Abnormal behavior of an acoustical liner with flow

Yves Aurégan, Maud Leroux, Vincent Pagneux
Laboratoire d’Acoustique de l’Université du Maine UMR CNRS 6613 Av. O. Messiaen, 72085 LE MANS Cedex 9, France,
e-mail: yves.auregan@univ-lemans.fr

An experimental investigation of the acoustical behaviour of a liner in a rectangular channel with grazing flow has been conducted. The liner consists of a ceramic structure of parallel square channels: 1 mm times 1 mm, 400 channels/inch square. The channels are rigidly terminated and ensure a locally reacting structure. The depth of the channels is 65 mm. Without flow the liner reacts classically: there is a very large decreasing in the transmission coefficient around the frequency where the impedance is minimum. With flow this behaviour is changed and a hump appears at this resonance frequency. The transmission coefficient can be bigger than 1 (up to 3 for a Mach number of 0.3). This effect has already been observed by Ronneberger et al. in a cylindrical geometry. This behaviour can be important for practical applications and can be explained by the appearance of hydrodynamic waves above the liner. Furthermore the mean flow pressure drop induced by this liner is deeply affected by its acoustical behaviour. When an acoustical wave is added, at the hump frequency of the liner, the pressure drop can increase by a factor 3 when the Mach number is 0.3. This effect is explained by a modification of the turbulent boundary layer induced by the acoustical wave.

1 Introduction

Acoustically treated ducts are widely used in ducts with flow to reduce the emitted noise. Nevertheless, the calculation of the acoustical propagation in such devices is difficult because of the complexity of the sound-flow interactions. The coupling between acoustics and flow vorticity can be specially important in the vicinity of a treated wall.

This paper displays such an important coupling between sound and flow. The effect is first demonstrated experimentally. The flow affects the sound transmission through a lined duct in such a way that the acoustical output level can be bigger than the input level. But also the static pressure drop, induced by the wall friction, can be substantially increased by the effect of an acoustical wave. This effect is then theoretically linked to the existence of hydrodynamic unstable waves near the lined wall.

2 Experimental results

2.1 Experimental setup

A view of the experimental setup is given in Fig. 1. A precise description of the setup can be find in [1]. The acoustical measurement zone is rectangular (15 mm × 100 mm). The acoustical pressure measurement is performed with two series of four microphones located upstream and downstream of the liner. The use of 2×4 microphones allows an overdetermination of the transmitted and reflected waves on both sides of the liner. The acquisition of the signal is made in sweep sinus mode and an average is made over 1000 cycles of the signal.

This experimental apparatus allows the measurement of the scattering matrix in pressure of the liner, for the plane waves with mean flow. The incident wave at the upstream side of the liner is denoted $p_{1}^{+}$ and the reflected wave $p_{1}^{-}$. On the downstream side, the waves are denoted $p_{2}^{+}$ and $p_{2}^{-}$ (see Fig. 2). The scattering matrix for the plane waves relates the scattered pressure amplitudes $p_{2}^{+}$ and $p_{2}^{-}$ to the incident pressure amplitudes $p_{1}^{+}$ and $p_{1}^{-}$ by

$$
\begin{pmatrix}
    p_{2}^{+} \\
    p_{2}^{-}
\end{pmatrix}
= 
\begin{pmatrix}
    T^{+} & R^{-} \\
    R^{+} & T^{-}
\end{pmatrix}
\begin{pmatrix}
    p_{1}^{+} \\
    p_{1}^{-}
\end{pmatrix}
$$

where $T^{+}$ and $T^{-}$ are the anechoic transmission coefficients, $R^{+}$ and $R^{-}$ are the anechoic reflection coefficients. The method of measurement used in the present study is called "the 2 sources method" [2].

Pressure drop measurements have been performed with
the use of an EFFA GA064A5-20 differential pressure transducer, which has a 0–20 mbar range with less than 0.1% error on the measurement range. The sensor delivers to a multimeter a tension which is proportionnal to the differential pressure measured between two points on the wall in front of the liner. The first measurement point is located at \( G = 2 \) cm from the upstream part of the liner and the distance between the two measurement points is \( F = 6 \) cm.

2.2 Liner

The tested liner (see Fig. 2) is a ceramic structure of parallel and square channels: \( A = 1 \) mm \( \times \) \( A = 1 \) mm, 400 channel/inch\(^2\). This element is a part of a catalytic converter\(^1\). The channels are rigidly terminated and ensure a locally reacting structure. The depth of the channels is \( D = 65 \) mm and the length of the liner is \( L = 100 \) mm.

The impedance of this liner was measured in a conventional normal incidence impedance tube (see Fig. 3). By varying the level during these impedance measurements, the linearity of this ceramic liner was verified. The reduced impedance can be fitted in the frequency range of interest (i.e. 1000–1500 Hz) by

\[
Z_f = \frac{1}{j \phi \tan(Bf)}
\]

with the porosity \( \phi = 0.8 \), \( B = 1.3 \times 10^{-3} - 1.3 \times 10^{-4} j \), and \( f \) the frequency.

2.3 The acoustical hump effect

Without flow, the effect of a locally reacting liner is well known. There is a big decreasing in the transmission coefficient in the vicinity of the zero of the impedance (there is a quarter of wave length in the liner). This effect can be seen in Fig. 4.

The transmission coefficient in the flow direction \( T^+ \) is displayed in Fig. 5 for different Mach numbers \( M \). It can be seen that the general shape of the transmission is not greatly affected by flow except near the resonance frequency of the liner where a “hump” appears. For a Mach number big enough, the transmission coefficient can become bigger than 1, i.e. the transmitted sound is bigger than the incident sound. This effect is not a classical whistling (with a very sharp peak in frequency) but only a sound amplification. This hump effect has been previously seen by Brandes and Ronneberger [3] for a periodic sequence of resonators in a cylindrical duct.

This hump cannot be explained by classical computation. In Fig. 6, the experimental results are compared to the computation made with a code where a uniform flow was assumed and where no special care to the hydrodynamics modes was taken [1]. The agreement between the measured and the computed transmission is good everywhere except in the hump region. Thus, the hump can be associated to the effect of hydrodynamic modes on the acoustics.

2.4 The hydrodynamical hump effect

The pressure drop along the liner has been measured between two points distant of \( F = 6 \) cm (see Fig. 2). For a given flow, the pressure drop without any acoustical wave has been first measured. Then the pressure drop with an acoustical wave coming from the upstream duct
Figure 4: Transmission coefficient of the ceramic liner without flow: experimental results (symbols) and computed from the impedance measured in the impedance tube (line).

Figure 5: Transmission coefficient of the ceramic liner in the flow direction for different Mach numbers $M$.

Figure 6: Transmission coefficient of the ceramic liner in the flow direction for $M = 0.3$: experimental results (symbols) and computed from the impedance measured in the impedance tube (line).

Figure 7: Statical pressure drop as a function of the frequency of an incident wave on the upstream side with flow.

3 Hydrodynamical modes

3.1 Computation method

The problem under consideration is described Fig. 8: sound propagation with shear flow in a 2D duct is considered with varying admittance $Y_w(x)$ at the wall. This problem can be modeled by using the dimension-

less Euler equations (2) and (3) and the dimensionless continuity equation (4):

$$ju + M \frac{\partial M}{\partial x} + \frac{\partial M}{\partial y} v = -\frac{\partial p}{\partial x},$$  \hspace{1cm} (2)
The axial velocity $u$ and the pressure $p$ are expanded on the function basis $\Psi_n$ where $\Psi_n = 1$ if $n = 1$, $\Psi_n = \sqrt{2} \cos \left( \pi (n-1)y \right)$ if $n > 1$. The transverse velocity $v$ is expanded on the function basis $\Phi_n = \sqrt{2} \sin \left( \pi ny \right)$:

\[
u = \sum U_n(x) \Psi_n(y),
\]
\[v = \sum V_n(x) \phi_n(y),
\]
\[p = \sum P_n(x) \Psi_n(y).
\]

By projecting the Eqs. (2) and (4) on $\Psi_n$ and Eq. (3) on $\Phi_n$, the following equations are obtained:

\[
\frac{dP}{dx} + M_0 MP \frac{dU}{dx} = -j U - M_0 dMV,
\]
\[

\frac{j V + M_0 MV}{dx} = PV P,
\]
\[

\frac{j P + M_0 MP}{dx} = - \frac{dU}{dx} - YCLP + VUV,
\]

The matrices $MP$, $dM$, $MV$, $PV$, $CL$ and $VU$ are given by:

\[
MP = \int_0^1 f(y) \Psi_n(y) \Psi_m(y) dy;
\]
\[
dM = \int_0^1 \frac{d f(y)}{dy} \phi_n(y) \Psi_m(y) dy;
\]
\[
MV = \int_0^1 f(y) \phi_n(y) \phi_m(y) dy;
\]
\[
PV = \int_0^1 \Psi_n(y) \frac{d \phi_n(y)}{dy} dy;
\]
\[
CL = [\Psi_n(0) \Psi_m(0)];
\]
\[
VU = \int_0^1 \phi_n(y) \frac{d \Psi_m(y)}{dy} dy.
\]

The Eqs. (5), (6) and (7) can be formally written:

\[
\left( \begin{array}{c}
\frac{dX}{dx} \\
\frac{dP}{dx}
\end{array} \right) = M \left( \begin{array}{c}
X \\
P
\end{array} \right),
\]

where $X = \left( \begin{array}{c}
U \\
V
\end{array} \right)$ and $M = \left( \begin{array}{cc}
M_1 & M_2 \\
M_3 & M_4
\end{array} \right)$.

Because of evanescent modes, the Eq. (8) is numerically unstable, thus the impedance matrix $Z$, such as $P = Z X$, is used. From Eq. (8), the evolution equation of this matrix is found to be:

\[
\frac{dZ}{dx} = M_3 + M_4 Z - Z M_1 - Z M_2 Z.
\]

Assuming that the impedance is known in the downstream duct (anechoic conditions), the Eq (9) can be integrated in the $-x$ direction. Thus the impedance matrix $Z$ can be known everywhere. From Eq. (8) an evolution equation for $X$ can be found to be $dX/dx = (M_1 + M_2 Z)X$. By integration from a given source, the vector $X$ and $P = Z X$ can be found for all $x$.

3.2 Results

The calculations are made for a given truncation number $N_t$ in the expansion of $u, v, p$ and for a given flow profile defined by

\[
f(y) = \frac{2 n_M + 1}{2 n_M} \left( 1 - y^{2 n_M} \right),
\]

where $n_M$ is the parameter which controls the flow shape (parabolic for $n_M = 1$ to flat with a thin boundary layer for $n_M$ big). In the following, $N_t = 7$ and $n_M = 2$.

The first result is plotted in Fig. 9. The reduced wavenumbers $K = kc_0/\omega$ of the liner modes are plotted for different values of the frequency (from 500 Hz to 3000 Hz). Those values of the reduced wavenumbers are very easy to compute because they are the eigenvalues of matrix $M$ in Eq. (8). The number of modes which is computed is $3 N_t - 1$. Part of these modes ($2 N_t$) are acoustical modes going in the downstream direction ($A_n^+$ in Fig. 9) and in the upstream direction ($A_n^-$ in Fig. 9). The other modes ($N_t - 1$) are hydrodynamical modes. Some of these hydrodynamical modes are stable ($HS$ in Fig. 9), i.e. they go with flow being neutral or attenuated. One of these modes ($HI$ in Fig. 9) is unstable, i.e. it goes with flow being exponentially increasing. This mode is responsible of the hump in the transmission coefficient (see Fig. 10). The result of this calculation is in good agreement with the experimental results but, if the value of $N_t$ or $n_M$ increases, this agreement is destroyed: the maximum value of the transmission coefficient in the hump increases to very large values. It means that either the computation is not accurate enough, or some nonlinear effect appears in the hydrodynamical unstable wave.

4 Concluding remarks

The described abnormal behavior exhibits two linked properties: the transmission coefficient can exceed one
and the mean flow pressure drop is influenced by acoustics. It would be of interest to be able to isolate the kind of properties of the liner that is responsible for this behavior and to see if it may happen with usual wall treatment. From a theoretical point of view, this phenomenon could be used as a test for the numerical code of acoustical propagation in flow. Further studies are being done to better understand the role of hydrodynamical unstable modes.

References

