Linear sound amplification and absorption in a corrugated pipe

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Introduction

Corrugated pipe

- Globally flexible and locally rigid
- Whistling/ sound amplification problem

General view of offshore flexible riser system

The structure of flexible riser
Introduction

Unsteady flow-acoustic coupling in cavities

- Flow separation
- Receptivity
- Shear layer
- Shear layer-edge interaction
- Upstream influence

Key parameters: $Ma$, $L/D$, $\theta/L$

- Rossiter's mode (aeroacoustic feedback loop) vs. Acoustic resonance
- Shear layer: K-H instability has been considered necessary
- Shear layer-edge interaction: both monopole and dipole have been assumed.

Sound amplification and absorption

- Predict sound amplification and absorption
- Investigate the role of K-H instability
- Identify sound sources (monopole/dipole)
Numerical model

1. Solve the transverse modes in each duct segment

Numerical model

\[ \frac{\partial}{\partial x} (M \frac{\partial u}{\partial x}) + \frac{dM}{dx} v = - \frac{\partial p}{\partial x} \]

\[ \frac{\partial}{\partial y} (M \frac{\partial v}{\partial y}) \]

\[ \frac{\partial}{\partial z} (M \frac{\partial r}{\partial z}) = \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial r}{\partial z} \right) \]

2. Mode match

Scattering matrix

\[ S = \begin{pmatrix} T^+ & R^- \\ R^+ & T \end{pmatrix} \]

3. Computation of the wavenumbers using the Floquet-Bloch theorem

The governing equations, the boundary conditions, and the geometry are W-periodic along x.

Floquet-Bloch theorem:

\[ \phi(x, r) = \hat{\phi}(x, r) e^{-i\omega x} \]

Scattering matrix:

\[ M_1 \begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = M_2 \begin{pmatrix} C_1'' \\ C_2'' \end{pmatrix} \]

where

\[ M_1 = \begin{pmatrix} T^+ & 0 \\ -R^+ & I \end{pmatrix}, \quad M_2 = \begin{pmatrix} I & -R^- \\ 0 & T \end{pmatrix} \]

Generalized transfer matrix eigenvalue problem
**Experiment**

![Diagram of acoustics experiment](image)

- **Air flow**
- **Anechoic termination**
- **Measurement pipe**
- **Tested pipe**
- **Anechoic termination**

**Experimental results**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_0$</td>
<td>0.0184</td>
</tr>
<tr>
<td>$R$</td>
<td>15 mm</td>
</tr>
<tr>
<td>$D$</td>
<td>4 mm</td>
</tr>
<tr>
<td>$L$</td>
<td>4 mm</td>
</tr>
<tr>
<td>$T$</td>
<td>8 mm</td>
</tr>
<tr>
<td>Total length</td>
<td>2 m</td>
</tr>
</tbody>
</table>

**Imaginary part of wavenumber**

![Graph of imaginary part of wavenumber](image)
Comparison

Mean velocity profiles for calculation

- Parallel mean flow
- Different shear layer thickness $\theta$

Numerical results

1. Fields

- $p$
- $v$
- $u$
Numerical results

2. K-H instability of the shear layer

Mean velocity profile

Instability characteristics of the shear layer
Instability is not a necessary condition for sound amplification in the corrugated pipe.

Numerical results

- Amplitude of the coefficients of the hydrodynamic modes.

Unstable
- \(\theta = 0.02, Sr = 0.39\)

Stable
- \(\theta = 0.03, Sr = 0.39\)

v fields
Numerical results

Growth in the displacement of the shear layer

Unstable
($\theta = 0.02$, $Sr = 0.39$)

Stable
($\theta = 0.03$, $Sr = 0.39$)

3. Sound amplification/absorption

Abs
Re
Im

$|a|/|\lambda| \approx 1$

$|a|/|\lambda| \approx 1.5$

$|a|/|\lambda| \approx 2$
Conclusions

- Flow-acoustic interaction leads to sound amplification and absorption in a corrugated pipe.
- The linear model can predict the phenomenon. It is shown that $S_r$ is decided by the hydrodynamic wave speed, the amplitudes of sound amplification/absorption are influenced by the thickness of the shear layer.
- $K$-H Instability is not a necessary condition for sound amplification.

Next

- For more accurate calculations, a real base flow (Evolution of the mixing layer, Non-parallel effect due to the recirculation) is needed in solving the LEEs.
- Investigate the sound source/sink mechanism.

Thank you for your attention.