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Cryogenics

Cryogenics 45 (2006) 739–746

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Experimental and theoretical investigation on frequency characteristic of loudspeaker-driven thermoacoustic refrigerator

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Received 29 June 2005; received in revised form 19 September 2005; accepted 30 September 2005

Abstract

In this paper, the frequency characteristic of loudspeaker-driven thermoacoustic refrigerator (TAR) and the influence factors were investigated theoretically and experimentally. Experimental results under different conditions show that a narrow transmission frequency band of TAR exists to generate strong oscillation and cause significant thermoacoustic effect, and TAR achieves optimum performance at the resonance frequency. The frequency characteristic is determined by combined network elements of TAR. Then the analytical model was built to validate the experimental results. It provides a coupling method for design of loudspeaker-driven TAR system.

Keywords: Loudspeaker-driven thermoacoustic refrigerator; Frequency characteristic; Electro-acoustic efficiency; Transmission frequency band; Quality factor

1. Introduction

As a new cooling technology with advantages of simplicity, lower manufacturing cost, higher reliability and environmental safety, a loudspeaker-driven thermoacoustic refrigerator (TAR) consists of electro-dynamic acoustic source, resonator, heat pump stack and the two heat exchangers between its two ends. Interaction of the oscillating gas at the surfaces of the stack channels brings out heat transport effect when acoustic wave generated by the loudspeaker makes gas oscillate in periodical compressing and expanding style. Subsequently, temperature difference between the two ends of the thermoacoustic stack is generated [1]. Also, if it is described by network language, TAR can be regarded as being composed of the mechanical and electric impedance of loudspeaker, acoustic compliance of cavity and the front input acoustic impedance on the loudspeaker piston. The combined coupling action of these network elements makes heat transfer process take place between the surfaces of the stack channels and oscillating gas, and realizes the cooling effect [2,3].

It is just the network elements that highlight the frequency characteristic of the loudspeaker-driven TAR. The TAR can exert the thermoacoustic effect only within the range of transmission frequency band and achieve the optimum performance at eigenfrequency, i.e., resonance frequency. But it can not make thermoacoustic effect within other frequency bands. And the eigenfrequency is determined by the combined action of all the network elements. Thus, the frequency is an important parameter for the loudspeaker-driven TAR and becomes a crucial factor to determine the performance of the TAR system. It is not the same as the thermo-driven TAR in that the frequency is generated in the self-organized oscillating process, therefore it always works at resonance frequency once it begins to oscillate when the temperature gradient between the two

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doi:10.1016/j.cryogenics.2005.09.004

ends of the stack exceeds the critical value. But for the loudspeaker-driven TAR, the working frequency is given by a wave generator connected with the loudspeaker. Thus, one should tune operating frequencies to ensure that the system works at the eigenfrequency and exerts the thermoacoustic effect.

The purpose of the paper is to investigate the frequency characteristic of the loudspeaker-driven TAR experimentally and theoretically, and obtain the factors influencing the characteristic. It is helpful for design of loudspeaker-driven TAR, choice of the parameters of the system and couple among all the elements of TAR.

2. Experimental investigation

2.1. Experimental apparatus and measurement

A schematic diagram of the experimental apparatus is illustrated in Fig. 1. The system was driven by a loudspeaker, B4N Hi-end, whose technologic parameters are shown in Table 1.

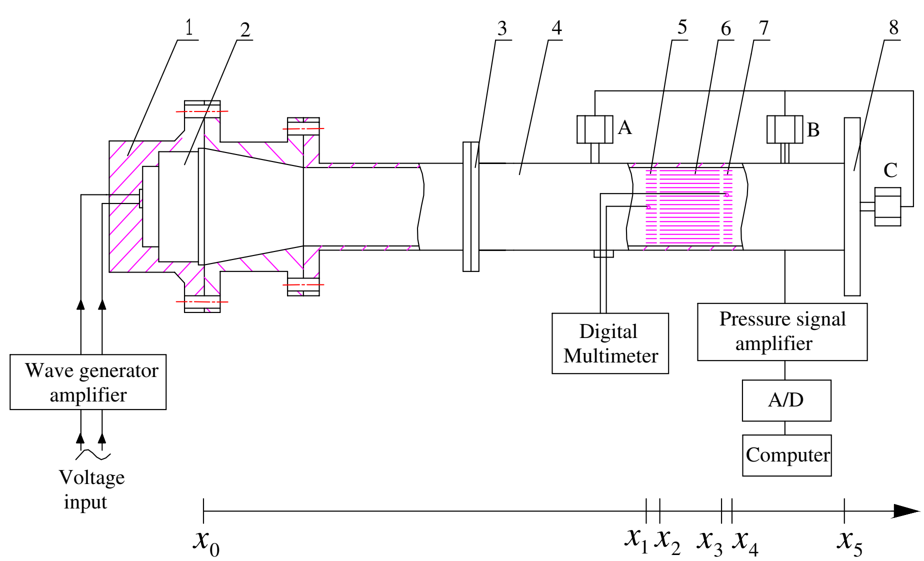
The length and inner diameter of the resonator are 437.5 mm, 38 mm.The lengths of hot heat exchanger, thermoacoustic stack and cold heat exchanger are 2 mm, 46 mm, 2 mm, respectively. The distance of hot heat exchanger from the rigid termination is 95 mm. The pin-array stack was used as heat pump stack, whose structure, dimensions and design process are given in literature [4].

In experimental measurement, the amplitude of oscillating pressure was measured by sensors A, B, C. The distances of sensors A, B from the rigid termination are 195 mm, 85 mm, respectively. The system was filled with nitrogen at a mean pressure $p_m = 1$ MPa. The temperature differences generated across the heat pump stack were measured by the thermocouples placed in contact with the left end and right end of the heat pump stack. The wave generator amplifier (DF1212E) connected with the loudspeaker was used to launch sound wave, supply the required voltage, amplify power and adjust the working frequencies. The oscillating pressure and temperature difference across the heat pump stack were measured at the different frequencies.

2.2. Experimentally measured results

2.2.1. Influence of stack elements on frequency characteristic

Under the conditions above, the measured amplitude of oscillating pressure and temperature difference across the



1. Acoustic source cavity; 2. Electrodynamic loudspeaker; 3. Flange; 4. Resonator; 5. Cold heat exchanger; 6. Thermoacoustic stack; 7. Hot heat exchanger; 8. Rigid acoustic termination

Fig. 1. A schematic diagram of experimental apparatus.

Table 1
Main technologic parameters of B4N Hi-end electro-dynamic sound source

Parameters	Force factor BL	Electrical resistance R_e	Electrical inductance L_e	Moving mass M_m
Value	5.1 N/A	6.5 Ω	0.00316 H	6.8 g
	Mechanic resistance R_m	Mechanic compliance C_m	Area of piston S_a	Volume of cavity V
Value	2.8 kg/s	1114.1 μm/N	0.006 m^2	350 mm^3

pin-array stack at different working frequencies are shown in Fig. 2(a) and (b).

In Fig. 2(a), A, B, C denote measured amplitude of oscillating pressure by sensors A, B, C, respectively, shown in Fig. 1. From Fig. 2(a), it can be seen that the transmission frequency band is 250–270 Hz and resonance frequency is 265 Hz, where the transmission frequency band can be defined as frequency range $f_1 \leqslant f \leqslant f_2$, when the amplitude of oscillating pressure reaches $1/\sqrt{2}$ times of the maximum value. Noting that in experimental response, the transmission frequency bands of oscillating pressure measured by the three pressure sensors are equal, only f_1 and f_2 of oscillating pressure measured by sensor C are shown in Fig. 2(a).

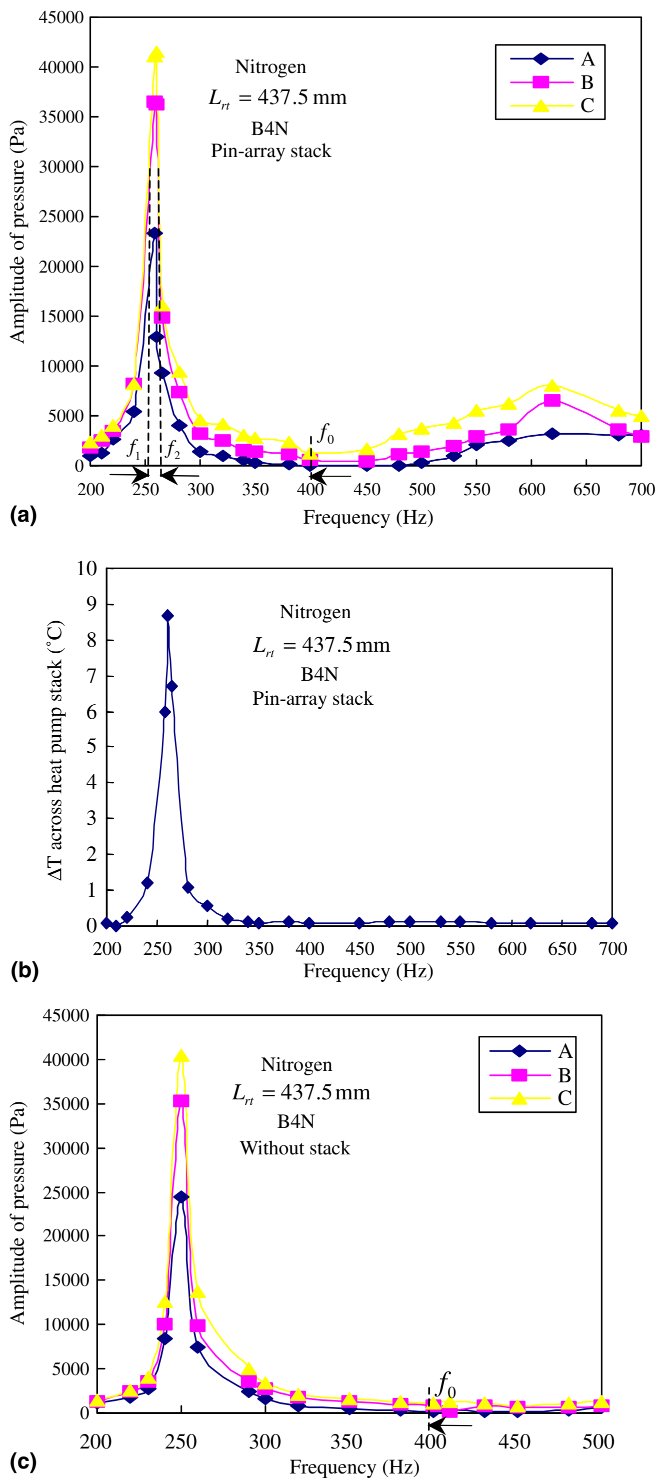


Fig. 2. Measured results about influence of stack elements on frequency characteristic of TAR when the system was filled with nitrogen. The position of resonance frequency calculated with the formula $f_o = c/2L_{rt} = 400$ Hz is denoted in the figure.

In order to compare the influence of stack geometries on the frequency characteristic, parallel plate stack, whose spacing between two plates is 0.8 mm, was used to substitute for the pin-array stack to remake the experiment under the same conditions.

The measured results show that the oscillating pressure characteristic is almost the same as in Fig. 2(a), and the measured temperature difference is a little lower than that in Fig. 2(b).

Then, in order to further demonstrate the influence of stack elements on the frequency characteristic, the stack was taken out and the oscillating pressure characteristic of the resonator without stack was measured under the conditions above. The measured results are shown in Fig. 2(c).

From Fig. 2(c), it can be seen that the resonance frequency of the TAR system is about 250 Hz. Comparing Fig. 2(a) and (c) shows that the stack elements can make some influences on the frequency characteristic. The transmission frequency band shifts left and resonance frequency decreases when no stack is in the resonator, whereas influence of stack geometries on the frequency characteristic is very small.

Moreover, Fig. 2(a) and (c) shows that the system can make strong oscillation and thermoacoustic cooling effect only within the transmission frequency band $f_1 \leqslant f \leqslant f_2$. And the amplitude of oscillating pressure and temperature difference reach peaks at the resonance frequency. When a working frequency is larger than f_2, or smaller than f_1, the thermoacoustic effect is very weak. Also, the transmission frequency band is very narrow. Thus, the measured results evidently show that the frequency is a very important parameter for loudspeaker-driven TAR and the resonance frequency does not simply satisfy the formula $f_0 = c/2L_{rt}$ (see Fig. 2), where c is the sound speed of gas and L_{rt} is the resonator length.

2.2.2. Influence of gas on frequency characteristic

In order to compare the influence of gas on the frequency characteristic of the TAR, helium was substituted for nitrogen to remake the experiment under the same conditions as that in Fig. 2(c). The measured amplitude of oscillating pressure at different working frequencies is shown in Fig. 3.

It can be seen that the transmission frequency band shifts right, resonance frequency is about 700 Hz and the transmission frequency band is a little wider than that of nitrogen.

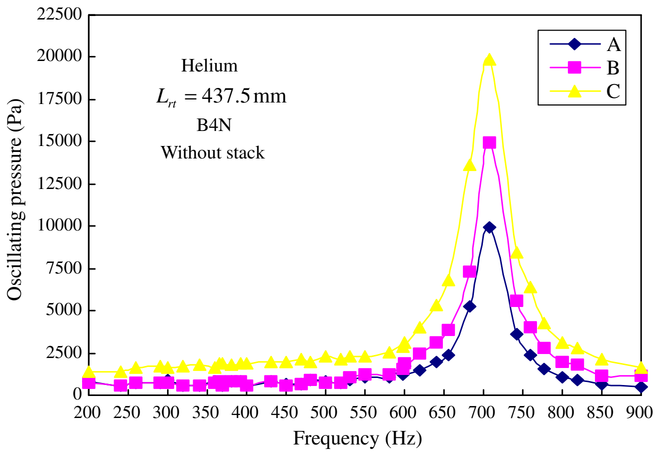


Fig. 3. Measured frequency characteristic of TAR without stack when the system was filled with helium. The frequency calculated with the formula $f_o = c/2L_{rt}$ is 1378 Hz.

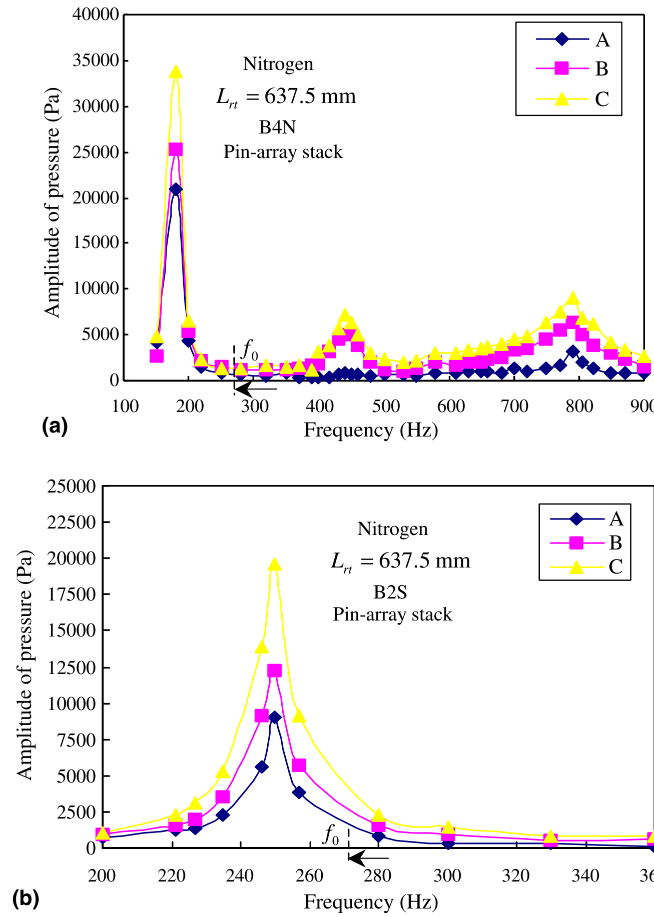


Fig. 4. Measured results about influence of loudspeaker on frequency characteristic of the system with pin-array stack when the system was filled with nitrogen and the resonator length is 637.5 mm. The frequency calculated with the formula $f_o = c/2L_{rt}$ is 272.5 Hz.

From Figs. 2(c) and 3, it follows that adopting mixed gas is a feasible way to shift transmission frequency band to make it between that corresponding to nitrogen and corresponding to helium. Thus, the range of transmission frequency band can be adjustable.

Also, comparing Figs. 2(c) and 3 show that the amplitude of oscillating pressure under the condition of helium is lower than that under the condition of nitrogen at the same frequency. It can be simply explained below.

The sound intensity, sound energy transmitted per unit of time through a unit area, can be written as $I_E = P^2/\rho c$, where P, ρ are pressure and gas density, respectively. The product ρc of nitrogen is about double of that of helium. Thus, from the expression, the amplitude of oscillating pressure under the condition of helium is lower than that under the condition of nitrogen at the same transmitted energy and the same frequency.

2.2.3. Influence of system length on frequency characteristic

Theoretically, the system length is a main factor to affect the frequency characteristic. In order to validate the influence, a short tube of 200 mm length was added and connected between the flanges in the resonator. The measured results of oscillating pressure under the conditions of nitrogen and pin-array stack are shown in Fig. 4(a), where the measured resonance frequency is 190 Hz.

Comparing Figs. 4(a) and 2(a) show that the system length can make great influence on frequency characteristic. Transmission frequency band shifts left as the length increases, namely the resonance frequency decreases with the increase of the system length. But the frequency characteristic does not simply satisfy the relation $f_0 = c/2L_{rt}$.

2.3. Influence of loudspeaker on frequency characteristic

In order to demonstrate the influence of loudspeaker on frequency characteristic, the B4N Hi-end loudspeaker was replaced by B2S Hi-end, whose main technological parameters are shown in Table 2.

Under the same conditions as that in Fig. 4(a), the measured results of oscillating pressure at different frequencies are shown in Fig. 4(b). Temperature difference between the

Table 2
Main technologic parameters of B2S Hi-end electro-dynamic sound source

Parameters	Force factor BL	Electrical resistance R_e	Electrical inductance L_e	Moving mass M_m
Value	2.5 N/A	6.5 Ω	0.0022 H	1.6 g
	Mechanic resistance R_m	Mechanic compliance C_m	Area of membrane S_a	Volume of cavity V
Value	1.5 kg/s	689 μm/N	0.003 m^2	82 mm^3

two ends of the stack was too small to be measured because of B2S Hi-end loudspeaker's limitation. The maximum electrical power of the loudspeaker is 20 W. The output electrical power is 15 W when the input voltage is 15 V. Thus, the converted sound power is too small to make temperature difference across the stack.

Comparing Fig. 4(a) and (b) shows that the transmission frequency band shifts right and resonance frequency increases to be 248 Hz when the loudspeaker is B2S.

From these experimental results, it can be seen that the loudspeaker-driven TAR makes thermoacoustic effect only within the range of transmission frequency band and the performance is optimum at the resonance frequency. The resonance frequency is determined by composite network parameters of all parts of the system. And the frequency characteristic is affected by the gas because the induced network parameters are different due to different gas.

3. Theoretical analysis of frequency characteristic

In order to analyze the experimental phenomenon and validate the experimental results, the dependence of frequency characteristic on the elements of network was investigated theoretically.

3.1. Analytical model

As noted in the previous section, the loudspeaker-driven TAR can be regarded as consisting of network elements of all components of the system, shown in Fig. 5(a).

In Fig. 5(a), E, I, F, u, P, U are voltage, electric current, force, velocity, pressure and volume velocity, respectively; R_e, L_e, M_m, C_m, R_m are indicated in Tables 1 and 2; B is magnetic flux density; L is effective coil length; S_a is area of the loudspeaker piston; C_a is acoustic compliance of the acoustic source cavity and it can be calculated as $C_a = V/\rho c^2$ (V is volume of the cavity, indicated in Tables 1 and 2); Z_a is the front input impedance on the piston, $Z_a = P(x_0)/U(x_0)$, and it can be calculated by the built model [5]

$$\begin{bmatrix} p(x_0) \\ U(x_0) \end{bmatrix} = H \begin{bmatrix} p(x_5) \\ U(x_5) \end{bmatrix} \quad \text{with } H = \prod_{i=1}^{i=5} H_i \tag{1}$$

and the boundary condition is $Z_a(x_5) = \infty$, where H_i ($i = 1$–5) are transfer matrix of network equation of the resonator, hot heat exchanger, stack, cold heat exchanger and resonator, respectively, whose expressions are given in Appendix; $Z_a(x_5)$ is acoustic impedance of the rigid termination; x_i ($i = 0$–5) are denoted in Fig. 1.

According to the electro-mechano-acoustic analogy relation, $Z_{\text{mechano}} = S_a^2 Z_{\text{acoustic}}$ and $Z_{\text{electro}} = (BL)^2/Z_{\text{mechano}}$, Fig. 5(a) can be simplified as Fig. 5(b), which is an equivalent electrical circuit including all the network parameters of the system.

In Fig. 5(b), Z_m is mechanic impedance, and Z_A is serial impedance of acoustic compliance of the cavity C_a and the front input impedance Z_a. They can be written as

$$Z_m = R_m + iwM_m + \frac{1}{iwC_m} \tag{2}$$

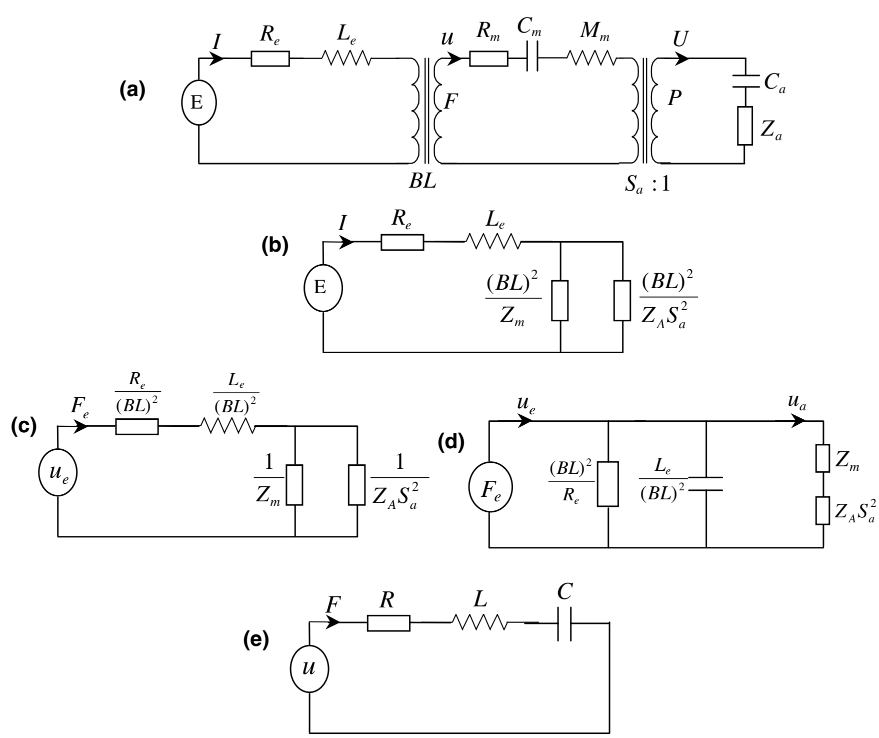


Fig. 5. A network figure of loudspeaker-driven TAR.

$$Z_A = Z_a + \frac{1}{iwC_a} \quad (3)$$

Furthermore, in terms of the relation $E = BLu$ and $F = BLI = u(BL)^2/Z$, Fig. 5(b) can be resulted in Fig. 5(c) and (d). Finally, the equivalent electrical circuit can be simplified as Fig. 5(e), where R, L, C are composite resistance, inductance and compliance.

According to the defined expression of electro-acoustic efficiency [6,7], it can be expressed as

$$\eta_{ea} = \frac{\mathrm{Re}(F_a u_a^*)}{\mathrm{Re}(F_e u_e^*)} \quad (4)$$

where $*$ denotes complex conjugation; F_e is the total force and F_a is the force on the piston due to the acoustical oscillation of the gas in the resonator; u_e is the total velocity and u_a is the velocity of the piston.

And, according to circuit principle, the quality factor can be written as

$$Q = \omega_r L/R \quad (5a)$$

and

$$Q = \omega_r/\mathrm{BW} \quad (5b)$$

where ω_r is resonance angular frequency.

Thus, width of transmission frequency band BW can be obtained as

$$\mathrm{BW} = \frac{R}{L} \quad (6)$$

Thus, according to electro-circuit principle in Fig. 5(d), the electro-acoustic efficiency of the system can be calculated by adopting the calculation model and the definition.

3.2. Calculation results

3.2.1. Influence of stack elements and gas on frequency characteristic

According to the calculation model above, the electro-acoustic efficiencies of the TAR system driven by B4N were calculated when the pin-array and parallel plate were used as heat pump stack under the conditions of those in Figs. 2 and 3, respectively. The calculated results of electro-acoustic efficiencies at different frequencies are depicted in Fig. 6.

Calculation results demonstrate that gas and stack elements can make great influence on the frequency characteristic, whereas the influence of stack geometries can be negligible because the influence of stack geometries on the network parameters of the stack is very small, the resonance frequency under the condition of helium is much higher than that of nitrogen and the transmission frequency band is wider. This is because different physical parameters due to different gases make different composite network parameters, then cause different quality factors and transmission frequency bands. The results are as the same as the experimental data.

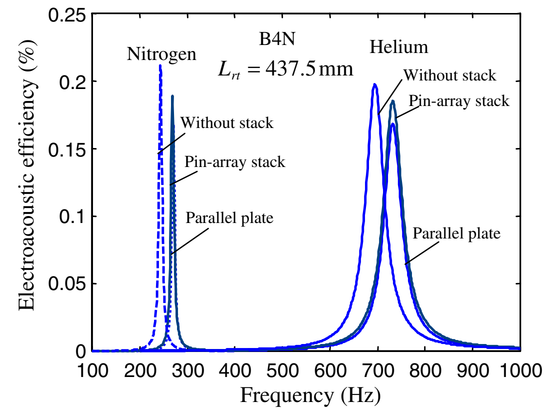


Fig. 6. Analytical results about influence of gas and stack elements on frequency characteristic.

3.2.2. Influence of system length on frequency characteristic

Electro-acoustic efficiencies of TAR with pin-array stack driven by B4N, filled with nitrogen at a mean pressure $P_m = 1$ MPa, as a function of working frequency and resonator length $L_{rt} = 0.4$–0.9 m (calculation step is 0.05 m) were calculated, shown in Fig. 7(a). As an example, the electro-acoustic efficiencies of TAR system with pin-array

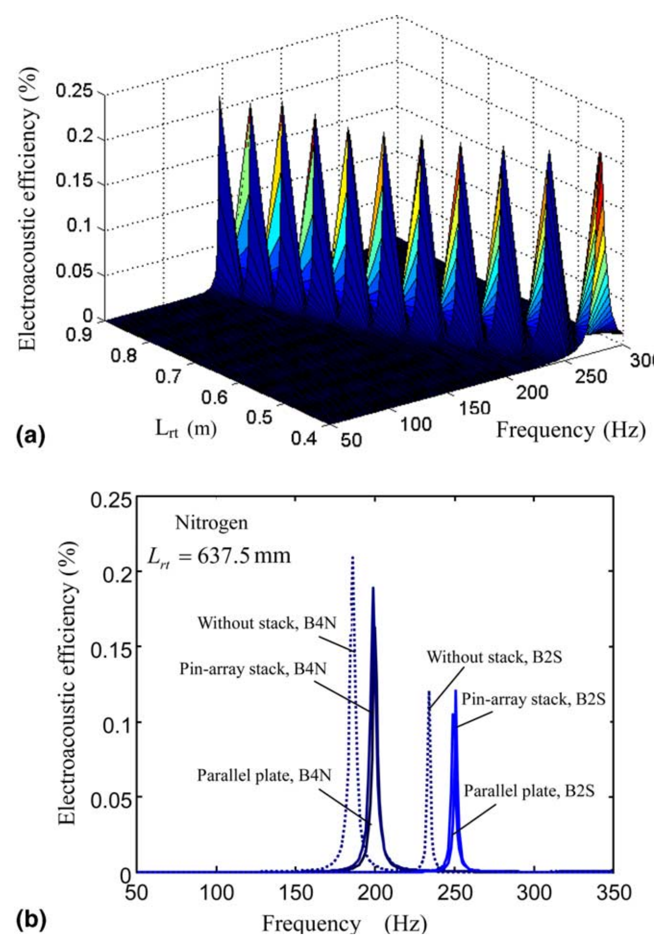


Fig. 7. Analytical results about influence of system length and loudspeaker on frequency characteristic.

stack, parallel plate and without stack, driven by B4N, at $L_{rt} = 637.5$ mm, are shown in Fig. 7(b).

Fig. 7(a) shows that the system length can make great influence on the resonance frequency. The resonance frequency decreases as the system length increases. Also, comparing Figs. 6 and 7(b) show that the transmission frequency band keeps almost unchangeable. The results are in agreement with the experimental data.

3.2.3. Influence of electro-dynamic loudspeaker on frequency characteristic

When the resonator length is 637.5 mm, the calculated electro-acoustic efficiencies of TAR with pin-array stack, respectively driven by B4N and B2S are shown in Fig. 7(b).

Calculated results demonstrate that loudspeaker can make some influences on resonance frequency. Different loudspeakers cause different resonance frequencies due to different network parameters of loudspeakers.

The calculation results above can explain the experimental phenomenon, namely, existence of an optimum frequency makes TAR generate strong oscillation and cause significant thermoacoustic effect, and the frequency is determined by network parameters of TAR including mechanical and electric impedance of loudspeaker, acoustic compliance of cavity and the front input acoustic impedance on the loudspeaker piston. Moreover, the transmission frequency band is very narrow.

3.3. Method to increase transmission frequency band

For loudspeaker-driven TAR, the adjustable range and adaptability can increase if the transmission frequency band can be widened. In Fig. 5(c) and (e), the decrease rate of equivalent composite resistance R is slower than that of equivalent composite inductance L as the force factor BL increases, thus, in terms of Eq. (5a), the quality factor Q decreases with the increase of BL. Furthermore, according to Eqs. (5b) and (6), the transmission frequency band can be wider with the increase of BL.

The electro-acoustic efficiencies were calculated under different technological parameters of loudspeaker. The calculated results, shown in Fig. 8, demonstrate that the only way to widen transmission frequency band is to increase force factor BL and the method can improve the electro-acoustic efficiency.

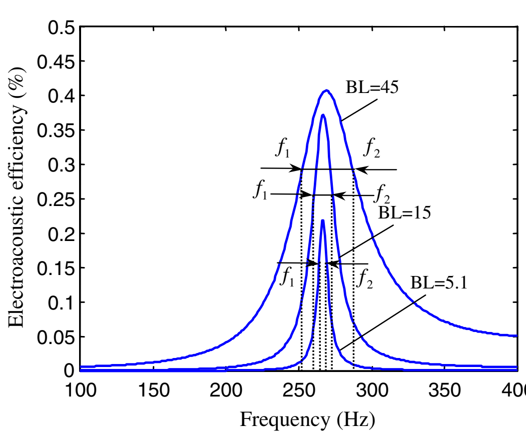


Fig. 8. Influence of BL on transmission frequency band.

4. Conclusions

It is evident from the experimental data under the different conditions that working frequency of loudspeaker-driven TAR is a very important parameter to characterize the thermoacoustic performance. TAR can cause strong thermoacoustic oscillation and exert the significant thermoacoustic effect only when it works within the range of transmission frequency band, and realize the optimum performance at resonance frequency. The frequency is determined by the combined action of all of the network elements including mechanical and electric impedance of loudspeaker, acoustic compliance of cavity and the front input acoustic impedance, in which the front input acoustic impedance is affected by gas, resonator lengths and stack elements. Moreover, the transmission frequency band is very narrow.

In order to verify the experimental results, the electro-mechano-acoustic network model was built and adopted to calculate electro-acoustic efficiency at different frequencies. The calculated results can validate the experimental results. It demonstrates the built model is feasible and it can be used to optimize the TAR system and provide a method to predict coupling of all the components of the system. Finally, the method to improve the transmission frequency band was also investigated.

Acknowledgments

The work was supported by the Chinese Doctoral Fund (No. 20030487046) and France Post-doctoral Fund (No. 2004-36700).

Appendix

According to literature [5], the transfer matrix of network equation of thermoacoustic stack in Eq. (1) is

$$H_3 = \begin{bmatrix} \frac{\gamma_2}{\gamma_1+\gamma_2} e^{-\gamma_2 x} + \frac{\gamma_1}{\gamma_1+\gamma_2} e^{\gamma_1 x} & z_c \cdot \frac{\gamma_1}{\gamma_1+\gamma_2}\left(e^{\gamma_1 x} - e^{-\gamma_2 x}\right) \\ \frac{1}{z_c} \frac{\gamma_2}{\gamma_1+\gamma_2}\left(e^{\gamma_1 x} - e^{-\gamma_2 x}\right) & \frac{\gamma_1}{\gamma_1+\gamma_2} e^{-\gamma_2 x} + \frac{\gamma_2}{\gamma_1+\gamma_2} e^{\gamma_1 x} \end{bmatrix} \quad \text{(A.1)}$$

where γ_1, γ_2 are acoustic propagation constants of the stack and z_c is characteristic impedance.

For heat exchangers (hot heat exchanger and cold heat exchanger) without temperature gradient, $\gamma_1 = \gamma_2$, Eq. (A.1) becomes

$$H_{2,4} = \begin{bmatrix} \cosh(\gamma_{h,c} x) & z_{h,c} \sinh(\gamma_{h,c} x) \\ \frac{1}{z_{h,c}} \sinh(\gamma_{h,c} x) & \cosh(\gamma_{h,c} x) \end{bmatrix} \quad \text{(A.2)}$$

where $\gamma_{h,c}$ are acoustic propagation constant of hot heat exchanger or cold heat exchanger and $z_{h,c}$ is characteristic impedance.

Furthermore, for resonator, the transfer matrix becomes

$$H_{1,5} = \begin{bmatrix} \cosh(\gamma_0 x) & z_{c0}.\sinh(\gamma_0 x) \\ \frac{1}{z_{c0}}\sinh(\gamma_0 x) & \cosh(\gamma_0 x) \end{bmatrix} \tag{A.3}$$

The expressions of acoustic propagation constants and characteristic impedances of stack, heat exchangers and resonator in the equations above are given in literature [5].

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