the structural intensity of one particular source without the contributions of others sources.

The purpose of this paper is to introduce the Structural Holography method. The first part presents the theoretical background of the method. A numerical simulation of displacement fields generate by few sources for an infinite plate is presented in a second part. The structural intensity for each source is computed by removing the contribution of others source. Finally, some results are presented for a simply supported plate.

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INTRODUCTION

Source identification is a very important diagnostic tool in noise and vibration control. Many methods have become an increasingly powerful research tool and allow today to understand and predict the structure behavior. The method presented in this paper is called Structural Holography and allows to identify and separate sources present on a plate by remote measurements of the displacement. The goal is to reconstruct an entire two-dimensional wave field from few one dimensional sensor lines (the holograms). Therefore, this method offers a drastic reduction of measurements and allows to characterize the vibratory phenomenon when a direct measurement is impossible.

Structural Holography is based on the method applied in the acoustic domain, introduced by Williams and Maynard called Near-field Acoustical Holography (NAH). NAH is a powerful method to reconstruct the velocity distribution of a vibrating plate from the acoustic pressure hologram measured from a microphone array in the near-field [1]. Since few years, NAH, which was initially devoted to stationary sources, has evolved and hereafter can be used through RT-NAH (Real-Time Near-field Acoustic Holography) for continuously visualizing nonstationary acoustic fields [2] or through TDH (Time Domain Holography) for reconstructing sound data blocks in the time domain [3]. NAH can be also coupled with other techniques to identify vibration sources from radiated noise measurements [4].

In the vibratory domain, many methods were developed to localize and quantify the sources present on plates. Pavic [5] and Noiseux [6] have developed the structural intensimetry, defined with acoustic analogy, to analyze the flexural wave propagation in simple structures. Gavric showed the potentiality of numerical calculation of structural intensity [7, 8] and demonstrated that this approach can be used for modal models obtained from the experimental modal analysis [9]. Another method for the localization of source is the force analysis technique (FAT) also known as the RIFF technique introduced by Pézerat [10,11], which uses a finite difference scheme to discretize the equation of motion and localize the force distribution acting on a structure like beams [12], plates [11] and shells [13,14].

The present work aims at presenting another method to identify and separate sources present on plates. The separation source is a technique already applied in the wave number domain by Ming-Te Cheng [15] to separate the incident and scattered sound fields. This method is based on the principle that any waveform can be decomposed using a two-dimensional spatial Fourier transform into wave components that propagate in a known manner [15]. Although structural holography method presented in this paper is also performed in the wave-number domain, the approach is different. Indeed, structural holography provides source separation in the case of multi-source excitations by considering them as direct or back waves with only two measurement lines of the displacement.

The purpose of this paper is to introduce the structural Holography method. The first part presents the theoretical background of the method. The far-field and near-field formulations are introduced. The separation source principle is presented and applied on a noisy displacement field numerically simulated, generated by 2 sources. The study object is a simply supported plate. To increase the accuracy of the identification source, the structural intensity approach is then applied to the reconstructed displacement field.

FORMULATION OF THE PROBLEM

Structural Holography is based on the equation governing the forced flexural vibration of a thin plate:

$$\frac{Eh^{3}(1+j\eta)}{12(1-v^{2})} \quad \left(\frac{\partial^{4}w(x,y,t)}{\partial x^{4}} + \frac{\partial^{4}w(x,y,t)}{\partial y^{4}} + 2\frac{\partial^{4}w(x,y,t)}{\partial x^{2}\partial y^{2}}\right) \\ +\rho h \frac{\partial^{2}w(x,y,t)}{\partial t^{2}} = F(t)\delta(x,y),$$
(1)

where *E* is the Young modulus, v the Poisson's ratio, *h* the thickness of the plate, η the damping, *F*(*t*) the punctual force and *w*(*x*,*y*,*t*) the transverse displacement. For y > 0, where no external force is applied, the equation becomes in the harmonic regime e^{jwt} :

$$\nabla^4 \operatorname{w}(x,y) - k_f^4 \operatorname{w}(x,y) = \mathbf{0},$$

(2)

where $k_f^4 = \frac{\omega^2 \rho h}{B}$ and $B = \frac{Eh^3(1+j\eta)}{12(1-\nu^2)}$. By applying the spatial Fourier transform (SFT) to equation (2) over the x direction, the wavenumber spectrum $W(k_x, y)$ is obtained. The SFT of this displacement field is then:

$$W(k_x, y) = SFT[w(x, y)] = \int w(x, y)e^{jk_x x} dx.$$
(3)

Process to obtain the Structural Holography formulation is the same as Near-Field Acoustical Holography (NAH). However, the application of the SFT is only performed on 1 dimension in vibratory domain (2 dimensions for NAH). This operation yields a 4th order differential equation:

$$\frac{\partial^4}{\partial y^4} W(k_x, y) - 2k_x^2 \frac{\partial^2}{\partial y^2} W(k_x, y) + (k_x^4) - k_f^4) W(k_x, y) = 0,$$

which admits the general solution :

$$W(k_{x}, y) = C_{Reva}(k_{x})e^{(\sqrt{k_{x}^{2}+k_{f}^{2}})y} + C_{eva}(k_{x})e^{(-\sqrt{k_{x}^{2}+k_{f}^{2}})y} + C_{Rmix}(k_{x})e^{(\sqrt{k_{x}^{2}-k_{f}^{2}})y} + C_{mix}(k_{x})e^{(-\sqrt{k_{x}^{2}-k_{f}^{2}})y}.$$
(5)

This solution has 4 coefficients which is a major difference with NAH. Unlike in acoustic domain, structural Holography requires 4 holograms to identify the 4 unknowns of <u>equation (5)</u>. Coefficients $C_{Reva}(k_x)$ and $C_{eva}(k_x)$ characterize purely evanescent waves for all values of k_x . Coefficients $C_{Rmix}(k_x)$ and $C_{mix}(k_x)$ define propagating waves for $k_x \leq k_f$ and evanescent waves for $k_x > k_f$. The letter R characterizes the back-waves.

To reconstruct the displacement with structural holography method, one must follow few step describe below:

- Measure displacements with on the holograms
- Apply a STF on the holograms to identify the coefficient
- Compute the wave number spectrum W(k_x, y) on the entire plate with eq.(5)
- Apply a ISTF on the wave number spectrum to reconstruct the displacement *w*(*x*,*y*)

However, structural holography is the general solution of a STF application on the <u>equation (2)</u>. As has been shown above, this formula is valid only in zones where no external forces is applied. Therefore, this method cannot back-propagate the displacement beyond the force F. This limit is visible because it generates a very high increase of the amplitude of the reconstructed displacement. Therefore, all results present in this paper showing the reconstructed displacement before this phenomenon. Beyond, the fields are not illustrated.

(6)

(7)

Near-Field and Far-Field Formulation

In the near-field and with the presence of back waves, the formulation with 4 coefficients is used and the matrix form to identify the coefficients is:

$$\begin{pmatrix} e^{(k_x^+)y_{h_1}} & e^{-(k_x^+)y_{h_1}} & e^{(k_x^-)y_{h_1}} & e^{-(k_x^-)y_{h_1}} \\ e^{(k_x^+)y_{h_2}} & e^{-(k_x^+)y_{h_2}} & e^{(k_x^-)y_{h_2}} & e^{-(k_x^-)y_{h_2}} \\ e^{(k_x^+)y_{h_3}} & e^{-(k_x^+)y_{h_3}} & e^{(k_x^-)y_{h_3}} & e^{-(k_x^-)y_{h_3}} \\ e^{(k_x^+)y_{h_4}} & e^{-(k_x^+)y_{h_4}} & e^{(k_x^-)y_{h_4}} & e^{-(k_x^-)y_{h_4}} \end{pmatrix} \begin{pmatrix} C_{Reva}(k_x) \\ C_{eva}(k_x) \\ C_{Rmix}(k_x) \\ C_{mix}(k_x) \end{pmatrix}$$

$$= \begin{pmatrix} W(k_x, y_{h1}) \\ W(k_x, y_{h2}) \\ W(k_x, y_{h3}) \\ W(k_x, y_{h4}) \end{pmatrix}$$

when $k_x^+ = \sqrt{k_x^2 + k_f^2}$, $k_x^- = \sqrt{k_x^2 - k_f^2}$, $W(k_x, y_{hi})_{\text{with i} = 1,...,4}$ is respectively the wave number spectrum on the four holograms and y_{hi} the position of hologram i on y axis. As the evanescent waves exist only in the near-field, these coefficients can disturb the results if the holograms are positioned in the far field. In that case, taking into account the purely evanescent waves in the determination of the coefficients would lead to large errors. To avoid this behavior, the C_{Reva} and C_{eva} coefficients are not taking into account when the holograms are positioned in the far field. Therefore, equation (5) becomes:

$$W(k_x, y) = C_{Rmix}(k_x)e^{(k_x^{-})y} + C_{mix}(k_x)e^{-(k_x^{-})y}.$$

Thus, <u>equation (7)</u> is simplified and provides analytical formulation of the two coefficients:

$$C_{Rmix}(k_x) = \frac{-W(k_x, y_{h2}) + W(k_x, y_{h1})e^{-(k_x^-)d}}{e^{(k_x^-)y_{h1}}(e^{-(k_x^-)d} - e^{(k_x^-)d})},$$
(8)

$$C_{mix}(k_x) = \frac{W(k_x, y_{h2}) - W(k_x, y_{h1})e^{(k_x^-)d}}{e^{-(k_x^-)y_{h1}}(e^{-(k_x^-)d} - e^{(k_x^-)d})}.$$
(9)

Source Separation Principle

The structural holography can be used to separate the sources. They are considered as forward waves or back waves and the hologram must be placed between the sources to separate. The source separation principle with holograms positioned in far-field is illustrated in figure 1. In a first step, the holograms are positioned on the reference displacement $w_{sum} = w_{Fi} + w_{Fj}$ and the coefficients are identified with equations (8) and (9) (step (B)). We assume that the displacement w_{Fi} is the forward waves and w_{Fj} the back-waves. In this case, the wave number spectrum $W_{Fi}(k_{x^3}, y)$ is computed on the entire plate with the coefficient C_{mix} only (equation (10)) and by the ISFT, the singular behavior of the displacement w_{Fi} is obtained (step (C)). It is the same protocol with C_{Rmix} to compute $W_{Fj}(k_{x^3}, y)$ (equation (11)) and to reconstruct the displacement field w_{Fi} (step (D)).

$$W_{i}(k_{x}, y) = C_{mix}(k_{x})e^{-(k_{x}^{-})y},$$

$$W_{j}(k_{x}, y) = C_{Rmix}(k_{x})e^{(k_{x}^{-})y}.$$
(10)

(11)

Filtering

The presence of noise in the measured signals causes measurement uncertainties. These small data errors are essentially located in the high wavenumber domain, amplified by back-propagation. Moreover, structural holography requires to apply the spatial Fourier transform to the holograms to identify the coefficients present in the general solution (eq. 5). As the Fourier transforms are performed with discrete data and over a finite hologram, numerical difficulties are occurred in the calculation of coefficients.

For these reasons, in order to not introduce noise during the back propagation operation, it is necessary to filter part of the evanescent waves before processing [16]. Therefore, before computing the wavenumber spectra on the entire plate with structural holography, it is necessary to apply signal processing techniques such as filtering in the wave-number domain on the coefficients. This treatment overcomes numerical difficulties caused by the noise and the use of the Spatial Fourier Transform [17].

The purpose of filtering in the wave-number domain is to remove the noise in the wave number spectrum computed with structural holography. To achieve this goal, the choice of a proper filter size (k_x bandwidth) is crucial [18]. If the filter size is too large, the noise may not be removed and the accessible distance for back-propagation can decrease. If the filter size is too small, useful information may be removed along with noise, and the results may not lead to the source positions [17].

In this paper, the wave-number domain filter used in the numerical experiment is a filter based on the Tukey window illustrated in figure 2 [19]:



Figure 2. Filter in the wave number domain based on the Tukey window applied to the coefficients.



Figure 2. Separation source principle with the holograms in the far-field

NUMERICAL EXPERIMENT OF THE SOURCE SEPARATION PRINCIPLE ON A SIMPLY SUPPORTED PLATE

The aim of this part is to separate two sources using the structural holography. In order to demonstrate the applicability of the method on a complex structure, the source separation is applied on a simply supported plate. As in all inverse problems, small errors in the measurements could introduce a high noise level on the results. Thus we propose to simulate the uncertainties of the measured displacements with an adding noise Δ_w on magnitude and Δ_ϕ on phase:

$$w^{noisy}(x,y) = w^{exact}(x,y) + \Delta_w e^{j\Delta \phi}$$

where Δ_w is a Gaussian random real number with SNR = 12 dB.

The numerical experiment is realized on a 1 mm thick aluminum plate excited by two harmonic point forces F_1 and F_2 at frequency 2200 Hz. A damping of 2 % is applied. The same 2D grid is used for all calculation:

N points in the x direction (length $L_x = 1.3$ m) separated by $\Delta_x = 0.01$ m

- N points in the y direction (length $L_v = 1.1$ m) separated by $\Delta_v =$ 0.01 m
- $\Delta_{kx} = (2 \pi)/L_x = 4.83 \text{ rad/m}$ $k_{max}^x = \pi /\Delta_x = 314.16 \text{ rad/m}$

The noisy reference displacement is the sum of two displacements noted $w_{sum} = w_{F1} + w_{F2}$, illustrated and schematized in figure 3. The excitation F_1 is located at the point $x_{F1} = 0.4$ m and $y_{F1} = 0.4$ m. The excitation F_2 is located at the point $x_{F2} = 0.8$ m and $y_{F2} = 0.6$ m. The force F_2 is higher than F_1 with $F_2 = 2 \times F_1 = 2$ N/m. As the method is used in the far-field, just 2 holograms are necessary. They are located at distances of $y_{h1} = 50$ cm and $y_{h2} = 51$ cm.





By using only 2 holograms, the coefficients C_{mix} and C_{Rmix} are identified with the equations (8) and (9). With the coefficients, it is now possible to apply the separation source principle.

The displacement w_{F2}^{rec} back propagated on the distance y = 5 cm , is presented as a function of x position in figure 4.A and is compared with the reference displacement w_{F2} of the simply supported plate only excited by the force F_2 . This result illustrates that the method allows to back-propagate the displacement w_{F2} with a good accuracy on the entire axis x. The 2D reconstructed displacement (figure 4.B) provides accurate localization of the force and an effective separation of source F_2 . Hence, the reconstructed displacement corresponds with the reference displacement w_{F2} (figure 4.C).





Figure 4. Back-propagated displacement on y = 5 cm (blue) and reference displacement (red) (A), 2D Reconstructed displacement W_{F2}^{rec} with structural holography (B) and reference displacement w_{F2} (C)



Figure 5. Back-propagated displacement on y = 5 cm (blue) and reference displacement (red) (A), 2D Reconstructed displacement W_{F1}^{Iec} with structural holography (B) and reference displacement w_{F1} (C)



Figure 5 (cont). Back-propagated displacement on y = 5 cm (blue) and reference displacement (red) (A), 2D Reconstructed displacement W_{F1}^{rec} with structural holography (B) and reference displacement w_{F1} (C)

As for w_{F2}^{rec} , the displacement w_{F1}^{rec} back propagated on the distance y = 5 cm, is presented as a function of x position in figure 5.A and is compared with the reference displacement w_{F1} of the simply supported plate only excited by the force F_1 . In comparison with the reconstructed displacement w_{F2}^{rec} , a good reconstruction of w_{F1} is realized only around the position of the force. Furthermore, from the observation of the complete 2D reconstructed displacement (figure 5.B), it is difficult to localize the force and to compare with the reference displacement (figure 5.C). Therefore, a post-treatment is applied to the reconstructed displacement to verify if structural holography separates the sources F_1 and F_2 , despite the low amplitude of the force F_1 versus F_2 ($F_2 = 2 \times F_1$). It is the subject of the next section.

STRUCTURAL INTENSITY APPROACH

The reconstructed displacement presented on the above part does not allow to localize and identify precisely the behavior of the source F_1 applied to the plate. Although the separation method works correctly for F_2 , an approach with structural intensity formulation on the two reconstructed displacement fields, to increase the performances of the source identification is proposed. The interest in evaluation of structural intensity arises for practical reasons, because net energy flow distribution offers information on energy transmission paths, positions of sources and sinks of mechanical energy [8].

Structural intensity in the plate here means the energy flow per length unit in a given direction. By expressing the shear forces and bending and twisting moments, a complex intensity of flexural waves is obtained:

$$I_{x} = \frac{B}{2\omega} Im\{\frac{\partial}{\partial x} (\nabla^{2} v)v * -(\frac{\partial^{2} v}{\partial x^{2}} + \mu \frac{\partial^{2} v}{\partial y^{2}})\frac{\partial v^{*}}{\partial x} - (1 - \mu)\frac{\partial^{2} v}{\partial x \partial y}\frac{\partial v^{*}}{\partial y},$$

$$I_{y} = \frac{B}{2\omega} Im\{\frac{\partial}{\partial y} (\nabla^{2} v)v * -(\frac{\partial^{2} v}{\partial y^{2}} + \mu \frac{\partial^{2} v}{\partial x^{2}})\frac{\partial v^{*}}{\partial y} - (1 - \mu)\frac{\partial^{2} v}{\partial x \partial y}\frac{\partial v^{*}}{\partial x},$$

$$(12)$$

$$(12)$$

$$(12)$$

$$(12)$$

$$(13)$$

with μ and ν respectively the normal displacement and velocity. To evaluate the influence of each term of <u>equations (12)</u> and <u>(13)</u> of the flexural wave intensity, a model based on the plane wave summation in different positions is used [20]:

$$v(x, y) = \sum_{m} v_{m}(x, y) = \sum_{m} V_{m} exp[-jk(x\cos\alpha_{m} + y\sin\alpha_{m})]$$
(14)

Pascal shows that the expression of the flexural wave intensity using the nabla operator in 3 dimensions, with $\frac{\partial}{\partial z} \equiv 0$ can become:

$$I = \frac{B}{2\omega} Im\{\nabla(\nabla^2 v)v^* - \nabla^2 v\nabla v^* - \frac{1-v}{2}\nabla \times \nabla v^* + \frac{1-v}{2}\nabla v \nabla v^*\}$$
(15)

The far-field model described by <u>equation (15)</u> where the plane waves interfere with each other satisfy $\nabla^2 v = -k^2 v$, which leads to equality of the first two terms of the expression (<u>15</u>):

$$I \approx \frac{B}{2\omega} Im \left\{ 2k_f^2 \nu \nabla \nu^* - \frac{1-\nu}{2} \nabla \times \nabla \times (\nu \nabla \nu^*) \right\}$$
(16)

If <u>equation (14)</u> represents the superposition of independent waves, interference phenomena disappear and the terms become null. Therefore, it yields the simplified formula for the far-field and free-field:

$$I \approx \sqrt{BM_s} Im\{v\nabla v^*\}$$
(17)

where Im(...) denotes the imaginary part, M_s is the mass per unit of area and v(x, y) is the velocity field as function of x and y. This formula cannot take into account the reflections created by discontinuities or plate edges.

We choose this equation because only the irrotational part is present in equation (17) and the irrotational part of the structural intensity vectors is very useful to localize energy sources and sinks. The rotational part indicates how the energy loops and this is not present to the energy flow at large distances from the source, while the irrotational intensity indicates how the energy flows from the source towards the structural far field. Therefore, for a hologram position in the far field, just take the irrotational intensity gives a better view of structural energy because the masking effects of energy loops that are related to the rotational intensity are not considered [21]. Moreover the good performance of this approximation was proved by Pascal [16].

NUMERICAL EXPERIMENT ON SIMPLY SUPPORTED PLATE

The aim of this part is to apply structural intensity method on the reconstructed displacement by structural holography. The studied displacement are the separated field presented in the section "Numerical experiment of the source separation principle on a simply supported plate".

The structural intensity is applied on W_{F1}^{rec} and compared with only the field w_{F1} . Thus, the singular behavior of the displacement w_{F1} is identified and the structural intensity is computed with the <u>equation</u> (<u>17</u>). The results obtained with structural holography are compared with the reference structural intensity (figure 6).

Results show that Structural Holography is able to separate the forces F_1 and F_2 in presence of high noise. Use of the structural intensity on the reconstructed displacement allows a good localization and identification of the force F_1 (illustrated figure 6.B and 6.D). Perturbation are presents due to the reflections on the edges. The phase changes visible on the reference I_{xF1} is not present on the computed structural intensity I_{xF1}^{rec} . As structural holography is based on an equation without second members, the reconstructed displacement cannot be computed after this limit. Therefore, the amplitude of the reconstructed displacement increases beyond the source F_1 .

The structural Intensity as a function of $y I_{yF1}^{rec}$ allows a good localization of the force F_1 but the amplitude is lower than the reference I_{xF1} . We assume that the hologram position on the axis x does not allow the measure of all phenomena on the dimension y. That why the performance is better for I_{xF1}^{rec} .

It is the same observations for the structural intensity on the reconstructed displacement W_{F2}^{rec} . However, results are better than for W_{F1}^{rec} . The structural intensity computed for the displacement W_{F2}^{rec} are illustrated figure 7. The comparison with the reference shows differences. Reference displacement is more noised than reconstructed displacement. It is explained by the filter applied in the wave number domain on the coefficient which filter the measurement uncertainties present in the high wave number domain and decrease the noise. It is the same observation for the structural intensity on the reconstructed displacement W_{F2}^{rec} . However, results are better than for W_{F1}^{rec} . The structural intensity computed for the displacement W_{F2}^{rec} are illustrated figure 7.B-7.D.



Figure 6. Structural intensity I_{xF1} (a) and I_{yF1} (c) of the reference displacement $w_{F1}(x, y)$ and structural intensity I_{xF1}^{rec} (b) and $y_{F1}(d)$ of the reconstructed displacement field computed with the separation source method by structural holography W_{F1}^{rec}



Figure 6 (cont). Structural intensity I_{xF1} (a) and I_{yF1} (c) of the reference displacement $w_{F1}(x, y)$ and structural intensity I_{xF1}^{rec} (b) and I_{yF1}^{rec} (d) of the reconstructed displacement field computed with the separation source method by structural holography W_{F1}^{rec}





Figure 7. Structural intensity I_{xF2} (a) and I_{yF2} (c) of the reference displacement w_{F2} (x, y) and structural intensity I_{xF2}^{rec} (b) and I_{yF2}^{rec} (d) of the reconstructed displacement field computed with the separation source method by structural holography W_{F2}^{rec}

CONCLUSIONS

A method to separate the source and reconstruct the displacement field of a structure by remote measures has been developed. The method presented in this paper and called "Structural Holography" offers a drastic reduction of measurements and is particularly interesting when a direct measurement of the velocity field is not possible. The objective of the present work is to use this technique, based on the same principle than Near-field Acoustic Holography, to reconstruct the displacement field on the whole surface of a plate by measuring vibrations from one dimensional sensor lines (the holograms).

The separation technique is applied to a numerical experiment involving a simply supported plate excited by two forces with noise. The results show the feasibility of the technique but the source identification is difficult. To increase the identification performance, an approach based

on structural intensity is used on the reconstructed displacement field. This post-treatment allows a precise localization of the force position and a good estimation of the intensity. Although the force F_1 is lower than F_2 , the position of the force is very clearly identified.

Therefore, with only two holograms and in presence of noise, structural holography allows to separate the source and provides the vibration behavior near the reference force studied. Moreover, applying the equation of structural intensity on the reconstructed displacement provides more information on the source (localization and identification).

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CONTACT INFORMATION

Corentin.chesnais@insa-lyon.fr

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