Out-of-plane elastic waves in phononic monolayer granular membranes

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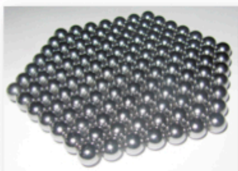
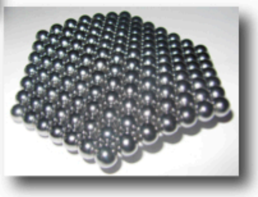
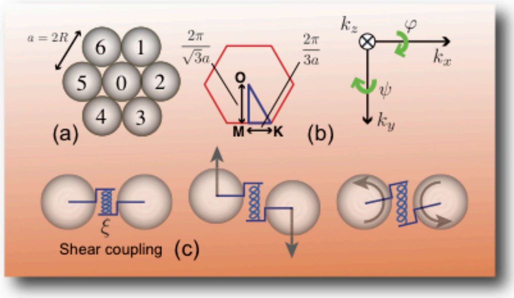
- ➪ Monolayer membranes composed of cohesive nanoscale spheres are now possible to build
- ➪ The periodic micro-structure of such membranes allows their use as phoxonic devices (both photonic and phononic) for sufficiently high frequency elastic waves and visible light
- ➪ Theoretical description of the vibrational properties of these phononic granular membranes





- ➪ Dispersion relations and band gaps are presented for **different contact properties**
- ➪ Results show that **complete band-gaps exist**
- ➪ Absolute or partial **band-gaps can open or close**, depending on the contact properties
- ➪ Applications in testing with acoustic waves the elasticity of recently developed granular membranes composed of nano-particles
- ➪ Design of devices aiming at **wave propagation control** on membranes

Geometry of the problem



$$\alpha = \frac{1}{4} k_x a$$

$$\beta = \frac{\sqrt{3}}{4} k_y a$$

$$\Omega^2 = \omega^2/(4\omega_0^2)$$

$$\omega_0^2 = \xi/m$$

Equations of motion for the central particle 0 with shear coupling

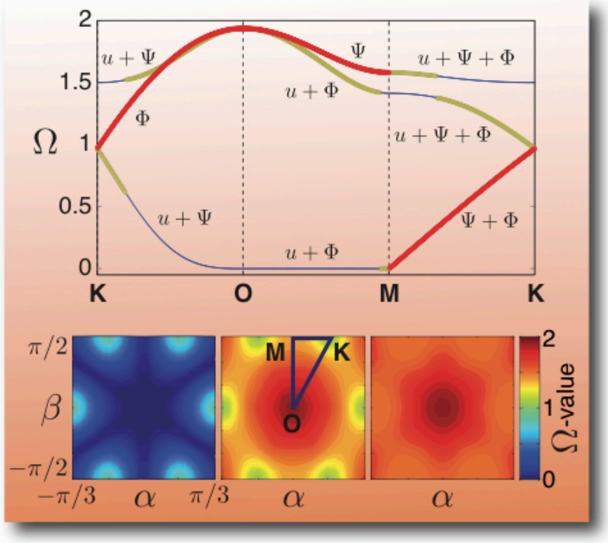
$$m\ddot{u}_0 = -\xi\left[(\delta u_1 + \delta u_4) + (\delta u_3 + \delta u_6) + (\delta u_2 + \delta u_5)\right]$$

$$I\ddot{\varphi}_0 = \frac{\sqrt{3}}{2}\xi R\left[(\delta u_1 - \delta u_4) - (\delta u_3 - \delta u_6)\right]$$

$$I\ddot{\psi}_0 = \frac{R}{2}\xi\left[(\delta u_1 - \delta u_4) + (\delta u_3 - \delta u_6)\right] + R\xi\left[\delta u_2 - \delta u_5\right]$$

δu_i elongation of the spring at the contact between the central and i-particle

Dispersion curves and iso-surfaces for the 3 eigenvalues (shear coupling)



The system of equations of motion, after plane wave substitution, reads $\mathbf{S}\overrightarrow{\nu} = 0$

with $\mathbf{S}$ the 3x3 dynamical matrix and

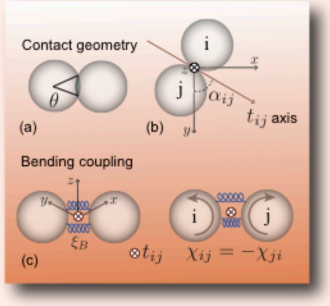
$$\overrightarrow{\nu} = \begin{pmatrix} u \\ \Phi \\ \Psi \end{pmatrix} e^{i\omega t - ik_x x_0 - ik_y y_0} e^{-ik_x \Delta x - ik_y \Delta y}$$

➪ Eigenvalues and eigenvectors of $\mathbf{S}$

- Pure rotational modes exist
- In symmetry directions, coupled displacement-rotation modes exist
- A zero frequency (soft) mode reveals the possibility for displacement-rotation compensation

Influence of the bending rigidity

Bending coupling



Bending parameter

$$p_B = \frac{\theta^2}{2}\frac{\xi_B}{\xi}$$

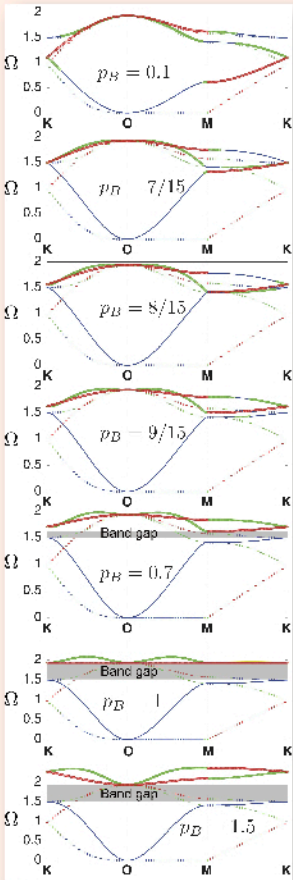
Correction to the dynamical matrix **S** when bending is introduced :

$$\Delta S_{22} = -\frac{3}{4} p_B \left[\sin^2(\alpha+\beta) + \sin^2(\alpha-\beta)\right],$$

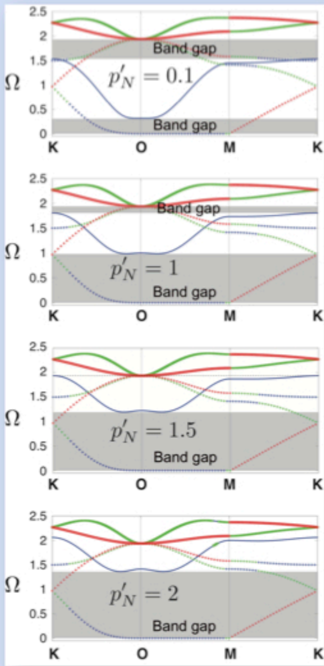
$$\Delta S_{23} = \Delta S_{32} = -\frac{\sqrt{3}}{4} p_B \left[\sin^2(\alpha-\beta) - \sin^2(\alpha+\beta)\right]$$

$$\Delta S_{33} = -p_B \left\{\left[\sin^2(\alpha+\beta) + \sin^2(\alpha-\beta)\right]/4 + \sin^2(2\alpha)\right\}$$

- A complete band gap appears above $\Omega = 3/2$ for $p_B > 3/5$ and saturates $(3/2 \leq \Omega \leq \sqrt{15}/2)$ for $p_B = 1$
- The zero frequency mode disappears



Influence of the normal rigidity with the substrate



Coupling parameter

$$p'_N = \xi'_N/\xi$$

ξ'_N normal rigidity with the substrate

Correction to the dynamical matrix **S** when normal rigidity with the substrate is introduced : $\Delta S'_{11} = -p'_N$

- When the membrane is on a rigid substrate it is composed rather of interacting oscillators than of oscillating masses
- Complete band gaps exist from zero frequency up to $\Omega^2 = p'_N$

- Complete forbidden band gaps for the elastic waves exist
- The high frequency complete band gap may disappear in the membranes supported by a substrate
- More complex phenomena are expected if the shear rigidity of the contacts between the particles and the substrate is taken into account ➪ coupling with in-plane modes

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