Modes coupling of shear acoustic waves polarized along a one-dimensional corrugation on the surfaces of an isotropic solid plate

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This paper aims at providing an analytical model, suitable to highlight the mode coupling due to scattering on small one-dimensional irregularities (parallel ridges) of the surfaces of isotropic solid plates, when shear horizontal waves polarized along the ridges propagate perpendicularly to them. An impedancelike boundary condition at the interface between the teeth (the ridges) and the inner plate (bounded outwardly by the ridges) accounts for the inertia of the teeth to describe the roughness. Using the integral formulation, the displacement field is expressed as a coupling between eigenmodes of the inner regular-shaped plate. © *2008 American Institute of Physics*.
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The characterization of the roughness of the solid plates before applying adhesive joints (essential for the prediction of the adhesive properties), is the subject of studies presented in the literature using both the Rayleigh and Lamb waves (see Refs. 1–10, and references contained therein). The attenuation phenomenon of the Lamb waves when propagating along the plate has been explained using the phase screen approximation, but only when assuming average parameters for characterizing the roughness (neglecting the important effects due to the spatial periods).[11,12] A more sophisticated model was also provided in anisotropic plates, taking into account both the power spectrum density and the slopes of the rough profile, and using a small perturbation method; but this model fails to take into account all the modes created by the scattering of the incident mode on the corrugation.[10] Conversely, analytic solutions for describing the acoustic coupling in fluid-filled rough waveguides were given, using Green's theorem and a perturbation method in the frame of a modal analysis[13,14] (but extension to solid plate has not been provided yet).

The aim of this paper is to provide an analytical model, more tractable than the previous ones, suitable to describe the mode coupling due to scattering phenomena on small one-dimensional (1D) irregularities (parallel ridges) of the surfaces of solid isotropic plates, when a shear horizontal (SH) wave propagates along the plate.

The setup considered is a homogeneous solid plate in vacuum or surrounded by an inviscid fluid, bounded by two parallel surfaces having 1D shape perturbations, is characterized by its density ρ and its shear second Lamé coefficient (μ). An inner plate with regularly shaped surfaces $x_3=d$ and $x_3=0$ is defined as being surrounded by the 1D corrugations (the regularly shaped surfaces bound inwardly the perturbed surfaces, see Fig. 1). These corrugations are assumed to be small deviations (from the regularly shaped surfaces) which are parallel to the x_2-axis (they do not depend on x_2-coordinate). They are characterized by a weak variation of the (equivalent) height $h(x_1,x_3=0,d)$, denoted below $h_1(x_1)$ and $h_2(x_1)$ at $x_3=0$ and $x_3=d$, respectively. They are subsequently called "small teeth" when considering the (x_1,x_3) plane. In the impedancelike model presented here, the effects of the roughness are described by the inertia of each teeth, which is accounted for modeling the boundaries $x_3=0$ and $x_3=d$ of the inner waveguide as non homogeneous reactive surfaces represented by a local operator (impedancelike operator in the frequency domain). Then, the solutions are given in the frame of a modal theory (using a unique set of the Neumann eigenmodes of the regularly shaped surface that bounds inwardly the perturbed surface of the waveguide), using a method relying on integral formulation and modal analysis. It is worth noting that the height and the length of the corrugation are here assumed such that the perturbation induced by this corrugation on the behavior of the wave propagating along the x_1-axis is very small.

It is assumed that an incident propagating wave coming from $x_1 \to -\infty$ or a source which is set at the input of the considered domain (at the entrance $x_1=0$ of the corrugated waveguide) is such as it creates a shear horizontal displacement field $\hat{u}_2(x_1,x_3;t)$ with a polarization parallel to the x_2-axis (the ridges) with a given profile in the x_3-direction, the SH wave propagating along the plate (perpendicularly to the ridges).

As mentioned above, the irregularities distributed along the walls of the waveguide considered are here represented by the behavior of small "teeth" (small volumes of material, equivalent heights h_q, $q=1,2$) placed side by side, on the

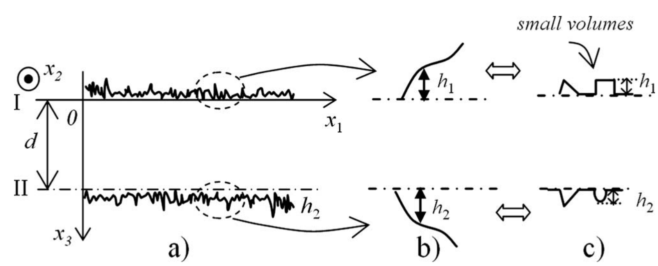

FIG. 1. Sketch of a waveguide with surfaces (set at $x_3=0$ and $x_3=d$) having small deviations from the regular shape. (a) General view, (b) zoom on a small part of the corrugation, and (c) modeling of the corrugation by a repartition of small volumes.

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outer side of the smooth boundaries [respectively, at $x_3=0$ (interface I, $q=1$) and $x_3=d$ (interface II, $q=2$)] of a regular-shaped guide (Fig. 1). These small elements of material are here considered as localized masses m_{v_q}, $q=1,2$, per unit of area (local reaction), which are put in motion following the x_2-axis by the shear strength applied on them at the interfaces $x_3=0$ and $x_3=d$. According to the direction of the x_3-axis, the shear stress acting on the mass m_{v_q} is equal to $T_{23}=(-1)^q\mu\partial_{x_3}\hat{u}_2$, where ∂_{x_3} stands for $\partial/\partial x_3$.

Applying Newton's law to each localized mass per unit of length following x_2 and per unit of length following x_1 ($m_{v_q}=\rho h_q$, $q=1,2$) thus leads to

$$\rho h_q \partial_{tt}^2 \hat{u}_2 = (-1)^q \mu \partial_{x_3}\hat{u}_2, \quad q=1,2, \quad \forall x_1>0, \quad \forall t, \tag{1}$$

where ∂_{tt}^2 stands for $\partial^2/\partial t^2$. Note that when the wall is regularly shaped ($h_q=0$) Eq. (1) represent a Neumann boundary condition for the displacement $\hat{u}_2$.

The harmonic (angular frequency ω) shear displacement field in the plate, polarized along the x_2-axis, denoted $\hat{u}_2(x_1,x_3;t)=\hat{U}_2(x_1,x_3)\exp(i\omega t)$, is governed by the set of equations including the propagation equation and the boundary conditions, which takes the following form:

$$\begin{cases} \bullet\ (\partial_{x_1x_1}^2+\partial_{x_3x_3}^2+k_T^2)\hat{U}_2(x_1,x_3)=0, & 0\leqslant x_3\leqslant d,\quad \forall x_1>0, \quad (2a)\\ \bullet\ \partial_{x_3}\hat{U}_2(x_1,x_3)|_{x_1,x_3=0}=-h_1k_T^2\hat{U}_2(x_1,x_3=0), & \forall x_1>0,\quad x_3=0, \quad (2b)\\ \bullet\ \partial_{x_3}\hat{U}_2(x_1,x_3)|_{x_1,x_3=d}=+h_2k_T^2\hat{U}_2(x_1,x_3=d), & \forall x_1>0,\quad x_3=d, \quad (2c)\\ \bullet\ \text{a given source strength or incoming wave at} & x_1=0,\quad 0\leqslant x_3\leqslant d, \quad (2d)\\ \bullet\ \text{Sommerfeld radiation condition when} & x_1\to+\infty, \quad (2e)\end{cases}$$

where $k_T=\omega/V_T$, $V_T=\sqrt{\mu/\rho}$ being the speed of the shear waves in the homogeneous solid constituting the guide.

The solution is expressed approximately as an expansion on the eigenfunctions $\psi_m(x_3)$, namely,

$$\hat{U}_2(x_1,x_3)\cong\sum_r \hat{A}_r(x_1)\psi_r(x_3), \tag{3}$$

where the eigenfunctions $\psi_m(x_3)$ are solution of the homogeneous Helmholtz equation subject to Neumann boundary conditions in the two dimensional waveguide bounded by the regularly shaped, parallel, and plane surfaces set at $x_3=0$ and $x_3=d$ on the inner side of the perturbed surfaces (Fig. 1):

$$\psi_m(x_3)=\sqrt{(2-\delta_{m0})/d}\cos(k_m x_3), \tag{4a}$$

the eigenvalues k_m being given by

$$k_m=m\pi/d \quad m\in N. \tag{4b}$$

Multiplying Eq. (2a) by the eigenfunctions $\psi_m(x_3)$, and integrating over the range $0\leqslant x_3\leqslant d$, and then integrating the second term (which includes the operator $\partial_{x_3x_3}^2$) by parts, yields, reporting, respectively, expressions (2b), (2c), (3), (4a), and (4b) for $\partial_{x_3}\hat{U}_2$ on the boundaries, $\hat{U}_2$, and ψ_m,

$$(\partial_{x_1x_1}^2+k_{x_1m}^2)\hat{A}_m(x_1)=-\sum_r \hat{\Gamma}_{rm}(x_1)\hat{A}_r(x_1), \tag{5}$$

where

$$k_{x_1m}^2=k_T^2-k_m^2 \tag{6a}$$

$$\hat{\Gamma}_{rm}(x_1)=k_T^2[|\beta_{rm}|\zeta_1(x_1)+\beta_{rm}\zeta_2(x_1)], \tag{6b}$$

$$\text{with}\quad \beta_{rm}=(-1)^{m+r}\sqrt{(2-\delta_{m0})(2-\delta_{r0})} \tag{6c}$$

$$\text{and}\quad \zeta_q(x_1)=h_q(x_1)/d, \quad q=1,2. \tag{6d}$$

The right hand side of Eq. (5) highlights the modal couplings due to the nonhomogeneities of the boundaries of the plate.

Note that if the term $r=m$ is extracted from the sum over the quantum number r, Eq. (5) has the same form as that found in [Eq. (25) of Ref. 13], the solution of which highlighting the fact that the scattering of the primary wave $\exp(-ik_{x_1m}x_1)$, created by the acoustic source, from the perturbed surface distributes the acoustic energy in the same mode (monomode approach) through secondary waves propagating, respectively, in the same direction as the primary wave and in the opposite direction.

The approximate integral solution of the problem stated above can be obtained by successive approximations of Eq. (5), using at each stage the integral formulation with an appropriate Green's function denoted $G_m(x_1;x_1')$, namely, here[15]

$$G_m(x_1,x_1')=\exp(-ik_{x_1m}|x_1-x_1'|)/(2ik_{x_1m}). \tag{7}$$

Using an iterative method to express the amplitude of each mode $\hat{A}_m(x_1)$, which assumes that the coupling function $\hat{\Gamma}_{rm}(x_1)$ is a small quantity of the dimensionless parameter ζ_q [see Eqs. (6b) and (6d)], thus the nth-order solution of Eq. (5) for $\hat{A}_m(x_1)$ is written as follows:

$$\hat{A}_m^{[n]}=\hat{A}_m^{(0)}+\hat{A}_m^{(1)}+\cdots+\hat{A}_m^{(n-1)}+\hat{A}_m^{(n)}, \tag{8}$$

where $\hat{A}_m^{[n]}$ denotes the nth-order perturbation expansion for $\hat{A}_m$, $\hat{A}_m^{(0)}$ the zero order approximation, $\hat{A}_m^{(1)}$ the first order correction term, and so on.

The amplitude of each mode $\hat{A}_m$ being governed by Eq. (5), the nth perturbation expansion $\hat{A}_m^{[n]}$ satisfies the following approximate equation:

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$$(\partial^2_{x_1x_1} + k^2_{x_1m})\hat{A}^{[n]}_m(x_1) = -\sum_r \hat{\Gamma}_{rm}(x_1)\hat{A}^{[n-1]}_r(x_1), \quad (9)$$

the zero order approximation leading to the following expression of Eq. (2d):

$$\hat{A}^{(0)}_m(x_1) = \hat{Q}_m G_m(x_1;0), \quad (10)$$

where the factor $\hat{Q}_m$ gives the strength of the mth modal contribution to the incoming acoustic pressure wave. The nth order term $\hat{A}^{[n]}_m$ of the perturbation expansion is therefore governed by Eq. (9), namely,

$$\hat{A}^{[n]}_m(x_1) = \sum_r \int_{-\infty}^{+\infty} G_m(x_1;x'_1)\hat{\Gamma}_{rm}(x'_1)\hat{A}^{[n-1]}_r(x'_1)dx'_1, \quad n \geqslant 1. \quad (11)$$

As a result, the example treated here is for a plate of thickness $d=0.005$ m and for a periodically corrugated surface only on the boundary $x_3=0$ (the other surface being smooth at $x_3=d$), with a spatial period $\Lambda=0.002$ m and with $N=100$ periods along the x_1 axis. Each tooth is $\Lambda/2$ long following x_1, and has the same mass $(\rho h_1 \Lambda/2)$ per unit length in the x_2-direction, i.e., each tooth has the same equivalent height $h_1=25\times10^{-6}$ m $(\zeta_1=h_1/d=0.005)$. The solid constituting the waveguide is glass ($V_T=3485$ m s^{-1}, $\mu=60.7$ GPa). The acoustic source is assumed to create only the mode $m=1$ (incoming wave). Four modes are here taken into account to illustrate the modal couplings: the first three modes $(r=0,1,2)$ are propagative for the chosen driven frequency $(fd/V_T=1.28, \lambda/\Lambda=1.95)$ and the last one $(r=3)$ is evanescent. The convergence of the amplitudes of the modes labeled r created by the corrugation depends on this frequency: here $n=5$ [see Eqs. (8) and (9)] is sufficient to ensure truncation errors that can be ignored.

Using the dispersion curves of the waveguide, together with the curves associated with the phonon relation[10,13,14,16]

$$k_{x_1m} + k_{x_1r} \pm 2\pi/\Lambda = 0, \quad (12)$$

to emphasize the interference processes between modes r and m, it is possible to predict that both a strong coupling and an strong self-coupling appear, respectively, with the coupled mode $r=0$ and the mode $m=1$ created by the source.

This is the situation presented on Fig. 2 showing the modulus of the amplitudes calculated at the fifth order perturbation expansion (normalized by the modulus of the incident field A^0_m). The different periodic oscillations which appear are directly linked to the phonon relation (12), as indicated on Fig. 2 (periods $L^{(\varepsilon)(\sigma)}_{mr\ell}=2\pi/K^{(\varepsilon)(\sigma)}_{mr\ell}$), such that

$$K^{(\varepsilon)(\sigma)}_{mr\ell} = (\ell 2\pi/\Lambda + \varepsilon k_{x_1m} + \sigma k_{x_1r})/2, \quad \varepsilon,\sigma = \pm 1, \quad \ell = 0,1. \quad (13)$$

In conclusion, the first motivation for this study was to characterize the effect of a 1D corrugation (parallel ridges) of the surfaces of a plate on SH waves polarized along the ridges and propagating along the plate (perpendicularly to the ridges), having in mind to provide a more tractable model than those available in the literature for the characterization of bounded roughened surfaces.

The approximate model used permits to highlight the mode coupling due to scattering phenomena on the irregularities of the surfaces of solid homogeneous plates. Results on periodic corrugation emphasize phonon relationships which govern the behavior of the oscillations of the amplitude of coupled modes.

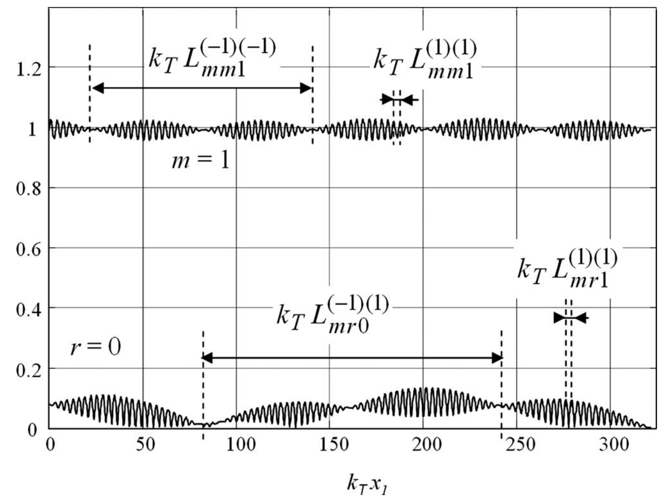

FIG. 2. Modulus of the normalized amplitude of the pressure variation (fifth order). Lower curve: $\hat{A}^{[5]}_{r=0}(x_1)/A^0_m$ for the mode $r=0$; higher curve: $\hat{A}^{[5]}_{m=1}(x_1)/A^0_m$ for the mode $m=1$ (the only mode generated by the source), when $fd/V_T=1.28$, $d/\Lambda=2.5$, the length ℓ of the corrugation is such as $k_T\ell=321.7$ ($\ell\approx51.3\lambda$, $N=100$ teeth), and the heights of the crenels are such as $\zeta=h/d=0.005$; the interface $x_3=d$ is smooth.

Despite the model does not include the shape profile of the boundary (but only the height of the profile), it remains suitable to express the intermodal coupling which occurs in such situations. The advantage of the procedure provided here, whereby these phenomena can be expressed, is to provide a very simple analytical solution which would be useful in real situations.

It is worth noting that the problem considered here is analytically exactly the same as the one which governs the acoustic pressure field in a fluid-filled waveguide with rigid corrugated walls, when the corrugation is modeled by an impedancelike operator which accounts for the compressibility of the fluid entrapped inside the teeth (considered as small cavities).

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