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Modeling of a one-dimensional acoustic device composed of a planar beam and fluid gap with discontinuity in thickness

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Precise modeling of 1D acoustic devices (passive or active, miniaturized or not) containing a planar beam loaded by a fluid gap and cavities is of interest in a variety of applications (transducers, acoustic filters, metamaterials, etc.). An analytical approach presented herein enables the description of the vibration of the planar elastically supported rigid beam of rectangular crossection surrounded by very thin slits and loaded by the fluid gap, which is divided in three parts of different thicknesses (the central part being the thinner one), the thermoviscous losses originating in the fluid being taken into account. Comparing to the case of the fluid gap of uniform thickness, such a geometry provides more parameters which can be adjusted in order to achieve the required behavior (resonant or damped, etc.). The analytically calculated beam displacement is presented and compared to the numerical solution provided by finite element method (a reference against which the analytical results are tested).

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1. INTRODUCTION

Acoustic and electroacoustic devices containing vibrating planar beams are getting attention recently[1,2] due to the straightforward fabrication process (comparing to thin membranes where controlling of the mechanical tension can be nontrivial task). The discontinuity in thickness of the fluid gap loading the moving part of the device has been found to be advantageous[3] in terms of optimization of the device behavior (gain, bandwidth) providing more geometrical parameters to handle. Another advantage of this geometry is an improved compactness of the device comparing to the case where the fluid gap of the uniform thickness has the same length as the beam and the lateral cavities are set outside the periphery of the beam.[2]

Different models of similar devices composed of vibrating parts and acoustic elements filled with thermoviscous fluid has been published in the past few years, the coupling of modes of the displacement of the moving parts and the acoustic pressure field being taken into account[2–5] or not.[2,6] Since the moving part of the device, the elastically supported 1D planar beam, is supposed herein to be sufficiently rigid to move in piston mode, the classical solution of the wave equation for the acoustic pressure in the fluid gap[7] is used in order to express the acoustic load of the beam. The thermoviscous losses originating in the narrow parts of the acoustic system behind the beam (fluid gap and thin lateral slits surrounding the beam) and the effect of the acoustic short circuit between the front and back side of the beam via the lateral slits are taken into account through the complex wavenumber and the source part of the wave equation governing the acoustic pressure in the fluid gap and through the formulation for the volume velocities entering to the cavities.[2]

2. THEORETICAL MODEL

The geometry of the system described herein is shown in figure 1. The rigid elastically supported beam of length L, width b and thickness h_b is loaded by the acoustic system containing in its central part the fluid gap of length l_3 and thickness h_g opened at both ends ($x = 0$ and $x = l_3$) to the cavities described by their lengths l_1 and l_2 and volumes V_1 and V_2. The beam is surrounded by thin lateral slits of thickness h_s which enable the piston-like movement of the beam.

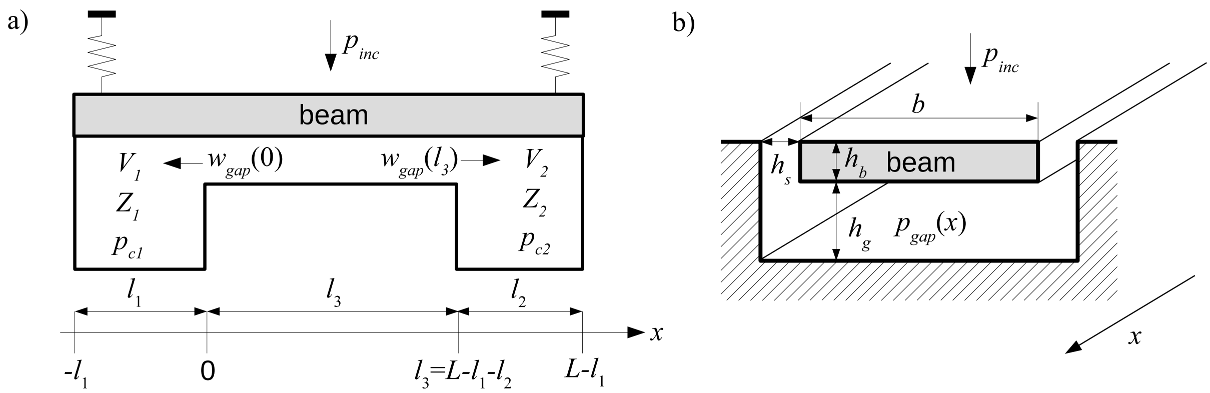

Figure 1: Geometry of the system: a) side view, b) cross-sectional view of the device cut at $x = l_3/2$

A. ANALYTICAL SOLUTION

i. Equations governing the acoustic pressure inside the device

The acoustic pressure in the fluid gap (between $x = 0$ and $x = l_3$) is given by the following equation[2]

$$\left(\frac{\partial^2}{\partial^2 x} + \chi^2\right) p_{gap}(x) = -(U_1 p_{inc} + U_2 \eta), \tag{1}$$

where p_{inc} is the incident pressure, η is the displacement of the beam, the complex wavenumber χ and the coefficients on the right-hand side of the equation (1) U_1 and U_2 being given by[2]

$$\begin{aligned}
\chi^2 =& -2\frac{F_{v,s}}{F_{v,g}}\frac{h_s}{h_g h_b b} + \frac{\omega^2}{c_0^2}\frac{\gamma-(\gamma-1)F_{h,g}}{F_{v,g}},\\
U_1 =& 2\frac{F_{v,s}}{F_{v,g}}\frac{h_s}{h_g\, h_b\, b},\\
U_2 =& \frac{\rho_0\omega^2}{F_{v,g}h_g}\left[1+\frac{h_s}{b}K_{v,s}\right],
\end{aligned} \tag{2}$$

where ω represents the angular frequency, c_0 is the adiabatic speed of sound and γ is the ratio of the specific heats (see table 1), the mean value of the particle velocity profile across the slit $F_{v,s} = 1 - K_{v,s}$ and across the air gap $F_{v,g}$ and the mean value of the temperature variation profile across the air gap $F_{h,g}$ are given in.[2] The classical solution of the equation (1) takes the following form

$$p_{gap}(x) = A\cos(\chi x) + B\sin(\chi x) - \frac{1}{\chi^2}(U_1 p_{inc} + U_2\eta). \tag{3}$$

The mean value of the velocity field across the lateral slits is given by[2]

$$\overline{v}_{s,y}(x) = -\frac{1}{i\omega\rho_0}\frac{p_{inc}-p(x)}{h_b}F_{v,s} + \frac{1}{2}i\omega\eta(x)K_{v,s}, \tag{4}$$

where i is the imaginary unit, ρ_0 is the fluid density and $p(x)$ is the acoustic pressure inside the device (in the gap and in the cavities)

$$p(x) = \begin{cases} p_{c1} & x \in (-l_1, 0) \\ p_{gap}(x) & x \in (0, l_3) \\ p_{c2} & x \in (l_3, l_3 + l_2). \end{cases}$$

The acoustic pressure in the cavities can be expressed using the impedances of the cavities $Z_{1,2} = \rho_0 c_0^2/(i\omega V_{1,2})$ as

$$p_{c1} = w_{tot1}Z_1,\ p_{c2} = w_{tot2}Z_2, \tag{5}$$

where the total volume velocities entering to the cavities take the following form

$$\begin{aligned}
w_{tot1} =& -i\omega\eta l_1 b - w_{gap}(0) - 2\overline{v}_{s,y}(0)h_s l_1\\
w_{tot2} =& -i\omega\eta l_2 b + w_{gap}(l_3) - 2\overline{v}_{s,y}(l_3)h_s l_2,
\end{aligned} \tag{6}$$

with $w_{gap}(x) = -\partial_x p_{gap}(x) F_{v,g} h_g b/(i\omega\rho_0)$ and $p(0) = p_{c1}$, $p(l_3) = p_{c2}$ in the last terms. Reporting the solution (3) into the expression for the volume velocity at the outputs of the air gap $w_{gap}(0)$ and $w_{gap}(l_3)$, the acoustic pressures in the cavities after some rearrangement become

$$\begin{aligned}
p_{c1} =& \frac{Z_1}{1+C_1 l_1 Z_1}\left[C_1 l_1 p_{inc} + C_2 l_1\eta + C_3 B\chi\right]\\
p_{c2} =& \frac{Z_2}{1+C_1 l_2 Z_2}\left[C_1 l_2 p_{inc} + C_2 l_2\eta - C_3(B\chi\cos(\chi l_3) - A\chi\sin(\chi l_3))\right],
\end{aligned} \tag{7}$$

where

$$
\begin{aligned}
C_1 =& \frac{2h_s F_{v,s}}{i\omega\rho_0 h_b},\\
C_2 =& -i\omega(b + K_{v,s}h_s),\\
C_3 =& \frac{F_{v,g}h_g b}{i\omega\rho_0}.
\end{aligned}
\tag{8}
$$

The following system of equations expressed from the continuity of the acoustic pressure at the boundaries of the fluid gap

$$
\begin{aligned}
p_{gap}(0) =& p_{c1},\\
p_{gap}(l_3) =& p_{c2},
\end{aligned}
\tag{9}
$$

gives the integration constants A and B of the solution (3)

$$
\begin{aligned}
A =& \eta H + p_{inc} I,\\
B =& \eta E/D + p_{inc} F/D,
\end{aligned}
\tag{10}
$$

with

$$
\begin{aligned}
H =& C_4 + EC_6/D,\\
I =& C_5 + FC_6/D,\\
C_4 =& U_2/\chi^2 + Z_1 C_2 l_1/(1 + Z_1 C_1 l_1),\\
C_5 =& U_1/\chi^2 + Z_1 C_1 l_1/(1 + Z_1 C_1 l_1),\\
C_6 =& Z_1 C_3 \chi/(1 + Z_1 C_1 l_1),\\
D =& C_6 \cos(\chi l_3) + \sin(\chi l_3) + \frac{Z_2 \chi C_3}{1 + Z_2 C_1 l_2}\left[\cos(\chi l_3) + C_6 \sin(\chi l_3)\right],\\
E =& \frac{Z_2 C_2 l_2}{1 + C_1 l_2 Z_2} + \frac{Z_2 \chi C_3 C_4}{1 + C_1 l_2 Z_2}\sin(\chi l_3) - C_4 \cos(\chi l_3) + \frac{U_2}{\chi^2},\\
F =& \frac{Z_2 C_1 l_1}{1 + C_1 l_2 Z_2} + \frac{Z_2 \chi C_3 C_4}{1 + C_1 l_2 Z_2}\sin(\chi l_3) - C_5 \cos(\chi l_3) + \frac{U_1}{\chi^2}.
\end{aligned}
\tag{11}
$$

The acoustic pressures in the cavities can then be expressed as

$$
\begin{aligned}
p_{c1} =& \eta O + p_{inc} P\\
p_{c2} =& \eta Q + p_{inc} S,
\end{aligned}
\tag{12}
$$

with

$$
\begin{aligned}
O =& \frac{Z_1 C_2 l_1 + C_3 \chi Z_1 E/D}{1 + C_1 l_1 Z_1},\\
P =& \frac{Z_1 C_1 l_1 + C_3 \chi Z_1 F/D}{1 + C_1 l_1 Z_1},\\
Q =& \frac{Z_2 C_2 l_2 - C_3 \chi Z_2 \left[E \cos(\chi l_3)/D - H \sin(\chi l_3)\right]}{1 + C_1 l_2 Z_2},\\
S =& \frac{Z_2 C_1 l_2 - C_3 \chi Z_2 \left[F \cos(\chi l_3)/D - I \sin(\chi l_3)\right]}{1 + C_1 l_2 Z_2}.
\end{aligned}
\tag{13}
$$

ii. The displacement of the beam

The equation governing the motion of the beam takes the form[2]

$$
\left[-\omega^2 M + i\omega R_c + K_c\right]\eta = b\left[l_1 p_{c1} + \int\limits_0^{l_3} p_{gap}(x)dx + l_2 p_{c2} - L p_{inc}\right] - 2\int_{-l_1}^{L}\left[\frac{dF_{s,y}(x)}{dx}\right]dx, \tag{14}
$$

where M is the mass of the beam, K_c is the stiffness of the elastic support of the beam and R_c is the damping coefficient. The second term in the right-hand side of the equation describing the effect of the lateral slits, is given by[2]

$$\int_{-l_1}^{L}\left[\frac{dF_{s,y}(x)}{dx}\right]dx = -\frac{1}{2}K_{v,s}h_s\left[L\,p_{inc} - l_1p_{c1} - l_2p_{c2} - \int_0^{l_3} p_{gap}(x)dx\right] + i\omega\eta\Pi_sL, \qquad (15)$$

where $\Pi_s = -k_v\mu h_b\cot(k_vh_s)$. The integral of the acoustic pressure in the fluid gap over the gap length can be expressed as follows

$$\int_0^{l_3} p_{gap}(x)dx = \eta J + p_{inc}N, \qquad (16)$$

where

$$\begin{aligned} J =& \frac{H}{\chi}\sin(\chi l_3) - \frac{E}{D\chi}\cos(\chi l_3) - \frac{U_2l_3}{\chi^2},\\ N =& \frac{I}{\chi}\sin(\chi l_3) - \frac{F}{D\chi}\cos(\chi l_3) - \frac{U_1l_3}{\chi^2}. \end{aligned} \qquad (17)$$

Reporting equations (12) and (16) to equation (14) and (15) gives the final expression for the beam displacement

$$\eta = p_{inc}\frac{(K_{v,s}h_s - b)(L - l_1P - l_2S - N)}{-\omega^2M + i\omega R_c + K_c + (K_{v,s}h_s - b)(l_1O + l_2Q + J + i\omega 2L\Pi_s)}. \qquad (18)$$

B. NUMERICAL SOLUTION

The numerical implementation (FEM) taking into account the effects of viscous and thermal boundary layers in thermoviscous fluids is usually based on a linear formulation which includes either a system of three coupled equations involving three quantities, the particle velocity $\vec{v}$, temperature variations τ and acoustic pressure p,[8] or a system of two coupled equations for the particle velocity and temperature variations, the acoustic pressure being calculated subsequently from these two quantities.[9] The 3D simulation, where the acoustic field has been calculated using the first approach and coupled with the classical formulation for the displacement of the beam in rigid body mode,[10] has been performed using the software Comsol Multiphysics[11] resulting in the numerical solution for the displacement of the beam against which the analytical results have been tested.

3. RESULTS AND DISCUSSION

The displacement of the beam of the dimensions given in table 1 (left column) and for $M = \rho_m h_b L b$ with $\rho_m = 2329\ \mathrm{kg/m^3}$ being the density of the material of the beam, $K_c = 1363.25\ \mathrm{N/m}$ and $R_c = 0$ calculated using the properties of the fluid (air in this case) given also in table 1 (right column) is presented in figure 2 (magnitude and phase). The comparison shows very good agreement between the analytical result obtained from the equation (18) (full blue line) and the numerical result obtained using the finite element method (black points) up to very high frequencies (1 MHz).

The acoustic pressure sensitivity $\sigma = U_0\eta/(p_{inc}h_g)$ of the device used as an electrostatic acoustic receiver with polarization voltage $U_0 = 30$ V and incident acoustic pressure $p_{inc} = 1$ Pa is shown in figure 3 for different values of a) the thickness of the fluid gap h_g (left figure) and b) the thickness of the lateral slits h_s (right figure). Unlike the usual behavior of the electrostatic transducers, only the level of the sensitivity changes here with the varying air gap thickness while the shape of the frequency dependent sensitivity curve remains almost unchanged. Decreasing the thickness of the lateral slits, by contrast, makes the sensitivity curve more flat and for h_s smaller than 2 μm the shape of the curve becomes similar to the one of conventional MEMS microphones.

Parameter	Value	Unit
Beam length L	3×10^{-3}	m
Beam width b	0.4×10^{-3}	m
Beam thickness h_b	50×10^{-6}	m
Left cavity length l_1	$L/4$	m
Right cavity length l_2	$L/4$	m
Left cavity thickness h_{c_1}	150×10^{-6}	m
Right cavity thickness h_{c_2}	150×10^{-6}	m
Fluid gap length l_3	$L - l_1 - l_2$	m
Fluid gap thickness h_g	50×10^{-6}	m
Thickness of the slits h_s	10×10^{-6}	m

Parameter	Value	Unit
Static pressure P_0	101330	Pa
Static temperature T_0	293.15	K
Density ρ_0	1.204	kg/m^3
Adiabatic speed of sound c_0	343.2	m/s
Shear dynamic viscosity μ	1.814×10^{-5}	Pa s
Bulk dynamic viscosity μ_B	1.088×10^{-5}	Pa s
Thermal conductivity λ_h	25.77×10^{-3}	W/(mK)
Ratio of specific heats γ	1.400	-
Specific heat coefficient at constant pressure per unit of mass C_P	1005	J/(kgK)

Table 1: Dimensions of the system and parameters of the thermoviscous fluid (air).

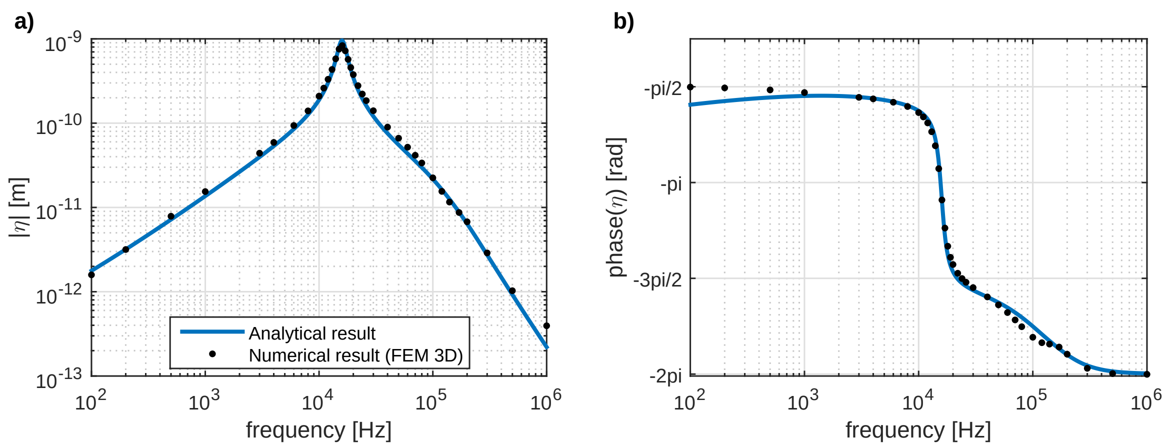

Figure 2: Analytically (full blue line) and numerically (black points) calculated displacement of the beam of dimensions given in the table 1: a) magnitude and b) phase.

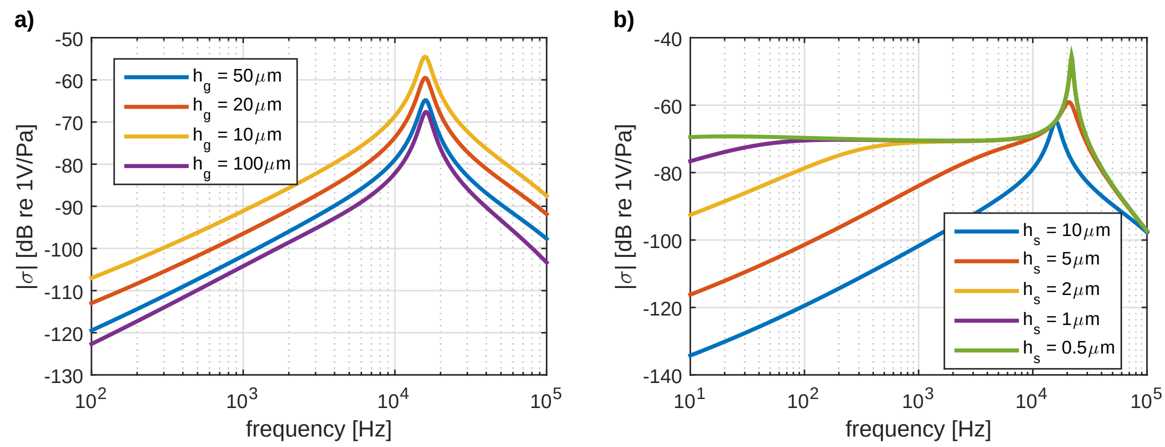

Figure 3: Magnitude of the sensitivity of the transducer for different values of a) the fluid gap thickness h_g and b) the thickness of lateral slits h_s.

4. CONCLUSION

An analytical model of 1D acoustic device composed of a rigid elastically supported planar beam and a fluid gap with discontinuity in thickness, taking into account the thermoviscous losses in the fluid gap and the effect of the lateral slits surrounding the beam, has been developed. A good agreement between the present analytical results and the reference numerical 3D simulation for the displacement of the beam has been observed. Additionally, the acoustic pressure sensitivity of the device used as an acoustic receiver has been calculated for different values of the fluid gap thickness and the thickness of the lateral slits, showing the strong influence of the latter on the shape of the frequency response of the receiver.

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