Taxation of early retirement windows and delaying retirement: the French experience

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Abstract

This paper investigates the effect of the 2003 French pension reform on hiring, firing and employment rates among older workers. This reform increased the mandatory retirement age and simultaneously it set a tax levied on early retirement windows payed by firms to their older workers, to encourage them to leave their job early. We use a matching model with endogenous job destruction extended to account for a mandatory retirement age and we calibrate the model with data drawn from the French Labor Force Surveys for the years 2002 and 2003. We show that in the case of a high tax rate, delaying retirement raises job separation rates, which partially offsets its positive effect on job finding rates. Consequently, the combination of an increase in the retirement age and a taxation on early retirement windows may have negative effects on the employment rate among older workers.

Key words: Delaying retirement, early retirement windows, job matching models, employment protection
J23, J63, J65

1. Introduction

Effective retirement ages had fallen significantly over the last 30 years in most OECD countries. This pattern may lead to fiscal sustainability problems, especially for countries with Pay-As-You Go pension schemes, in which pensions of retired individuals are financed through contributions of

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workers. The creation of early retirement schemes in the 1970's may be an explanation of the decline in employment among elderly people observed in Europe during the same period. Zaidmann (2000) has shown that in France these schemes led to a consensus between older workers, firms and government. This phenomenon may be due to two main reasons. First, early retirement schemes were partly financed by the government so they could be treated by firms as a layoff subsidy (Hutchens, 1999, Tuulia and Uusitalo 2005). Second, the government encouraged early retirement to make more room for young workers in the labor market in a setting of high youth unemployment (Zaidman, 2000).

Since the employment rate among workers aged more than 55 in France was in the early 2000's one of the lower in the European Union (29.9% in France with respect to an European average of 37.8%[1]), the French government implemented important changes to constrain early retirement. These changes aimed simultaneously at restricting the access conditions to publicly subsidized early retirement schemes for workers and firms and at increasing the share of early retirement expenses charged to employers. However, in spite of the increase of early retirement costs for firms, employers continue to encourage their older workers to leave early their job, offering them generous financial incentives called "early retirement windows".

We can discuss the motivations that lead employers to offer early retirement windows to their older workers rather than firing them. One potential explanation suggested by Amauger-Lattes and Desbarrats (2006) is that the legislation regarding separations among older workers implies a cumbersome and often costly procedure for employers. Studying more than 300 court rulings of the Court of Cassation during the period 1994-2004 regarding job separations for older workers, they highlight that in most cases, employers strike a mutual agreement with their older workers, offering them generous early retirement windows and reporting a "dismissal for serious misconduct", although it is not the case.

In the face of this widespread phenomenon, especially in the case of big firms, the French government set in 2003 a tax levied on the amount of early retirement windows payed by firms. The tax rate amounted to 23.85% in August 2003 and in August 2007 it raised to 50%. In addition, to deter firms from pushing their older workers into retirement too early, the 2003 reform

[1] Source: Eurostat

also led to an increase in the mandatory retirement age. Initially, this age was 60, which means that when a worker reached 60 and if her insurance period was sufficient to allow her to draw a full pension, an employer could push her into retirement paying her a low retirement allowance. In 2003, this age has been increased to 65 and since 2008 it is 70.

The goal of this paper is to investigate the effect of the combination of these two reforms on the hiring rate, the job destruction rate and the employment rate among older workers. Let us first define what job destruction means in this paper. As we allow employers to offer early retirement windows to their older workers to encourage them to leave their job and to avoid a layoff, the definition of a separation (or job destruction) differs across the age group of workers. On the one hand, for middle-age workers, a separation results from a layoff and the employer has to pay to the worker a severance pay, as defined in the French Employment Protection Legislation. On the other hand, for older workers close to retirement, a separation has not the same nature. Indeed, for this age group of workers, job destruction means that the employer encourages a worker to leave her job, before she reaches the mandatory retirement age, offering her early retirement windows.

In this setting, the tax set in 2003 and levied on the amount of early retirement windows payed by firms is a sort of age-dependent employment protection, given that it concerns separations for one specific age group of workers. In this paper, we put forward the idea that in the case of a high tax rate on early retirement windows, delaying retirement may raise the separation rate among the group of workers concerned by the tax. We refer to this effect as the "impatience effect". The idea is the following: as shown in the previous literature on age-dependent employment protection, especially on the Contribution Delalande in the French case[2] (Behaghel, 2007; Behaghel et al., 2008; Hairault et al., 2007; Cheron et al., 2008), a high tax rate deters employers from laying their older workers off after a shock, even though their filled jobs imply negative profits for firms. Indeed, as long as their firing cost exceed the expected losses, employers prefer waiting until their older workers reach the mandatory retirement age. This is the well-known labor-hoarding effect of the tax. In this paper, we point out that in this setting, delaying retirement leads to an increase in expected losses incurred by firms, making

[2]The Contribution Delalande is a tax payed by firms who fire workers aged more than 50

them more impatient to get rid of their older workers. Consequently, an increase in the retirement age may encourage firms to offer generous early retirement windows to their older workers to force them to leave, rather than waiting until they reach the new mandatory retirement age.

Highlighting this so-called impatience effect, our paper gains new insights into the effect of an increase in the retirement age on job creation, job destruction and employment. The effect of an increase in the legal retirement age on employment among older workers has been already considered in the literature. Previous studies pointed out that delaying retirement may have a positive "horizon effect" on job creation, when labor is treated as a quasi-fixed factor that implies fixed costs (Oi, 1962). These costs result from either a bilateral monopoly problem (Hutchens, 1986) or from an accumulation of specific human capital through training (cf Hashimoto 1981) and imply that firms are more reluctant to hire a worker close to the retirement age. Furthermore focussing on job destruction, Aubert et al. (2006) showed that there is an age-bias technological change, so employers are less likely to retain an older worker in the case of a shock on her job, if her employment duration is too short. Extending the model of Mortensen and Pissarides (1994) in order to account for the life cycle of the worker with a bounded retirement age, Chéron et al. (2007) have drawn similar conclusions and argue that an increase in the retirement age may have a positive effect on hiring rates and may reduce firing rates by lengthening the employment duration of older workers.

In this respect, the impatience effect that we highlight in this paper may offset the horizon effect in the case of high age-dependent employment protection, leading therefore to a rise in separation rates among the age group concerned by the tax. While previous studies on age-dependent employment protection (Behaghel, 2007; Behaghel et al., 2008; Hairault et al., 2007; Cheron et al., 2008) showed theoretically and empirically that an age-specific firing tax may have a strong negative effect on job creation among the protected age group and also that it may raise separation rates among the previous cohort of workers not concerned by the tax, these works did not pay any attention to the effect of delaying retirement on job creation, job destruction and employment in the case of a high age-specific separation cost.

In this paper, we determine a critical value of the rate of the tax on early retirement windows, above which the impatience effect is higher than the horizon effect. If the tax rate is higher than this critical value, an increase in the mandatory retirement age raises job destruction among older workers.

In that case, the positive horizon effect of delaying retirement on job finding rates among the age group of workers concerned by the tax is attenuated. In addition, as an increase in the retirement age reduces transitions from unemployment to retirement, it exerts a negative effect on employment among older workers, if the horizon effect is not sufficiently strong. Consequently, the higher the tax rate on early retirement windows, the lower the horizon effect with respect to the impatience effect and the stronger the negative effect of postponing retirement on employment among the protected age group.

Calibrating our model using data drawn from the French Labor Force Survey for the years 2002-2003, we provide a numerical illustration of these findings. We show that the change in the job separation rate for older workers after an increase in the retirement age strongly depends on the tax rate. Consequently, the effect of delaying retirement on employment among the protected age group is sensitive to the level of taxation of early retirement windows.

The remainder of the paper is structured as follows. In the next section, using the data drawn from the French Labor Force Survey for the period 2001-2009, we provide a brief presentation of the patterns in job destruction rates among older workers between 2001 and 2009, investigating to what extent separation rates have been affected by the 2003 and 2008 reforms. Then, using data from the Survey on Health Ageing and Retirement in Europe (SHARE), we show the incidence of early retirement windows in the job destruction among older workers in France. In section 3, we describe in detail the theoretical model, following the specification of Behaghel (2007). In section 4 we present our main theoretical findings on the effect of an increase in the mandatory retirement age on hiring rates and firing rates of middle-age and older workers in a setting of partial employment protection. In the section 5, we describe our quantitative analysis based on the French Labor Force Surveys for the years 2002 and 2003 and we present our results. Section 6 concludes.

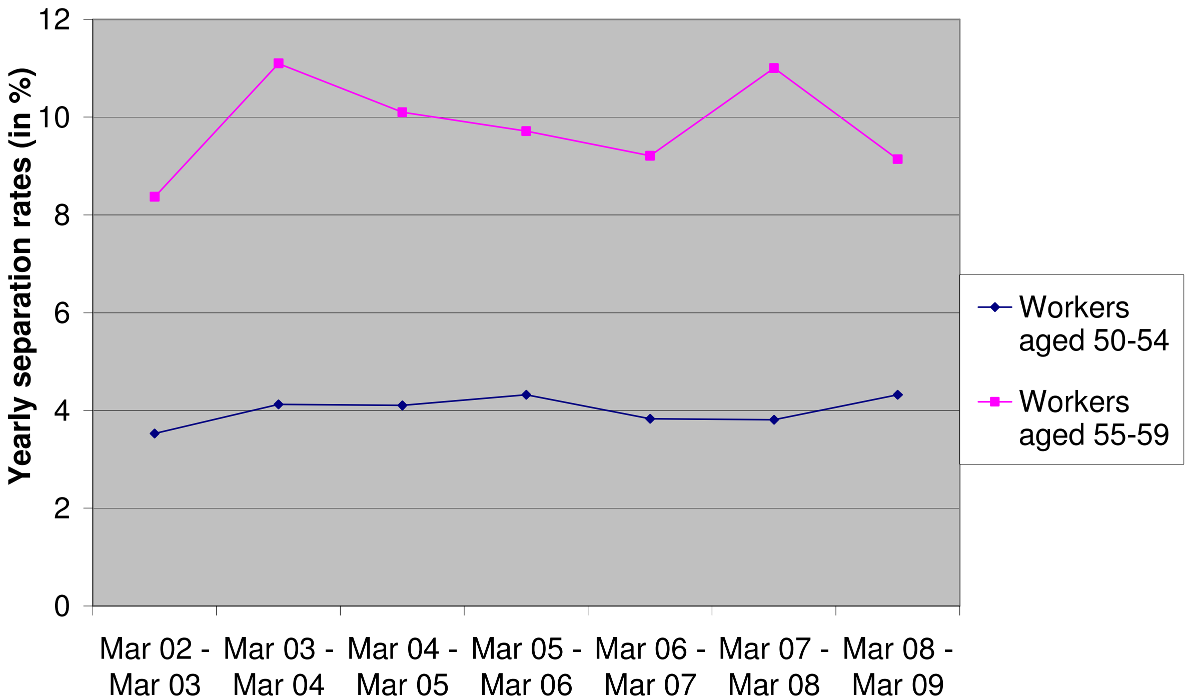
2. Early retirement in France from 2001 to 2009

As mentioned in the introduction, in France access conditions to publicly subsidized early retirement schemes have been severely restricted for workers and firms since the early 2000's. This policy has had two main effects: on the one hand, it has divided by 10 the number of yearly entries in such early retirement schemes. On the other hand, it has led to a sharp increase in the number of older unemployed without job-search requirements but who remain on unemployment benefits until they reach the retirement age (DARES, 2010). As this so-called "unemployment tunnel" has been used by firms since the early 2000's as a new way to get rid of their older workers, unemployment spells can be treated as pre-retirement periods in the case of older workers. In addition, as stated by Amauger-Lattes and Desbarrats (2006), separations result in most cases from a mutual agreement between older workers and employers, for which firms accept to pay early retirement windows to encourage their workers to benefit from the unemployment tunnel.

When investigating the effect of the 2003 reform on separation rates among older workers, we would expect that setting a tax on early retirement windows payed by firms would reduce separations, especially for workers close to the retirement age. Furthermore, as this reform led to an increase in the mandatory retirement age, the horizon effect would imply a stronger decrease in the transition rate from employment to unemployment or inactivity. However, using data from the French Labor Force Survey for the period 2002 to 2009, we observe the reverse story. Indeed, the figure 1 shows that the yearly separation rate among the workers aged 55-59 rose from 8.37% for the period between March 2002 and March 2003 to 11.1% for the period March 2003-March 2004. At the same time, the separation rate among the workers aged 50-54 displays a more slightly increase from 3.53% to 4.12%. Similarly, when examining the effect of the August 2007 increase in the tax rate from 23.85% to 50% along with an increase in the mandatory retirement age from 65 to 70 on separation rates among older workers, we also remark a sharp increase in transition rates out of employment among workers aged 55-59 from 9.21% for the period between March 2006 and March 2007 to 11% for the period March 2007-March 2008. At the same time, the job destruction rate among the workers 50-54 remained unchanged. These stylized facts leave some room for other theoretical explanations than the well-known horizon effect of an increase in the retirement age (Hairault et al., 2007) or the labor-hoarding effect of age-dependent employment protection (Behaghel, 2007; Behaghel et al., 2008; Hairault et al., 2007; Cheron et al., 2008).

However, for our theoretical framework to be relevant, we have to investigate the incidence of early retirement windows in the job destruction rate among older workers. While this pattern has been already considered in the

Figure 1: Yearly separation rates among older workers in France from 2002 to 2009



Note: The yearly separation rate is the probability that a worker employed in March of year t is out of employment in March of year t+1

Source: French Labor Force Survey (2002-2009)

American case[3], too few studies examined this firms behavior in the European case. Using data from the 1997 International Social Survey Program, Dorn and Sousa-Poza (2007) investigate the incidence of involuntary early retirement for 19 countries[4], asking retired respondents if they retired early "by choice" or "not by choice". Their analysis covers the early retirement of individuals aged between 45 and 64 who retired between 1983 and 1997. Providing some descriptive statistics, they show that in some European countries like Germany or Portugal, more than half of all retired respondents state that they retire early "not by choice". In France this proportion amounts to 41%, which is also very high.

To provide a decomposition of the different reasons that led older workers to leave their job, we use the wave 2006 of the Survey on Health Ageing and Retirement in Europe (SHARE). This data provides some information for eleven countries[5] on the factors of early exit. We include in our sample only French retired respondents aged between 57 and 69 in 2006, who were not entitled to a public or private pension when they left their activity. We obtain a sample made up of 182 individuals. We remark in the table 1 that in France early exit stems from 2 main reasons. First, almost one third of the sample report that they left their job after they had been given early retirement windows from their employer. Second, around 30% of the individuals report that they left early their job due to their bad health status.

Even though the incidence of early retirement windows in separations among older workers may be overestimated owing to a justification bias, we observe that the fraction of individuals reporting an early exit due to financial incentives offered by their employer is strongly higher than the fraction of individuals reporting an exit due to a simple layoff. So the table 1 provides some empirical evidence of the specific nature of the separations regarding older workers, justifying therefore our theoretical framework.

[3]See for instance Brown (1999)

[4]Canada, Cyprus, Denmark, France, Germany, Great Britain, Hungary, Italy, Japan, the Netherlands, New Zealand, Norway, Poland, Portugal, Slovenia, Spain, Sweden, Switzerland and USA

[5]Austria, Germany, Sweden, Netherlands, Spain, Italy, France, Denmark, Greece, Switzerland and Belgium

Table 1: Reasons for early retirement in France

Reasons for early exit	**Number of observations**	**Frequency (in %)**
Early retirement windows	71	39.01
Layoff	10	5.49
Own ill health	54	29.67
Ill health of a relative	13	7.14
Retire at same time as spouse	11	6.04
To spend more time with family	13	7.14
To enjoy life	10	5.49
Total	182	100

Note: Our sample contains all French retired individuals aged between 55 and 69 in 2006 who were not entitled to a public or private pension when they left their activity
Source: SHARE (waves 2004 and 2006)

3. The model

3.1. The economy

Following the specification of Mortensen and Pissarides (1994), we consider an economy in which firms produce one type of good using only one factor of production: the labor. For a sake of simplicity, we assume that a firm can not employ more than one worker and the number of jobs is endogenous. Following Behaghel (2007), we consider two age groups of workers: the middle-age workers (group C_1) and the older workers (group C_2). A middle-age worker may switch to the next age group at a Poisson arrival rate η_1. Similarly, an older worker may reach the mandatory retirement age at a Poisson arrival rate η_2.

In our model the Poisson arrival rate η_2 is a key parameter to determine the horizon of older workers denoted by H. We define H in the following way:

$$H = \int_0^{+\infty} t\eta_2 e^{-\eta_2 t} dt = \frac{1}{\eta_2}$$

In the remainder of the paper we assimilate an increase in the mandatory retirement age to a decrease in η_2.

We consider an economy "à la" Mortensen Pissarides (1994) with endogenous job creation and destruction. In this model, workers and firms with vacant jobs meet each other according to a matching function $m(u_i, v_i)$, that represents the number of matches as a function of the unemployment rate u_i among the group C_i and the vacancy rate v_i targeted to job-seekers belonging to the group C_i. Here, we assume that matching markets are segmented by age, so firms can target their job offer toward one particular age group. To test the robustness of our theoretical findings, we will discuss in the end of the section 5 the implications of an alternative specification with only one matching market for both types of workers.

We also assume that the matching function is increasing, concave in each argument and linear homogeneous. Let θ_i be the labor market tightness namely the number of vacancies per worker, so we can define the Poisson arrival rate $q(\theta_i)$ of a match for an employer posting a vacancy targeted to job seekers belonging to the group C_i.

$$q(\theta_i) = \frac{m(u_i, v_i)}{v_i} = m(\frac{1}{\theta_i}, 1) \quad i = 1, 2 \tag{1}$$

Therefore $q(\theta_i)$ is a decreasing function of the tightness θ_i. Furthermore we can define the Poisson arrival rate $p(\theta_i)$ of a match for a job seeker belonging to the group C_i:

$$p(\theta_i) = \frac{m(u_i, v_i)}{u_i} = \theta_i q(\theta_i) \quad i = 1, 2 \tag{2}$$

So $p(\theta_i)$ is an increasing function of the tightness θ_i. Consequently, θ_i is an endogenous key variable to determine the job-finding rates of each age group of workers in our economy.

3.2. The firms' behaviour

In our model each firm has one job that can be either filled and producing or vacant and searching. As long as the job is vacant, firms pay a cost c of maintaining a vacancy. When the vacancy is matched with a worker, her idiosyncratic productivity $y\epsilon$ is drawn randomly from the fixed distribution $G(\epsilon)$ with $\epsilon \in [0, \overline{\epsilon}]$. The firm hires the worker if ϵ is higher than the productivity threshold ϵ_i^c. Consequently, ϵ_i^c is an other endogenous key variable to determine the hiring rate of each age group of workers.

Let J_v^i be the value to an employer of posting a vacancy targeted on workers belonging to the group C_i. At steady-state, we obtain the following

Bellman equations:

$$rJ_v^i = -c + q(\theta_i)[\int_0^{\overline{\epsilon}} \max\{J_i(x), 0\} dG(x) - J_v^i] \quad i = 1, 2 \tag{3}$$

where $J_i(\epsilon)$ is the asset value of a job filled by a worker belonging to the age group C_i with a productivity level ϵ. Under the free-entry condition, the flow value to the employer from opening a new vacancy is equal to zero at steady-state equilibrium. Therefore we get :

$$\int_{\epsilon_i^c}^{\overline{\epsilon}} J_i(x) dG(x) = \frac{c}{q(\theta_i)} \quad i = 1, 2 \tag{4}$$

This first condition implies that the mean search cost must be equal to the value to an employer of a filled job. An increase in this value encourages therefore employers to open more vacancies.

When a job is filled, a worker belonging to the age group C_i starts producing an output $y\epsilon$, where ϵ is the random component of the productivity, and he receives a productivity-contingent wage $w_i(\epsilon)$. Then the job can be hit by an idiosyncratic shock at a Poisson arrival rate λ. In that case, a new random productivity level ϵ is drawn according to a cumulative distribution function $G(x)$ and the employer has no other choice either to close down the job or to keep the worker. Existing filled jobs are destroyed if the productivity level falls below a productivity threshold ϵ_i^d. Consequently, a job occupied by a worker belonging to the age group C_i may be destroyed at a Poisson arrival rate $\lambda G(\epsilon_i^d)$. We assume that λ does not differ across age groups so ϵ_i^d is a key endogenous variable to determine job destruction among each age group of workers.

In our model, we allow employers to offer early retirement windows to their older workers to encourage them to leave their job and to avoid a layoff. As we mentioned in the introduction, this firm's behavior is the response of employers to the tightening of access conditions to publicly financed early retirement schemes and an increasingly part of employers offer early retirement windows since the early 2000's. Consequently, in our model, we consider that the separation cost for an employer depends on the age of the worker. If a job occupied by a middle-age worker breaks up, her employer has to pay her a severance pay denoted by f_1 according to the French Employment Protection Legislation. However, if a job occupied by an older worker breaks up, before she reaches the mandatory retirement age, her employer has to offer to her early retirement windows denoted by f_2. Reproducing the 2003 French pension reform, we set a tax levied on the amount of the early retirement window payed by the employer at a rate τ. However, once an older worker reaches the mandatory retirement age, the job automatically breaks up and the separation cost for the employer is only f_r, that is a low retirement allowance.

In addition, recall that in our model a middle-age worker may switch to the next age group at a Poisson arrival rate η_1. In that case, if her random component of productivity ϵ is lower than the productivity threshold ϵ_2^d, her job breaks up. So for each age group of workers C_i $(i \in 1, 2)$, the value to an employer of hiring a worker with a random component of productivity level equal to ϵ is defined by the following Bellman equations:

$$\begin{aligned} rJ_1(\epsilon) = y\epsilon - w_1(\epsilon) + \lambda[\int_0^{\overline{\epsilon}} \max\{J_1(x), -f_1\} dG(x) - J_1(\epsilon)] \\ +\eta_1[\max\{J_2(\epsilon), -f_2(1+\tau)\} - J_1(\epsilon)] \end{aligned} \tag{5}$$

And:

$$\begin{aligned} rJ_2(\epsilon) = y\epsilon - w_2(\epsilon) + \lambda[\int_0^{\overline{\epsilon}} \max\{J_2(x), -f_2(1+\tau)\} dG(x) - J_2(\epsilon)] \\ +\eta_2[(J_v^2 - f_r) - J_2(\epsilon)] \end{aligned} \tag{6}$$

3.3. Rent-sharing rules

We assume that the wage is set to split the match surplus between the firm and the worker at all times and in fixed proportions, as in the case of a standard Nash wage bargaining. The worker's share is β. In a setting of employment protection, we have to consider two rent-sharing rules. Indeed, when a worker is matched with a vacancy, no severance payment has to be paid if negotiation fails. However, following the standard model of Mortensen and Pissarides (1994), we assume that wages are renegotiated continuously so that the wage received by a worker accounts for the employment protection he will benefit from in the case of a layoff. So, we may define a first rent-sharing rule when the worker is hired, that determines a potential wage w_i^0 in the following way:

$$w_i^0(\epsilon) = argmax\{[W_i(\epsilon) - U_i]^{\beta}[J_i(\epsilon)]^{(1-\beta)}\} \quad i = 1, 2 \tag{7}$$

where $W_i(\epsilon)$ is the flow value to a worker belonging to the age group C_i from employment and U_i is the flow value to an unemployed worker belonging to the age group C_i. Solving the program (7), we get the following first rent-sharing rule:

$$W_i(\epsilon) - U_i = \beta S_i^0(\epsilon) = \beta[J_i(\epsilon) + W_i(\epsilon) - U_i] \tag{8}$$

where $S_i^0(\epsilon)$ is the match surplus from a job creation targeted to workers belonging to the age group C_i. As the wage is assumed to be renegotiated immediately, we get the new maximization program:

$$w_i(\epsilon) = argmax\{[W_i(\epsilon) - U_i - f_i]^{\beta}[J_i(\epsilon) + f_i(1+\tau_i)]^{(1-\beta)}\} \quad \text{with } \tau_1 = 0 \text{ and } \tau_2 = \tau \tag{9}$$

Solving (9) we get the following rent-sharing rule:

$$W_i(\epsilon) - U_i - f_i = \beta S_i(\epsilon) = \beta[J_i(\epsilon) + \tau_i f_i W_i(\epsilon) - U_i] \tag{10}$$

We remark that a job filled by a middle-age worker breaks up if $J_1(\epsilon) \leq -f_1$ which implies $W_1 \leq U_1 + f_1$. In a similar way, a job filled by an older worker breaks up if $J_2(\epsilon) \leq -f_2(1+\tau)$, which implies $W_2 \leq U_2 + f_2$.

Let us first define the flow value from employment to a worker of the group C_i. When he is hired and as long as her job is not hit by an idiosyncratic shock, she receives the productivity-contingent wage $w_i(\epsilon)$. When a shock occurs at a Poisson arrival rate λ, the productivity level changes and the match may be dissolved. In that case the worker receives a payment from her employer f_i (f_1 corresponding to a severance payment and f_2 corresponding to the amount of the early retirement window received). If she remains employed despite the shock, she receives a new wage $w_i(\epsilon)$, which changes the value of her job $W_i(\epsilon)$. Furthermore, a middle age-worker occupying a job with a random component of productivity level equal to ϵ may switch to the next age group at a Poisson arrival rate η_1 and if her job does not break up she benefits from the discounted income flows an older worker derives from the same job. Similarly, an older worker may reach the mandatory retirement age at a Poisson arrival rate η_2 and then her job breaks up and she receives a retirement allowance f_r. In that case, the worker is retired and she benefits from a pension P discounted over an infinity of time. So for each age group of workers, the flow value from employment to a worker satisfies the following Bellman equations:

$$rW_1(\epsilon) = w_1(\epsilon) + \lambda[\int_0^{\bar{\epsilon}} \max\{W_1(x), U_1 + f_1\} dG(x) - W_1(\epsilon)]$$

$$+\eta_1[\max\{W_2(\epsilon), U_2 + f_2\} - W_1(\epsilon)] \tag{11}$$

and:

$$rW_2(\epsilon) = w_2(\epsilon) + \lambda[\int_0^{\bar{\epsilon}} \max\{W_2(x), U_2 + f_2\}dG(x) - W_2(\epsilon)]$$

$$+\eta_2[(f_r + U_3) - W_2(\epsilon)] \quad \text{where } rU_3 = P \tag{12}$$

Furthermore, we determine the present flow value from unemployment to a worker belonging to the age group C_i, denoted by U_i by the following equation:

$$rU_i = z_i + p(\theta_i)[\int_0^{\bar{\epsilon}} \max\{W_i^0(x), U_i\}dG(x) - U_i] + \eta_i(U_{i+1} - U_i) \quad i = 1,2 \tag{13}$$

where z_i is the non-labor income received by an unemployed worker belonging to the age group C_i. It is noteworthy that an older unemployed worker who retires does not receive any retirement allowance f_r.

In the appendix (7.1), we determine the wage equations for each age group of workers:

$$w_1(\epsilon) = (1-\beta)z_1 + \beta(y\epsilon + c\theta_1 - \eta_1 f_2\tau) + f_1(r + \eta_1) - \eta_1 f_2 \tag{14}$$

$$w_2(\epsilon) = \beta y\epsilon + (1-\beta)z_2 + \beta c\theta_2 + (r + \eta_2)[f_2(1 + \beta\tau)] - \eta_2 f_r \tag{15}$$

We observe first that the wage received by a worker belonging to the group C_i is an increasing function of the non-labor income z_i and of the probability to be matched with a job $p(\theta_i)$, given that these variables raise the worker's threat point, allowing her to extract a higher share of the match surplus from wage bargaining. Furthermore, we remark that the wage of a middle-age worker decreases with the amount of firing costs f_2 and that the wage received by an older worker decreases with the amount of retirement allowance f_r. These results are consistent with Lazear's findings (1990) that show that the higher the employment protection of a worker the lower his wage has to be in the beginning of his career.

3.4. Job destruction and job creation at steady-state

As we already mentioned in the subsection (3.2), in the case of a shock on a job, the employer has no other choice either to retain the worker with the new value of ϵ, the random component of productivity drawn from a distribution $G(x)$, or to close down the job. In the appendix 7.2, we determine two productivity thresholds, below which existing matches are dissolved.

$$y\epsilon_1^d = z_1 + \frac{\beta c}{1-\beta}\theta_1 - \lambda \int_{\epsilon_1^d}^{\bar{\epsilon}} S_1(x)dG(x) - \eta_1 \max\{S_2(\epsilon_1^d), 0\} + \eta_1 f_2 \tau \quad (16)$$

and:

$$y\epsilon_2^d = z_2 + \frac{\beta c}{1-\beta}\theta_2 - \lambda \int_{\epsilon_2^d}^{\bar{\epsilon}} S_2(x)dG(x) - (r + \eta_2) f_2 \tau \quad (17)$$

We observe that the productivity threshold is less than the opportunity cost of employment, composed of non-labor income z_i and of the expected gain to search for a job. Indeed, the third term on the right-hand side of (16) and (17) represents the option value of retaining an existing match despite a shock. This labor-hoarding phenomenon is due to the fact that firms are faced with a positive cost of maintaining a vacancy and therefore they accept to incur a loss in anticipation of a future improvement in the value of the match's product[6].

Regarding the effects on employment protection in the case of the workers belonging to the group C_1, we draw the same conclusions as Lazear (1990): any severance payment arrangement is neutral on the firing decision of firms through an optimal labor contract, in which a worker is willing to pay a fee when she signs the contract to buy the protection of her job. In a similar way, the amount of retirement allowance f_r is also neutral on the firing decision of firms regarding the older workers.

Furthermore in the case of the older workers, we observe that the amount of early retirement window payed by the firm f_2 has a negative impact on the threshold productivity ϵ_2^d. This may be due to the fact that a third agent, the government, receives a part of this payment through a tax at a rate τ, which implies that the worker is not given the whole payment when he is

[6]This type of labor-hoarding behavior has been well investigated by Mortensen Pissarides (1994)

fired: in that case, following the Lazear's theory (1990), firing incentives are distorted[7]. Indeed, as firms expect that firing older workers is more costly, they are more reluctant to close down their jobs. However, we also observe a threshold effect regarding the younger cohort of workers, in the sense that firms have interest to lay middle-age workers off before they switch to the next age group [8].

Regarding hiring rate of each age group of workers, we determine the productivity threshold ϵ_i^c below which the employer does not recruit an unemployed worker belonging to the group C_i, so $S_i^0(\epsilon_i^c) = 0$. Using (8) and (10), we deduce:

$$S_i(\epsilon) = S_i^0(\epsilon) + f_i\tau_i \quad \text{with } \tau_1 = 0 \text{ and } \tau_2 = \tau \tag{18}$$

Using the expression (18) we determine the productivity threshold ϵ_i^c as a function of ϵ_i^d:

$$\epsilon_i^c = \epsilon_i^d + (r + \eta_i + \lambda) f_i \tau_i \quad i = \{1, 2\} \tag{19}$$

Consequently, an increase in the firing costs f_i reduces the hiring rate of the workers belonging to the group C_i, only if the tax rate τ_i is higher than 0. Furthermore, the free-entry condition (4) and the two rent-sharing rules (8) and (10) imply:

$$(1 - \beta) \int_{\epsilon_i^c}^{\overline{\epsilon}} S_i^0(x) dG(x) = \frac{c}{q(\theta_i)} \tag{20}$$

At steady-state equilibrium, θ_2 and ϵ_2^d solve the following equation system:

$$\begin{cases} \frac{c}{q(\theta_2)} = & (1-\beta) \int_{\epsilon_2^c}^{\overline{\epsilon}} [\frac{y(x-\epsilon_2^d)}{(r+\eta_2+\lambda)} - f_2\tau] dG(x) \\ y\epsilon_2^d = z_2 + \frac{\beta c}{1-\beta}\theta_2 - \frac{\lambda}{(r+\eta_2+\lambda)} \int_{\epsilon_2^d}^{\overline{\epsilon}} [y(x-\epsilon_2^d)] dG(x) - (r+\eta_2) f_2 \tau \\ y\epsilon_2^c = & y\epsilon_2^d + (r+\eta_2+\lambda) f_2\tau \end{cases}$$

[7]There may be several other cases in which severance payment would be non neutral on firing decision. For instance, if we had considered a wage posting case or if we had introduced risk aversion, the amount of early retirement windows would have also affected separation rates

[8]For similar results, see Behaghel (2007), Hairault et al. (2007) and Chéron et al., (2008)

As the first and the second equation describe respectively a downward-sloping and an upward-sloping curve, there exists one unique solution (θ_2, ϵ_2^d) to this problem.

Regarding the workers belonging to the age group C_1, there may be two cases, depending on whether their job may break up when they are ageing or not. In the case 1, the worker keeps his job even though he switches to the next age group. It implies that the reservation productivity ϵ_1^d is higher than ϵ_2^d. Determining the match surplus in appendix 7.3, we show that $(\tilde{\epsilon_1^d}, \tilde{\theta}_1)$ solves the following equation system:

$$\begin{cases} \frac{c}{q(\tilde{\theta_1})} = & (1-\beta)\frac{(r+\eta_1+\eta_2+\lambda)}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}\int_{\tilde{\epsilon_1^d}}^{\overline{\epsilon}} y(x-\tilde{\epsilon_1^d})dG(x) \\ y\tilde{\epsilon_1^d} = & z_1 + \frac{\beta c}{1-\beta}\tilde{\theta}_1 - \frac{\lambda(r+\eta_1+\eta_2+\lambda)}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}\int_{\tilde{\epsilon_1^d}}^{\overline{\epsilon}} y[x-\tilde{\epsilon_1^d}]dG(x) - \eta_1 y\frac{\tilde{\epsilon_1^d}-\epsilon_2^d}{(r+\eta_2+\lambda)} + \eta_1 f_2\tau \end{cases}$$

In the case 2, the worker does not keep necessarily his job when he switches to the next age group. It implies that $\epsilon_1^d \leq \epsilon_2^d$. Determining the match surplus in appendix 7.3 we show that $(\hat{\epsilon_1^d}, \hat{\theta}_1)$ solves the following equation system:

$$\begin{cases} \frac{c}{q(\hat{\theta_1})} = \frac{(1-\beta)}{(r+\eta_1+\lambda)}\int_{\hat{\epsilon_1^d}}^{\overline{\epsilon}} y(x-\hat{\epsilon_1^d})dG(x) + (1-\beta)\frac{\eta_1}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}\int_{\epsilon_2^d}^{\overline{\epsilon}}[y(x-\epsilon_2^d)]dG(x) \\ \\ y\hat{\epsilon_1^d} = z_1 + \frac{\beta c}{1-\beta}\hat{\theta}_1 - \frac{\lambda}{(r+\eta_1+\lambda)}\int_{\hat{\epsilon_1^d}}^{\overline{\epsilon}}[y(x-\hat{\epsilon_1^d})]dG(x) - \frac{\lambda\eta_1}{(r+\eta_1+\lambda)}\int_{\epsilon_2^d}^{\overline{\epsilon}}[\frac{y(x-\epsilon_2^d)}{(r+\eta_2+\lambda)}]dG(x) + \eta_1 f_2\tau \end{cases}$$

In the appendix 7.4 we show that the equilibrium solves either the first system or the second. Furthermore as in each system the two first equations describe respectively a downward-sloping and an upward-sloping curve, we deduce that there exists one unique solution, that may be either the couple $(\tilde{\epsilon_1^d}, \tilde{\theta}_1)$ or the couple $(\hat{\epsilon_1^d}, \hat{\theta}_1)$.

3.5. Unemployment at steady-state equilibrium

We determine the equilibrium values of the unemployment rate u_i and the vacancy rate v_i among each group of workers, using two sets of steady-state conditions. The first implies that, for each age group C_i of workers, the labor force N_i is constant so we get:

$$N_0\eta_0 = N_1\eta_1 = N_2\eta_2 \tag{21}$$

These two equations characterize N_1 and N_2 for exogenous values of N_0, η_0, η_1 and η_2. The second condition implies that for each age group, the flow of workers out of unemployment equals the flow of workers back into unemployment:

$$\lambda G(\epsilon_i^d)(1-u_i)N_i + N_{i-1}\eta_{i-1}u_{i-1} = p(\theta_i)u_i N_i[1-G(\epsilon_i^c)] + \eta_i u_i N_i \qquad (22)$$

So combining (21) and (22), we determine the unemployment rate u_i in the following way:

$$u_i = \frac{\lambda G(\epsilon_i^d) + u_{i-1}\eta_i}{\lambda G(\epsilon_i^d) + p(\theta_i)[1-G(\epsilon_i^c)] + \eta_i} \qquad (23)$$

As $p(\theta_i)$ is an increasing function of θ_i and therefore of the vacancy rate v_i, we find the expression of the Beveridge curve [9] for each generation of workers, that is an inverse relation between vacancy and unemployment rate. Furthermore, defining the labor market tightness at steady-state equilibrium, we find an other relation between u_i and v_i such that $v_i = \theta_i u_i$. Therefore the equilibrium unemployment exists and is unique, at the intersection between the Beveridge Curve and the increasing curve whose equation is $v_i = \theta_i u_i$.

4. The effect of the 2003 French pension reform on hiring and job separation rates by age group of workers

To study the effect of a tax on early retirement windows payed by firms combined with an increase in the mandatory retirement age on hiring and separation rates by age, we make some assumptions regarding the functional forms of the matching function and of the distribution of the component ϵ of the productivity levels. First, we assume that matching function is Cobb-Douglas such that:

$$m(u_i, v_i) = u_i^{\alpha} v_i^{1-\alpha}$$

where α is the elasticity of the matching function. Furthermore, we assume that ϵ_i follows an uniform distribution in the interval $[0, 1]$.

In this section, we study in a first subsection the effect of an increase in the tax rate τ on hiring and job separation rates of older workers. Then, we examine the effect of an increase in the mandatory retirement age in a setting of a taxation of early retirement windows, to investigate the effect of the combination of both these reforms on transition rates among older

[9] see notably Blanchard et al. (1989)

workers. In a second subsection, we investigate the effect of an increase in the tax rate τ on job finding and separation rates among middle-age workers, then we examine the effect of a combination of this tax with an increase in the mandatory retirement age on these rates.

4.1. A qualitative analysis for older workers

Under the assumptions defined in the beginning of this section, we determine the job creation condition C^2 and the job destruction condition D^2 such that:

$$\begin{cases} C^2(\theta_2, \epsilon_2^d, \tau, \eta_2) = \frac{(1-\beta)y}{2(r+\eta_2+\lambda)}(1-\epsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y})^2 - c\theta_2^\alpha = 0 \\ D^2(\theta_2, \epsilon_2^d, \tau, \eta_2) = z_2 + \frac{\beta c}{(1-\beta)}\theta_2 - \frac{\lambda y}{2(r+\eta_2+\lambda)}(1-\epsilon_2^d)^2 - (r+\eta_2)f_2\tau - y\epsilon_2^d = 0 \end{cases}$$

Let C_i^2 and D_i^2 be respectively the partial derivatives of C^2 and D^2 with respect to their i-th argument. Differentiating this equations system with respect to ϵ_2^d, we obtain:

$$\begin{cases} \frac{\partial \epsilon_2^d}{\partial \tau} = \frac{C_3^2 D_1^2 - D_3^2 C_1^2}{D_2^2 C_1^2 - C_2^2 D_1^2} \\ \frac{\partial \epsilon_2^d}{\partial \eta_2} = \frac{C_4^2 D_1^2 - D_4^2 C_1^2}{D_2^2 C_1^2 - C_2^2 D_1^2} \end{cases}$$

We show in the appendix (7.5) that an increase in τ reduces job separation rates among older workers, which is consistent with existing literature on age-dependent employment protection (Behaghel, 2007; Behaghel et al., 2008; Hairault et al., 2007; Cheron et al., 2008).

Then, we examine the effect of a decrease in η_2, that is an increase in the mandatory retirement age, on separation rates. In absence of taxation of early retirement windows, delaying retirement reduces unambiguously the separation rate among older workers through a labor-hoarding effect, as shown in previous studies (Chéron et al., 2007). The underlying intuition is that an increase in the mandatory retirement age raises the option value of the employer to retain an existing match despite a shock, given that the employer expects a higher duration of this job.

The new theoretical result that we highlight in this paper is about the effect of an increase in the mandatory retirement age in a setting of a taxation of early retirement windows, as this is the case for the French 2003 and 2008 reforms. In that case, we show that delaying mandatory retirement age has two offsetting effects on the productivity threshold ϵ_2^d. On the one hand, it reduces job destruction through the well-known labor-hoarding effect of the tax. On the other hand in the case of a high tax rate τ, an increase in the mandatory retirement age may encourage employers to dismiss their older workers, offsetting therefore the dissuasive effect of the tax through a new effect that we refer to as the "impatience effect". The idea is the following: in the case of a high taxation of early retirement windows, employers have interest to retain their older workers, even though the present value of their job is negative after a productivity shock. Indeed, as long as the loss in profits does not exceed the separation costs due to the tax, employers prefer waiting until their worker reach the mandatory retirement age. In this setting, when the government raises the mandatory retirement age, the horizon along which firms will incur losses in profits is longer and employers could be more impatient to dismiss their older workers. In that goal, they will offer to them early retirement windows to force them to leave. This impatience effect will therefore raise job separations among older workers, offsetting the initial labor-hoarding effect of the tax.

Consequently, the effect of postponing retirement on job separations among older workers is ambiguous and depends widely on the tax rate τ. Indeed the higher the tax rate, the more likely employers to accept to incur important loss in profits, waiting until their workers reach the mandatory retirement age and the stronger the impatience effect after an increase in the mandatory retirement age. So we define a sufficient condition under which the impatience effect is higher than the labor-hoarding effect, which implies that delaying retirement will raise separation rate among older workers. We represent this sufficient condition with a critical value of the tax rate denoted by τ^c, above which the impatience effect will dominate the labor-hoarding effect.

Proposition 1. *For values of the tax rate τ sufficiently high such that $\tau > \tau^c = \frac{2\lambda y(1-\epsilon_2^d)^2}{f_2[2(r+\eta_2)]^2}$, then $\partial\epsilon_2^d/\partial\eta_2 < 0$.*

Proof: Computing the partial derivative $\partial\epsilon_2^d/\partial\eta_2$, we deduce the following condition:

$$\frac{\partial\epsilon_2^d}{\partial\eta_2} < 0 \Leftrightarrow D_4^2 < 0$$

In the appendix (7.5), we determine D_4^2, and we find that delaying retirement may lead to raise the productivity threshold ϵ_2^d if:

$$\tau > \frac{2\lambda y(1-\varepsilon_2^d)^2}{[2(r+\eta_2+\lambda)]^2 f_2} \tag{24}$$

Consequently, as $\tau^c > \frac{2\lambda y(1-\varepsilon_2^d)^2}{[2(r+\eta_2+\lambda)]^2 f_2}$, it is a sufficient condition under which $\partial\epsilon_2^d/\partial\eta_2 < 0$. ∎

First, we observe that the critical value τ^c falls when the amount of the early retirement window f_2 is high. Indeed, this tax is levied on the whole amount of financial incentives payed by employers to older workers. Consequently, the higher the amount of early retirement window the more likely an increase in mandatory retirement age will offset the labor-hoarding effect of the tax.

Second, the critical value τ^c is increasing with λ, which means that the incidence of the impatience effect on the separation rate is reduced for high values of λ, that is when the persistence of idiosyncratic shocks is low. Indeed, in that case, employers expect a higher probability to benefit from a future improvement in the value of the match's product and have therefore more interest to retain existing matches after a shock. As the labor-hoarding effect is reinforced through an increase in λ, the impatience effect is less likely to dominate.

Lastly, we remark that τ^c is a decreasing function of the productivity threshold ϵ_2^d. So, in the setting of a high job separation rate of older workers, delaying retirement may lead to more dismissals among this age group of workers when early retirement windows are taxed.

Furthermore, a rise in the productivity threshold ϵ_2^d may lead to reduce the hiring rate of older workers, all other things being equal and may therefore attenuate and even offset the positive horizon effect due to an increase in the mandatory retirement age. So we have to determine to what extent a combination of a tax on early retirement windows and of an increase in the mandatory retirement age affects the tightness θ_2. Differentiating the job destruction condition (D^2) and the job creation condition (C^2) with respect to θ_2 we get:

$$\begin{cases} \frac{\partial\theta_2}{\partial\tau} = \frac{D_3^2 C_2^2 - C_3^2 D_2^2}{D_2^2 C_1^2 - C_2^2 D_1^2} \\ \\ \frac{\partial\theta_2}{\partial\eta_2} = \frac{D_4^2 C_2^2 - C_4^2 D_2^2}{D_2^2 C_1^2 - C_2^2 D_1^2} \end{cases}$$

We show in the appendix (7.5) that an increase in the tax rate τ reduces θ_2, which is consistent with previous theoretical and empirical findings (Behaghel, 2007; Behaghel et al., 2008; Hairault et al., 2007; Cheron et al., 2008). Furthermore, in absence of age-dependent employment protection, delaying retirement exerts a positive horizon effect on job finding rates of older workers, as shown by Chéron et al. (2007).

Once again, our paper differs from these previous works given that we address the case of an increase in the retirement age in a setting of high age-dependent employment protection. As we have showed that for a high tax rate τ, such that $\tau \geq \tau^c$, this reform may increase ϵ_2^d through an impatience effect, it may reduce the present value to an employer of a job filled by an older worker, making firms more reluctant to hire older workers. Consequently, we highlight the fact that at some values of the tax rate τ an increase in the mandatory retirement age reduces job creation among older workers. So we determine a second critical value of the tax, denoted τ^{cc}, above which delaying retirement may reduce the tightness in the labor market for older workers.

Proposition 2. *For a high value of the tax rate τ, such that $\tau > \tau^{cc}$ where $\tau^{cc} = \frac{y\{(1-\varepsilon_2^d)[(r+\eta_2)+\lambda\varepsilon_2^d]+(1-\varepsilon_2^d)^2\}}{f_2(r+\eta_2+\lambda)[\lambda(1-\varepsilon_2^d)+(r+\eta_2+\lambda)]}$, then $\partial\theta_2/\partial\eta_2 > 0$.*

Proof: see appendix (7.5) ∎

In addition, recall that the job finding rate of older workers also depends on the productivity threshold $\epsilon_2^c = \epsilon_2^d + (r + \eta_2 + \lambda)(f_2\tau/y)$. It is noteworthy that even though $\tau > \tau^c$, which implies $\partial\epsilon_2^d/\partial\eta_2 < 0$, ϵ_2^c does not necessarily rise after an increase in retirement age. Indeed, calculating the derivative of ϵ_2^c with respect to η_2 we get the following expression :

$$\frac{\partial\epsilon_2^c}{\partial\eta_2} = \frac{\partial\epsilon_2^d}{\partial\eta_2} + (f_2\tau/y) \tag{25}$$

This expression may be negative only if $-\frac{\partial\epsilon_2^d}{\partial\eta_2} > (f_2\tau/y)$. The right-hand side of this inequality is due to the fact that after an increase in retirement age, the expected duration of a job is higher, so an employer may be less reluctant to hire an unemployed worker aged 55-59 years.

Let Π_2 be the job finding rate among older workers, we define Π_2 in the following way :

$$\Pi_2 = \theta_2^{1-\alpha}[1 - \epsilon_2^d - (r + \eta_2 + \lambda)\frac{f_2\tau}{y}]$$

Determining the partial derivative of Π_2 with respect to η_2, we obtain the following expression :

$$\frac{\partial \Pi_2}{\partial \eta_2} = \theta_2^{-\alpha}[(1-\alpha)\frac{\partial \theta_2}{\partial \eta_2}(1-\epsilon_2^c) - \theta_2(\frac{\partial \epsilon_2^d}{\partial \eta_2} + \frac{f_2 \tau}{y})]$$

Although this expression seems to be complicated, its sign depends strongly on the tax rate τ. It shows that contrarily to the findings of Chéron et al. (2007), delaying retirement may have ambiguous effect on job finding rates of older workers, if we account for different levels of age-dependent employment protection. In our case, we highlight three cases. In the first case, the tax rate is so low that $\tau < \tau^c$ and an increase in the mandatory retirement age raises unambiguously the job finding rate of an unemployed older worker. In the second case, the tax rate belongs to the interval $[\tau^c, \tau^{cc}]$. In this case, an increase in mandatory retirement age leads to a rise in ϵ_2^d but also in the tightness θ_2. Consequently if the impatience effect is sufficiently high, then $\partial \Pi_2 / \partial \eta_2 > 0$ which means that delaying mandatory retirement age reduces the job finding rate of an unemployed older worker. But it is the reverse story if the horizon effect dominates the impatience effect. Last but not least if the tax rate τ is higher that the critical value τ^{cc}, then delaying retirement increases the productivity threshold ϵ_2^d and reduces simultaneously the tightness θ_2, leading unambiguously to a fall in the job finding rate of an older job-seeker.

To summarize the new theoretical findings that we highlight in this paper, we could say that there are some cases of high age-dependent employment protection, in which delaying retirement may increase separation rate and reduce hirings among older workers. In other words, the positive effect of delaying retirement may be undermined when such a reform is combined with a high level of taxation of separation cost regarding older workers.

4.2. A qualitative analysis for middle-age workers

As mentioned in the previous section, we have to distinguish two cases: the case 1, where $\max\{S_2(\epsilon_1^d), 0\} = S_2$ and the case 2 where $\max\{S_2(\epsilon_1^d), 0\} = 0$. In the case 1, the tightness θ_1 and the productivity threshold ϵ_1^d are defined at equilibrium by the following equations system:

$$\begin{cases} C^1(\theta_1, \epsilon_1^d, \tau, \eta_2, \epsilon_2^d) = -c\theta_1^\alpha + \frac{(1-\beta)(r+\eta_1+\eta_2+\lambda)y}{2(r+\eta_1+\lambda)(r+\eta_2+\lambda)}(1-\epsilon_1^d)^2 \\ D^1(\theta_1, \epsilon_1^d, \tau, \eta_2, \epsilon_2^d) = -y\epsilon_1^d + z_1 + \frac{\beta c \theta_1}{(1-\beta)} - \frac{\lambda(r+\eta_1+\eta_2+\lambda)y}{2(r+\eta_1+\lambda)(r+\eta_2+\lambda)}(1-\epsilon_1^d)^2 - \eta_1 y \frac{\epsilon_1^d - \epsilon_2^d}{(r+\eta_2+\lambda)} + \eta_1 f_2 \tau \end{cases}$$

And in the case 2, the tightness θ_1 and the productivity threshold ϵ_1^d are defined at equilibrium by the following equations system:

$$\begin{cases} C^1(\theta_1, \epsilon_1^d, \tau, \eta_2, \epsilon_2^d) = -c\theta_1^\alpha + \frac{(1-\beta)y}{2(r+\eta_1+\lambda)}[1-\epsilon_1^d]^2 + \frac{(1-\beta)\eta_1 y}{2(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d]^2 \\ D^1(\theta_1, \epsilon_1^d, \tau, \eta_2, \epsilon_2^d) = -y\epsilon_1^d + z_1 + \frac{\beta c\theta_1}{(1-\beta)} - \frac{\lambda y[1-\epsilon_1^d]^2}{2(r+\eta_1+\lambda)} - \frac{\eta_1 y}{2(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d]^2 + \eta_1 f_2 \tau \end{cases}$$

We observe that in the case 1, the job creation condition does not depend on ϵ_2^d, contrarily to the case 2. This difference is noteworthy particularly when we study the effect of an increase in the tax rate τ on the productivity threshold ϵ_1^d. We show in the appendix (7.6) that in the case 1, an increase in τ raises ϵ_1^d if the following condition holds:

$$\eta_1 f_2 > -\frac{\eta_1 y}{(r+\eta_2+\lambda)}\frac{d\epsilon_2^d}{d\tau} \tag{26}$$

The term on the left-hand side of the inequality represents the direct positive effect of an increase in τ on the firing rate of middle-age workers, already highlighted by Behaghel (2007). Indeed, if it is more costly for an employer to get rid of an older worker, he has interest to lay his worker off before he switches to next age group. The right-hand side of the inequality represents the indirect effect of an increase in τ. Indeed, given that an increase in the tax rate reduces job destruction among older workers, so it raises the present value to the employer of a job filled by a middle-age worker who will switch to the next age group. Consequently, in the case of a shock on a job, firms have interest to retain the existing match.

Furthermore, in the case 2, we obtain in the appendix (7.6) a sufficient condition such that an increase in τ raises ϵ_1^d:

$$\eta_1 f_2 \alpha c\theta_1^{\alpha-1} > -\frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d]\frac{d\epsilon_2^d}{d\tau}[\alpha c\theta_1^{\alpha-1} - \beta c] \tag{27}$$

The term on the left-hand side of the inequality represents the direct effect and the term on the right-hand side corresponds to the indirect effect. Contrarily to the condition (26), the direct effect and the indirect effect have not necessarily opposite signs. Indeed, if the bargaining power of workers is too high, such that $\beta > \alpha\theta_1^{\alpha-1}$, then the inequality (27) is necessarily true. The idea is the following. If β is too high, then the decrease in ϵ_2^d raises θ_1, leading to a strong increase in wage and it may encourage employers to lay more middle-age workers off. However, if we assume that β is relatively low, such that $\beta < \alpha\theta_1^{\alpha-1}$, then in the case 1 or in the case 2, an increase in τ leads to a rise in ϵ_1^d provided that the direct effect offsets the indirect effect (through the fall in ϵ_2^d).

In addition, we show in the appendix (7.6) that in the case 1, the effect of an increase in the mandatory retirement age on ϵ_1^d depends on the sign of the following expression:

$$\frac{d\epsilon_1^d}{d\eta_2} > 0 \Leftrightarrow \frac{2\eta_1(r+\eta_1+\lambda)y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2}(\lambda\alpha c\theta_1^{\alpha-1} - \beta c)$$

$$+\alpha c\theta_1^{\alpha-1}[\eta_1 y\frac{(\epsilon_1^d - \epsilon_2^d)}{(r+\eta_2+\lambda)^2} + \frac{\eta_1 y}{r+\eta_2+\lambda}\frac{d\epsilon_2^d}{d\eta_2}] > 0 \tag{28}$$

The first term of this sum corresponds to the direct effect, that reduces job destruction among middle-age workers if $\beta < \lambda\alpha\theta_1^{\alpha-1}$. Indeed if the bargaining power is too high, then delaying mandatory retirement age may raise the job finding rate of middle-age workers, increasing therefore wages and encouraging employers to close down more jobs hit by a shock. However, if β is relatively low, a decrease in η_2 raises the option value of retaining an existing match in the case of a shock. This discussion around the role of β has been already considered in the paper of Chéron et al. (2007).

However, as in this paper we address the case in which $\tau \neq 0$, we exhibit a second term corresponding to the indirect effect that depends on the tax rate τ and may attenuate the direct effect. Indeed, we have previously shown that a decrease in η_2 raises job destruction among older workers if $\tau > \tau^c$. In that case, a firm employing a middle-age worker expects her job is more likely to break up when she will switch to the next age group, which makes her employer more reluctant to retain her job in the case of a negative productivity shock.

It is the same story in the case 2. Indeed, we show in the appendix (7.6) that the effect of a decrease in η_2 on ϵ_1^d depends on the sign of the following expression:

$$\frac{2(r+\eta_1+\lambda)\eta_1 y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2}[1-\epsilon_2^d]^2[\alpha c\theta_1^{\alpha-1} - \beta c]$$

$$+\frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d][\alpha c\theta_1^{\alpha-1} - \beta c]\frac{d\epsilon_2^d}{d\eta_2} \tag{29}$$

We find again both well-known direct and novel indirect effects. Consequently, under the condition $\beta < \alpha\theta_1^{\alpha-1}$, delaying retirement raises the option value to the employer to retain an existing match, but this direct effect may be attenuated if $\tau > \tau^c$.

Now, we investigate the effect of an increase in τ on the tightness θ_1 in the case 1. We show in the appendix (7.6) that an increase in τ reduces θ_1 under the following condition:

$$\eta_1 f_2 > -\frac{\eta_1 y}{r+\eta_2+\lambda}\frac{d\epsilon_2^d}{d\tau} \tag{30}$$

This condition ensures that an increase in τ raises job destruction among middle-age workers. Consequently, as $\epsilon_1^d = \epsilon_1^c$, a rise in ϵ_1^d reduces the job finding rate of middle-age workers.

We draw similar conclusions in the case 2. Indeed in the appendix (7.6), we show that an increase in τ reduces θ_1 under the following condition:

$$\frac{-(1-\beta)y}{(r+\eta_1+\lambda)}[1-\epsilon_1^d]\eta_1 f_2 + \frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d][\alpha c\theta_1^{\alpha-1} - \beta c]\frac{d\epsilon_2^d}{d\tau} < 0 \tag{31}$$

So provided that β is sufficiently low, such that $\beta < \alpha\theta_1^{\alpha-1}$, then the hiring rate of middle-age workers decreases with τ only if the direct effect through the increase in ϵ_1^c offsets the indirect effect through the fall in ϵ_2^d.

In addition, we have to determine the effect of an increase in the mandatory retirement age on the tightness θ_1. In the appendix (7.6), we show that it has a positive direct effect on θ_1. Nevertheless, a decrease in η_2 may lead to more separations among older workers and therefore to an increase in ϵ_1^d, if the tax rate τ is higher than τ^c. In that case, this indirect effect raises ϵ_1^c and attenuates therefore the direct positive effect of an increase in the mandatory retirement age on θ_1.

To sum up our findings, we have shown that in the case of an high taxation of early retirement windows, an increase in the mandatory retirement age may encourage employers to close down more jobs hit by idiosyncratic shocks which attenuates the positive horizon effect on job creation. To investigate the magnitude of these effects and to study their impact on employment rate among both age groups of workers, we implement a numerical illustration.

5. A numerical illustration

5.1. The effects of an increase in the mandatory retirement age on hiring and separation rates for both age groups of workers

For this numerical illustration, we consider two age groups of workers : the workers aged 55-59 years and the workers aged 50-54 years. Considering that one period is a year, we set initially $\eta_1 = \eta_2 = 0.2$, so the average duration of each age group equals 5 years and the mandatory retirement age is set to 60 before the 2003 French pension reform.

This numerical illustration aims at determining the magnitude of the effects of an increase in the mandatory retirement age raising from 60 to 65, namely a decrease in η_2 from 0.2 to 0.1, on the job finding and separation rates among each group of workers, for different values of the tax rate τ. The values chosen for this numerical illustration are reported in the table 2.

A first set of parameters $\Phi_1 = \{\alpha, \beta, \lambda, r, z_1, z_2, f_1, f_2\}$ is based on external information. Following the standard literature (Mortensen and Pissarides, 1999; Chéron et al. 2007) the elasticity of the matching function α is set to the extensively-used value $\alpha = 0.5$. The bargaining power of workers β is set to 0.5 so that the Hosios' condition (1990) holds. The annual interest rate r is set to 3%. The amounts of unemployment benefits z_1 and z_2 are computed from the values of the average wages of each group of workers. Using empirical results of Aubert (2005) drawn from a firm-level survey called DADS (*Déclaration Annuelle des Données Sociales*) for the year 2001, we set the gross hourly wage of a worker aged 50-54 to 16 euros and the gross hourly wage of a worker aged 55-59 to 17 euros. We determine therefore the gross yearly average wage for each group of workers, considering a basis of 35 hours per week. We obtain $w_1 = 29121$ euros and $w_2 = 30941$ euros.

To determine the amounts of unemployment benefits z_1 and z_2, we have to set a replacement rate, that is a percentage of net earnings in work. We use the measure constructed by the OECD, that is an average of the net replacement rates for different categories of workers, different family situations and different durations of unemployment. According to the OECD, for the period 2002-2004, the French average net replacement rate is around 57%. In this respect, as typical-case calculations relate to a 40-year-old worker, we have also to account for the fact that workers aged more than 50 benefit from more generous unemployment compensations. Following Langot et al. (2010), we consider a 6% (11.5%) premium on unemployment benefits received by workers aged 50-54 (55-59) relative to the previous age group.

Consequently, we end up with a replacement rate equal to 60.4% (67.4%) for workers aged 50-54 (55-59), so we set $z_1 = 17589.08$ euros and $z_2 = 20854.23$ euros.

We have also to calibrate age-dependent separation costs. In the case of a layoff of a worker aged 50-54 years, the employer has to give him severance payment according to the French Employment Legislation. We have shown previously that this payment had no effect on the hiring or firing behavior of firms. Regarding the 55-59 years old, we assume that employers prefer dismissing a worker invoking the serious misconduct and paying him early retirement window f_2 as highlighted by Amauger-Lattes and Desbarats (2006). A little is known about this payment so we set it in a first step to a yearly gross wage ($f_2 = 30941$ euros). Then, we will perform some sensitivity analysis to check whether our results are robust to different values of f_2. Similarly, as we expect that our results are sensitive to the value of the Poisson arrival rate of idiosyncratic shock λ, we set first this parameter to $\lambda = 0.2$ and in a second step, we will implement some sensitivity analysis with other values of λ.

Lastly, we choose to calibrate a second set of parameters $\Phi_2 = \{y, c\}$ to reproduce some stylized facts about hiring and separation rates among workers aged 55-59 years. According to the French Labor Force Survey for the period March 2002- March 2003, the job finding rate of an unemployed worker aged 55-59 equals 5.82%. In addition, the job separation rate for this cohort of workers equals 8.37%. Solving our model using these values, we obtain an average productivity of job y equal to 43677.31 and a cost of maintaining a vacancy c equal to 55793.35 euros. Using the French Labor Force Survey for the period March 2002-March 2003, we remark that these values allow us to match in a satisfying way the observed job finding and job destruction rates among the workers aged 50-54. Indeed, solving the equations system composed of the job creation and the job destruction condition for the workers aged 50-54 setting $c = 55793.35$ and $y = 43677.31$, we obtain a job finding rate equal to 20.02% and a job destruction rate equal to 3.63% for this group while the observed rates equal respectively 15.95% and 3.53% according to the French Labor Force Survey (cf figure 2). In addition, the expression of the steady-state unemployment rate (23) shows that the employment rate among the group C_1 of workers depends on the employment rate among the previous cohort of workers. According to the French Labor Force Survey, we set the employment rate among workers aged 45-49 years to 79.32%. From the steady-state expression (23), we compute employment rates and we see that the values obtained are close to those observed from the data. Indeed, we find an employment rate equal to 82.2% among the 50-54 years old and equal to 65.11% among the workers aged 55-59, while the observed rates equal respectively 75.8% and 55.68%.

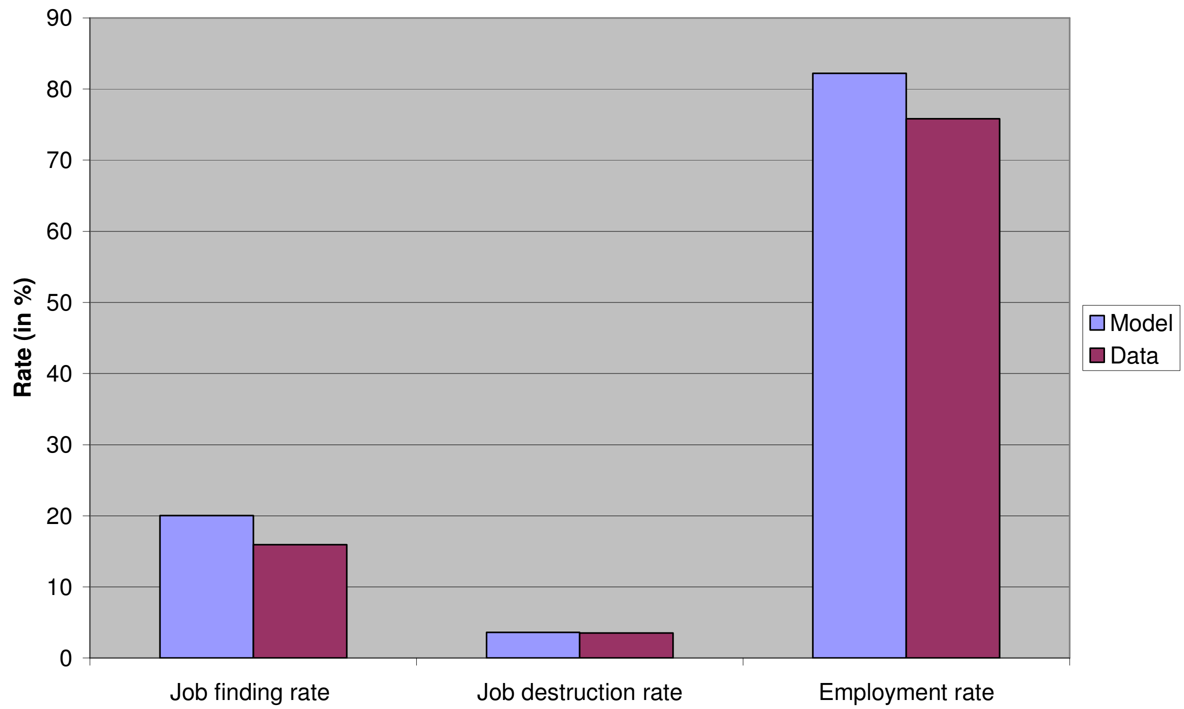
Table 2: Values of parameters

Elasticity of the matching function	α	0.5
Bargaining power	β	0.5
Poisson arrival rate of shocks	λ	0.2
Annual interest rate	r	0.03
Workers aged between 50 and 54		
Unemployment benefits		
(as a fraction of the net average wage)	z_1	60.4%
Probability to switch to the next age group	η_1	0.2
Workers aged between 55 and 59		
Unemployment benefits		
(as a fraction of the net average wage)	z_2	67.4%
Early retirement windows (as a fraction of the average yearly wage)	f_2	100%
Probability to reach the mandatory retirement age	η_2	0.2
Calibrated values		
Cost of maintaining a vacancy	c	55793.35
Average productivity	y	43677.31

From this numerical illustration, we seek to highlight the effect due to a tax on early retirement windows offered by firms, when the mandatory retirement age rises. First, we observe in the figure 3 that in absence of taxation of early retirement windows, our results regarding the effects of an increase in retirement age on job flows are consistent with those of Chéron et al. (2007). Indeed, for both age groups of workers, delaying retirement age exerts a strong positive effect on the job finding rates (+38.5% for workers aged 55-59 and +21% for workers aged 50-54) through the well-known horizon effect, and it reduces separation rates (-3% for workers aged 55-59 and -23% for workers aged 50-54) through a labor-hoarding effect, even though this effect is quite lower for the workers aged 55-59 than for workers aged 50-54.

All the interest of our numerical illustration lies on the way the effect of delaying retirement on job flows is affected when the level of age-dependent employment protection varies. Focussing first on separation rates, we see that the labor-hoarding effect is attenuated when the tax rate τ rises, due to

Figure 2: Job creation, job destruction and employment among the workers aged between 50 and 54 over the period March 2002-March 2003

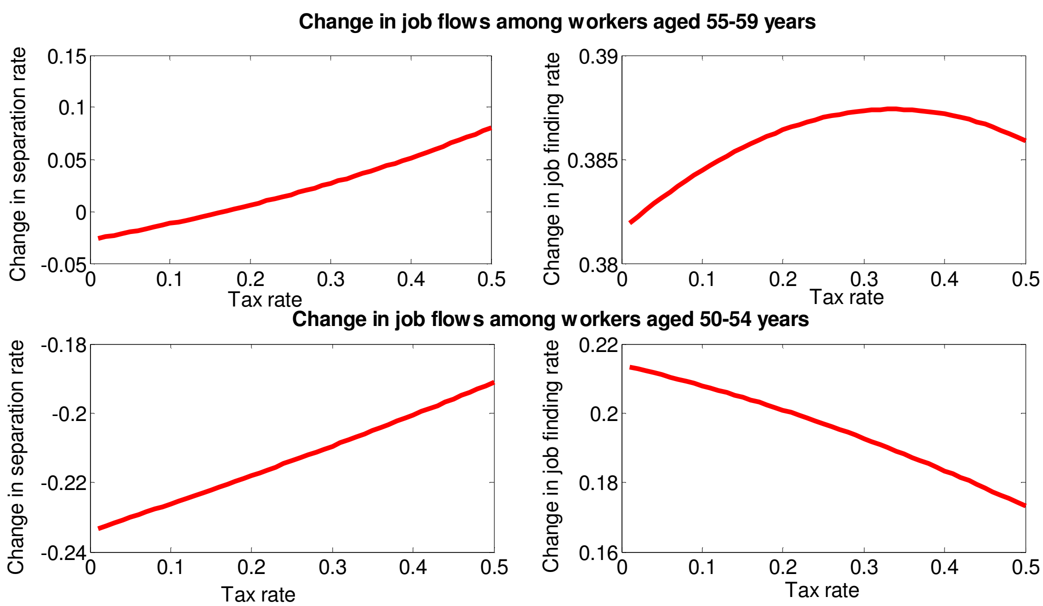


Lecture: The job finding rate is the fraction of workers aged 50-54 unemployed in March 2002 who find a job in the next twelve months.

The separation rate is the fraction of workers aged 50-54 employed in March 2002 who get into unemployment in the next twelve months.

Source: French Labor Force Survey (2002-2003)

Figure 3: The effect of a decrease in η_2 from 0.2 to 0.1 on job flows among each age group of workers for different values of the tax rate τ



Lecture: Each graph represents the relative variation in job flows after a decrease in η_2 from 0.2 to 0.1 as a function of the tax rate τ.

the impatience effect that we highlighted throughout this paper. As regards the workers aged 50-54, this labor-hoarding remains still higher than the impatience effect, even for higher values of the tax rate τ. Consequently, for this age group of workers, the impatience effect has a low incidence on the quantitative results of delaying retirement on the separation rate. However, regarding the workers aged 55-59, we remark that for high values of the tax rate τ, delaying retirement may lead to raise the separation rate, rather than reducing it. For this age group of workers, the labor-hoarding effect is more than offset by the impatience effect, so the latter has a strong incidence on the predicted effects of an increase in the retirement age on the separation rate.

In addition, as regards the workers aged 50-54, the positive horizon effect due to an increase in the retirement age on the job finding rate is also attenuated when the tax rate τ rises. Indeed, as the labor-hoarding effect of delaying retirement on the separation rate among this age group of workers is decreasing with τ, the increase in the job finding rate, that depends negatively on the productivity threshold ϵ_1^d, is also lower for a high level of taxation of early retirement windows. Nevertheless, once again for this age group of workers, the horizon effect remains still positive even for high values of the tax rate so the impatience effect has a low incidence on the predicted effects of delaying retirement on job creation among workers aged 50-54. When turning to the job finding rates among workers aged 55-59, it can appear to be surprising to obtain inverted U-shaped curve. We would expect that the higher the impatience effect the lower is the horizon effect. However, recall that the effect of a decrease in η_2 on the job creation threshold ϵ_2^c is ambiguous and is defined by the expression (25). We observe that the higher the tax rate τ, the higher the second term of the derivative and therefore delaying retirement may reduce the threshold productivity ϵ_2^c while it raises the threshold ϵ_2^d. It could correspond to the increasing part of the curve. Then, when the impatience effect is too strong for high values of τ, leading to a substantial increase in ϵ_2^d with the retirement age, the first term of the derivative in the expression (25) is higher than the second one, implying that ϵ_2^c is increasing with the retirement age, which attenuates the positive horizon effect. This could correspond to the decreasing part of the curve. Nevertheless, even for high values of the tax rate τ, the horizon effect remains still strongly positive. We conclude that the positive horizon effect due to an increase in the retirement age on the job finding rate of workers aged 55-59 years is very strong whatever the value of the tax rate τ.

To sum up our numerical findings, we show that in the case of taxation of early retirement windows, the impatience effect may have a strong incidence but only on the predicted effects of an increase in the mandatory retirement age on the separation rates among workers aged 55-59. This impatience effect could be therefore one explanation of the increase in the separation rate among workers aged 55-59 observed in France between the periods March 2002-2003 and March 2003-2004, as mentioned in the section 2.

5.2. The effects of an increase in retirement age on employment rates among both age groups of workers

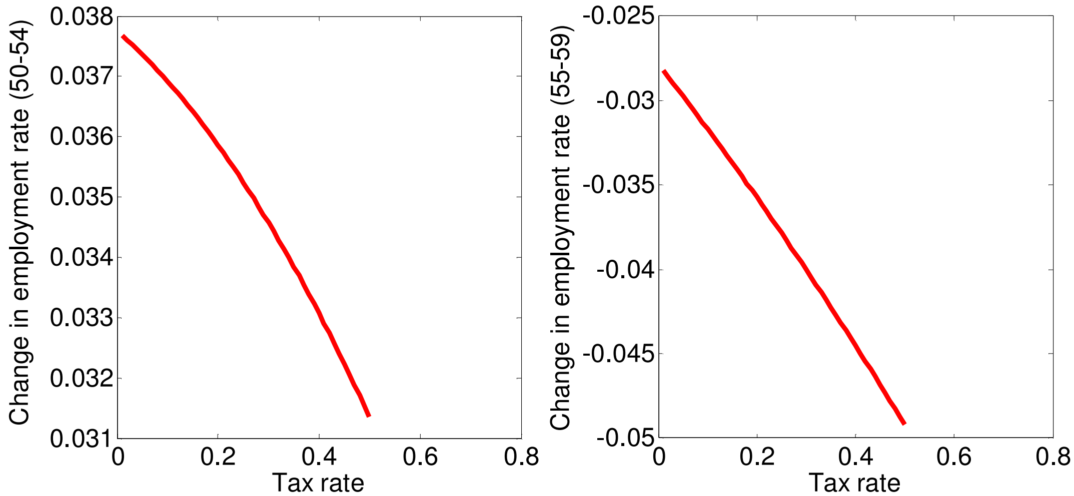
Using the expression (23) of the steady-state unemployment rate among the age group of workers C_i, we deduce the effects of an increase in retirement age on employment rates and we investigate to what extent a taxation on early retirement windows alters this effect.

Focussing first on the workers aged 50-54 years, we observe in the figure 4 that in the absence of taxation of early retirement windows, a decrease in η_2 from 0.2 to 0.1 leads to a rise in employment rate by around 3.8%. Even though the positive effect of delaying retirement on employment rate among this age group of workers is attenuated as the tax rate rises, it remains strongly positive even for high values of τ. So, we conclude that the impatience effect that we highlighted in this paper does not alter significantly the impact of postponing retirement age on employment rate among workers aged 50-54.

However, when we investigate the effect of an increase in retirement age on employment rate among the workers aged 55-59 years, we see that the tax rate τ plays a more important role. Using the expression (23), we observe that a decrease in η_2 exerts a direct negative effect on employment rate among workers aged 55-59 years. Indeed, when the mandatory retirement age raises, it leads to reduce flows out of unemployment regarding job seekers aged 55-59 years. In addition, a decrease in η_2 may have indirect effects on the employment rate. First, it may have a positive effect on employment through an increase in the job finding rate of workers aged 55-59 years. But this horizon effect may be partially offset by the impatience effect for high values of the tax rate τ. Indeed, in this setting, delaying retirement age leads to a rise in job destruction among the workers aged 55-59 years, which affects negatively employment rate among this group of workers.

We see in the figure 4 that in the absence of a taxation of early retirement windows, a decrease in η_2 from 0.2 to 0.1 reduces by around 3% the

Figure 4: The effect of a decrease in η_2 from 0.2 to 0.1 on employment rates among each age group of workers



Lecture: Each graph represents the relative variation in steady-state employment rates after a decrease in η_2 from 0.2 to 0.1. as a function of the tax rate τ.

employment rate among workers aged 55-59 years. In that case, even though the direct effect of a decrease in η_2 on employment dominates, it is strongly attenuated by the horizon effect and therefore, the decrease in employment rate after an increase in retirement age is quite low. However, for a tax rate τ equal to 50%, the impatience effect exerts a supplementary negative impact on the employment rate. Consequently, after an increase in retirement age, the employment rate falls by 5%. So in the case of a high taxation of early retirement windows, the decrease in the employment rate that results from an increase in retirement age is twice higher than in the case where early retirement windows are not taxed. We deduce therefore that the combination of a rise in the mandatory retirement age and of a taxation of the financial incentives to retire payed by firms to their workers aged 55-59 years may have negative effects on employment among this age group of workers.

5.3. A sensitivity analysis

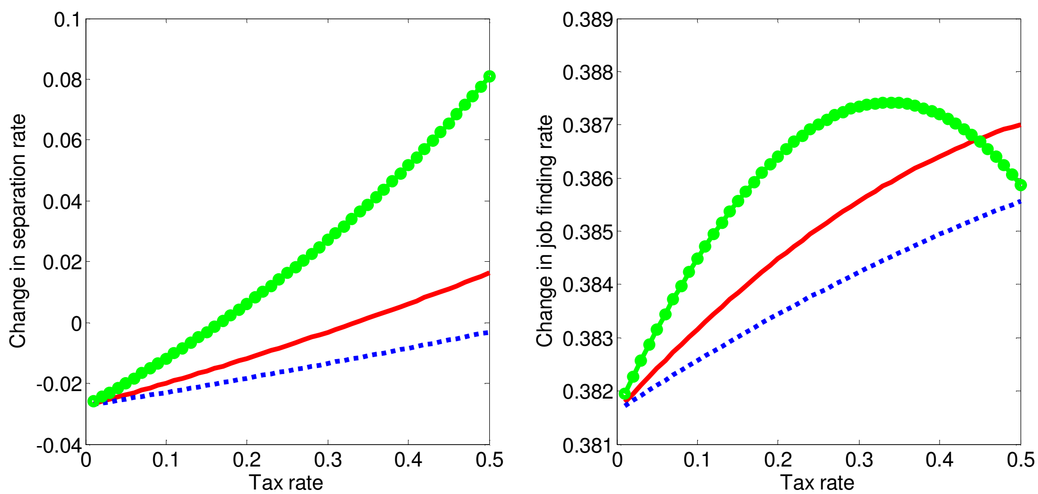
As for our numerical illustration, we set arbitrarily some parameters, especially the amount of early retirement windows f_2 and the arrival rate of idiosyncratic shocks λ, we have to check whether our results are robust to other values of these parameters. Recall that in the beginning of the subsection 4.1, we have determined a critical value of the tax rate τ^c, above which the impatience effect is higher than the labor-hoarding effect, leading to a rise in the separation rate among workers aged 55-59 after an increase in the retirement age. It is worth noting that this critical value is a sufficient condition but not necessary, which means that we could observe $\partial\epsilon_2^d/\partial\eta_2 < 0$ even though $\tau < \tau^c$. Nevertheless, studying the critical value of tax rate τ^c provides some information on the probability that the impatience effect will dominate over the labor-hoarding effect. Indeed, the higher τ^c and the lower this probability.

After some comparative statics in the subsection 4.1, we have shown that the incidence of the impatience effect on the predicted effects of delaying retirement on the separation rate among workers aged 55-59 should increase with the amount of early retirement windows f_2 and decrease with the arrival rate of idiosyncratic shocks λ. Considering different values of each of these two parameters, we can check whether our theoretical findings are consistent with our numerical results.

We investigate first the incidence of the impatience effect on job destruction among workers aged 55-59 for different values of f_2 and setting $\lambda = 0.2$. The figure 5 shows that in the benchmark case corresponding to the green circled curve, where f_2 is equal to one year of the average wage, the impatience effect starts to dominate over the labor-hoarding effect for a value of τ around 18%. Indeed, for higher values of τ, a decrease in η_2 from 0.2 to 0.1 leads to a rise in the separation rate in that case. However, for lower values of f_2, for instance in the case where f_2 is only the half of one year of the average wage, corresponding to the red curve, the impatience effect starts to dominate for a value of τ around 35%. In a similar way, in the case where f_2 represents only one third of the average yearly wage, that corresponds to the dashed blue curve, the impatience effect is still lower than the labor-hoarding effect even for a value of τ equal to 50%. Consequently, these findings show that the higher f_2 and the lower τ^c, which is in line with our theoretical predictions.

Proceeding in a similar way for λ, setting $f_2 = 30941$, we remark in the figure 6 that while in the benchmark case (red curve) where $\lambda = 0.2$, the impatience effect starts to dominate for a value of τ around 18%, this value is strongly higher (around 40%) when we set $\lambda = 0.4$ (blue dashed curve). Once again, it implies that the incidence of the impatience effect is lower when λ is high, which is consistent with our theoretical results.

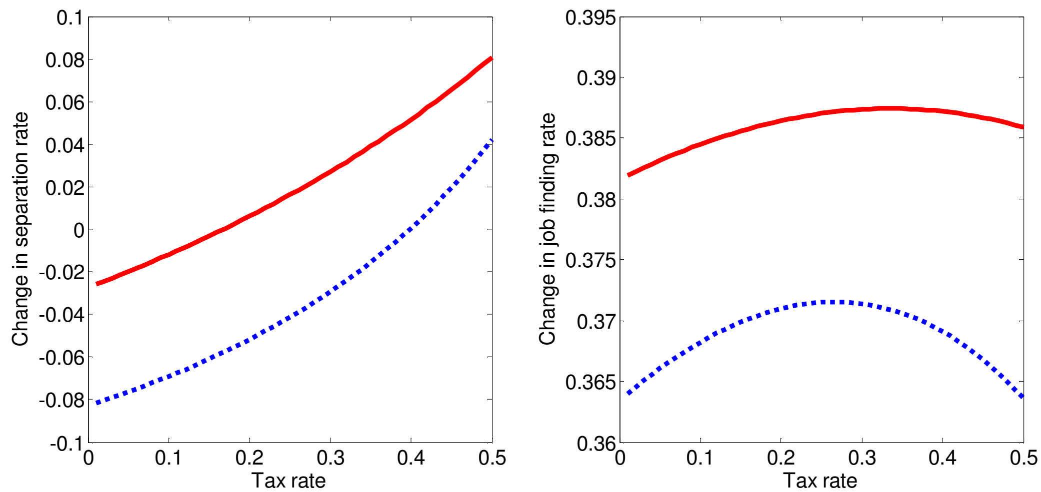
Figure 5: A sensitivity analysis for f_2



Lecture: Each graph represents the relative variation in job flows after a decrease in η_2 from 0.2 to 0.1. as a function of the tax rate τ.

The red curve corresponds to $f_2 = 0.5 * 30941$, the dashed blue curve corresponds to $f_2 = 0.3 * 30941$ and the circled green curve corresponds to $f_2 = 30941$

Figure 6: A sensitivity analysis for λ



Lecture: Each graph represents the relative variation in job flows after a decrease in η_2 from 0.2 to 0.1. as a function of the tax rate τ.
The red curve corresponds to $\lambda = 0.2$, the dashed blue curve corresponds to $\lambda = 0.4$

Regarding the robustness of our key theoretical result, that is an impatience effect in the case of taxation of early retirement windows that may lead to a rise in the separation rate among older workers after an increase in the retirement age, we could discuss some of the assumptions made throughout the paper. First, in our theoretical framework, we have considered segmented matching markets by age. However, it could be interesting to discuss how undirected search may affect our results. If we consider only one matching function for the two types of workers, it implies a change in the free-entry condition. Indeed, in the case of undirected search, vacancies and the two types of unemployed workers (according to the age group) meet each other according to a matching function denoted by $h(u_1 + u_2, v)$, that determines the number of hirings as a function of the number of unemployed of type i, with $i \in \{1, 2\}$, and of the number of vacancies v. In this case, θ represents the number of vacancies per job-seeker. The value to an employer of posting a vacancy, denoted by J_v, is now defined by the following Bellman equation:

$$rJ^v = -c+q(\theta)\{\gamma[\int_0^{\bar{\varepsilon}} \max\{J_2(x), 0\}dG(x)-J^v]+(1-\gamma)[\int_0^{\bar{\varepsilon}} \max\{J_1(x), 0\}dG(x)-J^v]\} \tag{32}$$

where γ represents the share of older unemployed workers, defined by flow equations (21) and (22). Consequently, the new free-entry condition is the following:

$$\frac{c}{q(\theta)} = \gamma \int_{\varepsilon_2^c}^{\bar{\varepsilon}} J_2(x)dG(x) + (1-\gamma) \int_{\varepsilon_1^c}^{\bar{\varepsilon}} J_1(x)dG(x) \tag{33}$$

In the appendix 7.7, we determine the new wage equations and productivity thresholds in the case of undirected search. We find that the destruction condition for older workers in the case of undirected search is the following:

$$y\varepsilon_2^d = z_2+\frac{\beta c\theta}{(1-\beta)\gamma}-\beta p(\theta)\frac{(1-\gamma)}{(1-\beta)\gamma}\int_{\varepsilon_1^c}^{\bar{\varepsilon}} J_1(x)dG(x)-\lambda\int_{\varepsilon_2^d}^{\bar{\varepsilon}} S_2(x)dG(x)-(r+\eta_2)f_2\tau \tag{34}$$

With respect to the destruction condition for older workers in the case of segmented markets by age, considering only one matching function implies an additional effect, that corresponds to the third term on the right hand side of the equation (34). However, the last term on the right-hand side of the expression remains unchanged, so in the case of a high value of τ, a decrease in η_2 implies an impatience effect, even though matching markets are not segmented by age. In addition, the higher γ, the lower the incidence of the additional effect due to undirected search. As delaying retirement has a positive effect on the employment rate among middle-age workers, while regarding employment among older workers, the positive horizon effect is offset by a direct negative effect through η_2, we would expect that delaying retirement may raise γ and therefore reduce the incidence of the additional effect due to undirected search on the impatience effect.

Furthermore, we could discuss briefly what would be the consequence of a substitution between early retirement windows and the tax rate τ. It is true that if the substitution was perfect, a rise in tau would imply a severe reduction in the amount of early retirement windows payed to older workers, so that the whole separation costs due by the firm remains unchanged. In that case, there would be no room for an impatience effect. Nevertheless, our theoretical findings still hold in the case of imperfect substitution. We could assume for instance that older workers constrain firms to offer a minimal lump-sum, given that separation results from a mutual agreement. In that case, a strong increase in tau combined with a minimum severance payment may lead to a rise in the separation costs for the firms, which may affect firing decision and leaves room for an impatience effect. The way this minimal amount of early retirement windows would be negotiated between older workers and their employers go beyond the scope of this paper. Nevertheless, as it is well-known that in France, job destruction among older workers result from mutual agreement between employers and employees, this issue could be a future premising field of research.

6. Conclusion

In this paper, we studied the effects of postponing retirement in a setting of an age-specific employment protection on the hiring and separation rates of older workers and also on employment of the elderly. Reproducing the 2003 French pension reform, we set a tax levied on early retirement windows payed by firms to their older workers to dismiss them. We provided some theoretical findings considering a matching model with endogenous destruction extended to account for a mandatory retirement age and in which we introduce age-dependent separation costs.

We highlighted that in the case of a high tax rate, delaying retirement may raise separations among the targeted age group of workers through an impatience effect. Indeed, a high tax rate discourages firms from dismissing older workers paying them financial incentives, so employers prefer waiting until their workers reach the mandatory retirement age. In this setting, delaying retirement forces employers to retain their workers for a longer time, and they could have interest to dismiss them before they reach the retirement age in spite of the cost induced by the tax. We pointed out that there exists a critical value of the tax rate above which the impatience effect offsets the labor-hoarding effect of postponing retirement, leading to a rise of the separation rate of older workers after an increase in the mandatory retirement age. Calibrating our data using the French Labor Force Surveys for the years 2002 and 2003, we showed that this critical value is negatively correlated to the amount of early retirement windows but it is positively correlated to the Poisson arrival rate of idiosyncratic shocks. Nevertheless, for reasonable value of parameters, we have shown that the impatience effect may have a strong incidence on the predicted effects of delaying retirement on the separation rate among older workers.

In addition, the impatience effect that we highlighted in this paper may partially offset the positive impact of an increase in the retirement age on the hiring rate of these workers. Theoretically, we determined a second critical value of the tax rate above which delaying retirement reduces the hiring rate among the group of workers targeted by the tax. However, we remarked in our numerical illustration that the extent to which the tax rate influences the effect of postponing retirement on the job finding rate is negligible for both age groups of workers.

However, the relative variation in the employment rate of the elderly after an increase in the mandatory retirement age strongly depends on the taxation level. Indeed, in the case of a high tax rate, delaying retirement may lead to more separations, which exerts a negative effect on the employment rate.

Similarly, regarding the previous cohort of workers, we have shown theoretically that the impatience effect may affect the impact of delaying retirement on the firing and the hiring rates but we found in our numerical illustration that the impatience effect has a lower incidence on the predicted effects of an increase in the retirement age on either job creation or job destruction. Consequently, the relative variation in the employment rate among this cohort of workers after an increase in the mandatory retirement age is not altered in a significant way by a taxation of early retirement windows.

However, these results have to be considered with caution. Indeed, in our model we consider an exogenous and constant amount of early retirement windows, while we can expect that the amount offered by employers to their workers may depend on several factors as the level of the tax, implying therefore a substitution between early retirement windows and the tax, or the characteristics of this worker. Too few information are still available on early retirement windows in France, however since the 2008 reform, employers who offer such financial incentives to their older workers have to indicate their names, their ages and the amount that they received prior to exit. Using this data, we could carry out an empirical study aiming at better understanding the factors that lead employers to offer such financial incentives and the determinants of the amount offered. It could also allow us to better grasp the way separation costs are negotiated between employers and older employees, through a mutual agreement. We leave this issue for further investigation.

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7. Appendix

7.1. The wage equations

Using the sharing rule (10) and rearranging terms, we obtain:

$$-(r+\eta_i+\lambda)(1-\beta)U_i = (r+\eta_i+\lambda)[\beta[J_i(\epsilon)+W_i(\epsilon)]-W_i(\epsilon)+f_i(1+\beta\tau_i)] \quad (35)$$

Let us first define the wage equation for the middle-age workers belonging to the age group C_1. Bellman equations (5) and (11) imply:

$$-(r+\eta_1+\lambda)(1-\beta)U_1 = \beta[y\epsilon+\lambda\int_{\epsilon_1^d}^{\overline{\epsilon}} S_1(x)dG(x)+\lambda U_1+\eta_1\max\{S_2(\epsilon),0\}+\eta_1(U_2-f_2\tau)]$$

$$-w_1(\epsilon)-\lambda\int_{\epsilon_1^d}^{\overline{\epsilon}}[W_1(x)-U_1-f_1]dG(x)-\lambda(U_1+f_1)-\eta_1\max\{W_2(\epsilon)-U_2-f_2,0\}-\eta_1(U_2+f_2)$$

$$+(r+\lambda+\eta_1)f_1 \quad (36)$$

Using the sharing-rule (10) we obtain:

$$\lambda\beta\int_{\epsilon_1^d}^{\overline{\epsilon}} S_1(x)dG(x) = \lambda\int_{\epsilon_1^d}^{\overline{\epsilon}}[W_1(x)-U_1-f_1]dG(x)$$

And:

$$\beta\max\{S_2(\epsilon),0\} = \max\{W_2(\epsilon)-U_2-f_2,0\}$$

Therefore we get:

$$-(r+\eta_1+\lambda)(1-\beta)U_1 = \beta y\epsilon - w_1(\epsilon) - U_1\lambda[1-\beta]$$

$$-(1-\beta)\eta_1(U_2+f_2)-\eta_1\beta[f_2(1+\tau)]+f_1(r+\eta_1) \quad (37)$$

Substituting the Bellman equation (13) into this expression, we obtain:

$$-(1-\beta)[p(\theta_1)[\int_0^{\overline{\epsilon}} \max\{W_1(x), U_1\}dG(x) - U_1] + z_1] = \beta\epsilon$$

$$-w_1(\epsilon) - (1-\beta)\eta_1 f_2 - \eta_1\beta[f_2(1+\tau)] - \lambda f_1 \tag{38}$$

Combining the rent-sharing rule (8) and the free-entry condition (4) we get:

$$\int_{\epsilon_i^c}^{\overline{\epsilon}} [W_1(x) - U_1]dG(x) = \frac{\beta}{(1-\beta)}\frac{c}{q(\theta_1)} \tag{39}$$

So, substituting this expression into (38), we deduce the following wage equation:

$$w_1(\epsilon) = (1-\beta)z_1 + \beta(y\epsilon + c\theta_1 - \eta_1 f_2\tau) + f_1(r+\eta_1) - \eta_1 f_2 \tag{40}$$

Then we determine the wage equations for the older workers belonging to the group C_2. Substituting Bellman equations (6) and (12) into the expression (10), we get:

$$-(r+\eta_2+\lambda)(1-\beta)U_2 = \beta\{y\epsilon + \lambda\int_{\epsilon_2^d}^{\overline{\epsilon}} S_2(x)dG(x) + \lambda(U_2 - f_2\tau) + \eta_2\frac{P}{r}\}$$

$$-w_2(\epsilon) - \lambda\int_{\epsilon_2^d}^{\overline{\epsilon}} [W_2(x) - U_2 - f_2]dG(x) - \lambda(U_2 + f_2) - \eta_2(\frac{P}{r} + f_r)$$

$$+(r+\eta_2+\lambda)f_2(1+\beta\tau) \tag{41}$$

The rent-sharing rule (10) implies:

$$\lambda\beta\int_{\epsilon_2^d}^{\overline{\epsilon}} S_2(x)dG(x) = \lambda\int_{\epsilon_2^d}^{\overline{\epsilon}} [W_2(x) - U_2 - f_2]dG(x)$$

Therefore we get:

$$-(r+\eta_2+\lambda)(1-\beta)U_2 = \beta y\epsilon - w_2(\epsilon) - U_2\lambda[1-\beta] - (1-\beta)\eta_2\frac{P}{r}$$

$$-\eta_2 f_r + (r+\eta_2)[f_2(1+\beta\tau)] \tag{42}$$

Substituting the Bellman equation (13) into this expression and rearranging terms, we obtain:

$$-(1-\beta)[z_2+p(\theta_2)[\int_0^{\bar{\epsilon}} max\{W_2(x),U_2\}-U_2]dG(x)] = \beta y\epsilon - w_2(\epsilon)$$

$$+(r+\eta_2)[f_2(1+\beta\tau)] - \eta_2 f_r \tag{43}$$

Using the sharing rule (8) and the free-entry condition (4), we deduce the following wage equation:

$$w_2(\epsilon) = \beta y\epsilon + (1-\beta)z_2 + \beta c\theta_2 + (r+\eta_2)[f_2(1+\beta\tau)] - \eta_2 f_r \tag{44}$$

7.2. The productivity thresholds

We have to determine the productivity threshold ϵ_i^d, below which the firm closes down the job filled by a worker belonging to the group C_i. First we define ϵ_1^d. Substituting the wage equation (14) into the expression (5) we get:

$$(r+\eta_1+\lambda)J_1(\epsilon) = (1-\beta)[y\epsilon - z_1] - \beta c\theta_1 - f_1(r+\eta_1+\lambda) + \eta_1 f_2(1+\beta\tau)$$

$$+\lambda\int_{\epsilon_1^d}^{\bar{\epsilon}}[J_1(x)+f_1]dG(x) + \eta_1\max\{J_2(\epsilon)+f_2(1+\tau),0\} - \eta_1 f_2(1+\tau)$$

Simplifying this expression we obtain:

$$(r+\eta_1+\lambda)S_1(\epsilon)(1-\beta) = (1-\beta)[y\epsilon - z_1] - \beta c\theta_1 - (1-\beta)\eta_1 f_2\tau$$

$$+\lambda(1-\beta)\int_{\epsilon_1^d}^{\bar{\epsilon}} S_1(x)dG(x) + \eta_1(1-\beta)\max\{S_2(\epsilon),0\} \tag{45}$$

Evaluating (45) at $\epsilon = \epsilon_1^d$ gives the following productivity threshold:

$$y\epsilon_1^d = z_1 + \frac{\beta c}{1-\beta}\theta_1 - \lambda\int_{\epsilon_1^d}^{\bar{\epsilon}} S_1(x)dG(x) - \eta_1\max\{S_2(\epsilon_1^d),0\} + \eta_1 f_2\tau \tag{46}$$

We proceed in a similar way to determine ϵ_2^d, substituting the wage equation (15) into the expression (6) we get:

$$(r+\eta_2+\lambda)J_2(\epsilon) = (1-\beta)[y\epsilon - z_2] - \beta c\theta_2 - f_2(1+\tau)(r+\eta_2) + (1-\beta)(r+\eta_2)f_2\tau$$

$$+\lambda\int_{\epsilon_2^d}^{\bar{\epsilon}}[J_2(x)+f_2(1+\tau)]dG(x) - \lambda f_2(1+\tau)$$

Simplifying this expression we obtain:

$$(r+\eta_2+\lambda)(1-\beta)S_2(\epsilon) = (1-\beta)[y\epsilon - z_2] - \beta c\theta_2 + (1-\beta)f_2\tau(r+\eta_2)$$
$$+\lambda(1-\beta)\int_{\epsilon_2^d}^{\bar{\epsilon}} S_2(x)dG(x) \tag{47}$$

So evaluating (47) at $\epsilon = \epsilon_2^d$, we get the following job destruction condition:

$$y\epsilon_2^d = z_2 + \frac{\beta c}{1-\beta}\theta_2 - \lambda\int_{\epsilon_2^d}^{\bar{\epsilon}} S_2(x)dG(x) - (r+\eta_2)f_2\tau \tag{48}$$

Furthermore, rent-sharing rules (8) and (10) imply:

$$S_i(\epsilon) = S_i^0(\epsilon) + f_i\tau_i \quad \tau_1 = 0, \tau_2 = \tau \tag{49}$$

So using (49) we can deduce the productivity threshold ϵ_i^c, such that:

$$\epsilon_i^c = \epsilon_i^d + (r+\eta_i+\lambda)f_i\tau_i \tag{50}$$

7.3. Match surpluses

Given that $S_2(\epsilon_2^d) = 0$, we get:

$$S_2(\epsilon) - S_2(\epsilon_2^d) = \frac{y(\epsilon-\epsilon_2^d)}{r+\eta_2+\lambda} \tag{51}$$

Furthermore, in the case where $\epsilon_1^d > \epsilon_2^d$, given that $S_1(\epsilon_1^d) = 0$, we obtain:

$$(r+\eta_1+\lambda)[\tilde{S}_1(\epsilon) - \tilde{S}_1(\epsilon_1^d)] = y(\epsilon-\epsilon_1^d) + \eta_1[S_2(\epsilon) - S_2(\epsilon_1^d)] \tag{52}$$

The expression (51) allows us to simplify this expression so we get:

$$\tilde{S}_1(\epsilon) = \frac{y(\epsilon-\epsilon_1^d)}{(r+\eta_1+\lambda)}(1+\frac{\eta_1}{r+\eta_2+\lambda}) \tag{53}$$

In the case where $\epsilon_1^d \leq \epsilon_2^d$, in a similar way we obtain:

$$\hat{S}_1(\epsilon) = \frac{y(\epsilon-\epsilon_1^d)}{(r+\eta_1+\lambda)} + \frac{\eta_1}{r+\eta_1+\lambda}\max\{\frac{y(\epsilon-\epsilon_2^d)}{(r+\eta_2+\lambda)}, 0\} \tag{54}$$

7.4. The equilibrium for the middle-age workers

In the case 1, where $\max\{S_2(\epsilon_1^d),0\} = S_2(\epsilon_1^d)$, an unique equilibrium $(\tilde{\epsilon_1^d},\tilde{\theta}_1)$ is defined by the following equation system:

$$\begin{cases} \frac{c}{q(\tilde{\theta}_1)} = & (1-\beta)\frac{(r+\eta_1+\eta_2+\lambda)}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}\int_{\epsilon_1^d}^{\bar{\epsilon}} y(x-\tilde{\epsilon_1^d})dG(x) \\ y\tilde{\epsilon_1^d} = z_1 + \frac{\beta}{(1-\beta)}c\tilde{\theta}_1 - \frac{\lambda}{(r+\eta_1+\lambda)}(1+\frac{\eta_1}{(r+\eta_2+\lambda)})\int_{\tilde{\epsilon_1^d}}^{\bar{\epsilon}} y(x-\tilde{\epsilon_1^d})dG(x) + \eta_1 f_2\tau - \eta_1 y\frac{(\tilde{\epsilon_1^d}-\epsilon_2^d)}{(r+\lambda+\eta_2)} \end{cases}$$

And in the case 2, where $\max\{S_2(\epsilon_1^d),0\} = 0$, an unique equilibrium $(\hat{\epsilon_1^d},\hat{\theta}_1)$ is defined by the following equation system:

$$\begin{cases} \frac{c}{q(\hat{\theta}_1)} = & \frac{(1-\beta)}{(r+\eta_1+\lambda)}\int_{\hat{\epsilon_1^d}}^{\bar{\epsilon}} y(x-\hat{\epsilon_1^d})dG(x) + (1-\beta)\frac{\eta_1}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}\int_{\epsilon_2^d}^{\bar{\epsilon}} y[x-\epsilon_2^d]dG(x) \\ y\hat{\epsilon_1^d} = z_1 + \frac{\beta}{(1-\beta)}c\hat{\theta}_1 - \frac{\lambda\int_{\hat{\epsilon_1^d}}^{\bar{\epsilon}} y(x-\hat{\epsilon_1^d})dG(x)}{(r+\eta_1+\lambda)} - \frac{\eta_1}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}\int_{\epsilon_2^d}^{\bar{\epsilon}} y(x-\epsilon_2^d)dG(x) + \eta_1 f_2\tau \end{cases}$$

So, we have to show that the equilibrium for the younger generation is either the couple $(\tilde{\epsilon_1^d},\tilde{\theta}_1)$ or the couple $(\hat{\epsilon_1^d},\hat{\theta}_1)$. In other words, we have to show that there can be neither 0 solutions nor 2 solutions to this problem. We borrow the proof of Behaghel (2007). Indeed, if the problem had 0 solutions, it would imply : $\tilde{\epsilon_1^d} < \epsilon_2^d < \hat{\epsilon_1^d}$. So subtracting the two job creation conditions each other we would get:

$$\frac{c}{q(\tilde{\theta}_1)} - \frac{c}{q(\hat{\theta}_1)} = \frac{(1-\beta)}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[(r+\eta_1+\eta_2+\lambda)\int_{\tilde{\epsilon_1^d}}^{\epsilon_2^d} y(x-\tilde{\epsilon_1^d})dG(x)$$

$$+\eta_1[y\epsilon_2^d - y\tilde{\epsilon_1^d}](1-G(\epsilon_2^d))] + \frac{(1-\beta)}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[(r+\eta_2+\lambda)\int_{\epsilon_2^d}^{\hat{\epsilon_1^d}} y(x-\tilde{\epsilon}_1)dG(x)$$

$$+(r+\eta_2+\lambda)[y\hat{\epsilon_1^d} - y\tilde{\epsilon_1^d}](1-G(\hat{\epsilon_1^d}))] > 0 \tag{55}$$

Therefore, we deduce that $\tilde{\theta}_1 > \hat{\theta}_1$.

Furthermore, subtracting the two job destruction conditions each other we would get:

$$\frac{\beta}{(1-\beta)}c(\tilde{\theta}_1 - \hat{\theta}_1) = (y\tilde{\epsilon_1^d} - y\hat{\epsilon_1^d}) + \lambda[\frac{\int_{\epsilon_1^d}^{\bar{\epsilon}}(y\hat{\epsilon_1^d} - y\tilde{\epsilon_1^d})dG(x)}{(r+\eta_1+\lambda)} + \frac{\eta_1 y\int_{\tilde{\epsilon_1^d}}^{\bar{\epsilon}}(\epsilon_2^d - \tilde{\epsilon_1^d})dG(x)}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}]$$

$$+\eta_1 y \frac{(\tilde{\epsilon_1^d} - \epsilon_2^d)}{(r+\eta_2+\lambda)}$$

$$\Leftrightarrow \frac{\beta}{(1-\beta)} c(\tilde{\theta_1} - \hat{\theta_1}) = (y\tilde{\epsilon_1^d} - y\hat{\epsilon_1^d})[1 - \frac{\lambda(1-G(\tilde{\epsilon_1^d}))}{(r+\eta_1+\lambda)}]$$

$$+\frac{\eta_1 y(\tilde{\epsilon_1^d} - \epsilon_2^d)}{(r+\eta_2+\lambda)}[1 - \frac{\lambda(1-G(\tilde{\epsilon_1^d}))}{(r+\eta_1+\lambda)}] < 0 \tag{56}$$

So we deduce that $\tilde{\theta_1} < \hat{\theta_1}$. This result is not possible given the previous result, so it is the proof that there exists at least one solution to the problem.

Furthermore, we proceed in a similar way to show that the problem does not admit two solutions. Indeed, if he admitted two solutions, it would imply: $\tilde{\epsilon_1^d} > \epsilon_2^d > \hat{\epsilon_1^d}$. In that case, the equation (55) would imply $\tilde{\theta_1} < \hat{\theta_1}$ and the equation (56) would imply $\tilde{\theta_1} > \hat{\theta_1}$, so this case is absurd. Consequently, the problem admits one unique solution: either the couple $(\tilde{\epsilon_1^d}, \tilde{\theta_1})$ if $\epsilon_1^d > \epsilon_2^d$, or the couple $(\hat{\epsilon_1^d}, \hat{\theta_1})$ if $\epsilon_1^d < \epsilon_2^d$.

7.5. Effects of the tax rate and the retirement age on hiring and separation rates among older workers

We determine the partial derivatives of the job creation condition:

$$\begin{cases} C_1^2 = -c\alpha\theta_2^{(\alpha-1)} < 0 \\ C_2^2 = \frac{-(1-\beta)}{(r+\eta_2+\lambda)}[y(1-\varepsilon_2^d) - (r+\eta_2+\lambda)f_2\tau] < 0 \\ C_3^2 = -(1-\beta)f_2[1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y}] < 0 \\ C_4^2 = \frac{-2(1-\beta)y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y})^2 - f_2\tau\frac{(1-\beta)}{(r+\eta_2+\lambda)}[1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y}] < 0 \end{cases}$$

Then we determine the partial derivatives for the job destruction condition:

$$\begin{cases} D_1^2 = \frac{\beta c}{(1-\beta)} > 0 \\ \\ D_2^2 = \frac{\lambda y}{(r+\eta_2+\lambda)}(1-\varepsilon_2^d) - y = \frac{-[(r+\eta_2)y+\lambda y \varepsilon_2^d]}{(r+\eta_2+\lambda)} < 0 \\ \\ D_3^2 = -(r+\eta_2)f_2 < 0 \\ \\ D_4^2 = \frac{2\lambda y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d)^2 - f_2\tau \end{cases}$$

We examine the effect of an increase in τ on ϵ_2^d:

$$\frac{d\varepsilon_2^d}{d\tau} = \frac{(C_3^2 D_1^2 - D_3^2 C_1^2)}{(D_2^2 C_1^2 - C_2^2 D_1^2)} < 0$$

Then we investigate the effect of a decrease in η_2 (namely an increase in the mandatory retirement age) on ϵ_2^d:

$$\frac{d\varepsilon_2^d}{d\eta_2} = \frac{(C_4^2 D_1^2 - D_4^2 C_1^2)}{(D_2^2 C_1^2 - C_2^2 D_1^2)}$$

We know that: $C_4^2 D_1^2 < 0$ and $C_1^2 < 0$. So if $D_4^2 < 0$ then $\frac{d\varepsilon_2^d}{d\eta_2} < 0$. We have to study the sign of D_4^2:

$$D_4^2 = \frac{2\lambda y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d)^2 - f_2\tau$$

Consequently, a sufficient condition such that $D_4^2 < 0$ may be expressed as follows:

$$\begin{aligned} \frac{2\lambda y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d)^2 - f_2\tau \quad &< \quad 0 \\ \Leftrightarrow \quad \tau &> \frac{2\lambda y(1-\varepsilon_2^d)^2}{[2(r+\eta_2+\lambda)]^2 f_2} \end{aligned}$$

Now, we determine the effect of an increase in τ on the tightness θ_2:

$$\frac{d\theta_2}{d\tau} = \frac{D_3^2 C_2^2 - C_3^2 D_2^2}{(C_1^2 D_2^2 - D_1^2 C_2^2)}$$

$$
\begin{aligned}
D_3^2C_2^2 - C_3^2D_2^2 &= \frac{(1-\beta)}{(r+\eta_2+\lambda)}[y(1-\varepsilon_2^d) - (r+\eta_2+\lambda)f_2\tau](r+\eta_2)f_2 - \\
&\quad (1-\beta)f_2[1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y}]\frac{(r+\eta_2)y+\lambda y\varepsilon_2^d}{(r+\eta_2+\lambda)}
\end{aligned}
$$

Factorizing this expression by $\frac{(1-\beta)}{(r+\eta_2+\lambda)}$ we deduce that its sign is the same as the sign of the following expression:

$$
[y(1-\varepsilon_2^d)-(r+\eta_2+\lambda)f_2\tau](r+\eta_2)f_2-f_2[1-\varepsilon_2^d-\frac{(r+\eta_2+\lambda)f_2\tau}{y}][(r+\eta_2)y+\lambda y\varepsilon_2^d]
$$

Simplifying and rearranging terms, we find that this expression equals:

$$
-f_2\lambda\varepsilon_2^d y[(1-\varepsilon_2^d) - \frac{(r+\eta_2+\lambda)f_2\tau}{y}] < 0
$$

Then we determine the effect of an increase in the mandatory retirement age on θ_2:

$$
\frac{d\theta_2}{d\eta_2} = \frac{D_4^2C_2^2 - C_4^2D_2^2}{(C_1^2D_2^2 - D_1^2C_2^2)}
$$

If $D_4^2 > 0$, then $\frac{d\theta_2}{d\eta_2} < 0$. However, if $D_4^2 < 0$, then :

$$
\begin{aligned}
D_4^2C_2^2 - C_4^2D_2^2 &= -[\frac{2\lambda y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d)^2 - f_2\tau]\frac{(1-\beta)}{(r+\eta_2+\lambda)}[y(1-\varepsilon_2^d)-(r+\eta_2+\lambda)f_2\tau] \\
&\quad -\{\frac{2(1-\beta)y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y})^2 \\
&\quad + f_2\tau\frac{(1-\beta)}{(r+\eta_2+\lambda)}[1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y}]\}\frac{[(r+\eta_2)y+\lambda y\varepsilon_2^d]}{(r+\eta_2+\lambda)}
\end{aligned}
$$

Factorizing by $\frac{(1-\beta)}{(r+\eta_2+\lambda)}[(1-\varepsilon_2^d)-\frac{(r+\eta_2+\lambda)f_2\tau}{y}]$ we deduce that this expression has the same sign as the following expression:

$$
\begin{aligned}
& -y[\frac{2\lambda y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d)^2 - f_2\tau] - f_2\tau\frac{[(r+\eta_2)y+\lambda y\varepsilon_2^d]}{(r+\eta_2+\lambda)} \\
& -\frac{2y}{[4(r+\eta_2+\lambda)]}(1-\varepsilon_2^d - \frac{(r+\eta_2+\lambda)f_2\tau}{y})\frac{[(r+\eta_2)y+\lambda y\varepsilon_2^d]}{(r+\eta_2+\lambda)}
\end{aligned}
$$

The sign of this expression may be ambiguous given that at $\tau > \tau^c$, $\frac{2\lambda y}{[2(r+\eta_2+\lambda)]^2}(1-\varepsilon_2^d)^2 - f_2\tau < 0$. We can define a sufficient condition such that $\frac{\partial\theta_2}{\partial\eta_2} > 0$:

$$\begin{aligned} f_2\tau\frac{[\lambda y(1-\varepsilon_2^d)]}{(r+\eta_2+\lambda)} &> \frac{y^2}{2(r+\eta_2+\lambda)^2}\{(1-\varepsilon_2^c)[(r+\eta_2)+\lambda\varepsilon_2^d]+(1-\varepsilon_2^d)^2\} \\ &\Leftrightarrow \quad \tau > \frac{y\{(1-\varepsilon_2^d)[(r+\eta_2)+\lambda\varepsilon_2^d]+(1-\varepsilon_2^d)^2\}}{f_2(r+\eta_2+\lambda)[\lambda(1-\varepsilon_2^d)+(r+\eta_2+\lambda)]} \end{aligned}$$

7.6. Effects of the tax rate and the retirement age on hiring and separation rates among middle-age workers

We consider first the case 1 where $\max\{S_2(\epsilon_1^d),0\} = S_2(\epsilon_1^d)$. We determine the effect of an increase in τ and of a decrease in η_2 on the productivity threshold ϵ_1^d. Differentiating this equations system we find the two following expressions:

$$\begin{cases} \frac{\partial\epsilon_1^d}{\partial\tau} = \frac{(C_3^1D_1^1-D_3^1C_1^1)+(C_5^1D_1^1-D_5^1C_1^1)\frac{d\epsilon_2^d}{d\tau}}{D_2^1C_1^1-C_2^1D_1^1} \\ \\ \frac{\partial\epsilon_1^d}{\partial\eta_2} = \frac{(C_4^1D_1^1-D_4^1C_1^1)+(C_5^1D_1^1-D_5^1C_1^1)\frac{d\epsilon_2^d}{d\eta_2}}{D_2^1C_1^1-C_2^1D_1^1} \end{cases}$$

We determine the partial derivatives for the job creation condition:

$$\begin{cases} C_1^1 = -\alpha c\theta_1^{\alpha-1} < 0 \\ \\ C_2^1 = \frac{-(1-\beta)(r+\eta_1+\eta_2+\lambda)y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}(1-\epsilon_1^d) < 0 \\ \\ C_3^1 = 0 \\ \\ C_4^1 = \frac{-2\eta_1(r+\eta_1+\lambda)(1-\beta)y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2}(1-\epsilon_1^d)^2 < 0 \\ \\ C_5^1 = 0 \end{cases}$$

Then we calculate the partial derivatives for the job destruction condition:

$$\begin{cases} D_1^1 = \frac{\beta c}{(1-\beta)} > 0 \\ \\ D_2^1 = \frac{-(r+\eta_1+\eta_2+\lambda)y[(r+\eta_1)+\lambda\epsilon_1^d]}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)} < 0 \\ \\ D_3^1 = \eta_1 f_2 > 0 \\ \\ D_4^1 = (1-\epsilon_1^d)^2 \frac{2\eta_1(r+\eta_1+\lambda)\lambda y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2} + \eta_1 y \frac{(\epsilon_1^d-\epsilon_2^d)}{(r+\eta_2+\lambda)^2} > 0 \\ \\ D_5^1 = \frac{\eta_1 y}{(r+\eta_2+\lambda)} > 0 \end{cases}$$

Consequently:

$$\frac{d\epsilon_1^d}{d\tau} > 0 \Leftrightarrow \eta_1 f_2 > -\frac{\eta_1 y}{(r+\eta_2+\lambda)} \frac{d\epsilon_2^d}{d\tau}$$

Furthermore:

$$\frac{d\epsilon_1^d}{d\eta_2} > 0 \Leftrightarrow \frac{2\eta_1(r+\eta_1+\lambda)y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2}(\lambda\alpha c\theta_1^{\alpha-1} - \beta c)$$

$$+\alpha c\theta_1^{\alpha-1}[\eta_1 y \frac{(\epsilon_1^d - \epsilon_2^d)}{(r+\eta_2+\lambda)^2} + \frac{\eta_1 y}{r+\eta_2+\lambda} \frac{d\epsilon_2^d}{d\eta_2}] > 0$$

We determine then the effect of an increase in τ and of a decrease in η_2 on the tightness θ_1. We obtain the following equations system:

$$\begin{cases} \frac{d\theta_1}{d\tau} = \frac{(D_3^1 C_2^1 - C_3^1 D_2^1) + (D_5^1 C_2^1 - C_5^1 D_2^1)\frac{d\epsilon_2^d}{d\tau}}{C_1^1 D_2^1 - D_1^1 C_2^1} \\ \\ \frac{d\theta_1}{d\tau} = \frac{(D_4^1 C_2^1 - C_4^1 D_2^1) + (D_5^1 C_2^1 - C_5^1 D_2^1)\frac{d\epsilon_2^d}{d\eta_2}}{C_1^1 D_2^1 - D_1^1 C_2^1} \end{cases}$$

Consequently:

$$\frac{d\theta_1}{d\tau} < 0 \Leftrightarrow \eta_1 f_2 + \frac{\eta_1 y}{r+\eta_2+\lambda} \frac{d\epsilon_2^d}{d\tau} > 0$$

Furthermore:

$$\frac{d\theta_1}{d\eta_2} < 0 \Leftrightarrow (D_4^1 C_2^1 - C_4^1 D_2^1) + D_5^1 C_2^1 \frac{d\epsilon_2^d}{d\eta_2} < 0$$

As $D_4^1 C_2^1 < 0$, $C_4^1 D_2^1 > 0$ and $D_5^1 C_2^1 < 0$, so the direct effect implies that a decrease in η_2 raises θ_1. The indirect effect through $\partial\epsilon_2^d/\partial\eta_2$ may reinforce the direct effect if $\tau < \tau^c$, but it may attenuate the direct effect if $\tau > \tau^c$.

In a second step, we consider the case 2 where $\max\{S_2(\epsilon_1^d), 0\} = 0$. We calculate the partial derivatives for the job creation condition:

$$\begin{cases} C_1^1 = -\alpha c \theta_1^{\alpha-1} < 0 \\ C_2^1 = \frac{-(1-\beta)y}{(r+\eta_1+\lambda)}[1-\epsilon_1^d] < 0 \\ C_3^1 = 0 \\ C_4^1 = -[1-\epsilon_2^d]^2 \frac{2(1-\beta)\eta_1 y(r+\eta_1+\lambda)}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2} < 0 \\ C_5^1 = \frac{-(1-\beta)\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d] < 0 \end{cases}$$

We also calculate the partial derivatives for the job destruction condition:

$$\begin{cases} D_1^1 = \frac{\beta c}{(1-\beta)} > 0 \\ D_2^1 = \frac{y[-(r+\eta_1)-\lambda\epsilon_1^d]}{(r+\eta_1+\lambda)} < 0 \\ D_3^1 = \eta_1 f_2 > 0 \\ D_4^1 = \frac{2(r+\eta_1+\lambda)\eta_1 y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2}[1-\epsilon_2^d]^2 > 0 \\ D_5^1 = \frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d] > 0 \end{cases}$$

We determine first the effect of an increase in τ and of a decrease in η_2 on the productivity threshold ϵ_1^d. Differentiating our equations system, we obtain the two following expressions:

$$\begin{cases} \frac{d\epsilon_1^d}{d\tau} = \frac{(C_3^1 D_1^1 - D_3^1 C_1^1) + (C_5^1 D_1^1 - D_5^1 C_1^1)\frac{d\epsilon_2^d}{d\tau}}{D_2^1 C_1^1 - C_2^1 D_1^1} \\ \frac{d\epsilon_1^d}{d\eta_2} = \frac{(C_4^1 D_1^1 - D_4^1 C_1^1) + (C_5^1 D_1^1 - D_5^1 C_1^1)\frac{d\epsilon_2^d}{d\eta_2}}{D_2^1 C_1^1 - C_2^1 D_1^1} \end{cases}$$

We determine a sufficient condition under which an increase in the tax rate

τ raises ϵ_1^d:

$$\frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d]\frac{d\epsilon_2^d}{d\tau}[\alpha c\theta_1^{\alpha-1}-\beta c]+\eta_1 f_2\alpha c\theta_1^{\alpha-1}>0 \quad (57)$$

Furthermore, $\frac{d\epsilon_1^d}{d\eta_2}$ has the same sign as the following expression:

$$\frac{2(r+\eta_1+\lambda)\eta_1 y}{[2(r+\eta_1+\lambda)(r+\eta_2+\lambda)]^2}[1-\epsilon_2^d]^2[\alpha c\theta_1^{\alpha-1}-\beta c]$$

$$+\frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d][\alpha c\theta_1^{\alpha-1}-\beta c]\frac{d\epsilon_2^d}{d\eta_2}$$

Then, we determine the effect of an increase in τ and of a decrease in η_2 on the tightness θ_1. We obtain the two following expressions:

$$\begin{cases} \frac{d\theta_1}{d\tau} = \frac{(D_3^1C_2^1-C_3^1D_2^1)+(D_5^1C_2^1-C_5^1D_2^1)\frac{d\epsilon_2^d}{d\tau}}{C_1^1D_2^1-D_1^1C_2^1} \\ \\ \frac{d\theta_1}{d\eta_2} = \frac{(D_4^1C_2^1-C_4^1D_2^1)+(D_5^1C_2^1-C_5^1D_2^1)\frac{d\epsilon_2^d}{d\eta_2}}{C_1^1D_2^1-D_1^1C_2^1} \end{cases}$$

Consequently, an increase in τ leads to a fall in the tightness θ_1 if the following condition holds:

$$\frac{-(1-\beta)}{(r+\eta_1+\lambda)}[1-\epsilon_1^d]\eta_1 f_2+\frac{\eta_1 y}{(r+\eta_1+\lambda)(r+\eta_2+\lambda)}[1-\epsilon_2^d][\alpha c\theta_1^{\alpha-1}-\beta c]\frac{d\epsilon_2^d}{d\tau}<0 \quad (58)$$

Furthermore, regarding the effect of a decrease in η_2 on θ_1, we draw similar conclusions as in the case 1.

7.7. Wage equations and productivity thresholds in the case of undirected search

Let us start from the equation (39). When substituting the rent-sharing rule and the free-entry condition in the wage equation. Undirected search implies:

$$\int_{\varepsilon_1^c}^{\bar{\varepsilon}}[W_1(x)-U_1]dG(x)=\frac{\beta}{(1-\beta)(1-\gamma)}[\frac{c}{q(\theta)}-\gamma\int_{\varepsilon_2^c}^{\bar{\varepsilon}}J_2(x)dG(x)] \quad (59)$$

So substituting this expression into (38) we get the following wage equation:

$$w_1(\varepsilon) = (1-\beta)z_1+\frac{\beta c\theta}{(1-\gamma)}-\beta p(\theta)\frac{\gamma}{(1-\gamma)}\int_{\varepsilon_2^c}^{\bar{\varepsilon}} J_2(x)dG(x)+\beta(y\varepsilon-\eta_1 f_2\tau)+f_1(r+\eta_1)-\eta_1 f_2 \tag{60}$$

In the same way, to determine the wage equation for older workers, the rent-sharing rule and the new free entry condition in the case of undirected search yield:

$$w_2(\varepsilon) = (1-\beta)z_2+\beta y\varepsilon+\frac{\beta c\theta}{\gamma}-\beta p(\theta)\frac{(1-\gamma)}{\gamma}\int_{\varepsilon_1^c}^{\bar{\varepsilon}} J_1(x)dG(x)+(r+\eta_2)[f_2(1+\beta\tau)]-\eta_2 f_\tau \tag{61}$$

First we determine ε_1^d in the case of undirected search:

$$\begin{aligned} y\varepsilon_1^d = z_1 + \frac{\beta c\theta}{(1-\beta)(1-\gamma)} - \beta p(\theta)\frac{\gamma}{(1-\beta)(1-\gamma)}\int_{\varepsilon_2^c}^{\bar{\varepsilon}} J_2(x)dG(x) \\ -\lambda\int_{\varepsilon_1^d}^{\bar{\varepsilon}} S_1(x)dG(x) - \eta_1 \max\{S_2(\varepsilon_1^d),0\} + \eta_1 f_2\tau \end{aligned} \tag{62}$$

Then we determine ε_2^d in the case of undirected search:

$$y\varepsilon_2^d = z_2+\frac{\beta c\theta}{(1-\beta)\gamma}-\beta p(\theta)\frac{(1-\gamma)}{(1-\beta)\gamma}\int_{\varepsilon_1^c}^{\bar{\varepsilon}} J_1(x)dG(x)-\lambda\int_{\varepsilon_2^d}^{\bar{\varepsilon}} S_2(x)dG(x)-(r+\eta_2)f_2\tau \tag{63}$$

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