Young in old out: a new evaluation

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Abstract

In this paper we examine the consequences of an increase in the legal retirement age on youth employment. Our contribution consists in treating work experience as a source of imperfect substitutability between young and older workers for some type of jobs, in the sense that young can apply only for simple jobs, requiring no experience. Using a theoretical matching model with heterogeneous jobs and workers, we point out that an increase in the legal retirement age may raise job-to-job flows from simple to complex jobs, making therefore firms more reluctant to open simple jobs, and reducing job opportunities for young workers. Then, we provide some empirical evidence for the Italian case, using a generalized propensity score matching approach.

Key words: retirement age, youth employment, propensity score, matching model, on-the-job search

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1. Introduction

Over the last three decades, most European countries experienced an ageing of their population and a fall in the labour force participation of older workers. In the face of steadily increasing dependency rates, some countries raised their legal retirement age to balance the budget of their pay-as-you-go retirement schemes. However, the efficiency of such a policy option depends

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strongly on its impact on the hiring rate and employment among every age group of workers and particularly the young workers.

At first sight, we can think that an increase in legal retirement age, encouraging firms to maintain their ageing workforce, could slow down the dynamics of new hirings, if firms were expected to squeeze out their older workers and to replace them by new young workers. However, this simplistic idea, that led to the creation of early retirement schemes, has been severely criticized over the past ten years. Following the idea of Bozio (2006) we may argue that young and older workers may be imperfectly substitutable in the labour market. Consequently, once an older worker retires, his job can not be filled by a young worker. We can discuss about what makes these two cohorts of workers imperfectly substitutable. Cadiou et al. (2002) highlight the fact that the low job finding rates among young workers and among older workers are not of the same nature. While the low hiring rate of senior workers is due to the fact that they are close to the retirement age, the low job finding rate for young workers stems from their lack of experience. Consequently, they can not fill a job requiring some professional experience to replace an older worker retired.

However, even though the two sorts of jobs for the young and the old generation of workers differ greatly in the set of skill and experience requirements, a reduction in exits of older workers due to an increase in the retirement age may have a negative impact on the hiring rate of young workers through a vacancy-chain effect (Contini and Revelli, 1997). The idea is the following: if we consider within a local labour market, that a vacancy opens up when an older worker leaves his job, there is a non-negative probability that it will be filled by an employed job-seeker (attracted by better opportunities or higher wage), who accumulated a sufficient level of experience to replace the older worker and whose job requires less professional experience. Therefore, after this job-to-job flow, a new vacancy opens up and may be filled by another young employed job-seeker, involving again job-to-job flows, that may result finally in hiring a young worker from out of the local labour market. So, holding job creation constant and considering a vacancy-chain pattern, retaining older workers in the labour force leads to fewer complex vacant jobs,

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1See also Chéron et al. (2007) for this question

2A local labour market is a group of firms belonging to the same sector of activity, located in the same area and whose jobs require the same educational level
i.e. vacant jobs requiring some professional experience, which reduces job to job flows and decreases the number of vacant jobs requiring no experience. As a result, job finding rates of young workers are negatively affected.

For this idea to be relevant, two conditions are needed. First, the economy has to be characterised by a large share of job-to-job transitions in the total number of separations. Second, job creation has to be uncorrelated with a variation in the exit rates of older workers. Regarding the first condition, recent empirical studies highlight the major role of job-to-job flows to explain the total labour market turnover. In the case of the US labour market, Nagypal (2003) reports that job to job transitions account for more than 50% of the total separations of workers with a college degree and for more than 30% of separations of workers without a college degree. Regarding the European case, Theodossiou and Zangelis (2007) report cross-country evidence on job to job transitions for 6 European countries and they find that job to job flows account for up to 17% of total separations in the different European economies considered. So it appears that workers in Europe are less mobile than in the US labour market. Nevertheless, in Italy, Contini and Rapiti (1994) show that job to job transitions are about 25% of total separations. The Italian labour market can therefore be characterized by a large share of job to job transitions.

The second condition requires job creation to be exogenous. As argued by Layard, et al. (1991), it seems unrealistic and they refer to this idea as a lump-of-labour fallacy. Indeed, job creation is expected to be affected by a change in the exit rate of older workers, so we can not view the number of jobs in the economy as a fixed stock. Using a theoretical matching model "à la" Mortensen-Pissarides (1994), Chéron et al. (2007) already pointed out that there exists a positive relation between the distance from retirement age and the hiring rate. This result implies that postponing retirement has a positive horizon effect on the hiring rate of each group of workers, including the young workers. Nevertheless, while it is clear that the distance from retirement strongly affects job finding rates of older workers, it is less straightforward that it plays a significative role in the determination of job opportunities for young workers.

In this paper we investigate the effect of an increase in the legal retirement age on youth employment, accounting for endogenous job creation and job-to-job flows to illustrate the mechanics of job openings dynamics. Our
theoretical contribution is to develop a simple matching model to highlight a new effect of an increase in the legal retirement age on the propensity of firms to hire young workers, at a local labour market level. The key assumption is to view new young entrants in the labour market as workers with no experience. Consequently, they can not be matched with complex jobs requiring some experience and they can interact only with employers who open simple vacancies, requiring no experience. As young workers compete with experienced workers, we build a matching model with heterogeneous jobs and workers, but in contrast to the standard literature\(^3\), we account for differences in work experience rather than differences in skills\(^4\). As we allow for job-to-job flows, some experienced workers may be mismatched with jobs requiring no experience. These workers are therefore looking for a complex job due to better wage offers. In this framework we show that delaying retirement may lead to an intensification of job-to-job flows from simple to complex jobs and reduces the flow value of a simple job to an employer. Consequently, it reduces the creation of simple jobs and exerts a negative effect on the job finding rate of young workers with no experience.

Then, in the second part of the paper, we provide some empirical evidence of negative effects of retaining older workers in the labour force on the hiring rate of young workers at a local labour market level. We examine the Italian case given that in this country job-to-job flows account for a large share of total worker flows. While the relation between elderly labour force participation and youth employment has been widely studied at a country-level (Bozio et al., 2008, Jouten et al., 2009), too few empirical studies have been carried out in a microeconomic perspective. However, as worker and job flows are strongly affected by firm-specific variables (Davis and Haltiwanger, 1999), it is interesting to investigate the effect of an exogenous decrease in exits of older workers on the hiring rate of young workers at the firm-level, controlling for all regressors that may affect job creation.

Exploiting a pension reform enacted by the Portuguese government in 1992, Portugal et al. (2009) examine the effect of a decrease in the exit rate of elderly, due to an increase in the legal retirement age for women, on worker flows of different age groups, using detailed matched employer-employee Por-

\(^3\)For similar specification see also Dolado et al., 2009; Gautier 2002; Mortensen and Pissarides 1999; Albrecht and Vroman, 1999

\(^4\)Once again, we examine the job openings dynamics at a local labour market level, so the educational level does not differ across workers or across jobs.
tuguese data. Their empirical methodology lies on a quasi-experiment given the fact that only women have been affected by the retirement reform. Using a difference-in-difference matching method (Heckman, 1997), they show that firms employing women treated by the reform hire one to two fewer workers for each senior retained after the increase in legal retirement age and this decrease in the hiring rate is particularly strong for younger workers.

Our paper is close to the spirit of their study. Indeed, our goal is to examine the effect of a fall in the exit rate of older workers on the hiring rate of young workers, using Italian matched employer-employee data from the Worker Histories Italian Panel and considering the period 1988-1989. We examine the period 1988-1989 given the fact that after 1990, the Italian labour market has been severely affected by changes in employment protection legislation (Kugler and Pica, 2008), then it has been hit by a severe recession and after 1997, job creation has been strongly influenced by the introduction of temporary contracts and of other atypical contracts by the Treu reform, making the Italian labour market more flexible (Sciulli, 2006). So, we choose to study the period 1988-1989 to be able to isolate the effect on hirings of young workers due to a fall in exits of older workers from effects due to other institutional effects or to economic fluctuations.

Nevertheless, our empirical study differs from the work of Portugal et al. for two reasons: first, we do not consider firms but rather local labour markets, using a synthetic-firm approach to account for job-to-job flows, that represent the starting point of our theoretical intuition. Second, our work consists in estimating the response at a local labour market level of the hiring rate of young workers associated with each value of a continuous treatment, i.e., the variation between 1988 and 1989 in the exit rate of older workers. As we do not exploit a quasi-experiment, we are faced with a selection bias, in the sense that local labour markets with different variation in exit rates of older workers may also differ in their characteristics likely to affect the accession rate of young workers. To address this econometric issue, we use a propensity score matching procedure generalized to a continuous treatment (Hirano and Imbens, 2004). This approach is an extension of the widely used propensity score methodology for binary treatments (Rosenbaum and

\footnote{Actually, we consider the absolute value of the decrease in the exit rate of older workers. Consequently, a high value of the treatment means that the observation unit experienced a severe fall in the exit rate of older workers between the two years.}
Rubin, 1983) that allows us to remove the selection bias on observables and to estimate to what extent the entry rate of young workers is affected by a decrease in the exit rate of older workers at a local labour market level. After correcting for the selection bias, we find that the higher the fall in exits of older workers, the less hirings of young workers within a local labour market.

The remainder of the paper is structured in the following way: in section 2, we develop our theoretical model and we present our theoretical findings. Then, in section 3, we present our empirical study, describing the type of data used, explaining our methodology and presenting our empirical results. Section 4 concludes.

2. The theoretical model: A labour market with heterogeneous jobs and workers

Following the wide literature about skilled and unskilled workers, we build a theoretical model with two sorts of workers and two kinds of jobs. But in the contrary of the standard literature, we distinguish workers according to their professional experience and not according to their educational level. To allow for job-to-job flows, we focus on a local labour market where all firms belong to the same sector of activity, are located in the same area and whose jobs require the same educational level. In our model, we consider the young workers, that we denote by $ne$ as subscript, who entry into the labour market with no professional experience and the workers with experience, denoted by $e$ as subscript. We assume that a number $\eta_0$ of young workers exogenously arrive in the labour market as unemployed. They can only apply for jobs requiring no experience. In the remainder of the paper, we denote this type of jobs by $S$ (simple job) as upscript. In our model, once a young is hired, he accumulates professional experience and can reach a sufficient experience level at a Poisson arrival rate $\lambda$ to apply for complex jobs, denoted by the upscript $C$. Workers have interest to apply for complex jobs to have better wage opportunities, given that in our model, the wage received by a worker occupying a simple job is assumed to be the minimum wage$^6$.

Both types of jobs may be broken up in the case of an idiosyncratic shock, according to a Poisson process with an arrival rate $\delta$. In that case, a worker

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$^6$See also Belan et al. (2007) for a similar specification
who occupied the job becomes unemployed but his experience level remains unchanged. Indeed, whatever the unemployment duration of a worker, his past working experience remains the same over time. Furthermore, as we introduce on-the-job search, workers with experience may always have interest to search for a simple job, implying a cross-skill equilibrium in contrast to the paper of Albrecht and Vroman (2002).

Following the Behaghel’s specification (2007), we consider a guillotine retirement age, introducing a finite horizon in the career of workers. We consider that a worker can retire as soon as he accumulates a sufficient experience level to apply for complex jobs. Let \( \eta \) be the Poisson arrival rate at which an experienced worker retires. The horizon of an experienced worker is therefore equal to \( \frac{1}{\eta} \). Consequently, a decrease in \( \eta \) lengthens the career of an experienced worker. In the remainder of the paper, we assimilate a decrease in \( \eta \) to an increase in the retirement age.

Our specification is twofold: first, it allows us to reproduce the life cycle of a worker, who arrives in the labour market with no experience and who has to accumulate a sufficient level of experience occupying simple jobs to apply for complex jobs. Second, we build an economy with overlapping labour markets (Sattinger, 2006) in which young workers can not be matched with jobs requiring experience. Through this theoretical framework, we introduce an imperfect substitution between a worker occupying a complex job who retires and a young worker. Our aim is to study the effect of an increase in the legal retirement age, assimilated to a decrease in \( \eta \), on the job finding rate of young workers.

In our model we assume that search is undirected, so workers have perfect information about the type of new vacancies but employers can not target their hirings to a special type of workers. In our economy, vacancies of type \( S \) and the two types of workers (with or without experience) meet each other according to a matching function denoted by \( h^S(u_{ne} + u_e, v^S) \), that determines the number of hirings for this kind of jobs as a function of the number \( u_i \) of unemployed of type \( i \), with \( i \in \{ne, e\} \), and of the number of

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7 The implications of a finite horizon in a matching model on job creation and job destruction have been already investigated by Cheron et al. (2006 and 2007).

8 However, in the case of complex jobs, employers are sure to be matched with only workers with experience, given that young workers can not apply for this type of jobs.
vacancies $v^S$ of type $S$. We assume that the matching function is increasing, concave and linear homogenous.

An employer opening a $S$-type vacancy may fill his job according to a Poisson process at an arrival rate $h^S(\frac{1}{\theta^S}, 1) = q(\theta^S)$. $q(\theta^S)$ is decreasing with the tightness on the simple job market, denoted by $\theta^S$, equal to the number of $S$-type vacancies over the number of job-seekers. We deduce therefore that an unemployed worker (whatever his type) may be matched with a simple job according to a Poisson process at an arrival rate $p(\theta^S)$, such that $p(\theta^S) = \theta^S q(\theta^S)$ . $p(\theta^S)$ is therefore increasing with $\theta^S$. As unemployed workers with no experience can be matched only with simple jobs, $p(\theta^S)$ corresponds to the job finding rate of young workers.

In a similar way, vacancies of type $C$ meet job seekers with experience according to a matching function denoted by $h^C(u_e + o_e, v^C)$, that determines the number of hirings for this kind of jobs as a function of the number $u_e$ of unemployed workers with experience, of the number $o_e$ of employed job seekers and of the number $v^C$ of $C$-type vacancies. Consequently, a $C$-type vacancy may be matched with a job seeker according to a Poisson process at an arrival rate $h^C(\frac{1}{\theta^C}, 1) = q(\theta^C)$. This arrival rate is decreasing with the tightness in the complex jobs market $\theta^C$, equal to the number of $C$-type vacancies over the number of job seekers with experience. We deduce therefore that an experienced job-seeker may find a complex job according to a Poisson process at an arrival rate $p(\theta^C) = \theta^C q(\theta^C)$ increasing with $\theta^C$.

The figure 1 represents the worker flows in our model. The dashed arrow indicates job-to-job flows.

To determine the labour force structure at steady-state, we solve in the appendix 1 the flow equations and we obtain the expression of the number of unemployed workers with no experience $u_{ne}$.

$$u_{ne} = \frac{\eta_0(\delta + \lambda)}{\lambda p(\theta^S)}$$ (1)

The expression (1) is quite intuitive. The youth unemployment increases with the number of new entrants into the labour market $\eta_0$ and decreases with their probability to find a simple job denoted by $p(\theta^S)$. Our theoretical model allows us to endogenize this probability as a function of the tightness $\theta^S$, depending on the simple-job creation behaviour of employers. Since this
Figure 1: Worker flows within a local labour market
behaviour may be strongly influenced by the horizon of workers, our approach allows us to investigate theoretically the channels through which an increase in the legal retirement age has an impact on hirings of young workers with no experience. In the next subsection, we represent the job creation decision of firms.

2.1. The firms’ behaviour

In our model, for the sake of simplicity, each firm can not open more than one job. As long as the job is vacant, the firm pays a search cost $c^i$ that depends on the type of job opened ($i \in \{S, C\}$). When a firm decides to open a S-type vacancy, it may be filled by a young worker with no experience according to a Poisson process at an arrival rate $\gamma q(\theta^S)$, where $\gamma$ is the share of unemployed workers with no experience over the whole unemployment, i.e. $\gamma = u_{ne}/(u_{ne} + u_e)$. In a similar way, a S-type vacancy may be filled by an experienced job seeker according to a Poisson process at an arrival rate $(1 - \gamma)q(\theta^S)$. At the steady-state equilibrium, the flow value $V^S$ to an employer from opening a S-type vacancy satisfies the following Bellman equation:

$$rV^S = -c^S + q(\theta^S)[\gamma(J^S_{ne} - V^S) + (1 - \gamma)(J^S_e - V^S)]$$  \hspace{0.5cm} (2)

where $J^S_i$ is the flow value to a firm of a simple job filled by a worker of type $i$, with $i \in \{ne, e\}$.

Furthermore, the flow value $V^C$ to a firm which opens a C-type vacancy is defined by the following Bellman equation:

$$rV^C = -c^C + q(\theta^C)(J^C - V^C)$$  \hspace{0.5cm} (3)

where $J^C$ is the flow value to an employer of a filled complex job. We assume that firms are free to entry the market. Consequently, at steady-state equilibrium all profits from opening a new vacancy are exhausted for the two sorts of jobs, which implies the following free-entry condition:

$$V^C = V^S = 0$$  \hspace{0.5cm} (4)

In the submarket of simple jobs, a firm opening a S-type job vacancy may either recruit a unexperienced unemployed worker or an experienced one. Let
$J_i^S$ be the present flow value from hiring a worker of type $i$, with $i \in \{ne, e\}$. We assume that both types of workers have the same productivity on the same type of job. However, an experienced worker can quit his job to find a complex job at a Poisson arrival rate $p(\theta^C)$ and can also retire at a Poisson arrival rate $\eta$, contrarily to workers with no experience.

Borrowing the idea of Belan et al. (2007), we assume that each worker occupying a simple job receives a minimum wage $w$. Consequently, the present values of expected profit to a firm from a simple job filled either with a worker with no experience or with an experienced worker satisfy the following Bellman equations:

$$r J_{ne}^S = y^S - w + \delta(V^S - J_{ne}^S) + \lambda(J_e^S - J_{ne}^S) \quad (5)$$

And:

$$r J_e^S = y^S - w + [\delta + \eta + p(\theta^C)](V^S - J_e^S) \quad (6)$$

Furthermore, when an experienced job seeker finds a complex job, he starts producing an output $y^C > y^S$ and he receives a wage $w^C > w$. This theoretical part aims at illustrating the mechanics of firm job opening dynamics after an increase in the retirement age, so we focus on the wage-posting equilibrium. The job of a worker occupying a complex job may break up for two reasons. First, it may be hit by an idiosyncratic shock at a Poisson arrival rate $\delta$ and the worker gets into unemployment. Second, the worker may retire at an arrival rate $\eta$. So, the flow value to a firm from a filled complex job satisfies the following Bellman equation:

$$r J^C = y^C - w^C + (\eta + \delta)(V^C - J^C) \quad (7)$$

2.2. Wage posting equilibrium

First, substituting the Bellman equation (7) in the expression (3) and using the free-entry condition, we obtain the following expression:

$$\frac{c^C}{q(\theta^C)} = \frac{(y^C - w^C)}{(r + \eta + \delta)} \quad (8)$$
The expression (8) implies that at the steady-state equilibrium, the average search cost of an employer who opens a complex job equates the present flow value of a complex job. Since the expected profit of a filled complex job to an employer is positively correlated with the horizon of the worker, delaying retirement, which equates in our model with a decrease in \( \eta \), will foster the creation of complex jobs and therefore will increase \( \theta^C \). This first result is in accordance with previous findings of Chéron et al. (2007) that highlight a positive horizon effect of an increase in the retirement age on job creation. However, in this paper, we pay more attention to the effect of delaying retirement on hirings of young workers. We mentioned previously that young workers can not apply for complex jobs, so we have to determine the effect of a decrease in \( \eta \) on the creation of simple jobs, that is on \( \theta^S \).

Combining the Bellman equation (2) and the free-entry condition we obtain the following expression:

\[
\gamma J_{ne}^S + (1 - \gamma) J_e^S = \frac{c^S}{q(\theta^S)} \tag{9}
\]

Substituting the equations (5) and (6) into the expression (9) we get the following relation between \( \theta^S \) et \( \theta^C \):

\[
\frac{c^S}{q(\theta^S)} = \frac{\gamma}{(r + \delta + \lambda)} [(y^S - w) + \frac{\lambda[y^S - w]}{r + \delta + p(\theta^C) + \eta}] + \frac{(1 - \gamma)(y^S - w)}{r + \delta + p(\theta^C) + \eta} \tag{10}
\]

The term on the left-hand side of the expression represents the average search cost of an employer who opens a S-type vacancy, so it is increasing with the tightness \( \theta^S \). The right-hand side of the expression corresponds to the present value of the average expected profit to a firm of a filled simple job. The higher the flow value of a filled simple job, the higher the tightness \( \theta^S \) and the higher the job finding rate of young workers \( p(\theta^S) \).

We find two main offsetting effects of an increase in the retirement age on the tightness for the submarket of simple jobs \( \theta^S \). On the one hand, a decrease in \( \eta \) raises the horizon of experienced workers and therefore increases the average duration of simple jobs, which encourages employers to open more simple jobs. On the other hand, a decrease in \( \eta \) has a negative effect
on $\theta^S$ through job-to-job flows. This is the key effect that we highlight in our model. Indeed, given that the creation of complex jobs is fostered by a decrease in $\eta$, the probability for an experimented worker occupying a simple job to be matched with a complex job is higher. Consequently, the average duration of simple jobs decreases and firms are more reluctant to open such jobs, reducing the job finding rate of young workers.

Nevertheless, recall that the share of unemployed workers without experience denoted by $\gamma$ is given by our flow equations at steady-state and therefore it is a function of both $\theta^S$ and $\theta^C$. So it is not so straightforward to examine the sign of the derivative of $\theta^S$ with respect to $\eta$. Assuming, as in the standard literature, that each matching function is Cobb-Douglas with an elasticity equal to $\alpha = 0.5$, we deduce in the appendix 2 a sufficient condition under which the effect through job-to-job flows dominates the horizon effect, which implies a negative effect of an increase in the legal retirement age on the accession rate of young workers.

**Proposition 1.** In the wage posting equilibrium, a decrease in $\eta$ reduces $\theta^S$, i.e \( \frac{\partial \theta^S}{\partial \eta} > 0 \) only if \( y^C - w^C > c^C (r + \delta + \eta)^2 \)

**Proof.** See Appendix 2

We have shown theoretically that an exogenous decrease in exit rates among older workers within a local labour market may reduce the tightness $\theta^S$ and therefore has a negative effect on the hiring rate $p(\theta^S)$ of young unemployed workers, provided that the flow value to an employer of a filled complex job is sufficiently high. Thus, when this condition holds, the horizon effect is offset by the indirect effect due to job-to-job flows. Given the expression of $u_{ne}$ we deduce that a fall in the exit rate $\eta$ may in some cases raise unemployment among young workers with no experience. Through our model, we show that job-to-job flows could be an explanation of the impact of delaying retirement on youth employment. In the second part of the paper, we want to test empirically this assumption. To allow for job-to-job flows, we focus on the Italian case, given that the Italian labour market is characterized by a large share of job to job transitions (Contini and Rapiti, 1994).
3. The empirical study

3.1. Selection bias removal using the Generalized Propensity Score

In the empirical part of this paper, we investigate to what extent retaining more older workers in the labour force may affect the hiring rate of young workers within a local labour market. In other words we estimate the response in terms of job opportunities for young workers associated with each dose of a continuous treatment, that corresponds to the absolute value of the decrease in the exit rate of older workers observed within a local labour market between two years, 1988 and 1989. As we do not exploit a quasi-natural experiment, we can not assume that the continuous treatment is exogenously assigned, so we cannot directly estimate a causal effect. To randomize the assignment to treatment, we use a propensity score matching procedure, generalized to a continuous treatment.

In order to formally describe the econometric framework we adopt, extra notation is required. Let $Y_i(t)$ denote a random variable that maps a particular potential treatment, $t$, to a potential outcome. We are interested in the average dose-response function, $\mu(t) = E[Y_i(t)]$. Following Hirano and Imbens (2004), we assume that $\{Y_i(t)\}_{t \in T_i}, T_i$, and $X_i, i = 1, \ldots, N$ are defined on a common probability space, that $T_i$ is continuously distributed with respect to Lebesgue measure on $T$, and that $Y_i = Y_i(T_i)$ is a well defined random variable.

In the randomized experiment underlying an observational study, the probabilities of assignment to treatments are not equal, but are rather functions of the covariates. Hence the template is an unconfounded assignment mechanism.

As in the binary treatment context, propensity score methods in a setting with continuous treatments rely heavily on the key assumption that adjusting for pre-treatment differences solves the problem of drawing causal inference. Formally, we make the weak unconfoundedness assumption, introduced by Hirano and Imbens (2004), which requires that the treatment assignment mechanism is conditional independent of each potential outcome given the pre-treatment variables: $Y_i(t) \perp T_i | X_i$ for all $t \in T$.

In our application we assume that unconfoundedness holds conditional on all the pre-treatment variables that we consider, arguing that these variables
are good proxies of factors that might affect the decrease in the exit rate of older workers, within a local labour market (Rubin, 2008).

Following Rubin (2008), we have to adjust for observables that may be correlated with some unobservable characteristics. So doing, balancing observables allows us to balance the unobservables over local labour market with different levels of elderly workers’ exit rate. Let us consider a set of $N$ synthetic firms characterized by a vector $X_i$ of observables, by a number $T_i$ equal to the absolute value of the decrease in the exit rate among older workers between 1988 and 1989 and by a number $Y_i(\gamma)$ equal to the hiring rate of young workers, associated with the treatment level $T_i = \gamma$. $T_i$ is our continuous treatment and $Y_i$ is our dependent variable. The unconfoundedness assumption may be formulated in the following way:

$$Y_i(\gamma) \perp T_i|X_i$$

Given unconfoundedness, we can apply matching methods based on the Generalized Propensity Score (GPS) with continuous treatment introduced by Imbens and Hirano (2004). The GPS is defined as the conditional density of the actual treatment given the observed covariates. Formally, let $r(\gamma, x) = f_T|X(\gamma|x)$ be the conditional density of the treatment given the covariates. Then, the GPS is $R_i = r(\gamma_i, X_i)$. The GPS is a balancing score, that is, within strata with the same value of $r(\gamma, x)$, the probability that $T = t$ does not depend on the value of $X$. In combination with the weak unconfoundedness assumption, this balancing property implies that:

$$f_T(\gamma|r(\gamma, X_i), Y_i(\gamma)) = f_T(\gamma|r(\gamma, X_i))$$

As a result, the GPS can be used to eliminate any biases associated with differences in the covariates. Formally, if the assignment to the treatment is weakly unconfounded, given pre-treatment variables $X_i$, then we can define in a second step a specification regressing the conditional mean of our dependent variable on the value of the treatment $T_i$ and on the value of the generalized propensity score $R_i$. In our analysis, we choose a simple linear model defined in the following way:

$$E(Y|T = \gamma, R = r) = b_0 + b_1 T + b_2 R + b_3 TR \quad (11)$$
where $\gamma$ and $r$ are specific values of respectively the treatment and the propensity score.

At last, we have to determine the average causality effect or dose-response function to evaluate the effect of several values of the treatment on the hiring rate of young workers. The dose-response function denoted by $\mu(\gamma)$ may be formulated in the following way:

$$
\mu(\gamma) = E[E(Y(\gamma)|r(\gamma, X)]
$$

(12)

In other words we have to estimate $\hat{\mu}(\gamma)$ such that:

$$
\hat{\mu}(\gamma) = \frac{1}{N} \sum_{i=1}^{N} \hat{b}_0 + \hat{b}_1 \gamma + \hat{b}_2 \hat{R} + \hat{b}_3 \hat{R} \gamma
$$

(13)

To obtain the dose-response function, we have to calculate $\hat{\mu}(\gamma)$ for different values of treatment, given that for each value $\gamma$, standard errors are computed through bootstrapping methods.

3.2. The data

For our empirical study, we use matched employer-employee data from the Worker Histories Italian Panel (WHIP) a database built by the University of Turin in cooperation with the Italian Social Security Administration (INPS). The original data set collects social security forms of a 1/90 random sample of employees every year. It provides accurate information about the employment spells of each worker. The data also includes also longitudinal records for firms employing the randomly selected workers in the sample. However, the data from the WHIP does not provide information about all individuals who work in a firm, so we can not observe the distribution of individual characteristics for each firm of the sample. To adress this issue, we carry out our study at the local labour market level, given that each observation unit is made up of a group of firms belonging to the same sector of activity, located in the same area and whose jobs require the same educational level.

We can discuss our synthetic-firm approach. Indeed, it leads us to underestimate the variance of each firm characteristic between firms given that each observation unit is a group of firms. To test the relevance of this approach, Contini et al. (2008) investigate the wage dynamics in the province of
Veneto, confronting the results obtained through the synthetic-firm method with those obtained through an analysis at the firm-level. They conclude that both approaches lead to similar results when the number of firms in each local labour market is quite small (between 50 and 95 firms per observation unit). Consequently, there is a trade-off when choosing the aggregation level between having the finest grid and having a sufficient number of firms in each observation unit.

Furthermore, studying the mobility of workers at a synthetic-firm-level allows us to account for the job-to-job flows. Using data from the WHIP, Leombruni and Quaranta (2005) show that job-to-job flows in the Italian labour market are observed within a sector, and within a geographical area. Consequently, we choose to aggregate our data by province and by sector of activity, and we end up with a sample of 517 synthetic firms. Studying the mobility of workers at a local labour market level, we account for the vacancy-chain pattern and we can expect that exits of older workers may imply hirings of young workers, holding job creation constant.

Our empirical analysis aims at investigating the effect of a decrease in the exit rate among older workers on the hiring rate of the young, during the period 1988-1989. Here, the exit rate of older workers is defined as the number of exits of older workers observed in a year over the stock of older workers employed in the start of the year. In addition, the hiring rate of young workers is defined as the number of hirings of young workers observed in a year over the stock of young workers employed in the end of this year. We choose this period given that since 1990, employment protection legislation has been severely changed and has had a strong effect on the Italian labour market (Kugler and Pica, 2008). Moreover, since 1993, the Italian labour market has been strongly affected by the economic recession. At last, since 1997, the Treu reform liberalized the use of temporary contracts and introduced new atypical contracts, involving a strong effect on Italian worker flows (Sciulli, 2006).

Consequently, we study the period 1988-1989 to isolate the effect on youth employment due to a slowdown in the exit rate among older workers from other effects due to institutional changes or to economic fluctuations. However, even during the period 1988-1989, job creation among young workers has been increased by the introduction of CFL (Contratto Formazione e La-
contracts (Tattara and Valentini, 2005). To control for this effect, we include in our set of regressors the share of CFL contracts observed in each local labour market.

Our interest lies on the effect of a decrease in the exit rate of older workers on the hiring rate of young workers. We treat the absolute value of the decrease in the exit rate between 1988 and 1989 as a continuous treatment $T_i = \gamma$. As a result, we discard observations that experienced between 1988 and 1989 an increase in the exit rate of older workers and also observations for which the exit rate of older workers remains constant during this period. So we focus on the average responses in terms of hirings of young workers for labour markets concerned by a decrease in the exit rate of older workers between 1988 and 1989. For some descriptive statistics, we group our observations into two categories: the group 1 is made up of synthetic firms, in which the value of the continuous treatment is lower than the median level, that is 13.5% and the group 2 is composed of synthetic firms, in which the value of the treatment exceeds 13.5%. We observe in the table 1 that the average hiring rate of young workers is higher in the group 1 (around 54.58%) than in the group 2 (around 51.57%). So a strong slowdown in the exit rate among older workers may reduce new hirings of young workers.

However given the endogeneity of our variable of interest, we can draw causal inference only after adjusting for pre-treatment differences in covariates between the synthetic firms.

To control for potential composition effects likely to influence the hiring rate of young workers or the exit rate of older workers observed within a local labour market, we use the set of information provided by our panel matched employer-employee data. We include first the age structure of each synthetic firms controlling for the share of younger workers and of older workers in each local labour market. We add also the share of women to test for gender-specific effects. We also introduce the average ratio between the average wage of young workers and the average wage of older workers in each synthetic firm. Indeed, this ratio may have a strong effect on the substitutability between

---

9The CFL is a fixed-term contract with a maximum duration of 24 months not renewable. This contract was introduced in 1984 to easy the entrance of youngsters aged between 15 and 29 into the labour market, providing fiscal benefits to firms hiring young people.
Table 1: Average hiring rate among workers aged 30 or less for different values of the continuous treatment.

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average hiring rate among young workers</td>
<td>54.58%</td>
<td>51.57%</td>
<td>53.07%</td>
</tr>
<tr>
<td>Number of observations</td>
<td>117</td>
<td>117</td>
<td>234</td>
</tr>
</tbody>
</table>

Note: The group 1 is composed of synthetic firms that experienced a decrease in the exit rate among older workers between 1988 and 1989 lower than 13.5% in absolute value. The group 2 is composed of synthetic firms that experienced a decrease in the exit rate among older workers between 1988 and 1989 higher than 13.5% in absolute value. Source: WHIP (waves 1988-1989)

young and older workers. We include also the share of CFL contracts signed in each local labour market, following empirical results supporting evidence of the strong positive impact of these contracts on youth employment (Tattara and Valentini, 2005).

Furthermore, the empirical study of Contini et al. (2002) show that the age or the size of the firm affect strongly the workers’ mobility. Consequently, in our set of regressors, we include the share of young firms (less than 8 years) and the share of small firms (less than 9 workers) in each local labour market. To control for a potential effect due to economic trends in each synthetic firm, we add in our set of explanatory variables the unemployment rate and a dummy equal to one if the local labour market is declining. We provide some descriptive statistics in the table 2.

3.3. Results

The first step consists in matching our observations calculating the Generalized Propensity Score (GPS in the remainder of the paper). The GPS

\[^{10}\text{It means that in this synthetic firm, the number of jobs in the end of the year is lower that the number of jobs observed in the beginning of the year.}\]
### Table 2: Descriptive statistics for the years 1988 and 1989

<table>
<thead>
<tr>
<th>Variable</th>
<th>Year 1988</th>
<th>Year 1989</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hiring rate of young workers</td>
<td>51.37%</td>
<td>53.07%</td>
</tr>
<tr>
<td>Exit rate of older workers</td>
<td>33.02%</td>
<td>13.49%</td>
</tr>
<tr>
<td>Share of young workers</td>
<td>38.75%</td>
<td>39.63%</td>
</tr>
<tr>
<td>Share of older workers</td>
<td>15.19%</td>
<td>14.38%</td>
</tr>
<tr>
<td>Share of women</td>
<td>24.55%</td>
<td>24.87%</td>
</tr>
<tr>
<td>Share of CFL contracts</td>
<td>22.1%</td>
<td>24.42%</td>
</tr>
<tr>
<td>Share of young firms (less than 8 years)</td>
<td>34.78%</td>
<td>34.07%</td>
</tr>
<tr>
<td>Share of small firms (less than 9 employees)</td>
<td>40.97%</td>
<td>41.15%</td>
</tr>
<tr>
<td>Ratio between the average wage of youngsters and that of older workers</td>
<td>82.24%</td>
<td>88.86%</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>10.85%</td>
<td>10.63%</td>
</tr>
<tr>
<td>Dummy equal to one if the synthetic firms is declining</td>
<td>29.91%</td>
<td>16.24%</td>
</tr>
<tr>
<td>Number of observations</td>
<td>234</td>
<td>234</td>
</tr>
</tbody>
</table>

Source: WHIP (waves 1988 and 1989)
methods are implemented in our application using the gspcore, dose-respons
STATA package (Bia & Mattei, 2008). A key assumption in the STATA im-
plemented version of the GPS methods is the normality of the treatment
variable conditional on the pre-treatment covariates. In our study we as-
sume that the Box–Cox transformation of the treatment has a Normal
distribution, given the covariates. Formally, let $BoxCox(T_i)$ denote the Box-
Cox transformation of the treatment variable, we define $BoxCox(T_i)$ in the
following way:

$$BoxCox(T_i) = \begin{cases} \frac{T_i^{\lambda} - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \log(T_i) & \text{if } \lambda = 0 \end{cases}$$

Using the Shapiro-Wilk test, we check that $BoxCox(T_i) | X_i \sim N(\beta_0 + 
\beta_1 X_i, \sigma^2)$, where $\beta_0$, $\beta_1$ and $\sigma^2$ are parameters to be estimated through a
maximum likelihood procedure. So for each observation, the generalized
propensity score is defined in the following way:

$$\hat{R}_i = \frac{1}{2\sqrt{\Pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}[BoxCox(T_i) - \beta_0 + \beta_1 X_i]\right)$$

Then, according to the distribution of the values of the treatment, that
is the absolute value of the decrease in the exit rate among older workers
between 1988 and 1989, we divide our sample into different groups. In our
study, we choose to break down our treatment into four intervals: [0;0.05],
[0.05;0.15], [0.15;0.3] and [0.3;1]. Therefore, at each time $t$, we divide our
sample into quartiles according to the value of the treatment.

In each of these groups, we calculate the GPS for a treatment level $T_i$ cor-
responding to the median treatment level for each group. We can determine
the GPS of each observation $i$ belonging to the group $j$, with $j = \{1, 2, 3, 4\}$
in the following way:

$$\hat{R}_i = \frac{1}{2\sqrt{\Pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}[BoxCox(T^j) - \beta_0 + \beta_1 X_i]\right)$$

where $T^j$ is the median treatment level for each group $j$. According to
the distribution of the GPS in each of these groups, we divide them into $K$
blocks. Here, we consider 5 blocks. Then, to assess the balancing property
of the GPS, we compare the distribution of covariates between two blocks $k$ and $k'$ with $k = \{1, 2, 3, 4, 5\}$. Each block $k$ is composed of synthetic firms presenting a specific treatment level corresponding to their group $j$ and a specific GPS corresponding to their block $k$. Our strategy is therefore to compare the distribution of covariates between two groups of synthetic firms with a similar GPS but a different treatment level.

Prior to evaluate the dose-response function, we have to ensure that the balancing property is checked. In the table 3, we report the values of the t-statistic when we carry out a test of difference in means for each explanatory variable and for different treatment levels after having adjusted by the GPS. The idea is to test for each group of observations, defined by a specific treatment interval, whether the observables are similar with those of synthetic firms of the other groups, after matching. We use the bold case values to indicate that the t-stat is higher than 1.96. In that case, balancing property does not hold. We remark in the table 3 that no t-stat values are higher than 1.96. We can conclude that our GPS matching procedure allows us to check the balancing property.

Then, we represent the dose-response function in the figure 2. Standard errors are bootstrap standard errors from 2000 replications.

This function describes to what extent a decrease in the exit rate among older workers has an effect on the hiring rate of young workers. Given that the treatment is equal to the absolute value of the decrease, a high treatment level corresponds to a strong fall in exits of older workers. We observe in the figure 2 that for a treatment level that ranges from 0.1 to 0.4, the treatment affects slightly the hiring rate of young workers. However, for a dose of treatment higher than 0.4, we remark that a slowdown in the exits of older workers has a negative effect on hirings of young workers.

Thus we show empirically that a strong decrease in exits of older workers reduces sharply hirings of young workers at a local labour market level, which provides some empirical evidence of our theoretical results. Nevertheless, we have to take these empirical findings with caution. Indeed, we matched our observations given a set of observables which allows us to check the balancing property only for the variables that we include. Therefore we did not correct for a selection bias on unobservables.
Table 3: Testing difference between means for different treatment interval after adjustment by the GPS

<table>
<thead>
<tr>
<th>Variables for the year 1988</th>
<th>t-stat for different treatment intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[0;0.05]</td>
</tr>
<tr>
<td>Share of young workers</td>
<td>-0.18</td>
</tr>
<tr>
<td>Share of older workers</td>
<td>0.369</td>
</tr>
<tr>
<td>Share of women</td>
<td>-1.15</td>
</tr>
<tr>
<td>Share of CFL contracts</td>
<td>-0.389</td>
</tr>
<tr>
<td>Share of young firms (less than 8 years)</td>
<td>-0.11</td>
</tr>
<tr>
<td>Share of small firms (less than 9 employees)</td>
<td>0.625</td>
</tr>
<tr>
<td>Ratio between the average wage of young and that of older workers</td>
<td>0.364</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>0.941</td>
</tr>
<tr>
<td>Dummy equal to one if the synthetic firm is declining</td>
<td>-0.218</td>
</tr>
<tr>
<td>Hiring rate of young workers</td>
<td>-0.011</td>
</tr>
</tbody>
</table>

Note: the treatment corresponds to the absolute value of the decrease in the exit rate among older workers between 1988 and 1989 for each synthetic firm.

Bold case values indicate that the t-statistic is higher than 1.96 in absolute value.

Source: WHIP (waves 1988 and 1989)
Figure 2: The dose-response function

Note: Dashed lines are bounds for 95% confidence intervals. These intervals are based on bootstrap standard errors from 2000 replications.
Furthermore, recall that our study has been carried out through a synthetic-firm approach. It implies that our observations have been built grouping in a same cell firms belonging to the same sector of activity and located in the same province. Our approach is therefore strongly influenced by the aggregation level chosen to build each synthetic firm.

4. Concluding remarks

In this paper, we study the effect of delaying retirement age on hirings of young workers. Our theoretical contribution is to view the work experience as a source of imperfect substitutability between young and older workers. Allowing for job to job flows and investigating the mechanics of job opening dynamics through a matching model with heterogeneous jobs and workers we point out two main effects of an increase in the retirement age on the hiring rate of young workers. First, it lengthens the horizon of an experienced worker and encourages job creation for both types of jobs. Second, the increase in complex job creation raises the probability for an experienced employed job seeker to find a complex job. Therefore, it reduces the average duration of a simple job, making therefore firms more reluctant to open simple jobs and reducing hirings of young workers with no experience. We show that the net effect of a decrease in the exit rate among older workers on the hirings of young workers is negative if the flow value to an employer of a filled complex job is sufficiently high.

Then, in the second part of the paper, we estimate empirically the effect of a slowdown in the exit rate among older workers on the hiring rate of young workers within a local labour market. We use matched employer-employee data drawn from the Worker Histories Italian Panel and we build the dose response function in terms of job opportunities of young workers as a response to a decrease in the exit rate of older workers. After removing the selection bias on observables, we observe that a strong decrease in the exit rate of older workers, that may result from an increase in the retirement age, affects negatively the hiring rate of young workers within a local labour market.

However, we have to take our results with caution given that our matching method does not allow us to correct for a selection bias on unobservables. Furthermore, our findings may depend on the aggregation level chosen to
build our synthetic firms. To test the robustness of our results, we should reproduce this study on very big firms assimilating job-to-job flows to promotions within the firm. One extension of this paper may come to mind. We could allow for both types of worker mobility, within a local labour market, through job-to-job flows and within a firm, through internal promotions. This specification could allow us to better grasp channels through which a decrease in the exit rate among older workers affects the hiring rate among young workers. We leave this idea for further investigation.

Acknowledgements

We would like to thank François Langot and Bruno Decreuse for their very helpful comments on the theoretical model. We are also grateful to participants at the "Theories and Methods in Macroeconomics" conference (Le Mans, May 2010), at the EPEE seminar (Evry, 2010) at the "Journées de Microéconomie Appliquée" (Angers, 2010) and at the "Journées Louis-André Gérard-Varet" (Marseille, 2010). Any remaining errors are ours.

A. Appendix 1

We find the six following steady-state flow equations:

\[(u_c + o_e)p(\theta^C) = E^C(\delta + \eta)\]  
\[(E^C + o_e)\delta = u_e[\eta + p(\theta^S) + p(\theta^C)]\]  
\[u_e p(\theta^S) + E^S_{ne} \lambda = o_e[\delta + \eta + p(\theta^C)]\]  
\[u_{ne} p(\theta^S) = E^S_{ne} (\delta + \lambda)\]  
\[E^S_{ne} \delta + \eta_0 = u_{ne} p(\theta^S)\]  
\[\eta_0 = \eta(u_c + o_e + E^C_e)\]

where \(u_c\) and \(o_e\) represent respectively the number of experienced job-seekers unemployed and employed, \(E^S_{ne}\) and \(E_c\) refer respectively to the number of workers with no experience occupying a simple job and to the number
of experienced workers occupying a complex job and \( \eta_0 \) represents the number of young workers who arrive in the labour market.

Combining equations (17) and (18) we obtain:

\[
u_{ne} = \frac{\eta_0(\delta + \lambda)}{\lambda p(\theta^S)} \tag{20}\]

Furthermore, combining the equations (15) and (19) we deduce the expression of unemployment for experienced workers:

\[
u_e = \frac{\delta \eta_0}{\eta[\eta + p(\theta^S) + p(\theta^C) + \delta]} \tag{21}\]

**B. Appendix 2**

As \( \gamma \) is given by our flow equations at steady-state, combining equations (20) and (21), we write \( \gamma \) in the following way:

\[
\gamma = \frac{(\delta + \lambda)\eta[\eta + p(\theta^S) + p(\theta^C) + \delta]}{\eta[\eta + p(\theta^S) + p(\theta^C) + \delta](\delta + \lambda) + \delta \lambda p(\theta^S)}
\]

We assume as in the standard literature that each matching function is Cobb-Douglas with an elasticity equal to \( \alpha = 0,5 \), such that:

\[
\begin{align*}
h^S(u_{ne} + u_e, v^S) &= (u_{ne} + u_e)^{\frac{1}{2}}(v^S)^{\frac{1}{2}} \tag{22} \\
h^C(u_e + o_e, v^C) &= (u_e + o_e)^{\frac{1}{2}}(v^C)^{\frac{1}{2}} \tag{23}
\end{align*}
\]

Substituting (23) into (8), we obtain the following expression:

\[
(\theta^C) = \left[\frac{(y^C - w^C)}{c^C(r + \eta + \delta)}\right]^2 \tag{24}
\]

Substituting (24) into (10), we find that at steady-state equilibrium, \( \theta^S \) satisfies the following equation:

\[
\begin{align*} 
L(\theta^S, \eta) &= \frac{\gamma}{(r + \delta + \lambda)}[(y^S - w) + \frac{\lambda[y^S - w]}{r + \delta + \frac{c^S(w^C)}{c^C(r + \eta + \delta) + \eta}] + \frac{(1 - \gamma)(y^S - w)}{r + \delta + \frac{(y^C - w^C)}{c^C(r + \eta + \delta) + \eta}] + \frac{c^S}{q(\theta^S)} &= 0 \tag{25}
\end{align*}
\]
Rearranging terms, we obtain the following equation:

\[
L(\theta^S, \eta) = \gamma(y^S - w)\left[\left(\frac{y^C - w^C}{c^C(r + \eta + \delta)}\right) + \eta \right] + (r + \delta + \lambda)(y^S - w) - \frac{c^S}{q(\theta^S)} = 0
\]

Differentiating \(L(\theta^S, \eta)\) we get:

\[
L_1 d\theta^S + L_2 d\eta = 0 \iff \frac{d\theta^S}{d\eta} = -\frac{L_2}{L_1}
\]

where \(L_i\) is the partial derivative of \(L(\theta^S, \eta)\) with respect to its \(i\)th element, with \(i \in \{1, 2\}\). We determine \(L_1 = \partial L(\theta^S, \eta)/\partial \theta^S\)

\[
L_1 = \frac{[(y^S - w)\left[\left(\frac{y^C - w^C}{c^C(r + \eta + \delta)}\right) + \eta \right] + (r + \delta + \lambda)(y^S - w)}{[r + \delta + \left(\frac{y^C - w^C}{c^C(r + \eta + \delta)}\right) + \eta](r + \delta + \lambda)} - c^S \alpha(\theta^S)^{\alpha-1}
\]

We have to determine the sign of \(\partial \gamma/\partial \theta^S\). We remark that:

\[
\frac{1}{\gamma} = 1 + \frac{\delta \lambda p(\theta^S)}{\eta[\eta + p(\theta^S) + p(\theta^C) + \delta](\delta + \lambda)}
\]

As the second term on the right-hand side of the equation is increasing with \(\theta^S\), we deduce that \(\gamma\) is decreasing with \(\theta^S\). Consequently, \(L_1 < 0\).

Furthermore, let \(E(J^S)\) be the the esperance of the expected income flows to an employer from a filled simple job such that:

\[
E(J^S) = \gamma(y^S - w)[p(\theta^C) + \eta] + (r + \delta + \lambda)(y^S - w)
\]

where \(p(\theta^C) = \frac{(y^C - w^C)}{c^C(r + \eta + \delta)}\). Consequently, \(L_2\) has the same sign as \(\partial E(J^S)/\partial \eta\).

We consider the logarithm of the expression to simplify calculation so:

\[
\ln[E(J^S)] = \ln[\gamma(y^S - w)[p(\theta^C) + \eta] + (r + \delta + \lambda)(y^S - w)]
- \ln[r + \delta + p(\theta^C) + \eta] - \ln(r + \delta + \lambda)
\]

So, differentiating this expression with respect to \(\eta\), we obtain:
\[
\frac{\partial \ln E(J_S)}{\partial \eta} = \gamma \left[ \frac{\partial p(\theta^C)}{\partial \eta} + 1 \right] + \frac{\partial_\gamma [p(\theta^C) + \eta]}{\gamma [p(\theta^C) + \eta] + (r + \delta + \lambda)} - \frac{\left[ \frac{\partial p(\theta^C)}{\partial \eta} + 1 \right]}{[r + \delta + p(\theta^C) + \eta]} \\
= \left[ \frac{\partial p(\theta^C)}{\partial \eta} + 1 \right] \left[ 1 \left[ \frac{1}{[p(\theta^C) + \eta] + (r + \delta + \lambda)} \right] - \frac{1}{p(\theta^C) + \eta + r + \delta} \right] \\
+ \frac{\partial_\gamma [p(\theta^C) + \eta]}{\gamma [p(\theta^C) + \eta] + (r + \delta + \lambda)}
\]

Recall that:
\[
\frac{1}{\gamma} = 1 + \frac{\delta \lambda p(\theta^S)}{\eta[\eta + p(\theta^S) + p(\theta^C) + \delta][\delta + \lambda]}
\]

Differentiating this expression with respect to \( \eta \), we obtain:
\[
\frac{\partial \left( \frac{1}{\gamma} \right)}{\partial \eta} = -\frac{\gamma \left( \frac{\partial p(\theta^C)}{\partial \eta} \right)}{\left( \delta + \lambda \right) \left( \eta[\eta + p(\theta^S) + p(\theta^C) + \delta][\delta + \lambda] \right)^2} - \frac{\frac{\partial \gamma}{\partial \eta}}{\gamma[\eta + p(\theta^S) + p(\theta^C) + \delta]}
\]

We observe that:
\[
p(\theta^C) + \eta \frac{\partial p(\theta^C)}{\partial \eta} = \frac{(y^C - w^C)}{c^C(r + \eta + \delta)} - \frac{\eta(y^C - w^C)}{c^C(r + \eta + \delta)^2}
\]

Substituting this expression into the previous one, we obtain:
\[
\frac{\partial \left( \frac{1}{\gamma} \right)}{\partial \eta} = -\frac{\gamma \left( \frac{\partial p(\theta^C)}{\partial \eta} \right)}{\left( \delta + \lambda \right) \left( \eta[\eta + p(\theta^S) + p(\theta^C) + \delta][\delta + \lambda] \right)^2} - \frac{1}{\gamma[\eta + p(\theta^S) + p(\theta^C) + \delta]} < 0
\]

In addition, given that \( \gamma < 1 + \frac{\lambda}{(r+\delta)} \), we deduce that:
\[
\left[ \frac{1}{[p(\theta^C) + \eta] + (r + \delta + \lambda)} \right] = \frac{1}{[p(\theta^C) + \eta + r + \delta]} < 0
\]

Consequently, \( \partial \ln(E(J_S)/\partial \eta) > 0 \) only if
\[
\left[ \frac{\partial p(\theta^C)}{\partial \eta} + 1 \right] < 0 \iff (y^C - w^C) > c^C(r + \eta + \delta)^2
\]

If this condition holds, it ensures that \( L_2 > 0 \) and therefore \( \partial \theta^S/\partial \eta > 0 \).
References


