Acta Acustica united with Acustica
Vol. 103 (2017) 158 – 163

DOI 10.3813/AAA.919041

PVDF Cylindrical Electroacoustic Emitting Transducer with Perforated Back-plate: Simplified Analytical Approach

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Summary

The objective of the paper is to provide a simplified analytical approach to describe the behaviour of a cylindrical emitting transducer with a piezoelectric polymer (PVDF) membrane which is loaded by a thin air-gap connected to a small central toroidal cavity through small holes in a back-plate. This paper departs from a previous one in that the presence of a perforated back-plate separates the volume behind the membrane in two parts (the air-gap and the central cavity); it departs also in that the construction of the analytical solutions is not made in terms of eigenmodes of both the membrane and the air-gap (as usual) in order to avoid difficulties due to the coupling of Dirichlet and Neumann eigenfunctions (exact solutions are used, involving integral formulation for one of them).

PACS no. 43.38.Fx

A previous work [1] focused on an analytical modelling of a cylindrical ultrasonic emitting transducer with a piezoelectric polymer (PVDF) membrane loaded by a closed backing cavity. The coupling between the displacement field of the membrane and the pressure field in the cavity was described in using the technique which relies on the sets of the appropriate eigenmodes of both the displacement field of the membrane and the pressure field in the cavity behind it, along with the general solution of the associated homogeneous propagation equations. This approach highlights the parameters governing the behaviour of the transducer and shows the effects of resonances along both the axial and the radial directions in the cavity. Theoretical results were experimentally validated by results of measurements of the radiated field in free space, in a large frequency range (typically up to 100 kHz).

Although this transducer could be well adapted for given applications (involving a narrow-bandwidth and cylindrical shape such as echolocation systems [2]), it has some shortcomings in terms of potentiality. More specifically, apart from the damping of the membrane, the damping of the transducer is fixed by the properties of the viscous and thermal boundary layers inside the cavity, which is not adjustable, except by using absorbing materials [3] which are not really reliable. It is then difficult to attenuate the effects of unsuitable resonances and to settle an expected bandwidth. Moreover, for given physical parameters, the only parameters which set the behaviour of the transducer are the length and the radius, which are otherwise imposed in practice. Consequently, the best way to get such adjustable characteristics should be the use of small holes or slits.

Therefore, the present paper is concerned with a transducer which departs from the previous one [1] in that a perforated back-plate separates the cavity into two parts: the central one (a small toroidal cavity) and the peripheral one (a thin air-gap behind the membrane), connected together through small holes as shown in Figure 1. A simplified analytical approach of the behaviour of this transducer is presented here, which should be appropriate to derive the potentiality of the device in terms of acoustic field emitted in free space, with a large frequency range (typically up to 100 kHz). Yet, this simplified approach provides exact analytical solutions for both the membrane oscillations and the pressure variations in the air-gap (using here the integral formulation) hitherto expressed only by multi-modal analysis, in order to avoid difficulties due to the coupling of Dirichlet and Neumann eigenfunctions. Actually, in the lower frequency range (up to 10 kHz) and for small dimensions, lumped circuit elements (that could be derived assuming lower order approximations) would provide accurate modelling in specific situations (such modelling is beyond the scope of this paper). The transducer (Figure 1) consists of a polyvinylidene fluoride (PVDF) membrane of height ℓ fixed at its upper and lower sides to a cylindrical structure composed of two discs of radius R fixed together with a cylindrical rod of radius R_{C_0}. A perforated

Received 9 December 2016,
accepted 9 December 2016.

† Died at age 80 in March 2015 in Praha, Czech Republic.

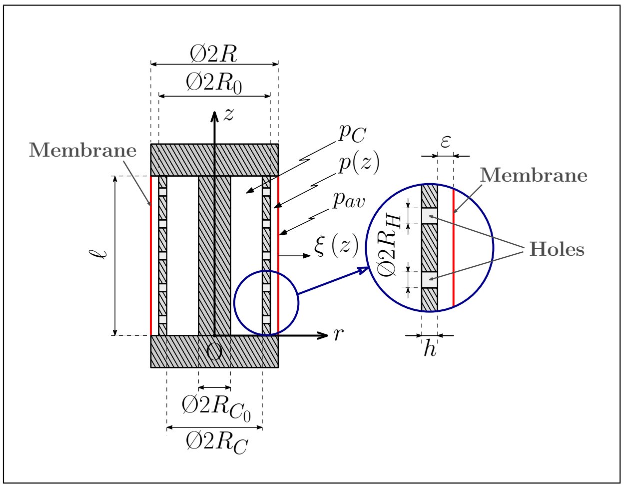

Figure 1. Cross sectional view of the transducer.

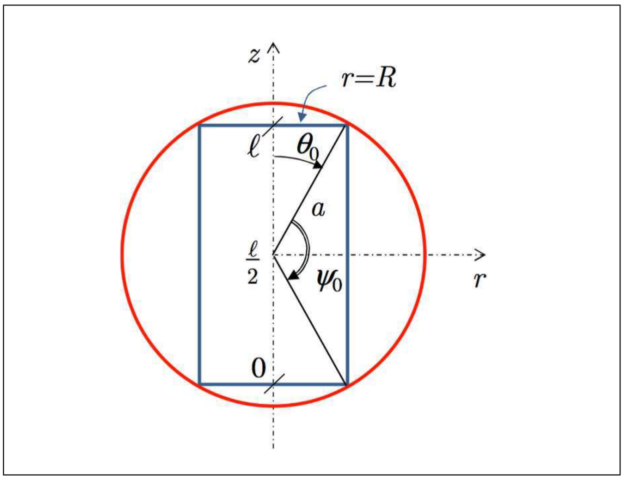

Figure 2. Approximate equivalent spherical geometry for the radiation impedance.

backing cylinder of internal and external radii $(R_0 - h)$ and R_0 respectively is placed close to the membrane, delimiting an air-gap of thickness ε which is acoustically connected to a back cylindrical cavity (of internal and external radii R_{C_0} and $R_0 - h$ respectively) through holes of radius R_H and length h. The cylindrical coordinate system (r, φ, z) chosen has its origin O at the bottom centre of the rod, and the $\vec{O}z$-axis is the axis of the whole cylindrical structure.

In what follows, pressure and displacement fields are assumed to be axisymmetric, so the problem considered does not depend on the azimuthal coordinate φ. The motion considered is harmonic, the time dependence being $e^{i\omega t}$. The properties of the external medium are given by the density ρ_0 and the adiabatic speed of sound c_0.

The analytic procedure whereby one expressed the acoustic field emitted by the cylindrical transducer of finite length ℓ and radius R can be simplified in a large extent if this cylindrical source is replaced by a spherical source of radius $a \cong \sqrt{\ell^2 + R^2}$ (Figure 2) whose both spherical caps (opposite each other and delimited respectively by the angles θ_0 and $\theta_0 + \psi_0$) are motionless and whose the central part behaves approximately as the cylindrical source. It is sufficient here just to meet this approximation because what is important in this work is the behaviour of the pressure field, as a function of the frequency, in the neighbouring of the median plane $z = \ell/2$. Therefore, assuming periodic motion, the sought-after expression for the complex amplitude of the radiated acoustic pressure is related approximately to the complex amplitude $\xi(z)$ of the displacement of the membrane by an expansion, involving Legendre polynomials $P_n(\cos\theta)$ with $\cos\theta = 2\frac{z}{\ell} - 1$ and spherical outgoing Bessel functions $h_n^-(k_0 r)$ with $k_0 = \omega/c_0$, which takes the following form, from using velocity continuity on the membrane at $r = a$ [4],

$$p_{rad}(r, z) \cong \sum_{n=0}^{\infty} A_n P_n(\cos\theta) h_n^-(k_0 r),$$

where $A_n = -\mathrm{i}\rho_0 c_0 V_n / h_n^{'-}(k_0 a)$, prime meaning the derivative with respect to the product $(k_0 r)$, and where V_n, defined as the coefficient of expansion of $v(\theta) \cong \mathrm{i}\omega\xi(z)$ on the Legendre polynomials

$$v(\theta) \cong \mathrm{i}\omega\xi(z) = \mathrm{i}\omega\xi\left[\frac{\ell}{2}(1+\cos\theta)\right] = \sum_{n=0}^{\infty} V_n P_n(\cos\theta),$$

is given by (thanks to the orthogonality property of the Legendre polynomials),

$$V_n = \mathrm{i}\omega(2n+1)\frac{1}{\ell}\int_0^{\ell} \xi(z) P_n\left(2\frac{z}{\ell} - 1\right) \mathrm{d}z$$

$$\text{with} \quad z = \frac{\ell}{2}(1+\cos\theta).$$

In considering only the first two terms in the series and noting that the second term tends to zero when the field is expressed at $z \cong \ell/2$, i.e. $\theta \cong \pi/2$, the acoustic pressure radiated reduces to

$$p_{rad} \cong \mathrm{i}\omega Z_{rad} \frac{\mathrm{e}^{-\mathrm{i}k_0 r}/(k_0 r)}{\mathrm{e}^{-\mathrm{i}k_0 a}/(k_0 a)} \bar{\xi}, \tag{1}$$

where

$$Z_{rad} = \frac{(p_{rad})_{r=a}}{\mathrm{i}\omega\bar{\xi}} = \mathrm{i}\rho_0 c_0 \frac{k_0 a}{1 + ik_0 a} \tag{2}$$

is the radiation impedance and $\bar{\xi} = (1/\ell)\int_0^{\ell} \xi(z)\,\mathrm{d}z$ is the mean value of $\xi(z)$ along the z-axis.

The PVDF membrane having tension T_z, thickness t_m, and mass per unit area M_S, supported on two rigid circular frames of radius R at its ends $z = 0$ and $z = \ell$ (Dirichlet boundary conditions), is driven by an harmonic piezoelectric force per unit area p_π, created by the electrical voltage V_r,

$$p_\pi = \left[\left(s_{11}^{E^*} d_{31} - s_{12}^{E^*} d_{32}\right) / (s_{E^*} R)\right] V_r, \tag{3}$$

where d_{31} and d_{32} are piezoelectric coefficients, and where $s_{11}^{E^*}$ and $s_{12}^{E^*}$ are complex compliance coefficients of the membrane, and where $s_{E^*} = \left(s_{11}^{E^*}\right)^2 - \left(s_{12}^{E^*}\right)^2$ (see [1] for details). It is loaded by both the external acoustic pressure $(p_{rad})_{r=a} = \mathrm{i}\omega Z_{rad}\bar{\xi}$ and the pressure field in the air gap $p(z)$ (assumed to be independent of the azimuthal coordinate). Thus, the set of equations for the complex amplitude $\xi(z)$ of the displacement field (positive when directed along the r-axis) can be written as follows, after substituting $\xi(z)$ for its mean value $\bar{\xi}$ along the z-axis in the piezoelectric reactive term,

$$T_z\left(\partial_{zz}^2 + K^2\right)\xi(z) = \tag{4a}$$
$$-\left\{p(z) + p_\pi - \left[\mathrm{i}\omega Z_{rad} + s_{11}^{E^*} t_m / \left(s_{E^*} R^2\right)\right]\bar{\xi}\right\},$$
$$\xi(z=0) = \xi(z=\ell) = 0, \tag{4b}$$

where $K^2 = (M_S/T_z)\,\omega^2$.

Substituting the pressure variation $p(z)$ in the air-gap for its mean value $\bar{p}$ along the z-axis leads readily to the solution

$$\xi(z) = \frac{\bar{p} + p_\pi - \left[\mathrm{i}\omega Z_{rad} + s_{11}^{E^*} t_m / \left(s_{E^*} R^2\right)\right]\bar{\xi}}{M_S\omega^2} \cdot \left[\cos(Kz) + \frac{1-\cos(K\ell)}{\sin(K\ell)}\sin(Kz) - 1\right], \tag{5}$$

where the first two terms between brackets (which depend on z) give the general solution of the homogeneous equation of the membrane and the last one is the solution when the movement is forced by the pressure fields p_π and $\bar{p}$. This result leads readily to the mean value $\bar{\xi}$ of $\xi(z)$ along the z-axis.

To determine the displacement field $\xi(z)$, it is necessary to find an expression of the pressure field in the air-gap $p(z)$ which actually involves the function $\xi(z)$, noting that, ultimately, it is the mean value $\bar{p}$ which is needed. To this purpose, the following assumptions are used: -i/ the pressure variation $p(z)$ is assumed to be independent of the radial coordinate r because the thickness $\varepsilon = R - R_0$ of the fluid-gap is much lower than the wavelength, -ii/ the global behaviour of the n_φ holes set at a given coordinate $z = z_{\nu_z}$ is described by the volume velocity $U_{\nu_z}(z)$ for the row (one of the n_z rings labelled ν_z) of the localised holes considered. Therefore, the set of equations which govern the pressure field $p(z)$ inside the air-gap (propagation equation and Neumann boundary conditions at $z = 0$ and $z = \ell$) takes the following form [5]:

$$\left(\partial_{zz}^2 + \chi^2\right)p(z) = \tag{6a}$$
$$\frac{\mathrm{i}\omega\rho_0}{\varepsilon F_v}\left[\mathrm{i}\omega\xi(z) - \frac{1}{2\pi R_0}\sum_{\nu_z=1}^{n_z} U_{\nu_z}(z)\delta(z - z_{\nu_z})\right],$$
$$\partial_z p(z=0) = \partial_z p(z=\ell) = 0, \tag{6b}$$

where the complex wavenumber χ accounts for the angular frequency ω of the field and the properties of the fluid, namely the density ρ_0 and the compressibility through the adiabatic speed of sound c_0, the heat capacity at constant pressure per unit mass C_p, the specific heat ratio γ, the shear viscosity coefficient μ, and the thermal conduction coefficient λ_h,

$$\chi^2 = \frac{\omega^2}{c_0^2}\frac{1 + (\gamma - 1)(1 - F_h)}{F_v}, \tag{7a}$$

with

$$F_{v,h} = 1 - \frac{2-\phi}{2}\frac{\tan(k_{v,h}\varepsilon/2)}{k_{v,h}\varepsilon/2}, \tag{7b}$$

where the porosity $\phi = n_\varphi n_z \pi R_H^2/(2\pi R_0 \ell)$ is the ratio of the total area of the holes to the back-plate area [6], $k_v = (1-i)\sqrt{\rho_0\omega/(2\mu)}$ and $k_h = (1-i)\sqrt{\rho_0 C_p\omega/(2\lambda_h)}$ being respectively the wavenumbers associated with the shear and entropic diffusion movements in the boundary layers on the back-plate and on the membrane, and where $\delta\left(z - z_{\nu_z}\right)$ is the Dirac delta function. As expected, accounting for that $\int_{R_0}^{R} r dr = \left(R^2 - R_0^2\right)/2 \cong R_0\epsilon$, the total volume velocity U_{Tot} of the $n_\varphi n_z$ holes in the fluid gap can be expressed in the following manner:

$$U_{Tot} = \int_{R_0}^{R} r\,\mathrm{d}r \int_0^{2\pi}\mathrm{d}\varphi\int_0^{\ell}\mathrm{d}z \cdot \left[\frac{1}{2\pi R_0\varepsilon}\sum_{\nu_z=1}^{n_z} U_{\nu_z}(z)\delta\left(z - z_{\nu_z}\right)\right]. \tag{8}$$

The solution $p(z)$ is written as the sum

$$p(z) = p_\xi(z) + p_u(z), \tag{9}$$

with

$$p_\xi(z) = \frac{\omega^2\rho_0}{\varepsilon F_v}\int_0^{\ell} G(z, z_0)\,\xi(z_0)\,\mathrm{d}z_0, \tag{10}$$

and

$$p_u(z) = \frac{\mathrm{i}\omega\rho_0}{\varepsilon F_v}\frac{1}{2\pi R_0}\sum_{\nu_z=1}^{n_z} G\left(z, z_{\nu_z}\right)U_{\nu_z}\left(z_{\nu_z}\right), \tag{11}$$

where the Green's function G, which satisfies the Neumann boundary conditions at $z = 0$ and $z = \ell$, takes the form [7]

$$G = \frac{-1}{\chi\sin(\chi\ell)}\cos(\chi z_<)\cos\left[\chi(z_> - \ell)\right], \tag{12}$$

$(z_<, z_>)$ designating the smaller/larger of z/z_0 respectively.

The mean value $\bar{p}_\xi$ of $p_\xi(z)$ along the z-axis is obtained first in replacing the volume velocity of the membrane by its mean value, i.e. in replacing $\xi(z)$ by its mean value $\bar{\xi}$, and then in taking the mean value of the Green's function which is equal to $\left[-1/(l\chi^2)\right]$

$$\bar{p}_\xi = -\frac{\omega^2\rho_0}{\varepsilon F_v\chi^2}\bar{\xi}. \tag{13}$$

It remains to express the volume velocity $U_{\nu_z}(z)$ for any row (ring labelled ν_z) of the holes, and then to express the mean value $\bar{p}_u$ of $p_u(z)$ along the z-axis.

Assume that -i/ the length $h = R_0 - R_C$ and the radius R_H of the cylindrical holes are much lower than the wavelength, -ii/ the thickness of the viscous boundary layer is much lower than the radius R_H of the holes (the effect of the thermal boundary layer are neglected), -iii/ the acoustic pressure $p_C(R_C)$ in the back-cavity at the entrance of the holes is expressed as the product of a uniform input impedance of the cavity Z_C (see details below) by the total volume velocity of the holes $p_C(R_C) = Z_C U_{tot}$ with $U_{tot} = \sum_{\nu_z=1}^{n_z} U_{\nu_z}(z)$. The volume velocity of the n_φ holes azimuthally distributed at a coordinate $z = z_{\nu_z}$ can then be written [4]

$$\begin{aligned} U_{\nu_z}(z) &= Y_h \left[p_C - p(z)\right] \\ &= Y_h \left[Z_C U_{tot} - p(z)\right], \end{aligned} \tag{14}$$

where the admittance Y_h is given by

$$Y_h = n_\varphi \frac{\pi R_H^2}{\mathrm{i}\omega\rho_0 h}\left[1 - \frac{2}{k_\nu R_H}\frac{J_1(k_\nu R_H)}{J_0(k_\nu R_H)}\right],$$

J_0 and J_1 being the Bessel functions of the first kind of order zero and one respectively. What is also needed below is

$$\begin{aligned} U_{Tot} &= \sum_{\nu_z=1}^{n_z} U_{\nu_z}(z) \\ &= \frac{Y_h}{n_z Y_h Z_C - 1}\sum_{\nu_z=1}^{n_z} p\left(z_{\nu_z}\right). \end{aligned} \tag{15}$$

The input impedance of the cavity Z_C is obtained readily from the expression of $p_C(r,z)$, solution of the following propagation equation

$$\left(\partial_{rr}^2 + \frac{1}{r}\partial_r + \partial_{zz}^2 + k_0^2\right) p_C(r,z) = 0, \tag{16}$$

and subjected to Neumann boundary conditions at the coordinates $z = 0$, $z = \ell$, and $r = R_{C_0}$,

$$\begin{aligned} p_C(r,z) &= \sum_{q=0,2,4,\ldots}^{\infty} A_q Q_q \cos\left(\frac{q\pi}{\ell}z\right) \\ &\quad \cdot \left[J_0\left(k_q r\right) + B_q N_0\left(k_q r\right)\right], \end{aligned} \tag{17}$$

with $B_q = -J_1\left(k_q R_{C_0}\right)/N_1\left(k_q R_{C_0}\right)$, $Q_q = \sqrt{1/\ell}$ if $q = 0$ and $\sqrt{2/\ell}$ otherwise, $k_q = \sqrt{k_0^2 - (q\pi/\ell)^2}$, N_0 and N_1 being the Bessel functions of the second kind of order zero and one respectively. When considering only one axial mode in the cavity (i.e. $q = 0$) the pressure field is uniform along the z-axis and depends only on the radial coordinate r. Then the input impedance takes the form

$$Z_C = \frac{\mathrm{i}\rho_0 c_0}{2\pi R_C \ell}\frac{J_0(k_0 R_C) + B_0 N_0(k_0 R_C)}{J_1(k_0 R_C) + B_0 N_1(k_0 R_C)}. \tag{18}$$

Note that this approximation prevents us from interpreting the behaviour of the transducer when resonances corresponding to the eigenvalues $q \neq 0$ occur, but in practice these resonances are removed out of the frequency domain of interest. Yet the first radial resonance of the backing cavity is accounted for in expression (18) (the other ones also but they are out of the frequency domain).

Finally, invoking Equations (14) and (15), expression (11) of $p_u(z)$ becomes successively

$$\begin{aligned} p_u(z) &= \frac{\mathrm{i}\omega\rho_0}{\varepsilon F_\nu}\frac{1}{2\pi R_0}\sum_{\nu_z=1}^{n_z} G\left(z, z_{\nu_z}\right) Y_h\left[Z_C U_{Tot} - p\left(z_{\nu_z}\right)\right] \\ &= \frac{\mathrm{i}\omega\rho_0}{\varepsilon F_\nu}\frac{Y_h}{2\pi R_0}\left[\frac{Z_C Y_h}{n_z Y_h Z_C - 1}\left(\sum_{\nu_z=1}^{n_z} p\left(z_{\nu_z}\right)\right)\right. \\ &\quad \left.\left(\sum_{\nu_z=1}^{n_z} G\left(z, z_{\nu_z}\right)\right) - \sum_{\nu_z=1}^{n_z} G(z, z_{\nu_z}) p(z_{\nu_z})\right]. \end{aligned} \tag{19}$$

Substituting $p(z)$ for its average value $\bar{p}$ and taking the mean value of $\bar{p}_u = (1/\ell)\int_0^\ell p_u(z)\,\mathrm{d}z$ lead to

$$\begin{aligned} \bar{p}_u &\cong \frac{\mathrm{i}\omega\rho_0}{\varepsilon F_\nu}\frac{Y_h}{2\pi R_0}\left[\frac{n_z Y_h Z_C}{n_z Y_h Z_C - 1} - 1\right] \\ &\quad \cdot \left[\frac{1}{\ell}\sum_{\nu_z=1}^{n_z}\int_0^\ell G\left(z, z_{\nu_z}\right)\mathrm{d}z\right]\bar{p} \end{aligned} \tag{20}$$

and then, noting that the mean value of the Green's function is equal to $\left[-1/\left(\ell\chi^2\right)\right]$,

$$\bar{p}_u \cong \frac{\mathrm{i}\omega\rho_0}{2\pi R_0 \varepsilon \ell F_\nu \chi^2}\frac{\bar{p}}{1/Y_h - n_z Z_C}. \tag{21}$$

Equations (21) and (13) show that $\bar{p}_u$ and $\bar{p}_\xi$ are proportional to $\bar{p}$ and $\bar{\xi}$ respectively, implying that $\bar{p} = \bar{p}_u + \bar{p}_\xi$ is a linear combination of $\bar{p}$ and $\bar{\xi}$, the mean displacement of the membrane $\bar{\xi}$ being itself a linear combination of $\bar{p}$ and p_π. This allows expressing readily the sought-after mean displacement field $\bar{\xi}$ of $\xi(z)$ (Equation 5) along the z-axis as a function of the equivalent piezoelectric pressure source p_π,

$$\begin{aligned} \bar{\xi} &\cong p_\pi\left[\frac{M_S\omega^2}{\frac{\sin(K\ell)}{K\ell} + \frac{[1-\cos(K\ell)]^2}{K\ell\sin(K\ell)} - 1} + \mathrm{i}\omega Z_{rad}\right. \\ &\quad \left. + \frac{s_{11}^{E^*} t_m}{s_{E^*} R^2} + \frac{\omega^2\rho_0\ell}{\varepsilon\ell\chi^2 F_\nu - \frac{\mathrm{i}\omega\rho_0}{2\pi R_0(1/Y_h - n_z Z_C)}}\right]^{-1}. \end{aligned} \tag{22}$$

Finally, accounting for the expression of this function p_π as a function of the input voltage V_r (Equation 3), the radiated acoustic pressure p_{rad} (Equation 1) is readily expressed as a function of the variable r, the input voltage V_r, and known parameters.

Figure 3 shows the theoretical magnitude and the phase of the emitted pressure at $r = 45\ mm$ in the frequency range ($3\ kHz$, $70\ kHz$) for the physical (available) and geometrical (of interest) parameters given in Table I and

Table I. Values of the physical and geometrical parameters of the transducer.

Parameter	Symbol	Value [units]
Inner radius of the back cavity	R_{C_0}	15 $[mm]$
Outer radius of the back cavity	R_C	19 $[mm]$
Inner radius of the air-gap	R_0	19.9 $[mm]$
Outer radius of the air-gap	R	20 $[mm]$
Height of the membrane	ℓ	8 $[mm]$
Radius of the holes	R_H	1 $[mm]$
Membrane thickness	t_m	25 $[\mu m]$
Membrane density	ρ_m	1800 $[kg/m^3]$
Piezoelectric coefficient	d_{31}	-15 $\times 10^{-12}$ $[C/N]$
Piezoelectric coefficient	d_{32}	-3 $\times 10^{-12}$ $[C/N]$
Compliance coefficient	s_{11}^E	2.25 $\times 10^{-10}$ $[m^2/N]$
Compliance coefficient	s_{12}^E	-1.25 $\times 10^{-10}$ $[m^2/N]$
Mechanical damping	$\tan(\delta_m)$	0.10 $[-]$
Permittivity	ε_{33}^T	1.06 $\times 10^{-10}$ $[F/m]$
Tension	T_z	8 $[N/m]$

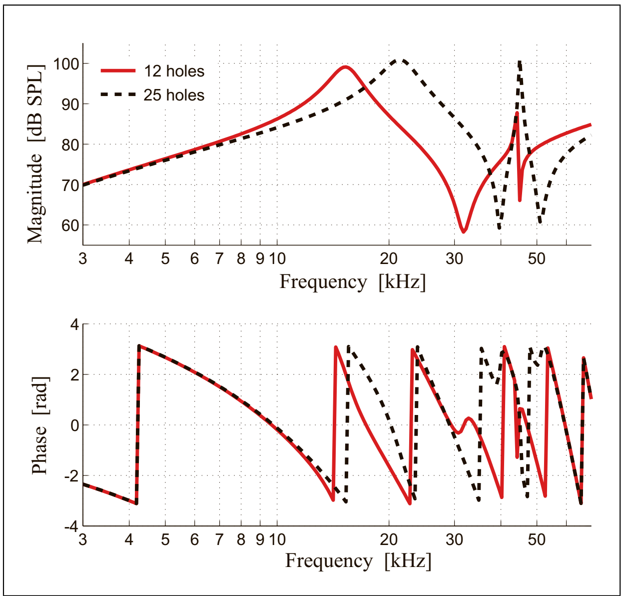

Figure 3. Magnitude (upper graph) $[dB(SPL)]$ and phase (bottom graph) $[rad]$ of the acoustic pressure radiated as a function of frequency $[kHz]$ (Equations 1 and 22). Solid line: 12 holes. Dashed-line: 25 holes.

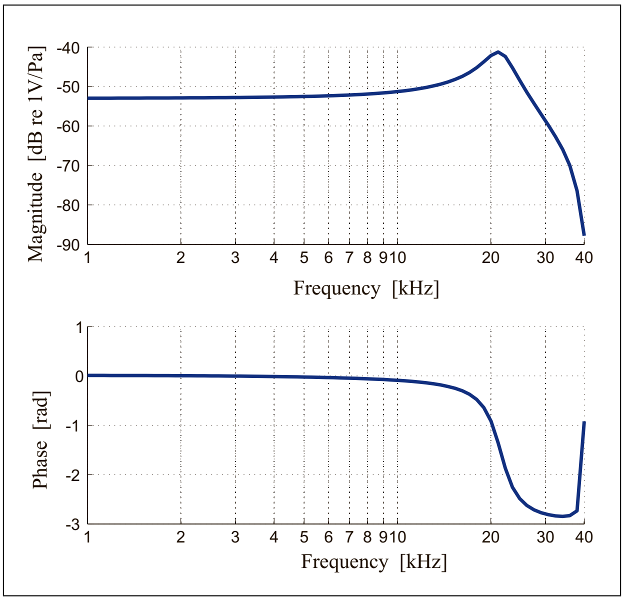

Figure 4. Magnitude (upper graph) $[dB\ re.\ 1\ V/Pa]$ and phase (bottom graph) $[rad]$ of the sensitivity of the receiver (with 25 holes) as a function of frequency $[kHz]$ (Equation 23).

for air parameters under standard conditions, the value of the supplied peak voltage V_r being real and equal to 3 volts. The solid line corresponds to the back-plate with 12 holes and the dashed-line to the back-plate with 25 holes, both sets of the holes being regularly distributed at the abscissa $z = \ell/2$. The first maximum of pressure amplitude corresponds to the first resonance of the loaded membrane and the following anti-resonance corresponds approximately to the first radial resonance of the backing cavity. The general shapes of these curves is coherent with those given for a transducer without back-plate in Reference [1] (see Figure 7), and the frequency bandwidth $(14\,kHz, 16\,kHz)$ for 12 holes and $(20\,kHz, 22\,kHz)$ for 25 holes are suitable for typical applications (e.g. [2]). The presence of a perforated back-plate adds significant flexibility when designing such kind of transducer, because it allows to adjust both the thermo-viscous damping and the frequency range by fitting the geometry and/or the dimensions of the holes and the air-gap, the external dimensions (length and radius) being otherwise imposed in practice. For example, in Figure 3, the abscissa of the maxima of the curves depends significantly on the number of holes.

Finally, it is worth noting that the transducer could be used alternately as an emitter and alternately as a receiver. When considering the transducer as a receiver, the quantity of interest is the sensitivity η, ratio of the output voltage and the incident pressure p_{inc}[4, chapter 11],

$$\eta \cong \frac{t_m/R}{d_{31} - \varepsilon_{33}^T s_{11}^E / d_{31}} \frac{\bar{\xi}}{p_{inc}}, \tag{23}$$

which is expressed from using Equation (22), p_π being replaced by p_{inc}.

This expression leads to the expected classical behaviour of the magnitude of the sensitivity (dB re. 1 V/Pa) and the phase (rad) display in Figure 4, in the frequency range (1 kHz, 40 kHz), with twenty five holes, the resonance occurring at the same frequency as in Figure 3 (21 kHz), and the sensitivity being constant in the lower frequency range (below 1 kHz).

Acknowledgements

Authors more than acknowledge the late Emeritus Professor Zdeněk Škvor, who is the one without whom this work would not have been carried out, like the earlier work presented in Reference [1].

This publication was supported by the European social fund within the framework of realizing theproject “Support of inter-sectoral mobility and quality enhancement of research teams at Czech Technical University in Prague”, CZ.1.07/2.3.00/30.0034.

References

[1] T. Lavergne, Z. Škvor, L. Husník, M. Bruneau: On the modeling of an emitting cylindrical transducer with a piezoelectric polymer membrane. Acta Acust. United Ac. **102** (2016) 705–713.

[2] M. A. Hersh, M. A. Johnson: Assistive technology for visually impaired and blind people. Springer-Verlag London Limited, 2008.

[3] A. Gallo: For converting between electrical and acoustic energy. Okt. 27 1998. US Patent 5,828,766.

[4] M. Bruneau, T. Scelo (translator and contributor): Fundamentals of acoustics. Wiley-ISTE, 2006.

[5] P. Lotton, M. Bruneau, Z. Škvor, A.-M. Bruneau: A model to describe the behaviour of a laterally radiating piezoelectric loudspeaker. Appl. Acoust. **58** (1999) 419–442.

[6] R. S. Grinnip: Advance simulation of a condenser microphone capsule. AES 117th Convention, San Francisco, 2004.

[7] P. M. Morse, K. U. Ingard: Theoretical acoustics. Princeton University Press, 1968.