Examination on the FEM part

November 5, 2015

1 Introduction

We are considering the following 1D acoustic cavity at circular frequency ω.

$$\Omega =]0; L[\,,\ \frac{\partial^2 p}{\partial x^2} + k^2 p = 0\,,\ k = \frac{\omega}{c}\,,\ c = \sqrt{\frac{K}{\rho}}. \tag{1}$$

k is the wave number, c is the sound velocity, K is the compressibilty and ρ is the density.

Question 1:

Explain why the weak form of this medium can be written:

$$\forall\, q\,,\ \frac{1}{\rho\omega^2}\left[\frac{\partial p}{\partial x}(L)q(L) - \frac{\partial p}{\partial x}(0)q(0)\right] - \frac{1}{\rho\omega^2}\int_\Omega \frac{\partial p(x)}{\partial x}\frac{\partial q(x)}{\partial x}\,dx + \frac{1}{K}\int_\Omega p(x)q(x)\,dx = 0 \tag{2}$$

2 Finite-Element discretization of a first problem

We are considering the following boundary conditions: a rigid wall in $x = L$ and a harmonic displacement of amplitude 1 in $x = 0$.

Question 2:
What is the analytical value of the pressure $p(x)$?

Question 3:
We are considering the finite element discretization of the problem with 1 linear finite-element.

- What is the number of degrees of freedom ?
- What is the methodology of the discretization ?
- What is the final linear system ?

Question 4:
Present the shape of the convergence curves in the case of a discretization by linear and quadratic elements.

3 Second problem: case of two fluids

We are now considering the case where Ω is composed by two fluids. The first one has properties ρ_1 and c_1 and corresponds to volume $\Omega_1 =]0; L/2[$, the second one has properties ρ_2 and c_2 and corresponds to volume $\Omega_1 =]L/2; L[$.

Question 5:
What is the new weak form on pressure ?

We are considering a discretization where each medium is discretised with 1 linear finite-element.

Question 6:
What is the final linear system if we consider the same boundary conditions than in the first problem.

4 Third problem: case of an excitation by a plane wave

We are now considering now an excitation by a plane wave in $x = 0$. An incident wave of amplitude 1 arrives on the medium and a reflected wave of amplitude R is reflected:

$$p_{inc}(x) = e^{-jkx} \, , \, p_r(x) = Re^{jkx} \, , \, x < 0 \tag{3}$$

A rigid wall is still considered in $x = L$.

Question 7:
What is the new weak form for p in Ω. Note that this new weak form will depend on R.

Question 8:
What is the new linear system if we consider the discretization by 1 linear element.

5 Fourth problem: case of a radiation condition in $x = L$

We are now considering a radiation condition in $x = L$ instead of a rigid wall.

Question 9:
What is the new weak form ?

6 A Matlab script

Explain what is doing the following script. Modify it to model the third problem.

```
L=1;
nb_elem=10;

nb_nodes=nb_elem+1;


frequence=300;
omega=2*pi*frequence;

c=340;
rho=1.2;

k=omega/c;

x=linspace(-L,0,300);
A=-(omega*rho)/(j*k*sin (k*L));
p_analytique=A *cos( k*x);

figure

plot(x,abs(p_analytique))


h_elem=L/nb_elem;

H_elem=[1 -1;-1 1]/h_elem;
Q_elem=(h_elem/6)*[2 1 ;1 2];

A_FEM=sparse(nb_nodes,nb_nodes);
F_FEM=sparse(nb_nodes,1);

for i_e=1:nb_elem
   dof=[i_e i_e+1];
   A_FEM(dof,dof)=A_FEM(dof,dof)+H_elem/(rho*omega^2)-Q_elem/(rho*c^2);
end

F_FEM(1)=-j/(omega);

P_FEM=A_FEM\F_FEM;

x_FEM=linspace(-L,0,nb_nodes);
hold on
plot(x_FEM,abs(P_FEM),'.','Markersize',25)
```