I. INTRODUCTION

The low frequency noise is very difficult to suppress or mitigate with devices having a thickness much smaller than the sound wavelength. There is a lot of research work about innovative acoustic materials efficient at low frequencies. Among these materials, some works have been devoted to cavities covered by membranes that are often called “drumlike silencers.”1–3 It has been shown that a large and broadband attenuation can be achieved at low frequencies if the membrane is carefully chosen.

When those devices are subjected to a grazing flow, as it occurs in real applications, their acoustical behavior can change. In particular, when the flow velocity is relatively high, instabilities could occur in the fluid-structure system. It has been revealed that the traveling wave flutter (TWF) is the main instability process for a flexible surface when irreversible transfer of energy from the flow to the motion of the flexible plate exceeds the rate of structural dissipation.4–6 The mechanism of such an instability was first discovered in water wave generation by wind.7 The instability can be stabilized by the structural damping, and thus has been classified as a class B instability according to the Benjamin-Landahl categorization.7–9 However, the influence of such instability on the sound propagation has not been investigated, which leads to the first objective of this paper.

The other goal of this work is to develop a simple model based on the multimodal method for the acoustic–flow–structural interaction (AFSI). The existing theories for this problem can be classified into two categories: the “structure-centric” and the wave-based approaches.10 In the “structure-centric” approach, the Galerkin method based on expanding the structure response as in vacuo modes of the elastic plate is used to study the flow–structure interaction (FSI) for flexible plates of finite length4,11 and the acoustic–structural interaction (ASI) of a drumlike silencer when no flow was present.2 The traveling wave approach is used in the cases with infinitely long plates where the stability properties are obtained from the dispersion equation of the fluid-structure system.12,13 The finite plate response in FSI has also been build by expressing the unsteady motion as a superposition of waves of the corresponding infinite system which are continuously rescattered at the plate edges.14 Recently, a numerical methodology based on the conservation element and solution element (CE/SE) method for AFSI of a flexible panel in a duct with uniform flow has been proposed.15

In Sec. II A, the multimodal method16–18 is used to compute the AFSI of a membrane-cavity configuration in a duct with shear flow. The case without flow is given in Sec. II B. The model is validated by comparison with available experimental results in Sec. III A. The interaction between the membrane motions and the shear flow and its influence on sound propagation are discussed in Sec. III B. It is shown that the neutral hydrodynamic modes could be destabilized in this coupling and that sound amplification can occur. The effects of flow velocity, boundary layer thickness, and structural damping on the flexural instability and sound amplification are also studied.

II. NUMERICAL MODEL

A. Calculations with a grazing shear flow

In a perfect flow, the linear propagation of small disturbances about a steady mean flow can be described by the linearized Euler equations (LEEs):

\[
\left(\frac{\partial}{\partial t} + M_0 \frac{\partial}{\partial x}\right) u + M_0 \frac{\partial f}{\partial y} v = -\frac{\partial p}{\partial x},
\]

where 

\[
\begin{align*}
\left(\frac{\partial}{\partial t} + M_0f \frac{\partial}{\partial x}\right) v &= - \frac{\partial p}{\partial y}, \tag{2} \\
\left(\frac{\partial}{\partial t} + M_0f \frac{\partial}{\partial x}\right) p &= - \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right). \tag{3}
\end{align*}
\]

All the quantities have been put in dimensionless form (in this paper, all the quantities with a star are quantities with dimensions while all the quantities without a star are dimensionless quantities) using:

\[
p = \frac{P}{\rho_0^*c_0^*}, \quad (x,y) = \left(\frac{x^*}{H^*}, \frac{y^*}{H^*}\right), \quad (u,v) = \left(\frac{u^*}{c_0^*}, \frac{v^*}{c_0^*}\right),
\]

\[
\omega = \frac{c^*}{c_0^*}H^*, \quad M_0 = \frac{U_0^*}{c_0^*}, \quad t = \frac{t^*c_0^*}{H^*},
\]

where \(H^*\) is the height of the channel, \(\rho_0^*\) is the mean density, \(c_0^*\) is speed of sound, the mean flow velocity is written \(U^*(y) = U_0^*(y)\), \(U_0^*\) is the average mean velocity in the duct and \(f(y)\) describes the flow profile, \(u^*\) and \(v^*\) are the velocity disturbance in, respectively, the \(x\)- and \(y\)-direction, \(p^*\) is the pressure disturbance, \(\omega^*\) is the angular frequency.

All variables are sought in the form:

\[
p = P(y) \exp(-i\omega t) \exp(i\omega t), \quad v = V(y) \exp(-i\omega t) \exp(i\omega t), \quad q = Q(y) \exp(-i\omega t) \exp(i\omega t),
\]

where \(i^2 = -1\), \(q = i\partial p/\partial x\), and \(k\) is dimensionless wavenumber. Equation (2) is written in term of these new variables as

\[
i(\omega - M_0f) p = - \frac{dP}{dy}.
\]

Removing the axial velocity from Eqs. (1) and (3) leads to

\[
(1 - M_0^2k^2)P + 2i\omega M_0fp - \omega^2P \frac{dp}{dy} = -2iM_0f \frac{df}{dy} \exp(\omega t).
\]

The duct-cavity system is split into three zones denoted by Roman number in Fig. 1. The mean flow profile is considered unaltered along the duct where \(y < 1\) with a zero mean velocity on the upper walls in zone I and III and on the lower surface of the membrane in zone II.

\[
\text{FIG. 1. (Color online) Sketch of a membrane-covered cavity in a duct with mean shear flow.}
\]

The normalized equation of the membrane displacement \(\delta\) is given by

\[
M_0 \frac{\partial^2 \delta}{\partial t^2} + D \frac{\partial \delta}{\partial t} - T \frac{\partial^2 \delta}{\partial x^2} + (p_+ - p_-) = 0,
\]

where \(M = M_0^*/(\rho_0^*c_0^*)H^*)\) is the mass per unit area of the membrane, \(D = D_0^*/(\rho_0^*c_0^*)\) is the damping of the membrane and \(T = T_0^*/(\rho_0^*c_0^*)H^*)\) its tension, \(p_+\) and \(p_-\) are the pressure on the upper and lower sides of the membrane.

To solve this problem of acoustic-flow-structural coupling, the multimodal method is used, where the disturbances in the ducts are expressed as a linear combination of transverse modes.\(^{16–18}\) Equations (5) and (6) are discretized in the \(y\)-direction by taking \(N_1\) equally spaced points in zone I and III, \(N_2\) equally spaced points in zone II. The spacing between interior points in all segments is

\[
h = (H + C)/N_2, \quad \text{and the first and last points are taken h/2 from the duct walls and the membrane. The centered finite difference method is used to solve the problem.}
\]

Employing a second order polynomial expansion in \(y\) for \(P(y)\) around \(y = 1\) in zone II, and using Eq. (5) and \(i(\omega - M_0f)\) \(\Delta = V\), we have

\[
p_+ = \frac{9P_{i+1} - 3h\omega^2 \Delta}{8}, \quad P_- = \frac{-3h\omega^2 \Delta}{8},
\]

where \(P_+\) and \(P_-\) denote the pressure on the upper and lower sides of the membrane. \(P_{i-1}, P_{i}, P_{i+1}\), and \(P_{i+2}\) are the pressure at the discrete points where \(y = 1 - 3h/2, y = 1 - h/2, y = 1 + h/2,\) and \(y = 1 + 3h/2\), respectively. Equations (8) and (9) give the relation between the membrane displacement and the pressure at the adjacent discrete points.

The following generalized eigenvalue problem, coming from Eqs. (5), (6), (8), (9), and using \(kP(Q) = -i\omega A = B\), respectively, to reduce the second order equations of \(k\) in Eqs. (6) and (8) into the first order equations, is formed in zone II:

\[
\begin{pmatrix}
1 - M_0^2f^2 & 2iM_0f & 0 & 0 & 0 \\
0 & iM_0f & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -8Ti & 0 \\
0 & 0 & 0 & -i & 0
\end{pmatrix}
\begin{pmatrix}
Q \\
V \\
P \\
\Delta \\
B
\end{pmatrix} = 0,
\]

where \(I\) is the identity matrix, \(f, f_2,\) and \(f_\delta\) are diagonal matrices with on the diagonal the values of \(f, f_2^*,\) and \(df/\partial y\) at
the discrete points in the ducts. \( Q \), \( V \), and \( P \) are the column vectors giving, respectively, the value of \( Q(y) \), \( V(y) \), and \( P(y) \) at the discrete points. \( M_i \) is a row vector of \( N_z \) zeros except \( M_i(N_j - 1) = 1 \), \( M_i(N_j) = -9 \), \( M_i(N_j + 1) = 9 \), and \( M_i(N_j + 2) = -1 \). \( M_2 = -6\alpha^2 - 8M\alpha^2 + 8D\alpha^2 \). \( D_i \) and \( D_2 \) are matrices for the first and second order differential operator with respect to \( y \). The boundary condition \( dp/dy = 0 \) on the duct walls are taken into account in the differential operator matrices. Additional corrections on Eq. (10) are needed because the first and second order derivatives of \( P \) with respect to \( y \) on the \( N_i \) and \( N_i + 1 \)st discrete points have the following forms:

\[
\begin{align*}
D_1 P_{N_i} &= (-P_{N_i - 1} + P_{N_i})/(2h) + \omega^2 \Delta/2, \\
D_1 P_{N_i + 1} &= (-P_{N_i + 1} + P_{N_i + 2})/(2h) + \omega^2 \Delta/2, \\
D_3 P_{N_i} &= (P_{N_i - 1} - P_{N_i})/h^2 + \omega^2 \Delta/h, \\
D_3 P_{N_i + 1} &= (-P_{N_i + 1} + P_{N_i + 2})/h^2 - \omega^2 \Delta/h.
\end{align*}
\]

Solving the eigenvalue problem Eq. (10) gives the wave-numbers and the corresponding eigenvectors. Since the mean flow velocity and its derivative are equal to zero for \( y > 1 \) in zone II, the last \( N_2 \) elements of \( V \) and the corresponding rows and columns in the matrices in Eq. (11) are skipped. Thus, \( 2N_2 + N_i + 2 \) modes are found in zone II, including \( N_2 \) acoustic modes propagating or decaying (evanescent modes) in the \(+x\) direction, \( N_2 \) acoustic modes propagating or decaying in the \(-x\) direction, \( N_i \) hydrodynamic modes propagating in the \(+x\) direction, and two additional modes due to the membrane propagating in the \( \pm x \) directions, respectively. In zones I and III, the variables \( \Delta \) and \( B \) are removed from the eigenvalue problem Eq. (10), and only wall boundary conditions need to be considered in the derivative matrices. Thus, \( 3N_i \) modes are found, including \( N_i \) acoustic modes propagating or decaying in the \(+x\) direction, \( N_i \) acoustic modes propagating or decaying in the \(-x\) direction, and \( N_i \) hydrodynamic modes propagating in the \(+x\) direction.

The \( n \)th eigenvector of Eq. (10) in zone II is \( (Q_n, V_n, P_n, \Delta_n, B_n) \), where \( Q_n \), \( V_n \), and \( P_n \) are the mode profiles for \( q \), \( v \), and \( p \), respectively, \( \Delta_n \) and \( B_n \) are the modal values of membrane displacement and its derivative with respect to \( x \). The eigenvectors have been normalized in such a way that the maximum of the pressure amplitude over the height of the channel is equal to 1. In zones I and III, the \( n \)th eigenvector is \( (Q_n, V_n, P_n, \Delta_n) \). In each zone, the column vectors giving, respectively, the value of \( Q(y) \), \( V(y) \), and \( P(y) \) and \( \Delta \) are written as a linear combination of the mode profiles (or values):

\[
\begin{align*}
Q_n(x) &= \sum_{n=1}^{N} C_n^q Q_n \exp(-ik_n^q x), \\
V_n(x) &= \sum_{n=1}^{N} C_n^v V_n \exp(-ik_n^v x), \\
P_n(x) &= \sum_{n=1}^{N} C_n^p P_n \exp(-ik_n^p x), \\
\Delta_n(x) &= \sum_{n=1}^{N} C_n^\Delta \Delta_n \exp(-ik_n^\Delta x), \quad (j = 2)
\end{align*}
\]

where \( C_n^j \) is the coefficient of the \( n \)th mode in zone \( j \). \( N = 2N_2 + N_i + 2 \) in zone II and \( N = 3N_i \) zones I and III.

Modes in each duct segment are then matched. At the interfaces between segments, the continuity of \( q \), \( v \), and \( p \) is applied. On the vertical solid walls inside the cavity, the \( x \)-direction velocity vanishes and so \( q = 0 \). Moreover, the displacement \( \delta \) is zero at the two ends of the membrane, where the plate is simply supported. These conditions can be put in the form of a large matrix that links all the incoming waves to the outgoing waves and to all the internal variables. From this large matrix, the scattering matrix is written:

\[
\begin{pmatrix}
C_1^+ \\
C_2^+
\end{pmatrix}
= S
\begin{pmatrix}
C_1^- \\
C_2^-
\end{pmatrix}, \quad (11)
\]

where vectors \( C_1^+ \) (resp. \( C_2^+ \)) contain the normalized duct mode coefficients for \( x = 0 \) (resp. \( x = L \)) for wave going in the flow direction (resp. for wave propagation opposite to the flow) and

\[
S = \begin{pmatrix}
T^+ & R^- \\
R^+ & T^-
\end{pmatrix}
\]

where \( T^+ (2N_1 \times N_i) \), \( R^+ (N_i \times 2N_1) \), \( T^- (N_i \times N_i) \), and \( R^- (2N_i \times N_i) \) are transmission and reflection matrices with and against the mean flow.

B. Calculations without flow

When there is no mean flow in the duct, the eigenvalue problem Eq. (10) reduces to

\[
\begin{pmatrix}
I & 0 & 0 & 0 \\
0 & I & 0 & 0 \\
0 & 0 & 0 & -8Ti \\
0 & 0 & -i & 0
\end{pmatrix}
\begin{pmatrix}
Q \\
P \\
\Delta
\end{pmatrix}
= \begin{pmatrix}
0 & \omega^2 I + D_2 & 0 & 0 \\
I & 0 & 0 & 0 \\
0 & M_1 & M_2 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
Q \\
P \\
\Delta
\end{pmatrix}. \quad (12)
\]

In zone II, \( 2N_2 + 2 \) modes are found, including \( N_2 \) acoustic modes propagating or decaying (evanescent modes) in the \(+x\) direction, \( N_2 \) acoustic modes propagating or decaying in the \(-x\) direction, and the two additional modes due to the membrane propagating in the \( \pm x \) directions, respectively. In zones I and III, the variables \( \Delta \) and \( B \) are removed from the eigenvalue problem and \( 2N_i \) modes are found, including \( N_i \) acoustic modes propagating or decaying in the \(+x\) direction, \( N_i \) acoustic modes propagating or decaying in the \(-x\) direction. The \( n \)th eigenvector of Eq. (12) in the zone II is \( (Q_n, P_n, \Delta_n) \). In zones I and III, the \( n \)th eigenvector is \( (Q_n, P_n) \).

At the interfaces between segments, the continuity of \( q \) and \( p \) is applied. On the vertical solid walls inside the cavity, the \( x \)-direction velocity vanishes and so \( q = 0 \) and the displacement \( \delta = 0 \) at the two ends of the membrane. The

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modal matching gives the transmission and reflection matrices: 
\[ T(N_1 \times N_1), \quad R(N_1 \times N_1) \].

**III. RESULTS**

A simple polynomial law with a unity average value has been used to give the velocity profiles of shear flow:

\[ f = f_0(1 - y^m) \quad \text{with} \quad f_0 = \frac{m + 1}{m}, \quad (13) \]

where the parameter \( m \) can be varied to change the thickness of the boundary layer, as shown in Fig. 2.

**A. Sound suppression**

The model described in Sec. II is validated by comparing its results to the benchmark data of Choy and Huang. They have studied the effect of a drumlike silencer in a duct without and with flow when the membrane has a high tension. In the experiment, both the duct height and the cavity depth are 100 mm, the cavity length is 500 mm. Mass per unit area of the membrane is 0.17 kg/m², and experiments were conducted for membrane with two different tensions: 7646.9 and 8213.4 N/m (\( T = 0.54 \) and \( T = 0.58 \)). Due to symmetry reasons, only the upper half of the experiment is modeled and the duct center plane is considered to be rigid. Comparisons of the transmission loss (TL) are made in Figs. 3 and 4, a general good agreement between the present model and the experiments is found. The discrepancy between the theoretical and the experimental results can be due to the two-dimensional assumption made in the model, where the flexural waves in the \( z \)-direction have not been taken into account.

Some acoustic properties of the drumlike silencer can also be observed from the present results calculated with different parameters. First, the dissipation in the structural motions mainly influences the peaks of TL in the case without flow, increasing the structural damping can smooth out the peaks, as shown in Fig. 3. Second, results in Fig. 4 indicate that the boundary layer thickness of the mean duct flow does not have significant effects on TL for the chosen parameter set. For a high-tension membrane subjected to a

![FIG. 2.](image-url) Mean velocity profiles near the membrane location \( y = 1 \) for parameter \( m \) being equal difference series from 5 to 40.

![FIG. 3.](image-url) Transmission loss of a drumlike silencer without flow. The experimental results are from Ref. 3 with \( T = 0.54 \).

![FIG. 4.](image-url) Transmission loss of a drumlike silencer with flow velocity \( M_0 = 0.045 \) and different boundary layer thickness controlled by the parameter \( m \). \( D = 0.0226 \) is used in the calculations. (a) \( T = 0.54 \) and (b) \( T = 0.58 \). The experimental results are from Ref. 3.
low speed flow, the flexural wave speed is much larger than the speed of the hydrodynamic waves. The hydrodynamic modes due to the shear in the flow are weakly involved in the modal coupling (further discussion on this point is given in Sec. III B). Thus, the main flow effect is the convection of the modes. This explains why the acoustic properties of the drumlike silencer are affected by the average flow velocity, but not sensitive to the velocity profile.

B. Sound amplification

In this section, we consider cases where the hydrodynamic modes participate in the modal coupling. The geometrical parameters are: \(H^* = 15\, \text{mm}, C^* = 15\, \text{mm}, L^* = 20\, \text{mm}\), the default parameters for the membrane are \(M^* = 1\, \text{kg/m}^2\), \(D^* = 10\, \text{kg} \cdot \text{m}^2/\text{s}^2\), and \(T^* = 100\, \text{N/m}\) \((M = 51.2821, D = 0.0226, \text{and } T = 0.0444)\), the default parameters for the mean flow are \(M_0 = 0.1\) and \(m = 10\). Calculations in this section are made with these default parameters except the ones put in legends or captions of the figures. To accurately describe the membrane–shear flow interaction and control the calculation time as well, the discrete points in the region \(0.8 < y < 1.2\) are five times denser than elsewhere. The distance between discrete points is \(h_2 = 1/2000\) for the points in the region \(0.8 < y < 1.2\) and \(h_1 = 1/400\) for points elsewhere. This points number guarantees results with relative error smaller than \(10^{-3}\) in the default case. However, for high \(M_0\) and/or high \(m\) in Figs. 7 and 8, smaller values \(h_1 = 1/800\) and \(h_2 = 1/8000\) are used to give converged results.

The eigenvalues in zone II are plotted in Fig. 5(a). The real parts of the wavenumbers are associated with the phase velocity of the waves, while the imaginary parts are the amplification or decay rates. We can see the propagating and evanescent acoustic modes in the +\(x\) and –\(x\) directions, which are denoted by A+ and A–, respectively. The neutral hydrodynamic modes result from the singularities of the Prandtl-Brown equation\(^{19}\) when \(\omega - U_k = 0\), thus a continuous hydrodynamic spectrum on the real axis, \(\omega/\sqrt{U_{\text{max}}} < k < \omega/\sqrt{U_{\text{min}}}\) is obtained. Since the mean flow velocity is zero on the lower surface of the membrane, the hydrodynamic continuum ranges from \(\omega/\sqrt{U_{\text{max}}}\) to infinity on the real axis. In the present discretized modal analysis, the continuous spectrum is represented by a set of discrete values \(k_n = \omega(n)(\gamma_j_{\text{min}})\). Figure 5 also shows an unstable hydrodynamic mode, which can appear under certain conditions in the present problem. Its wavenumber has an additional positive imaginary part such that the mode grows exponentially in the direction of propagation. The two additional modes due to the membrane are denoted by F+ and F– and they propagate in the +\(x\) and –\(x\) directions, respectively. These two modes are associated with the flexural waves in the membrane.

In the modal interaction analysis, we first compare the hydrodynamic modes in zone II to those in zone I with shear flow. Since both the mean flow profile and the discretization are unchanged along the duct, the hydrodynamic modes in zone II remain the same as those in zone I except one mode that is destabilized. In the case without flow, the two modes due to the membrane are symmetric about the origin. With shear flow, the real parts of F– and F+ almost maintain the no-flow value, while the imaginary part of F+ is modified. The wave speed of F+ is mainly determined by the properties of the membrane: \(c_p \approx \sqrt{T/M}\). The results presented in Fig. 5 show that the neutral hydrodynamic mode that is destabilized is the one with the same wave speed as F+ while F+ itself becomes more dissipative.

To avoid any confusion between the unstable hydrodynamic mode and the acoustic modes decaying in the –\(x\) direction, the Briggs-Bers causality criterion is used.\(^{20,21}\) To apply this criterion, an imaginary part is added to the frequency \(\omega\). The Briggs-Bers criterion states that if one of the mode crosses the real axis while the imaginary part of the frequency \(\text{Im}(\omega)\) ranges from –\(\infty\) to 0, it means that this mode is unstable. For the present \(\text{exp}(\text{i}(\omega t - kx))\) convention, the modes with wavenumbers that are in the lower complex plane when \(\text{Im}(k) \rightarrow -\infty\) propagate in the +\(x\) direction, while the modes with wavenumbers that are in the upper complex plane when \(\text{Im}(k) \rightarrow -\infty\) propagate in the –\(x\) direction. From the eigenvalues tracing in Fig. 6, one of the modes is crossing the real axis and this mode is therefore unstable.

The influence of the shear flow on the modal interaction is shown in Figs. 7 and 8. First, the onset of the unstable hydrodynamic mode can be observed as the flow velocity is increased. The velocity of \textit{in vacuo} flexural waves is \(\sqrt{T/M} = 0.03\). Thus, interaction can only happen when \(M_0 > 0.03\),\(^{6}\) such onset condition is in agreement with the results in Fig. 7. The imaginary part of the unstable hydrodynamic mode increases progressively with the increasing flow speed from \(M_0 = 0.04\) to \(M_0 = 0.3\), which suggests a progressive increase of the coupling. However, the slope decreases after \(M_0 = 0.13\), and it approaches zero around \(M_0 = 0.3\).
Further increasing the flow velocity up to $M_0 = 0.5$ barely affects the imaginary part of the unstable mode. The velocity profile also significantly affects the modal interaction, as shown in Fig. 8. For a given average flow velocity $M_0 = 0.1$, the unstable hydrodynamic mode has an increasing amplification rate as the profile parameter is increased from $m = 2$ to 14. However, $\text{Im}(k_{\text{unstable}})$ starts to decrease if the boundary layer thickness is further reduced. It is shown in Figs. 7(b) and 8(b) that the destabilized hydrodynamic mode and $F^+$ always has the same wave speed, which increases with increasing flow velocity but decreases with increasing $m$.

Increasing flow velocity and increasing $m$ both decreases the wave speed of $F^-$. The dependence on flow conditions indicates that the wave speeds of $F^+$ and $F^-$, modified by the coupling, are not exactly equal to the \textit{in vacuo} flexural wave speed of the membrane.

Figure 9 shows the effect of the structural dissipation on the modal interaction. The dissipation rates of $F^-$ and $F^+$ increase linearly with $D$ as expected. For increasing dissipation, the amplification rate of the unstable hydrodynamic mode decreases and approaches to zero, which demonstrates the stabilizing role of the structural damping.

Figure 10 presents the frequency influence on the modal interaction. At high frequencies, the hydrodynamic mode becomes stable. The wave speeds for the calculation in Fig. 10 is given in Fig. 11. The wave speeds of $F^-$ and $F^+$ significantly differ from the \textit{in vacuo} flexural wave speed when the unstable hydrodynamic mode occurs. When the system is stable at high frequencies, $F^-$ and $F^+$ have the same dissipation rate and their wave speeds approach to the \textit{in vacuo} flexural wave speed.

The above results show that the instability caused by the interaction between the shear flow and the membrane motion can occur. The acoustic modes do not participate in this interaction, because the acoustic wave speed is much larger than the flexural and flow velocities. However, the interaction between the flexural and hydrodynamic modes affects the scattering of the acoustic waves by the cavity covered by a membrane.

The transmission coefficients for a plane acoustic wave without and with flow are compared in Fig. 12. Without flow, $|T^+|$ has minimums resulting from the wave reflection by the membrane-cavity configuration. With shear flow, the

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FIG. 6. (Color online) Tracing of the eigenvalues: $\text{Re}(\omega)$ is kept constant $0.1372$ while $\text{Im}(\omega)$ runs from $-0.0055$ (results are denoted by the squares) to $0$ (results are denoted by the circulars).

FIG. 7. (Color online) (a) Wavenumbers in zone II for increasing mean flow Mach number $M_0$, (b) real, and (c) imaginary parts of flexural and unstable hydrodynamic modes as a function of $M_0$. The calculation is made with frequency $\omega = 0.1372$. The arrows in (a) denote the direction of the variation of the modes for increasing $M_0$. Note that in (b), $-\text{Re}(k_{\text{unstable}})$ is plotted.

FIG. 8. (Color online) (a) Wavenumbers in zone II for increasing velocity profile parameter $m$, (b) real, and (c) imaginary parts of flexural and unstable hydrodynamic modes as a function of $m$. The calculation is made with frequency $\omega = 0.1372$. The arrows in (a) denote the direction of the variation of the modes for increasing $m$. Note that in (b), $-\text{Re}(k_{\text{unstable}})$ is plotted.

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amplitude of the transmission coefficient can be greater than 1 in very sharp peaks. It means that the acoustical waves are amplified and can extract some energy from the flow–membrane interactions. On those peaks, a very large amplitude of the membrane motion is observed. For instance, in Fig. 12(b), the displacement of the membrane is on the order of half the height of the channel. This absurdly high value indicates that nonlinear effects must be taken into account in the vicinity of the peaks in order to make a correct prediction.

One wonders why high amplitude vibration and sound amplification occur only on narrow frequency bands, while instability exists over a broad frequency range as shown in Fig. 10. The reason is that, for a compact cavity, in addition to instability condition, sound amplification requires another condition: 

$$L/C^{2} > n_k f$$

where $L$ is the length of the cavity and $k_f$ is the wavelength of the flexural mode of the membrane.

FIG. 9. (Color online) (a) Wavenumbers in zone II for increasing structural dissipation, (b) real, and (c) imaginary parts of flexural and unstable hydrodynamic modes as a function of $D$. The calculation is made with frequency $\omega = 0.1372$. The arrows in (a) denote the direction of the variation of the modes for increasing $D$. Note that in (b), $-\text{Re}(k_f)$ is plotted.

FIG. 10. (Color online) (a) Wavenumbers in zone II for frequency sweep from $\omega = 0.0277$ to $\omega = 0.5821$ (100–2100 Hz), (b) real, and (c) imaginary parts of flexural and unstable hydrodynamic modes as a function of frequency. The arrows in (a) denote the direction of the variation of the modes for increasing $\omega$. Note that in (b), $-\text{Re}(k_f)$ is plotted.

FIG. 11. (Color online) Wave speeds as a function of frequency. Note that the wave speed of the hydrodynamic mode is only plotted over the frequency range where instability occurs.

FIG. 12. (Color online) (a) Transmission coefficient as a function of frequency calculated without and with shear flow. (b)–(e) Vibration shapes of the membrane calculated with shear flow at frequencies $\omega = 0.1372$ (495 Hz, first peak), $\omega = 0.1383$ (499 Hz, first minimum), $\omega = 0.2137$ (771 Hz, second minimum), and $\omega = 0.2764$ (997 Hz, second peak), respectively. In (b)–(e), the solid lines denote the displacement of the membrane, the dashed lines are the contributions of $F$– and $F^+$.
Such a match condition is required by the compactness of the cavity and it guarantees a small net volume flux when the flow-induced vibration of the membrane occurs.

The effects of the flow and structural damping on the sound transmission are shown in Fig. 13. Due to the large number of parameters affecting the acoustic–flow–structural interaction, it is difficult to find a law that predict the amplitude of the peak in $|T^+|$. However, the frequency for which the maximum occurs can be explained. Figure 14 gives the ratios of cavity length to wavelength $L/\lambda$ at frequencies near the peak frequency. The wave velocities of $F_-$ and $F_+$ differ from each other and also differ from the in vacuo flexural wave velocity as illustrated in Fig. 11. By setting an average wavelength by $L/\lambda_F = (L/\lambda_F^- + L/\lambda_F^+)/2$, the peak occurs at frequency where the $L \approx n\lambda_F$ condition is satisfied. This condition can be used to explain the shift of the peak frequency as $m$ is increased, shown in Fig. 13(b). As $m$ is increased from 5 to 20, Fig. 8(b) indicates that both $\lambda_F^-$ and $\lambda_F^+$ decreases and thus the peak frequency decreases when $m$ increases. Since the wavelengths of $F_-$ and $F_+$ barely change when the damping is increased [see Fig. 9(b)], the peak frequency almost remains the same value in cases of different dissipations, as shown in Fig. 13(c). On this Fig. 13(c), it can also be seen that the sound amplification can be suppressed by increasing the damping.

IV. CONCLUSION

The acoustic description of a device composed of a two dimensional (2D) cavity covered by a membrane located on the wall of a duct and subjected to a shear flow can be realized with a multimodal technique. This calculation method is validated by comparison with available experimental results. This method is particularly effective and allows us to analyze in detail the effect of the flow on the device. When high-tension membranes are used, as long as the mean flow velocity is smaller than the velocity of the in vacuo flexural waves of the membrane, the effect of flow is mainly a wave convection. In this case, the shear effect in the flow is weak and the flow can be considered as uniform without loss of accuracy.

The situation is drastically changed when the flow velocity is larger than the velocity of the in vacuo flexural waves of the membrane. In this case a strong coupling between the hydrodynamic modes and the membrane occurs. This coupling can lead to a convective instability of the device. The unstable mode has the same velocity as the wave velocity of the flexural waves in the membrane. For a given flow profile, the axial growth rate of the instability increases with the mean flow velocity but saturates at high velocities. For a given mean flow velocity, there is an optimum boundary layer thickness for the instability. Increasing the structural damping tends to stabilize the instability.

Sound amplification can be obtained when the system is unstable. The other condition for sound amplification is that the length of the cavity and the membrane is around an integer multiple of the wavelength of the flexural mode of the membrane, $L \approx n\lambda_F$. Sound amplification can be suppressed by increasing the structural damping. Also, both flow
velocity and velocity profile have influence on sound propagation when the shear flow is coupled with the membrane motion.

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