Sustainable sonic crystal made of resonating bamboo rods

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The acoustic transmission coefficient of a resonant sonic crystal made of hollow bamboo rods is studied experimentally and theoretically. The plane wave expansion and multiple scattering theory (MST) are used to predict the bandgap in transmission coefficient of a non-resonant sonic crystal composed of rods without holes. The predicted results are validated against experimental data for the acoustic transmission coefficient. It is shown that a sonic crystal made from a natural material with some irregularities can exhibit a clear transmission bandgap. Then, the hollow bamboo rods are drilled between each node to create an array of Helmholtz resonators. It is shown that the presence of Helmholtz resonators leads to an additional bandgap in the low-frequency part of the transmission coefficient. The MST is modified in order to account for the resonance effect of the holes in the drilled bamboo rods. This resonant multiple scattering theory is validated experimentally and could be further used for the description and optimization of more complex resonant sonic crystals.

I. INTRODUCTION

With the growing interest for sustainable development, new acoustic materials more respectful to the environment should be designed.1 In acoustics, the usual absorbing and insulating materials, that can be found in buildings, for example, come from the petrochemical industry and can be hazardous to one’s health. However, some more ecological and natural solutions exist involving natural porous materials or sonic crystals made of recycled materials.2

Phononic crystals have received much more attention from the acoustical community in the last decade because of their particular acoustic properties, including bandgaps.3 Numerous studies have been reported in the ultrasonics frequency range for steel rods embedded in water,4 epoxy,5 or air.6 Indeed, the acoustic waves cannot propagate through this periodic arrangement over some frequency bands depending on the characteristics of a unit cell.7 This is due to multiple scattering leading to destructive interferences between the rods.

In order to improve the width and central frequencies of the bandgaps for acoustic insulation, structure characteristics should be adapted. We denote by “suitable lattice” the lattice with the best desired characteristics in the audible frequency range designed in accordance with the bamboo mean radius. By acting on the characteristics of the individual cells, transmission loss related to the periodicity can be combined with additional resonant effects at low frequencies, improving the acoustic properties of the structure. Until now, research has mostly focused on split ring like resonators.8,9 This type of resonator is easy to build, can be very well calibrated, and can resonate for frequencies below the first bandgap frequency. Such structures are useful for acoustic insulation10 because of the transmission losses increasing due to these resonances at low frequencies. The bandgap can be enhanced, by tuning the resonators, to create “metamaterials” with more efficient or different properties, like those of an acoustic lens.11

In this paper, the sonic crystal is made with resonant bamboo. By drilling a hole in each initially closed cavity, a bamboo rod becomes a stack of Helmholtz resonators (HRs). The dimensions of bamboo rods used in this paper enable resonance of the HR at very low frequencies. The use of natural material induces dispersion on the dimensions of the scatterers hence on their resonance frequencies. This dispersion can be useful to design a wide frequency range acoustic insulator. The so-designed structure possesses a low transmission coefficient around 300 Hz. These frequencies have never been reached before with natural materials, to our knowledge.

The first part of the paper is devoted to the determination of suitable parameters for a bamboo rod sonic crystal efficient in the audible frequency range by use of the plane wave expansion (PWE) method. Previous investigations on airborne sonic crystals with resonant inclusions have been achieved by using a semi analytical method.12,13 Here, the experimental response of the sample is then compared with multiple scattering theory (MST) calculations.14 Finally, a careful study of the Helmholtz resonator associated bandgap (orientation of the neck, effect of the number of the lines) allows one to account for the resonant character of the scatterers in the MST calculations and to correctly predict the acoustic behavior of the sample.

II. DESIGN OF THE SONIC CRYSTAL

The phononic crystal under study is a periodic arrangement of rods in air, i.e., a sonic crystal (SC). Typically, rods are arranged in a square or triangular lattice.15 To determine whether or not a bandgap could exist, an important parameter...
is the filling ratio \((Fr)\), between the area of the scatterer and the area of the unit cell, given by

\[
Fr = \frac{\pi d^2}{2a^2 \sqrt{3}}
\]  

(1)

for a triangular arrangement where the unit cell is a regular hexagon. The diameter of a rod is \(d = 2R\) and the distance between two adjacent rod centers, i.e., the lattice periodicity, is \(a\), see Fig. 1. In order to create sufficiently strong destructive interferences between diffracted waves by each rod and to observe a forbidden band, it is usually observed that \(Fr\) yields between 0.4 and 0.6 (Ref. 16) for a square arrangement. In this range, the destructive interferences create an absolute bandgap where no wave can propagate in any direction inside the crystal. When the \(Fr\) is out of this range only a pseudo bandgap (bandgap in only one direction) is observed. A specific feature of the triangular arrangement is the possibility of creating absolute bandgap for \(Fr\) below 0.4 and have a full bandgap in a lower frequency range.\(^1\)

No wave can propagate through a SC in the frequency range of the bandgap, and consequently, such an arrangement is a good candidate to achieve acoustic shielding. The bandgap width depends on \(Fr\) and a bandgap appears when the half of the wavelength is equal to the separation distance between the successive rows. The central frequency of the first bandgap can then be easily calculated through \(f = c/\sqrt{3a}\) (where \(c\) is the speed of sound in the medium) as a first approximation. The determination of the bandgap can be achieved by using the PWE. A large contrast between the impedance of the fluid and the impedance of the rods is required for the energy to be strongly scattered in any directions and so for the SC to be efficient.

**A. Determination of the lattice periodicity**

The goal is to design a natural sonic crystal efficient in the audible frequency range. Because of their peculiar shape, bamboo rods are used. They are made of a wood tough enough for them to be approximated as infinitely rigid scatterers. Each rods is 2.60 m high with an external diameter \(d^0\) between 3.7 and 4.3 cm. As a first approximation, the mean diameter of all the bamboo rods can be used as the diameter of each scatterers for a PWE calculation. This mean diameter is \(d = 2R = 4\) cm, wherein \(R\) is the mean radius of all the bamboo rods. The PWE is used to determine the more suitable bandgap for a low frequency application (the widest with the lowest central frequency as possible with regards to the radius constraint) and so, the most suitable lattice shape and periodicity. The acoustic wave equation is rewritten in the reciprocal space (2D spatial Fourier transform) where the problem becomes a standard eigenvalue problem. This eigenvalue problem is solved numerically over the first Brillouin zone. A triangular lattice is chosen because such lattices enable absolute bandgap with a larger periodicity than a square lattice. The acoustic field is calculated for a unit cell along the First Brillouin zone with coordinates \(\Gamma = (0,0), X = (4\pi/a,0)\) and \(M = (\pi/a, \pi/\sqrt{3}a)\).

To ensure a good convergence of the results, the PWE is computed with 169 waves. When the periodicity increases, the bandgap central frequency decreases, but its width drastically shrinks and then the absolute character of the bandgap disappears. A lattice constant of 9 cm is chosen. The corresponding filling ratio is \(Fr = 0.18\). This value seems to be the best compromise between the bandgap width and a low bandgap central frequency. With these parameters the first absolute bandgap lies between 2000 Hz and 2550 Hz and particularly, the bandgap lies between 1600 and 2550 Hz in the \(\Gamma X\) direction (see Fig. 2). This configuration is used in the following.

The PWE is useful to determine bandgaps for a sonic crystal made of perfectly rigid cylinders but is limited to the study of infinite structures. PWE cannot take into account the diffraction effects on the boundaries of a finite SC. For finite structures of sufficiently large extend, the global behavior remains almost the same as the one of the corresponding and PWE results constitute a first good approximation. As ultrasound in water can easily be highly directive, diffraction issues due to the finite size of the experimental device can be easily avoided. However, measurements are quite sensitive to these diffraction issues in the audible frequency range. To account for the finite size of the sonic crystal and for the position of the microphone, another method should be used. The finite difference time domain method\(^3\) or multiple scattering theory\(^18\) (MST) are good candidates. The MST is used here to calculate the acoustic field transmitted through the SC, which
is compared with experimental results. The MST also account for the dispersion of rod radii. However, the rod position is fixed by the lattice periodicity, because accounting for it would require a large amount of (and complicated) distance measurements. Moreover, the PWE does not allow for internal resonance. Moreover, the MST is modified (see Sec. III C) to account for possible resonant effects of the scatterers.

**B. Determination of the transmission loss with multiple scattering theory**

MST, usually known as the Korringa–Kohn–Rostoker approach, was mainly developed for the calculation of electronic band structures. This method was widely used in elasticity and electromagnetism. The main idea is to separate the fields distributed in the 2D space into non-overlapped regions, each of them containing a rod. The total acoustic field is calculated for each region as being the sum of the diffracted field by the scatterer and the incident field on this scatterer, the latter being composed of the scattered field by the other scatterers (Graf’s addition theorem) and by the direct incident field. Let us consider a structure made of the other scatterers (Graf’s addition theorem) and by the scattered field by the scatterer and the incident field on this scatterer. Their coefficients also take the form

\[ \begin{align*}
B^m_n &= K^m_n + \sum_{j \in \mathbb{N} \neq n} \sum_{q \in \mathbb{Z}} S^m_{np} B^j_q, \\
A^m_n &= \left( -i \right)^m \exp(ikr^m \cos(\theta' - \theta) - im\phi'), \quad S^m_{np} = H^{(1)}_{m-n} (kr^m) e^{i(q-m)\phi'}
\end{align*} \]

where \( H^{(1)}_{m-n} \) is the first-kind Hankel function of order \( m \), \( J_m \) is the Bessel function of order \( m \), \( A^m_n \) are the coefficients of the scattered field by the \( n \)-th cylinder, \( B^m_n \) are those of the incident field impinging upon the \( n \)-th cylinder. The local incident field on the \( n \)-th cylinder is generated by the actual incident field \( p_{inc} \) as well as by the fields that are scattered by all other cylinders \( j, j \neq n \). Their coefficients also take the form

\[ B^m_n = K^m_n + \sum_{j \in \mathbb{N} \neq n} \sum_{q \in \mathbb{Z}} S^m_{np} B^j_q, \]

where \( K^m_n \) are the coefficients of the actual incident field, i.e., a planar incident wave in the cross-sectional plane, \( K^m_n = \left( -i \right)^m \exp(ikr^m \cos(\theta' - \theta) - im\phi') \), and \( S^m_{np} = H^{(1)}_{m-n} (kr^m) e^{i(q-m)\phi'} \) are translation terms, \( (r^m, \phi') \) being the coordinates of the \( j \)-th cylinder in the polar coordinate system associated with the \( n \)-th cylinder. Coefficients \( A^m_n \) and \( B^m_n \) are related together via the boundary condition on the \( n \)-th cylinder, i.e., \( A^m_n = D^m_n B^m_n \), wherein \( D^m_n \) is the scattering coefficient. This leads to the final linear system, which may be written in the matrix form, where \( B \) denotes the infinite column matrix of components \( B^m_n \)

\[ (I - DS)B = K, \]

wherein \( I \) is the identity matrix, \( S \) is the matrix of components \( S^m_{np} \), \( D = \text{diag}(D^m_n) \), and \( K \) is the infinite column matrix of components \( K^m_n \).

Once Eq. (4) is solved for \( B^m_n \), the field in the entire space can then be calculated. The infinite sum \( \sum_{m=\infty} \) over the indices of the modal representation of the diffracted field by a cylinder is truncated as \( \sum_{m=-M}^M \) such that \( M = \text{int} \left( \frac{4.05 \times (kr^m)^{1/3}}{kr^m} \right) + 10 \). In the former equation, \( \text{int}(a) \) represents the integer part of \( a \) and 10 is a security factor.

MST accounts for the properties (radius, position, and filling property) of each scatterer, for visco-thermal dissipation, and allows one to calculate the acoustic field at any location of the system. Nevertheless, only one incidence angle of a plane wave can be considered at each calculation. Each rod radius is measured and used in the model to be the closest to the real SC described in Sec. IIIC. The acoustic field behind the SC is averaged along the lateral direction on 17 points. The air medium can be considered as a perfect fluid (without dissipation) and Neumann type boundary condition can be used at the air-bamboo interface. The scattering coefficient \( D^m_n \) of the \( n \)-th bamboo rod accounted for in the calculation through \( A^m_n = D^m_n B^m_n \) takes the form (in the \( e^{-iut} \) temporal convention)

\[ D^m_n = \frac{J'_m(kR^n)}{H^{(1)}_m(kR^n)}, \]

where \( J'_m \) and \( H^{(1)}_m \) are the derivatives of, respectively, the Bessel and Hankel functions of the first kind with respect to the radial coordinate.

To validate the hypothesis of perfect fluid and Neumann type boundary condition, results calculated under this hypothesis and those calculated with an impedance condition are compared in Fig. 3. In this latter case, the scattering coefficient \( D^m_n \) takes the form

\[ D^m_n = \frac{J'_m(kR^n) - i\beta_m J_m(kR^n)}{H^{(1)}_m(kR^n) + i\beta_m H^{(1)}_m(kR^n)}, \]

where the characteristic surface admittance \( \beta_m \) expresses the thermo-viscous effects in the boundary layers. The expression of the characteristic surface admittance in this case is

\[ \beta_m = \frac{1 - i}{2} \frac{m^2}{k^2} \frac{1}{\sqrt{\gamma - 1}} \left[ \delta_x + (\gamma - 1) \delta_h \right], \]

where \( \delta_x \) and \( \delta_h \) are the volume and thickness of the bamboo cylinder, \( k \) is the wave number, \( m \) is the mode number, and \( \gamma \) is the specific heat ratio.

**FIG. 3.** Transmission coefficient of the SC calculated with MST and a lattice constant of 9 cm in normal wave incidence (TX). Neumann type boundary condition (---) and impedance conditions are used (-----) at the air-roof interface. The grey rectangle depicts the bandgap location calculated with PWE.
where \( \delta_v = \sqrt{2\nu_v/\omega} \) is the viscous skin depth (the air kinematic viscosity is \( \nu_v = 1.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1} \)), \( \delta_h = \sqrt{2\nu_h/\omega} \) is the heat skin depth (the air thermal diffusivity is \( \nu_h = 2.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1} \)) and \( \gamma = 1.4 \) the capacity ratio. In the frequency range of interest, dimensions of the skin depths are negligible in comparison with the dimensions of the crystal and of the rods. The visco-thermal effects have a small influence on the acoustic field below 6 kHz for our SC, see Fig. 3. The fluid can be considered as a perfect fluid and Neumann type boundary conditions are used in the following.

Results exhibit a bandgap in the transmission coefficient curve between 1600 and 2550 Hz in the \( \Gamma X \) direction that confirm the PWE estimation (see Fig. 2).

C. Experimental results

The sample is a SC made of \( N = 45 \) bamboos of radius ranging between 3.7 and 4.3 cm, i.e., \( R^n \in [3.7 \text{ cm}; 4.3 \text{ cm}] \), and 2.60 m high. The crystal consists of 9 rows of 5 columns (see Fig. 4).

This SC can be considered has a 2D SC because its height is larger than the dimensions in the other directions. All experiments are performed in an anechoic room. Figure 5 depicts the experimental set-up. A loudspeaker connected to the low frequency generator output of the spectrum analyzer (Stanford Research SR785) produces a chirp signal between 100 and 4000 Hz. The loudspeaker is placed far enough (2.80 m) from the SC for the incident wave to be considered as a plane incident wave in the \( \Gamma X \) direction. A Microphone B&K 1/4-inch records the transmitted field at 9 cm behind the SC (i.e., a lattice period) to minimize diffraction due to the finite nature of our sample. The transmitted signal is recorded on 17 points along the \( x_2 \) coordinate over two lattice periods around the central rod (one on each side), and is sent to the spectrum analyzer. Two measurements are conducted, with and without the sample. The ratio between the two recorded spectra averaged over the 17 points of measurement gives the transmission coefficient. Experimental results are compared to MST calculations in Fig. 6. A bandgap is observed between 1600 and 2550 Hz, with a transmission coefficient below 0.2, which corresponds to an attenuation of about 14 dB. MST captures well the specific features of the wave interaction by the SC: diffraction due to the finite size, small dispersion of the rod radii, etc. The transmission coefficient calculated when the rod radii dispersion is accounted for and when the radii of all the rods are equal to the mean radius value, i.e., \( R^n = R = 2 \text{ cm} \), are almost the same. This means that the radii dispersion weakly affects the first bandgap for this SC. Discrepancies between experimental and MST calculated transmission coefficient appear for frequencies higher than 4000 Hz. They can be attributed to the surface irregularities and to the position dispersion of bamboos that introduce disorder not accounted for in the MST calculation.

In Fig. 6 the calculated and experimental transmission coefficients are sometimes higher than 1. Passive structures cannot create energy and the transmission coefficient cannot be higher than 1. In our case the microphone is close enough to the device to record constructive interferences created by the multiple scattering inside the SC leading to transmission coefficient higher than 1. These constructive interferences are normally trapped inside the SC but can be recorded near the crystal boundaries. This effect disappears when the microphone is placed far enough from the SC but then diffraction phenomena related to the finite size of the sample...
appear and quickly degrade the experimental and numerical results. Another possibility would be to use an intensity probe to record the energy flow.

Experimental results are validated by PWE and MST calculations and prove the efficiency of a natural sonic crystal in the audible frequency range. Moreover, natural disorder does not influence the efficiency of the scattering effect in the considered frequency range.

### III. SONIC CRYSTAL WITH LOCALLY RESONANT SCATTERERS

The idea to improve the transmission losses of the so-designed sonic crystal is to combine bandgap with resonance phenomena. Current research use resonant scatterers like split ring or Helmholtz resonators. This kind of scatterer can resonate for frequencies below the first bandgap and the corresponding wavelength is larger than the scatterer radius or the lattice periodicity. Helmholtz resonators (HRs) are already used in architectural acoustics to reduce undesirable low frequency noise through perforated plates, for example.

In bamboos, the internodal regions of the stem are hollow and closed which creates a stack of closed cavities. By drilling a hole of 9 mm diameter in each cavity, bamboo becomes a stack of HRs. The holes were drilled at approximately a third of the internodal length of each cavity. This was empirically determined in order to mostly excite the HR for frequencies below the first bandgap and the corresponding wavelength is larger than the scatterer radius or the lattice periodicity. Helmholtz resonators (HRs) are already used in architectural acoustics to reduce undesirable low frequency noise through perforated plates, for example.

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For $R_h = 0.45$ cm, $R_n$ from 1.55–1.85 cm, and $l = 0.3$ cm, the resonance frequency is estimated between 230 and 380 Hz by Eq. (8). The thickness of the wall of each cavity, which is the neck length, is considered constant, so that the variation in the internodal radius is accounted for through $R_n = R_n - l$. Several experiments are performed to determine the influence of the Helmholtz resonators on the acoustic properties of the structure. A comparison between the transmission coefficient of the sonic crystal with and without HR is shown in Fig. 7. The neck of each rod is oriented along the same axis, i.e., $\alpha' = \alpha$, $\forall \alpha \in [0, 45]$, in front of the incident wave, i.e., $\alpha = 0$.

The HR resonance influence is clearly visible in the transmission coefficient around 300 Hz, which corresponds to the fundamental frequency estimated above. Higher order resonances are also excited and enhance the transmission losses around 500 and 800 Hz. The bandgap, between 1600 and 2550 Hz, due to the periodicity of the arrangement is not modified. Replacing rigid scatterers by resonant ones does not influence the bandgap associated to periodicity when keeping identical dimensions and when $f_{HR}$ is far enough from the bandgap location. Another peak in the transmission coefficient is added. The width of the HR “bandgap” is around 150 Hz, which is wider than the one of the peak induced by the resonance of a single HR, because of radius and cavity length spread in the natural Helmholtz resonator sonic crystal (HRSC).

#### B. Influence of the neck orientation of the Helmholtz resonator

In order to analyze the influence of the neck orientation on the structure behavior, the same experiment is repeated with $\alpha = 45^\circ$ and $90^\circ$. The experimental transmission coefficients for $\alpha = 0^\circ$, $45^\circ$, and $90^\circ$ are compared in Fig. 8.

The orientation of the HR neck does not seem to influence the transmission coefficient. Another measurement, which is not shown here, was performed with randomly chosen $\alpha'$, i.e., the neck are randomly oriented. No strong influence was again observed. Small discrepancies are visible for frequencies higher than 2600 Hz. These are due to surface irregularities and to the fact that bamboos are not straight. Indeed, by turning each bamboo around its axis, the distance between some of them might be changed of a few millimeters. In the HR frequency range (between 230 and 380 Hz),

![FIG. 7. Experimental transmission coefficient of a triangular lattice sonic crystal of 45 bamboo rods (---) and without (--- ---) Helmholtz resonators.](image)

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bamboos.  

The wavelength is much larger than the external radius of a rod, i.e., $kr^n \ll 1$, $\forall n \in [1, 45]$. The low frequency approximation of the scattering by the cylinders is also valid. Consequently, the scattering by the resonant rod can be considered as originated from a line scatterer and is then independent of the scatterer orientation.

These experiments confirm that creating a sonic crystal with this kind of resonator improves the acoustic insulation by adding more transmission losses in the low frequency range. Helmholtz resonators absorb sound at their resonance frequency, independently of the position of their neck.

The expected effect of the Helmholtz resonance on the solid rod crystal band structure is the opening of an hybridization gap around the resonator natural frequency, covering the whole Brillouin zone, i.e., independent of the wave vector. A PWE simulated band structure taking account for the Helmholtz resonator effects is in principle possible, but would not account for the three-dimensional character of this problem of finite size resonator stacks, and for the distribution of resonances.

C. Implementation of the resonant behavior in the MST

The resonance of the HR can be implemented in a straightforward manner in the MST, because the scattered field is independent from the angle $\varphi$, by multiplying the scattering coefficient by a Lorentzian band-cut filter function [see Eq.(5)] in order to account for the resonance effect of the holes in the drilled bamboo rods. The central frequency of this function is equal to the resonance frequency of the resonator. The scattering coefficients $D_m^n$ become

$$D_m^n = D_m^n \text{Rb}^n,$$

with

$$\text{Rb}^n = 1 - \Pi^n \left( \frac{Q}{1 - iQ \frac{\omega}{\omega_H^n} - \frac{\omega_H^n}{\omega}} \right),$$

wherein $\Pi^n$ is a rectangular function centered at $\omega_H^n$ and of width 40 Hz, $\omega_H^n = 2\pi f_H^n$ is the resonance frequency of $n$-th bamboo rod estimated by considering a constant internodal length $L = 24$ cm, and $Q$ is the quality factor. The resonance frequency $f_H^n$ depends on the internal radius [Eq. (8)] and is also dependent on the considered bamboo rod. The quality factor $Q$ was hypothesized independent from the rod. It was determined by adjusting, in the MST (now called RMST), the frequency width of the transmission coefficient deep due to the HR and amplitude around $\omega_H^n$ with experimental results measured in the one line configuration. The so-determined value is $Q = 20$. Experimental and simulated results for the whole configuration, i.e., with five lines, are presented in Fig. 9 and exhibit good agreement. This validates the way the HR resonances are implemented in the RMST.

The RMST predicts the bandgap associated with the HR well. In this HR bandgap, a transmission coefficient of 0.4 is reached which corresponds to a 8 dB attenuation. Nevertheless, the experimental HR bandgap is wider and of lower amplitude than the one predicted. This small discrepancy is due to the dispersion of the bamboo internal radius and internodal length, i.e., dispersion of the HR dimensions. While the variation of the internal radius is accounted for, the one of the cavity length is fixed at $L = 24$ cm. In practice, $L$ varies from 20 to 28 cm. This variation could modify substantially the resonance frequencies and enlarge the HR bandgap. Because of this enlargement, the amount of energy that is trapped is also lower, explaining the differences in the HR bandgap depth.

To improve the HRSC performances, the HR bandgap effect can be increased by adding resonators, i.e., by adding bamboos. The number of bamboos is raised from $N = 45$ to 84 by designing an HRSC of 7 lines of 12 bamboos each.

D. Influence of the crystal size

Experimental measurements are done with a crystal of 12 bamboos per line with increasing number of lines from 1 to 7. The SC bandgap is practically unchanged, so that special attention is paid to the HR deep in the transmission. Figure 10 depicts the transmission coefficient for frequencies around the HR frequency resonance for these configurations. The 7 lines crystal exhibits a transmission coefficient of 0.2, which corresponds to a 14 dB attenuation. The HR deep is also deeper than in the 45 bamboos configuration. Two minima, respectively at 290 and 340 Hz, are noticed. These are due to the
smaller radius additional bamboos. Indeed, the external radius of the added bamboos are smaller than those used in the 45 bamboo configuration. Their radius lies between 3.3 and 3.8 cm. The resonance contribution of the added bamboos is slightly different than the one of the bamboos used in the initial HRSC. The difference makes these resonances distinct, therefore leading to two peaks. If they were closer, a coupled mode would have been excited, with only one thinner peak of deeper amplitude. Nevertheless, the advantage is an enlargement of the HR deep. The figure shows that the difference between the resonance frequencies of two consecutive lines must be less than 20 Hz to couple these resonances.

The RMST shows good agreement with experimental results, Fig. 10. RMST predicts that a wider HR bandgap could have been obtained with a smoother radius variation along coordinate $x_1$.

IV. CONCLUSION AND DISCUSSION

The behavior of a triangular sonic crystal predicted with the PWE method and made with natural scatterers is studied for the audible frequency range. The scatterers are bamboo rods and are arranged periodically with a lattice constant of 9 cm. The multiple scattering by this sonic crystal induces a bandgap between 1600 and 2500 Hz. Because of the considered frequency range, the inherent disorder and surface irregularities of natural materials as well as viscothermal effects, do not influence the results and a quite efficient bandgap for a small sample (45 scatterers, 9 lines, and 5 rows) is observed. All experiments are validated with a multiple scattering theory algorithm. To improve properties of the SC, bamboos are drilled to transform the cylindrical rigid scatterers into stacks of Helmholtz resonators and create a locally resonant sonic crystal. These resonances add anomalies in the transmission coefficient at very low frequency (300 Hz) and do not affect the SC bandgap. A careful study of the behavior of the HR has enabled us to modify the MST to account for the resonant features of the HR. The transmission coefficient amplitude inside the HR bandgap is 0.4, which corresponds to a loss of 8 dB, for a 45 bamboo configuration and 0.2, which corresponds to a loss of 14 dB, for an 84 bamboo configuration.

This lack of efficiency is due to the too few number of resonators excited in the recorded zone.

However, the transmission losses due to the resonance effect seems to be widened and deepened by the careful choice of the distribution of HRs. The next step could be to design a sonic crystal with a gradient of properties, in particular, a gradient of HR resonances.


