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A New Method for the Determination of the Acoustic Center of Acoustic Transducers

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Summary

The concept of acoustic center is important in practical realization of free-field reciprocity calibration of microphones. The accuracy in the position of the acoustic center has a significant influence on the accuracy of the estimated free-field sensitivity of the microphones. The last experimental methods used for half inch microphones allows to obtain results only from 2 kHz (the current lower frequency limit). In order to broaden this frequency range (in the lowest frequency domain, from nearly 2 kHz to 400 Hz), an improved experimental method which increases the signal to noise ratio is presented here. Thus, today, experimental results, which are based on the inversely proportional distance law, are available from 400 Hz to 20 or 30 kHz.

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1. Introduction

Ideally, a point source of sound generates in an infinite region (free field) a sound pressure field whose the amplitude is inversely proportional to the distance from the source. For extended sources, the inverse distance law can be considered as true if the observation point is very far away from the source. Then, the source can be represented by a point source called "acoustic center".

The concept of acoustic center is important in practical realization of free-field reciprocity calibration of microphones [1]. The accuracy in the position of the acoustic center has a significant influence on the accuracy of the estimated free-field sensitivity of the microphones. The international standard IEC 61094-3 [1] defines the acoustic center of a transducer as follows: "For a sound emitting transducer, for a sinusoidal signal of given frequency and for a specified direction and distance, the point from which the approximately spherical wavefronts, as observed in a small region around the observation point, appear to diverge". Two notes are included: first "the acoustic center of a reciprocal transducer when used as a receiver is coincident with the acoustic center when used as a transmitter", second "this definition only applies to regions of the sound field where spherical, or approximately spherical wavefronts are observed". The international standard specifies the concept of equivalent point-transducer as: "transducer which, when located at the position of an acoustic center of a microphone, simulates the transmitting and receiving characteristics of that microphone for a given direction and range of distance".

Thereby, the acoustic center of a source depends on three main parameters: the frequency, the direction of the receiver, and its distance from the source. Several methods have been used for determining the acoustic center of a transducer: one of them relies on phase measurements [2], but most of them are based upon the measurement of the deviations of the amplitude of the sound pressure from the inverse distance law [3, 4]. These methods are summarized and discussed in reference [5].

Several analytical results on the position of the acoustic center have been published almost forty years ago. Using D.S. Jones works [6] (entitled "the scattering of a plane harmonic wave by a semi-infinite cylindrical rod"), Y. Ando determined the acoustic center of a "pipe horn loudspeaker" from the phase shift between two receiving positions [7, 8]. E. Matsui completed Ando's model by taking into account the influence of the receiver on the acoustic field, but his study focused only on obtaining the free-field correction to the sensitivity of the microphone [9] (not on the position of the acoustic center). More recently, works on the determination of acoustic center of microphones have been focused on experimental methods and and numerical method using the boundary element method [4, 10] which relies on the determination of the amplitude-based acoustic center (this is coherent with the method used experimentally).

The classical procedure for determining the acoustic center of microphones relies on the measurement of the acoustic transfer impedance between pairs of microphones

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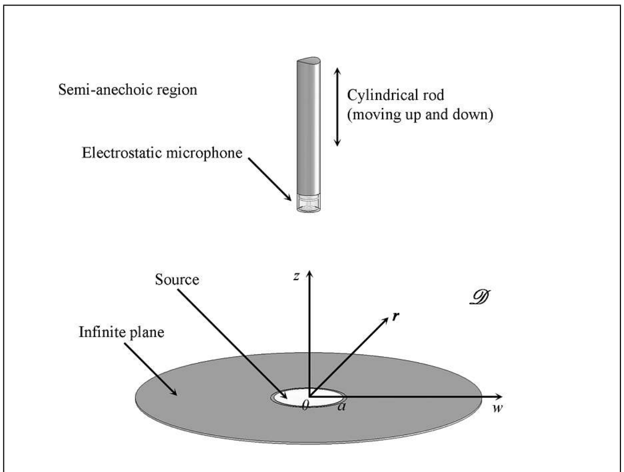

Figure 1. Setup used in the experimental determination of the acoustic center of acoustic transducers.

coupled in a free-field. The transfer impedance is measured using three microphones successively. This procedure gives the sum of the positions of the acoustic center of each pair of microphones, for three pairs, leading to the position of the acoustic center of each microphone. This method is characterized by a poor signal-to-noise ratio in the lowest frequency range (typically, for half inch microphones, measurements below 2 kHz are very difficult).

Thus the purpose of this paper is to present and discuss a new experimental method (for the determination of the acoustic center of any kind of acoustic transducers), based on the classical procedure using the inversely proportional distance law, whose main advantage is to improve the signal-to-noise ratio. This method allows to obtain experimental results not only from 2 kHz (the current lower frequency limit) but from 400 Hz.

2. Experimental method and experimental results

2.1. Modeling the method

The setup considered is a circular sound source (area $S = \pi a^2$) flush-mounted on an infinite baffle and coupled with the acoustic receiving transducer to be tested through a semi-infinite field. This receiving transducer is mounted on a semi-infinite rod in the axis of the source (Figure 1). The fluid is characterized by its density ρ_0, the adiabatic speed of sound c_0, and its sound attenuation coefficient γ due to atmospheric absorption.

If the emitter is modeled by a point source set at $\boldsymbol{r} = \boldsymbol{r}_s$ (the acoustic center), the harmonic sound pressure generated is given by the well-known expression, the time factor being here chosen as $e^{j\omega t}$

$$p(\boldsymbol{r}) = \frac{jk_0\rho_0c_0Q_0}{2\pi}\frac{e^{-jk_0|\boldsymbol{r}-\boldsymbol{r}_s|}}{|\boldsymbol{r}-\boldsymbol{r}_s|}e^{-\gamma|\boldsymbol{r}-\boldsymbol{r}_s|}, \quad \forall \boldsymbol{r} \in \mathbf{D}, \qquad (1)$$

where $k_0 = \omega/c_0$ is the adiabatic wavenumber and Q_0 is the volume velocity of the source. Along the z-axis (normal to the infinite baffle set at $z = 0$, the acoustic center being at the coordinate $z = z_s$), the modulus of expression (1) can be written as

$$|p(z)| = \frac{A}{|z - z_s|}e^{-\gamma|z-z_s|}, \quad \forall z > 0, \qquad (2)$$

where $A = k_0\rho_0c_0Q_0/(2\pi)$.

The acoustic transducer to be tested gives the amplitude of the sound pressure at its acoustic center d (defined positive in front of the microphone membrane). Therefore, for a position z of the receiving membrane, the modulus of the pressure measured at its acoustic center is given by

$$|p(z-d)| = \frac{A}{|z-(z_s+d)|}e^{-\gamma|z-(z_s+d)|}, \quad \forall z > 0. \qquad (3)$$

Assuming that the positions z of the receiving membrane are large compared with the sum $z_s + d$, and inverting Eq. (3), yield the equation that shows a linear dependence as a function of z

$$\frac{e^{-\gamma z}}{|p(z-d)|} \approx mz + b, \quad \forall z > 0, \qquad (4)$$

with $m = 1/A$ and $b = -(z_s + d)/A$.

So far as the attenuation factor and the acoustic center z_s of the source are known, measuring the modulus of the acoustic pressure as a function of the distance z yields the parameters b and m, then leads to the value of the acoustic center abscissa of the tested acoustic transducer using the following relation:

$$z_s + d = \frac{-b}{m}. \qquad (5)$$

The improvements in the measurement method of the acoustic center presented here permit to increase the signal to noise ratio and thus to broaden the frequency range (in the lowest frequency domain, from nearly 2 kHz to 400 Hz). The signal to noise ratio is increased because here the acoustic source is a one inch microphone flush-mounted in a quasi-infinite plane baffle which provides in the lower frequency range an acoustic level nearly 15 dB higher than the acoustic level obtained in the current technique (a half inch emitting microphone set at the end of a semi-infinite half inch cylindrical rod [3, 4]).

2.2. The acoustic center of the circular source flush-mounted in a quasi-infinite baffle

The ability to proceed to the measurement of the acoustic center of the receiving microphone implies knowing primarily the coordinate z_s of the acoustic center of the emitting microphone. This coordinate can be obtained from the appropriate, well-known approach presented below.

The sound field generated by a circular membrane flush-mounted in an infinite baffle and radiating in free space can be calculated using the Rayleigh integral [11]

$$p(\boldsymbol{r}) = \frac{jk_0\rho_0c_0}{2\pi}\iint_S \frac{e^{-jk_0|\boldsymbol{r}-\boldsymbol{r}_s|}}{|\boldsymbol{r}-\boldsymbol{r}_s|}v(\boldsymbol{w_0})\,dS, \quad \forall \boldsymbol{r} \in \mathbf{D}, \qquad (6)$$

where $v(\boldsymbol{w_0})$ is the normal velocity of the membrane (assuming an axial symmetry). Along the z-axis, Eq. (6) can be written as

$$p(z) = \mathrm{j}k_0\rho_0 c_0 \int_0^a \frac{\mathrm{e}^{-\mathrm{j}k_0\sqrt{z^2+w_0^2}}}{\sqrt{z^2+w_0^2}} v(w_0) w_0 \,\mathrm{d}w_0, \quad \forall z > 0. \quad (7)$$

Calculating the modulus of the inverse of the sound pressure $1/p(z)$ along the z-axis, on the range $z \in [z_{min}, z_{max}]$ of interest, and making a linear regression over this interval, give the origin abscissa b and the slope m of this linear regression. These results lead to the abscissa of the acoustic center z_s of the source (as described above in section 2.1). The results obtained for a one inch microphone (B&K 4144) used as a source, as functions of the frequency, at different distances ($z \to \infty$ and $z \in [0.2\text{ m}, 0.4\text{ m}]$) are given in Figure 2 when the velocity $v(w_0)$ of the membrane is uniform (dotted and dashed-dotted lines) or when it is given by (solid and dashed line)

$$v(w_0) = v_0 \left[1 - \frac{J_0(k_w w_0)}{J_0(k_w a)}\right], \quad (8)$$

where J_0 is the Bessel function of zero order, v_0 is a constant that defines the amplitude of the velocity and $k_w^2 = (\omega^2\sigma_s - \mathrm{j}\omega S Z_c)/T_m$ the axial wavenumber which involves the tension of the membrane T_m, its surface density σ_s, and the acoustic impedance of the backing cavity Z_c. For a one inch microphone B&K 4144, these parameters can be determined using the approach proposed by Rasmussen [12].

These theoretical results show that the distance between the abscissa of the membrane and that of its acoustic center is non negligible at the highest frequency range (beyond 5 kHz) when the velocity profile of the membrane is assumed to behave as a Bessel function (the input parameters of the model being determined from the results given in reference [12]). These theoretical results are in accordance with those given in [5], their accuracy being very good in the frequency range of interest here. For higher frequencies, these theoretical results would need to be confirmed using the experimental results for the velocity profile obtained with laser-vibrometer, see for example [13] (this is beyond the scope of this paper).

2.3. Experimental setup, measurement method

Measurements were made using the experimental setup described in Figure 3. The acoustic transducer to be tested is connected to a preamplifier mounted on the end of a long rod (having both the same section) which can moves automatically up and down along the vertical axis. The source, a one inch microphone (B&K 4144) is flush-mounted in the floor of a semi-anechoic room. Electric signals are conditioned by a Nexus amplifier and the reciprocity apparatus. The analyzer B&K PULSE provides the sinusoidal electrical signal to the source and allows to measure the transfer function $H_m(z, f) = u_r/u_g$ (modulus and phase) defined as the ratio between the output voltage u_r of the receiver (for a position z of the membrane) and the output voltage u_g of the generator. The environmental conditions are measured and recorded during the measurement.

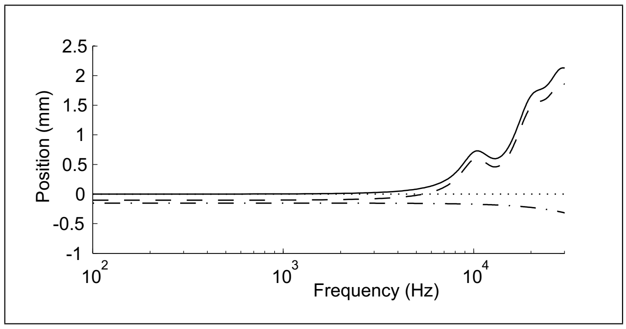

Figure 2. Theoretical abscissa of the acoustic center of a one inch microphone (B&K 4144) flush-mounted in a quasi-infinite baffle, in free space, as a function of the frequency.
Solid line: Bessel velocity distribution and $z \to \infty$;
dashed line: Bessel velocity distribution and $z \in [0.2\text{ m}, 0.4\text{ m}]$;
dotted line: uniform velocity distribution and $z \to \infty$;
dashed-dotted line: uniform velocity distribution and $z \in [0.2\text{ m}, 0.4\text{ m}]$.

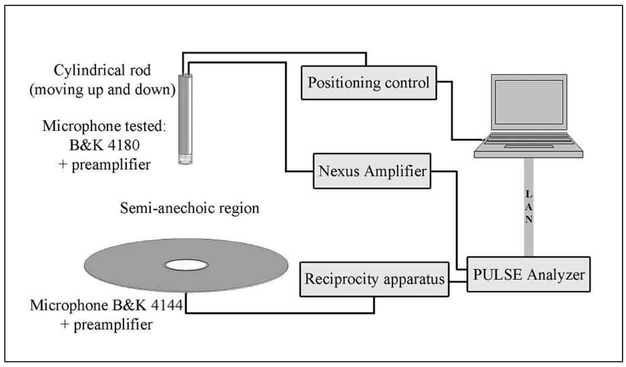

Figure 3. Schematic diagram of the measurement setup.

For comparisons with literature data, the acoustic transducer tested is a half inch laboratory standard microphone (B&K 4180). When the distance z remains in the interval $z \in [0.2\text{ m}, 0.4\text{ m}]$, the measured coherence function (indicator of the behaviour of the signal to noise ratio) between the signals u_r and u_g shows that the measurement method presented here fall down across the lower frequency range, up to 400 Hz (until now the lower frequency limit was nearly 2 kHz). Thus, measurements of the transfer function $H_m(z, f)$ were made at discrete frequencies from 400 Hz to 30 kHz for $z \in [0.2\text{ m}, 0.4\text{ m}]$. For several reasons, the values of $H_m(z, f)$ below 400 Hz are needed, in particular when calculating the Inverse Fourier Transform of this transfer function (see below). These missing values are given by patching known values, as presented in reference [14].

Using this method presented here, the measured transfer function $H_m(z, f)$ is proportional to the sound pressure $p(z - d)$ at the acoustic center of the transducer to be tested. However, many imperfections of the measurement setup prevent us from obtaining the experimental values of the transfer function with a good accuracy, and these

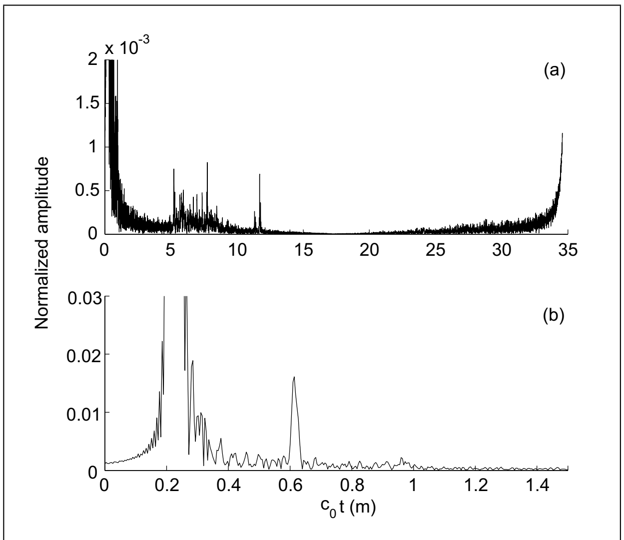

Figure 4. Envelope of the impulse response $h_m(z,t)$ ($h_m(z,t) = FFT^{-1}[H_m(z,f)]$) where $H_m(z,f)$ is measured at $z = 0.2$ m with the frequency step $\Delta f = 10$ Hz, for different scales.

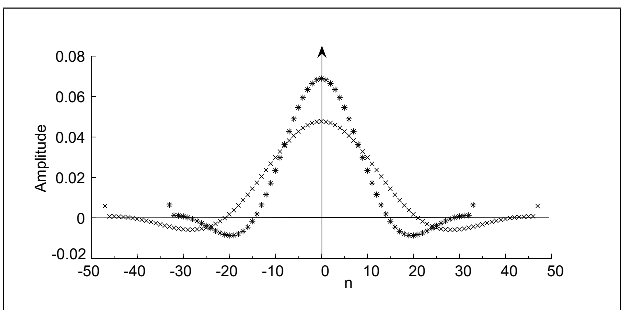

Figure 5. The spectral windows $W_1(f_n)$ (stars) and $W_2(f_n)$ (crosses) defined in Figure 6.

unwanted disturbances must be removed before proceeding to the calculation of the acoustic center. These disturbances are due to reflections from the walls of the semi-anechoic room, successive reflections between the floor and the tested transducer, small discontinuities on the floor (at the frontier between the emitting transducer and the baffle, and between the baffle and the floor), slight vibrations of the baffle (probably), among others as cross-talk, acoustic noise, and electric noise.

Most of these disturbances can be identified from the impulse response. Figure 4a and 4b (different scales) show the envelope of the impulse response $h_m(z,t)$, as function of the parameter c_0t, measured at a distance from the emitting microphone equal to $z = 0.2$ m, in the frequency range [400 Hz, 30 kHz]. This envelope is obtained by calculating successively the FFT^{-1} and the Hilbert transform of the measured transfer function $H_m(z,f)$, the missing low frequency values being completed by patching ideal values as mentioned above.

It appears that, in the time domain, most of the disturbances are shifted from the main impulse response and thus, a time selective window $w(t)$ may be applied to remove them [14, 15]. Many peaks appear in the domain $c_0t = [5\text{ m}, 12\text{ m}]$ (Figure 4a) which correspond to the reflections from the walls, the door, and the ceiling of the semi-anechoic room (they are easily removed because they are not superposed to the main peak). Successive reflections between the transducers, small discontinuities on the floor (listed above), and probably slight vibrations of the baffle appear near the useful signal (main peak) (Figure 4b). Note that the sampling of the frequency domain, with a frequency step Δf, and the FFT^{-1} operation yield to a periodic time domain, the period being given by $T = 1/\Delta f$. Then, to identify all reflections without aliasing, a frequency step $\Delta f = 10$ Hz is chosen here, leading to the period $c_0T \approx 34.5$ m (Figure 4a).

To obtain accurate results for the acoustic center, the measurement procedure benefits from the techniques currently available. Especially, the measurements are conducted so as several phenomena are accounted for and compensated in the experimental results, such as the sound attenuation γ (atmospheric absorption), the variations of the static pressure and the static temperature, and then the variations of the density ρ, the speed of sound c, and the wavenumber k. Thus, in practice, it is convenient to process the function $H(z,f)$ defined as

$$H(z,f) = \frac{H_m(z,f)}{(\rho c)f^2} e^{jkz} e^{\gamma z}, \tag{9}$$

whose associated impulse response is set around $c_0t = 0$, and to choose a quite fast frequency step (here $\Delta f = 172$ Hz) in order to reduce the measurement time (but remaining adapted to the dimensions of the semi-anechoic room in order to eliminate the echoes due to the aliasing).

As mentioned above, a time selective window $w(t)$ must be used to filter the impulse response. For doing that, it is suitable to filter the response $H(z,f)$ in the frequency domain with a spectral window $W(f)$, which is the Fourier Transform of the time window $w(t)$, leading to the filtered signal (* denoting the convolution product)

$$H_c(z,f) = H(z,f) * W(f). \tag{10}$$

The spectral window $W(f)$ is designed using a classic optimization method (Parks-McClellan algorithm). Starting from an ideal time selective window $w(t)$ and allowing given deviations, this algorithm provides a finite spectral window $W(f_n)$ which is given by an optimized number of requisite discrete values. Two filters $w_1(t)$ and $w_2(t)$ was used (see discussion below), shown in Figure 5 (in the frequency domain) and Figure 6 (in the time domain).

2.4. Experimental results

In Figure 7, which shows the acoustic center of a half inch microphone (B&K 4180) as a function of the frequency (dashed line), the filter used is the less selective one (filter w_1, Figure 6, dotted line), the full line showing the standardized results. The oscillations of the experimental results around the standardized ones come from the disturbances due to the acoustic energy diffracted in the vicinity of the emitting microphone. In support of this explanation,

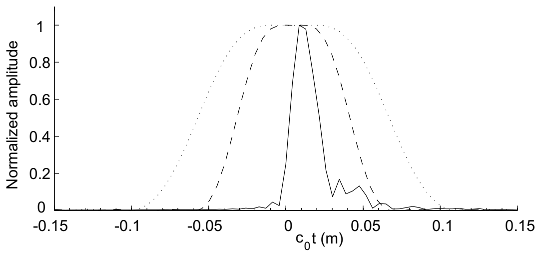

Figure 6. Time selective windows $w_1(t)$ (dotted line) and $w_2(t)$ (dashed line) and one example of the envelope of an impulse response $h(t)$ (solid line).

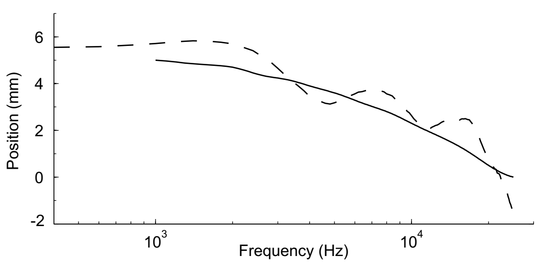

Figure 7. Acoustic center position of a condenser microphone B&K 4180; measured values, the transfer functions $H(z, f)$ being filtered with the filter W_1, (dashed line), IEC standard values [1] (solid line).

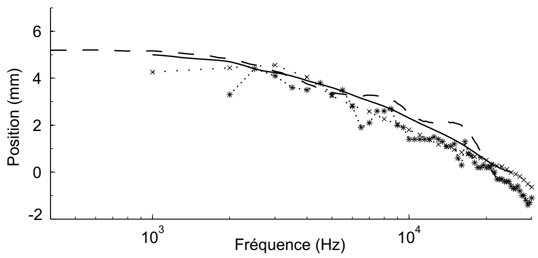

Figure 8. Acoustic center position of a condenser microphone B&K 4180; measured values, the transfer functions $H(z, f)$ being filtered with the filter W_2, (dashed line), IEC standard values [1] (solid line), and values obtained experimentally using the reciprocity method [3] (stars) and [4] (crosses).

in Figure 8 the acoustic center of the half inch microphone is represented (dashed line) after applying a most selecting filter (filter w_2, Figure 6, dashed line), showing a clear reduction of these oscillations. One can consider that in this last result most of the disturbances are eliminated. Finally, the results obtained experimentally here are in good agreement with several others presented in Figure 8, namely the standardized ones [1] (full line), and those obtained experimentally using the reciprocity method [3] (stars) and [4] (crosses).

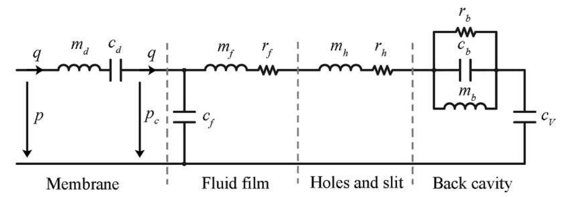

Figure A1. Lumped element circuit of electrostatic microphones.

Table A1. Values of the parameters for the electrostatic microphone B&K 4180 (LS2, half inch).

Component	Unit	B&K 4180
m_d	$\mathrm{kg\,m^{-4}}$	660
c_d	$10^{-12}\mathrm{m^3\,Pa^{-1}}$	0.068
c_f	$10^{-12}\mathrm{m^3\,Pa^{-1}}$	0.01
m_f	$\mathrm{kg\,m^{-4}}$	190
r_f	$10^6\mathrm{Pa\,s\,m^{-3}}$	93
m_h	$\mathrm{kg\,m^{-4}}$	75
r_h	$\mathrm{Pa\,s\,m^{-3}}$	$2100\sqrt{f}$
m_b	$\mathrm{kg\,m^{-4}}$	115
c_b	$10^{-12}\mathrm{m^3\,Pa^{-1}}$	0.19
r_b	$10^6\mathrm{Pa\,s\,m^{-3}}$	74
c_V	$10^{-12}\mathrm{m^3\,Pa^{-1}}$	0.952

3. Conclusion

The concept of acoustic center is important in practice, because it has a significant influence on the accuracy of the estimated free-field sensitivity of the microphones. Then, experimental investigations of the acoustic center must provide results achieving a good precision.

Several experimental methods have been used for determining the acoustic center of a transducer: one of them relies on phase measurement, but most of them are based upon the measurement of the deviations of the amplitude of the sound pressure from the inverse distance law. These last experimental methods allow to obtain results only from 2 kHz (the current lower frequency limit). In order to broaden this frequency range (in the lowest frequency domain, from 2 kHz to 400 Hz), an improved experimental method which increases the signal to noise ratio is given in the present paper. Thus, today, experimental results, which are based on the classical procedure using the inversely proportional distance law, are available from 400 Hz to 20 or 30 kHz.

Appendix

A1. Lumped element circuit to modeling the behaviour of electrostatic microphones [12]

The lumped element circuit used in the present paper to modeling the behaviour of the electrostatic microphone B&K 4180 (LS2, half inch) is the one given in reference

[12]. It is shown in Figure A1 where p and p_c are respectively the acoustic pressure in front of and behind the membrane, and q is the volume velocity of the membrane. The quantities m, r, and c (with subscripts) represent the masses, resistances, and compliances of the different parts of the microphone, respectively the membrane (subscript d), the fluid film behind the membrane (subscript f), the holes in the backing electrode (subscript h), and the rear cavity (subscripts b and V). The values of these parameters are calculated from the dimensions of the microphone, its quality factor, and its equivalent volume. They are given in Table A1. The tension T_m and the surface density σ_s are respectively given by

$$T_m = \frac{S_m^2}{8\pi c_d}, \tag{A1}$$

$$\sigma_s = \frac{3}{4} m_d S_m, \tag{A2}$$

where S_m is the surface of the membrane.

References

[1] IEC 61094-3: Measurement microphone. Part 3: Primary method for free-field calibration of laboratory standard microphones by the reciprocity technique. 1995.

[2] M. Vorländer, H. Bietz: Novel broad-band reciprocity technique for simultaneous free-field and diffuse-field microphone calibration. Acustica **80** (1994) 365–377.

[3] R. P. Wagner, V. Nedzeinistsky: Determination of acoustic center correction values for type LS2aP microphones at normal incidence. J. Acoust. Soc. Am. **104** (1998) 192–203.

[4] S. Barrera-Figueroa, K. Rasmussen, F. Jacobsen: The acoustic center of laboratory standard microphones. J. Acoust. Soc. Am. **120** (2006) 2668–2675.

[5] F. Jacobsen, S. Barrera-Figueroa, K. Rasmussen: A note on the concept of acoustic center. J. Acoust. Soc. Am. **115** (2004) 1468–1473.

[6] D. S. Jones: The scattering of a scalar wave by a semi-infinite rod of circular cross section. Phil. Trans. Roy. Soc. A **247** (1955) 499–528.

[7] Y. Ando: On the sound radiation from semi-infinite circular pipe of certain wall thickness. Acustica **22** (1970) 219–224.

[8] Y. Ando: Experimental study of the pressure directivity and the acoustic centre of the "Circular pipe horn loud speaker". Acustica **20** (1968) 366–369.

[9] E. Matsui: Free-field correction for laboratory standard microphones mounted on a semiinfinite rod. J. Acoust. Soc. Am. **49** (1970) (5 Part 2) 1475–1483.

[10] P. Juhl: A numerical investigation of standard condenser microphones. J. Sound Vib. **177** (1994) 433–446.

[11] M. Bruneau, T. Scelo (translator and contributor): Fundamentals of acoustics. ISTE, UK, USA, 2006.

[12] K. Rasmussen: The static pressure and temperature coefficients of laboratory standard microphones. Metrologia **36** (1999) 265–273.

[13] S. Barrera-Figueroa, K. Rasmussen, F. Jacobsen: On determination of microphone response and other parameters by a hybrid experimental and numerical method. J. Acoust. Soc. Am. **123** (2008) 3229.

[14] S. Barrera-Figueroa, K. Rasmussen, F. Jacobsen: A time-selective technique for free-field reciprocity calibration of condenser microphones. J. Acoust. Soc. Am. **114** (2003) 1467–1476.

[15] S. Barrera-Figueroa: New methods for transducer calibration: Free-field reciprocity calibration of condenser microphones. PhD thesis, Acoustic Technology of the Technical University of Denmark, 2003.